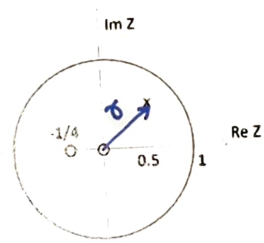
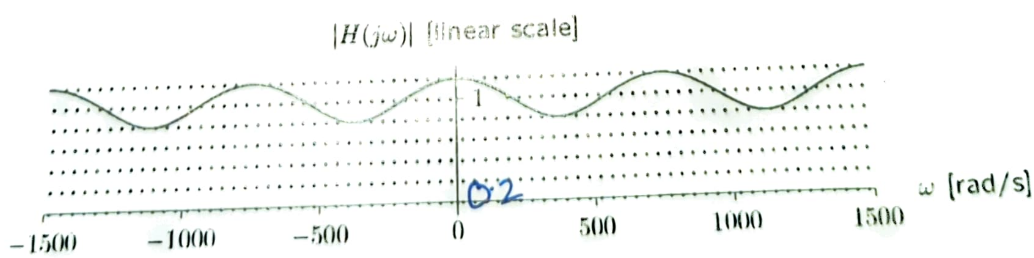
Signal Processing | **Mid Sem Makeup exam** | **October 2023**

1. A signal $x[n] = \delta(n+3) - \delta(n+1) + 2\delta(n) + 3\delta(n-2)$ with DTFT as $X(e^{j\omega}) = X_R(e^{j\omega}) + jX_I(e^{j\omega})$. [3+2+2+3] marks
   a. Compute $X_R(e^{j\omega})$ and $\int_{-\pi}^{\pi} X_I(e^{j\omega})\, d\omega$
   b. $DTFT\,(y[n]) = X_R(e^{j\omega})e^{j2\omega} + jX_I(e^{j\omega})$, find $y[n]$ without explicitly considering DTFT?
   c. Derive the DTFT of x[2n] in terms of DTFT of x[n], which is an arbitrary signal.
   d. For the given x[n] above, compute x[2n] and its DTFT and verify that the relation in part c holds.

2. An even sequence is one which satisfies x[n] = x[-n]. Assuming it has a rational z-transform X(z) answer the following. [3+4+3] marks
   a. What does even nature of signal imply for X(z)?
   b. Use your answer in part a to complete the partial pole-zero plot shown below for a real, even sequence and find X(z). What could be its ROC? Assume the pole is at radius r.
   c. If a signal g[n] = x[4-n]. How will the pole-zero plot of G(z) differ from that of X(z)? Explain.



3. An LTI system is represented by $h[n] = \delta[n - n_0] + \alpha\,\delta[n - n_1]$ with $n_1 > n_0$. The plot below shows the magnitude of the $H(z)$ when evaluated on the unit circle, i.e $|z| = 1, or\ z = e^{j\omega}$ where $\omega = \frac{2\pi}{T}$ is the angular frequency. [3+5+4] marks
   a. Assume α < 1 and sketch $h[n]$. Find the system function $H(z)$.
   b. Justify the oscillatory pattern in $|H(e^{j\omega})|$ by evaluating $H(z)$on the unit circle. Relate the variables α, n_0 , n_1 in $h[n]$ to the oscillations. If $n_0 = 0$, find n_1 and α.
   c. Draw the block diagram of this system.



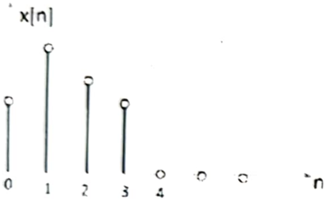
PTO for problem 4

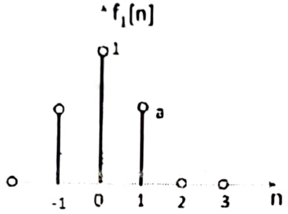
4. Upsampling a sequence x[n] by a factor of M is desired. A proposed method for this is :

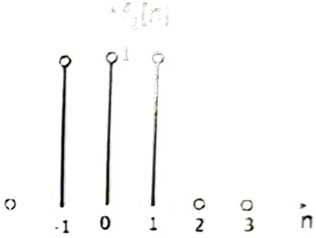
i) create a sequence $x_e[n]$ by introducing M-1 zeros between successive samples of x[n].

ii) convolve $x_e[n]$ with a suitable sequence f[n] to obtain the final result.

iii) A sample x[n] and 2 possible candidates for f[n] are shown below. [2x5 +2 marks]







a. Which of these sequences (f_1 or f_2) will give the best upsampled result? Why? What is the length of the final result with either of these sequences? Answer *without* doing any convolution.

b. Sketch the appropriate f[n] for upsampling with M = 4.

c. Identify the purpose of each of the steps in the proposed method.