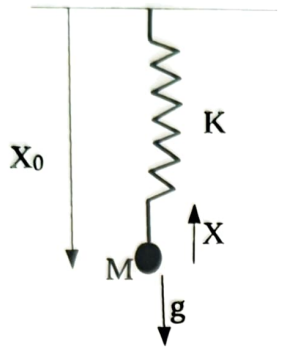
Quiz 2

(Science 1 : Classical Mechanics, each question carries 15 marks : 5×3)

Time: 1 Hour

Q1.

The following mass M is attached to a spring of spring constant K and conducting a simple harmonic oscillation in the vertical direction in presence of gravity. The position at rest is given by X_0 and the X is simple harmonic oscillation around X_0.

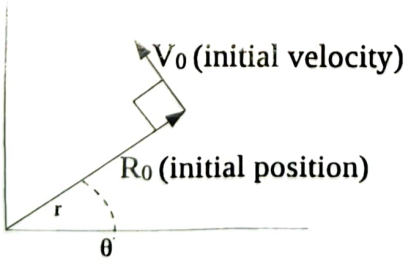


I. Write down the expression for the kinetic energy and potential energy.
II. Evaluate the Lagrangian L and Hamiltonian H of the system
III. Write down the Lagrangian equation of motion for X.
IV. Calculate the value of X_0 (position at rest) as function of M,g and K.
V. If total energy is E, evaluate the frequency and amplitude of the oscillation.

Q2.

A particle of mass M is moving under a central potential given by V(r)= -k/r on a plane as shown below with center of force at the origin. An initial velocity of V_0 is given to the system perpendicular to the initial position vector R_0 as shown in the figure below.

The values are $M = 10^{24}\,kg\,; k = 10^{43}\,m^3 kg\,s^{-1}\,; R_0 = 100\times 10^6\,km\,; V_0 = 10\,km\,s^{-1}$



I. Write down the Lagrangian $L(\dot{r}, \dot{\theta}, r, \theta)$ of the system and the generalized momenta P_r, P_θ corresponding to r and θ.
II. Write down the Hamiltonian $H(P_r, P_\theta, r, \theta)$
III. Calculate the initial angular momentum A around the center and initial total energy E of the system
IV. Show if the orbit would be bound or not bound and what is the maximum initial velocity V_0 to get bound orbit
V. Calculate the velocity V_0 for a circular orbit of radius R_0 .