Long Questions [2 questions, 50 points]

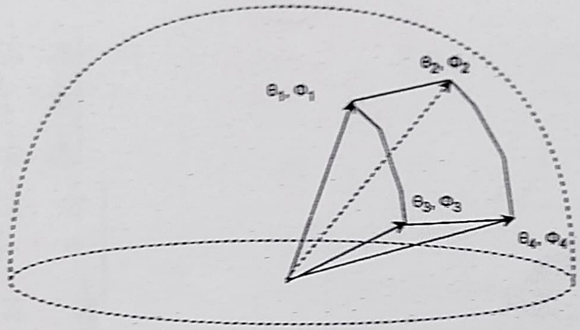
1. You are given an object whose center is located at (5, 5, 5), and its vertex list is given by (v_1, v_2, v_3 ... v_N). Answer the following questions:
   a. Rotate the first vertex $v_1 = [x_1, y_1, z_1]$ along the x-axis by 70 degrees using only matrix multiplications as transforms. Write in terms of "sin" and "cos".
   b. Given the sequence of matrix multiplications from (a), write down a single matrix multiplication that operates on all vertices together, and outputs all transformed vertices. Write it symbolically (no need to calculate), assuming suitable variables and combined transformation matrices.

[25 points]

2. The rendering equation reads:

$$L_o(\omega_o) = \int_{\Omega} f(\omega_o, \omega_i)\, L_i(\omega_i) \cos\theta \; d\omega_i$$

The integration domain Ω is pictured below, and is defined as a rectangular region on the sphere with spherical co-ordinates: $(\theta_1 = 30^o, \phi_1 = 30^o)$, $(\theta_2 = 30^o, \phi_2 = 60^o)$, $(\theta_3 = 60^o, \phi_3 = 30^o)$ & $(\theta_4 = 60^o, \phi_4 = 60^o)$.



Assume a diffuse BRDF ($f(\omega_o, \omega_i) = \frac{c}{\pi \cdot \cos\theta}$) and a constant incoming radiance ($L_i(\omega_i) = A$). Note that c & A both are constants.
Your task is to derive and expression for $L_o(\omega_o)$.

Hints: Recall the following, which may be useful for the derivation:

a. $d\omega = \sin\theta \; d\theta \; d\phi$
b. $\int \sin\theta \; d\theta = -\cos\theta$
c. $\int \cos\theta \; d\theta = \sin\theta$

[25 points]

Short Questions [4 questions, 12.5 points]

3. For a camera placed at $(0, 0, 0)$, looking at $(1, 1, 1)$ with the up vector defined as $(0, 1, 0)$, what is the camera matrix? Assume camera looks at -Z axis. **[12.5 points]**

a. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

b. $\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{bmatrix}$

c. $\begin{bmatrix} -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}} & 0 \\ -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$

d. $\begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} \end{bmatrix}$

4. The sensor width of the camera is 8mm and sensor height is 6mm. The focal length is 5mm. **[12.5 points]**

   a. What is the vertical field of view?

   b. What is the horizonal field of view?

5. Given the following set of points **[12.5 points]**

x	y	z
-2	10	3
15	-4	5
10	30	15
20	10	5
-15	-10	-20

We need to build a BVH on this scene, with the following build rules:
- At each level, the bounding box should be split along the longest axis.
- Once the longest axis is determined, the splitting plane is placed at it's center.

The equation of the first splitting plane for the first level is:

a. x + 2.5 = 0

b. x - 2.5 = 0

c. y - 10 = 0

d. z + 2.5 = 0

6. A scene contains a sphere centered at the origin with radius 1 and a single point light at position $(3, 0, 4)$ emitting radiance $(1, 1, 1)$.
Assume the surface has a diffuse BRDF with $c = (0.5, 0.5, 0.5)$.
What is the solution to the rendering equation at point $(0, 0, 1)$ which lies on the sphere? Assume the viewing direction to be $(0, 0, -1)$. **[12.5 points]**

a. $(\frac{2}{5\cdot\pi}, \frac{2}{5\cdot\pi}, \frac{2}{5\cdot\pi})$

b. $\frac{4}{125\cdot\pi}, \frac{4}{125\cdot\pi}, \frac{4}{125\cdot\pi})$

c. $(\frac{2}{125\cdot\pi}, \frac{2}{125\cdot\pi}, \frac{2}{125\cdot\pi})$

d. $(\frac{1}{50\cdot\pi}, \frac{1}{50\cdot\pi}, \frac{1}{50\cdot\pi})$