# MOBILE ROBOTICS - Monsoon 2024

## MIDSEM – Open Book, Notes and Laptop

## NO INTERNET

Total marks: 35 to be scaled to 20

1) Show by a simple example and apt figures that the Rotation Matrix, R, that rotates a vector in a frame to a new vector in the same frame is the same as the Rotation Matrix that describes a frame rotated by R with respect to the base/reference frame **(3 points)**

2) Often in SLAM and SFM algorithms the camera frames or the robot frames are represented with respect to the base frame by the 3 orthogonal axes. Show mathematically how would you represent the coordinate axes of a frame that enjoys a homogenous transform T with respect to the base frame in the base frame? **(2 points)**

3) What are the 3 mapping representations taught in class? **(1 point)**
   a) Rank them in terms of ease of representation **(1 point)**
   b) Rank them in terms of ease of retrieval or indexing **(1 point)**
   c) Rank them in terms of memory efficiency **(1 point)**
   d) Ease of doing navigation on these representations **(1 point)**
   e) Ease of doing pose estimation on these representations **(1 point)**

4) *Optimization using Levenberg-Marquardt / Gauss-Newton*: Consider the following non-linear least-squares problem where you want to fit the model

   $f(x; \theta) = \theta 1 \sin(\theta 2\ x)$ to the following 5 data points:

| x | y |
|---|---|
| 1.0 | 0.8415 |
| 2.0 | 0.9093 |
| 3.0 | 0.1411 |
| 4.0 | −0.7568 |
| 5.0 | −0.9589 |

The initial guess for the parameters is $\theta 1 = 1.0$ and $\theta 2 = 1.0$.

(a) Formulate the objective function F (θ) to be minimized. **(2 points)**

(b) Compute the Jacobian in its analytical form. You do not have to evaluate it **(3 points)**

(c) Write down the Gauss Newton update rule **(2 points)**

**5. Rotation Interpolation: Challenges with Euler Angles and the Benefits of Quaternions**

(a) Explain why Euler angles is a bad choice for interpolating between two orientations in 3D space? Justify with the issues that arise using examples. **(2 points)**

(b) How do quaternions help in smooth interpolation? Explain briefly about SLERP (Spherical linear interpolation). **(2 points)**

**6. Iterative Closest Point (ICP) for Point Cloud Alignment**

(a) Consider two point clouds P, Q with N correspondences pi , i = 1 . . . N and qi , i = 1 . . . N . Assume that both point clouds are centered and their centroids align.

Show that the expression for the rotation matrix R that aligns the point clouds is given by R = VUT where U, V are the left and right singular matrices of the covariance matrix **(3 points)**.

(b) What are the issues with the sum of squares objective function in vanilla Iterative Closest Point (ICP)?

$$E(\mathbf{x}) = \sum_i \mathbf{w}_i \|\mathbf{p}_i - \mathbf{q}_i\|^2$$

What improvements can be made to overcome these issues? **(2 points)**

7. Consider the following 2 methods for optimizing the transformation matrix in ICP- SLAM:

- Directly Optimizing the elements of the Transformation Matrix and reproject the updated matrix back to the Transformation Space

- Optimize for the Transformation Matrix using Lie Algebra

(a) Find and explain the number of parameters we are optimizing for in both cases.**(2 points)**

(b) Explain the structure of the Jacobian (what do the elements correspond to) in each case. **(4 points)**

(c) Explain why there is a need for Lie Group Optimization when the first method exists. **(2 points)**