International Institute of Information Technology, Hyderabad

(Deemed to be University)

Probability and Random Processes – Monsoon, 2023

Question cum Answer Booklet

Mid Semester Examination

Max. Time: 1.5 Hr **Max. Marks: 30**

Roll No:_________________ **Programme: _______________** **Date of Exam:______**

Room no: _______ Seat No: ______ **Invigilator's Signature:___________________**

Special Instructions about the exam

1. **Please use the Rough/Additional Sheets in this Booklet mainly for rough work.**
2. **Please mention the Additional Sheet no. under the corresponding Question if you use it as an additional sheet.**

Marks Table (To be filled by the Examiner)

Question No / Marks								Name of the Examiner who marked

General Instructions to the students

1. Place your Permanent / Temporary Student ID card on the desk during the examination for verification by the Invigilator.
2. Reading material such as books (unless open book exam) are not allowed inside the examination hall.
3. Borrowing writing material or calculators from other students in the examination hall is prohibited.
4. If any student is found indulging in malpractice or copying in the examination hall, the student will be given 'F' grade for the course and may be debarred from writing other examinations.

Best of Luck

Question 1. Let A and B be two events with probabilities $P(A) = \frac{3}{4}$ and $P(B) = \frac{1}{3}$.

(a) (3 Marks) Show that $\frac{1}{12} \leq P(A \cap B) \leq \frac{1}{3}$.

(b) (5 Marks) Construct a sample space Ω, events $A, B \subseteq \Omega$, and a probability law P such that the upper and lower bounds above are achieved with equalities.

Question 2 (5 Marks)**.** Consider a probabilistic model with sample space Ω and probability law P. Let $\{C_1, C_2, \ldots, C_n\}$ be a partition of Ω. For two events A and B, suppose we know that

- A and B are conditionally independent given C_i, i.e., $P(A \cap B|C_i) = P(A|C_i)P(B|C_i)$, for all $i \in \{1, 2, \ldots, n\}$;
- B is independent of C_i, i.e., $P(B \cap C_i) = P(B)P(C_i)$, for all $i \in \{1, 2, \ldots, n\}$.

Are A and B independent events?

Question 3. Let X be a discrete random variable with PMF

$$P_X(x) = \begin{cases} Kx^2, & \text{if } x \in \{-3, -2, -1, 0, 1, 2, 3\} \\ 0, & \text{otherwise,} \end{cases}$$

for some constant K.

(a) (2 Marks) Determine the value of K.

(b) (4 Marks) Let $Y = |X|$. Find the PMF of Y.

Question 4. Let X and Y be two independent random variables, each taking values 0 or 1 with equal probability $\frac{1}{2}$, and let $Z = X \oplus Y$ (i.e., $Z = 0$ if $X = Y$ and $Z = 1$ if $X \neq Y$).

(a) (3 Marks) Are the random variables X, Y, and Z (mutually) independent?

(b) (3 Marks) Are X, Y, and Z pairwise independent?

Question 5 (5 Marks)**.** Consider an exponential random variable X with PDF

$$f_X(x) = \begin{cases} \lambda \mathsf{e}^{-\lambda x}, & \text{if } x \geq 0, \\ 0, & \text{otherwise,} \end{cases}$$

where λ is a positive parameter. Compute $\mathrm{Var}(X)$.

ROUGH/ADDITIONAL SHEET - 1

Rough/Additional Sheet - 2

ROUGH/ADDITIONAL SHEET - 3

ROUGH/ADDITIONAL SHEET - 4