IIIT-H

EC5.205: Introduction to Coding Theory

Summer-2024

Exam: Quiz-1
Marks: 20

Date: 29-Jan-2024
Time: 8:45 am to 09:30 am

Instructions:

- Answering all the questions is compulsory.
- All steps should be justified in detail.
- Clearly state the assumptions (*if any*) made that are not specified in the questions.

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1. (6 points) Consider a binary linear block code with a parity check matrix given by

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}.$$

Suppose a transmitted codeword is sent over a binary symmetric channel with crossover probability $p = 0.9$. Decode the received vector $\mathbf{y} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \end{bmatrix}$ using the standard array decoding.

2. (6 points) For any two vectors $\mathbf{u}, \mathbf{v} \in \mathbb{F}_2^n$, let $\mathbf{u} \cap \mathbf{v}$ denotes the vector in $\mathbb{F}_2^n$, which has 1's precisely in those positions where both $\mathbf{u}$ and $\mathbf{v}$ have 1's. Let $w(\mathbf{u})$ denote the Hamming weight of $\mathbf{u}$. Then show that:

(a) $w(\mathbf{u}+\mathbf{v}) = w(\mathbf{u}) + w(\mathbf{v}) - 2w(\mathbf{u} \cap \mathbf{v})$

(b) $[w(\mathbf{u} \cap \mathbf{v})] \bmod 2 = (\mathbf{u}\mathbf{v}^T) \bmod 2$

$w(u+v) = $ (u=0, v=1) (u=1, v=0) $u=1, \quad v=1$

Using this show that if every codeword of C has weight divisible by four, then C is self-orthogonal. Can this code be self-dual? Justify.

3. (8 points) We have a binary symmetric channel with crossover probability $p < 1/2$. Suppose the transmitter is using $2n$-repetition code and the transmitted messages are equally likely. Obtain the most simplified version of the optimal decoding rule (i.e., the decoding rule which minimizes the probability of error). Calculate the average probability of error for this code when the optimal decoding rule is used. Show that the optimal decoding rule is not unique (i.e., *some* received vectors could be assigned one of multiple codewords as the estimate, without affecting the average probability of error).