Linear Algebra (UG1, Spring 2023)

Quiz [10 marks]; Time: 45 mins (+15 mins)

March 31, 2023

1 Instructions

Read questions carefully. Notations for vector addition and scalar multiplication are from class lectures. Provide appropriate reasoning for your claims.

Question A

Suppose V is a vector space defined over a real field $\mathbb{R}$. Let $\vec{v}, \vec{w} \in V$. Explain why there exists unique $\vec{x} \in V$ such that $\vec{v} + 3\vec{x} = \vec{w}$. [1 marks]

Question B

Let A be an $n \times n$ matrix over a field $\mathbb{F}$. Prove the following two statements:

1. If A is invertible and $AB = 0$ for some $n \times n$ matrix B, then $B = 0$. [1 marks]
2. If A is not invertible, then there exists an $n \times n$ matrix B such that $AB = 0$ but $B \neq 0$. [2 marks]

Question C

[$3 \times 2 = 6$ marks]

1. Prove that the union of two subspaces of V is a subspace of V if and only if one of the subspaces is contained in the other.
2. Find all possible solutions to the following homogeneous system of linear equations:

$$\begin{aligned} x_1 + \frac{1}{2}x_2 + \frac{1}{3}x_3 &= 0 \\ \frac{1}{2}x_1 + \frac{1}{3}x_2 + \frac{1}{4}x_3 &= 0 \\ \frac{1}{3}x_1 + \frac{1}{4}x_2 + \frac{1}{5}x_3 &= 0. \end{aligned}$$