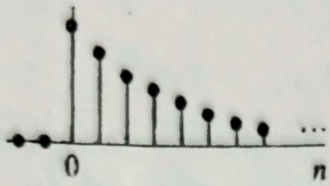
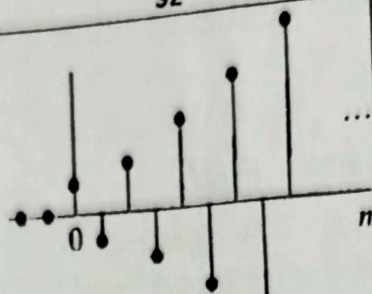
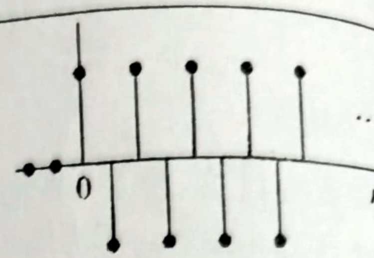
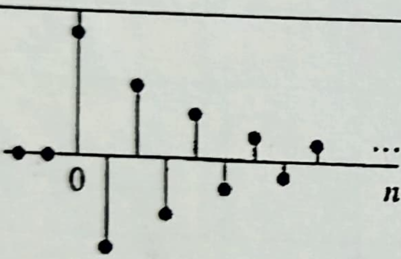
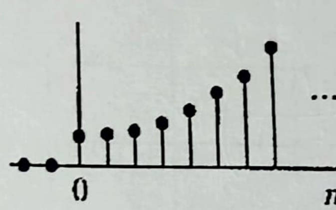
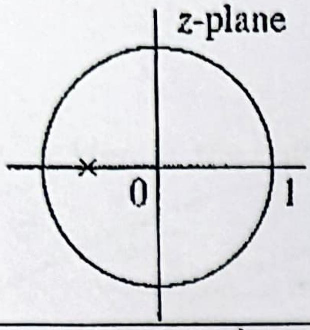
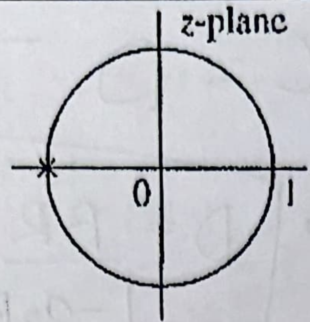
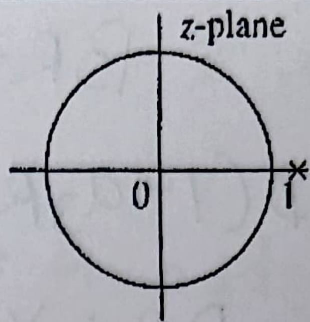
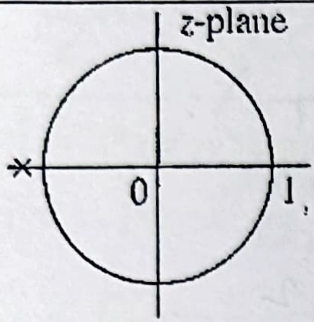
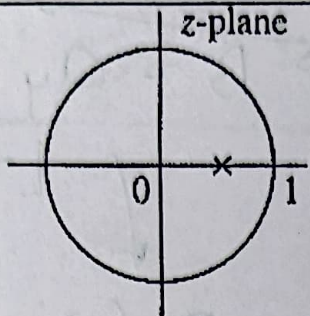
Q1.a: Match the given *causal* sequences S1 to S5 with the given pole-zero plots PZ1 to PZ5. Write your answer below each sequence. [5 Marks]

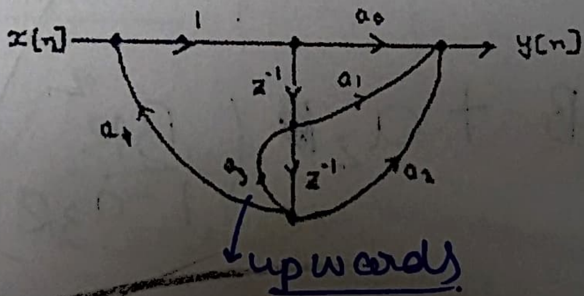
S1	S2	S3
 decreasing, $\lambda(<1)$ all +ve PZ5	 incresing $\lambda(>1)$ but alternating (-ve) PZ4	 const (but alternate) so 1 (-ve) PZ2
 λ (<1) decreasing alternating (-ve) PZ1	 incressing (>1) all(+ve) PZ3	5
S4	S5	

PZ1	PZ2	PZ3
		
		
PZ4	PZ5	

Q1.b: A signal flow graph is also used to represent a block diagram of a digital system. Here, signal flow direction is indicated by arrows; Adders are implied by nodes between signal flow lines; delays are indicated by Z^{-1} and scalars by variables/numbers. Use this to answer the following.

(i) Derive the transfer function for the digital filter shown below. [6 Marks]

(ii) Is this an FIR or IIR filter? Explain your answer. [4 Marks]



ANS:- $\boxed{H(z) = \dfrac{1 - z^{(M+1)}}{z^{m}(1-z)}}$

Q2. Consider the rational transfer function given below.

$$H(z) = \frac{z^{-(M+1)} - 1}{z^{-1} - 1}$$

a: Identify the poles and zeros of H(z). [3 Marks]

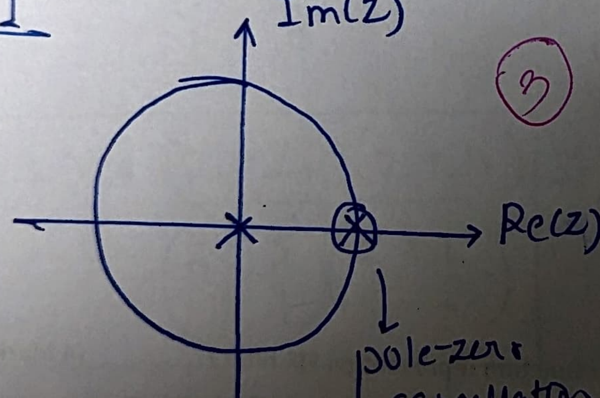
Thus IF M = even

i.e. $M = 0, 2, 4, 6 \ldots$ etc -

Pole Zero- Diagram:-

Pole:- $z = 0$, $z = 1$.

Zero: $z = 1$



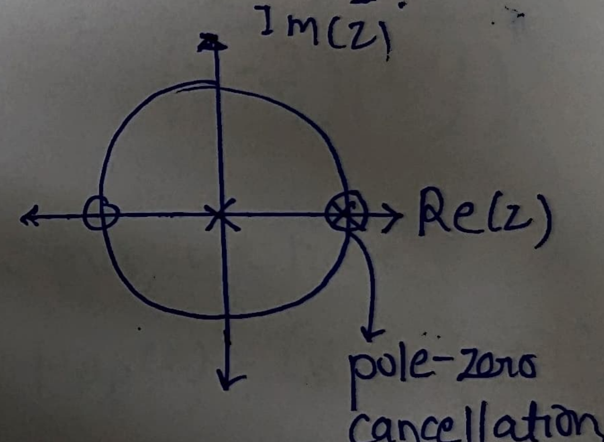
3

If M = odd

i.e. $M = 1, 3, 5, 7, \ldots$

Pole-Zero Diagram →

- Pole => $z = 1$, $z = 0$.
- Zero => $z = \pm 1$.



b: What type of filter (LPF/HPF, etc) does the above represent? Explain. [3 Marks] c: Show that this is an FIR filter. [5 Marks]

d: In general, when can a rational transfer function represent an FIR filter? [4 Marks]

In general; a Rational Transfer function of the form:

$$H(z) = \frac{\sum_{n=0}^{N-1} b_k z^{-n}}{\sum_{n=0}^{M-1} a_k z^{-n}}$$

where b_k, a_k are scalar filter coefficients.

- This represents an FIR filter when $a_0 = 1$ & $a_k = 0 \ \forall \ k = 1, 2, \cdots M-1$.

Thus, FIR filter rational transfer function is of the type:

$$\boxed{H(z) = \frac{\sum_{n=0}^{N-1} b_k z^{-n}}{1}}$$

Q3.a: Consider a stable LTI system with transfer function H(Z)

$+\frac{z}{2} - \frac{3z}{4}$

$$H(z) = \frac{1 + 4z^{-2}}{1 - \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2}}$$

(i) Factorise H(z) as a product of an allpass filter $H_{ap}(z)$ and a minimum phase filter $H_m(z)$.
Hint: pole-zero pairs of H_{ap} occur with mirror symmetry with respect to the unit circle; all poles and zeros of H_m are inside the unit circle. [5 Marks]

(ii) Plot the poles and zeros of your two systems and indicate their ROC. [4 Marks]

i) $H(z) = \dfrac{1+4z^{-2}}{1 - \frac{z^{-1}}{4} - \frac{3z^{-2}}{8}} = \dfrac{z^2+4}{z^2 - \frac{z}{4} - \frac{3}{8}} = \dfrac{z^2+4}{\left(z-\frac{3}{4}\right)\left(z+\frac{1}{2}\right)}$

5

$$\boxed{H(z) = \frac{(z-2i)(z+2i)}{\left(z-\frac{3}{4}\right)\left(z+\frac{1}{2}\right)}}$$

From this, we have

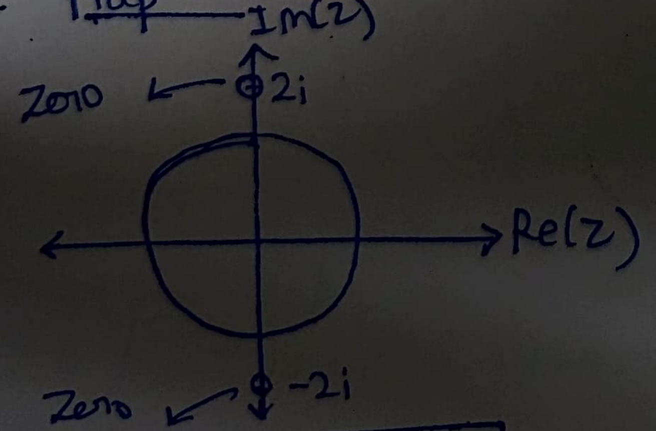
i) $H_{ap}(z) = \dfrac{(z-2i)(z+2i)}{1}$

& $H_m(z) = \dfrac{1}{\left(z+\frac{1}{2}\right)\left(z-\frac{3}{4}\right)}$

$\boxed{\therefore H(z) = H_{ap}(z) \cdot H_m(z)}$

, where Hap (z) & Hm (z) are given here.

ii). Hap (z):-

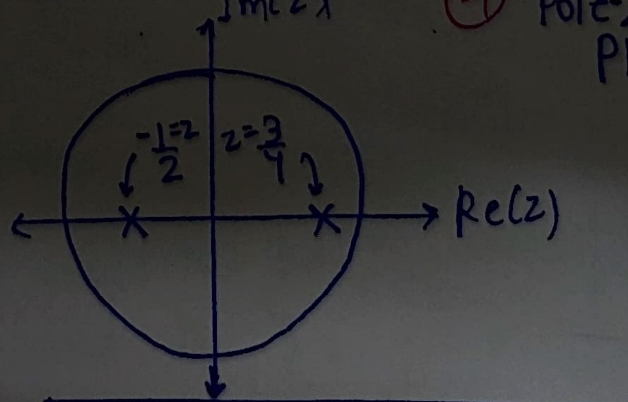


ROC:- $0 < |z| < \infty$

Entire z-plane is ROC

ii) Hm(z)

4 Pole-Zero Plot.

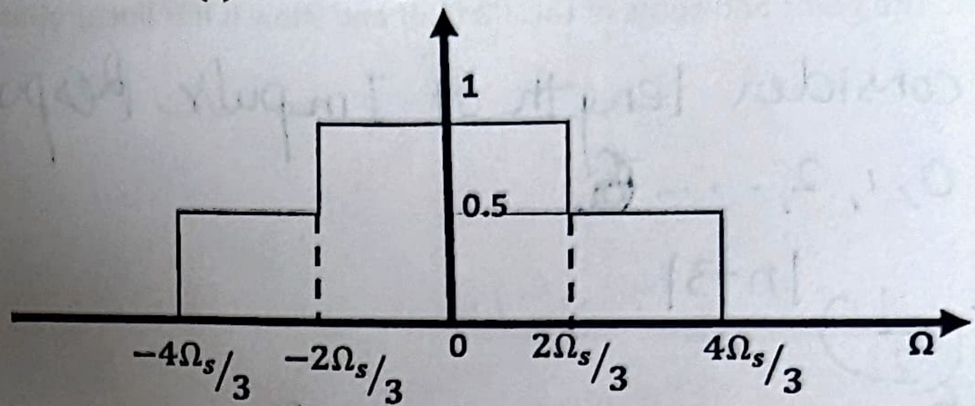


ROC:- $\frac{3}{4} < |z| < \infty$

Q3.b: Consider a IIR filter with impulse response $h[n] = \frac{1}{2}^{|n-3|}$.

(i) Find the poles and zeros of this IIR filter and show it has linear phase. [6 Marks]

Q4.a: Let's consider $H(J\Omega)$ is a CTFT of $h(t)$ with a magnitude spectrum as shown below with zero phase spectrum then



(i) Plot the amplitude spectrum of DTFT $H(e^{j\omega})$ of $h[n]$, where $h[n]$ is the discrete time signal obtained from $h(t)$ using sampling frequency F_s ($\Omega_s = 2\pi F_s$). [2 Marks]

(ii) Let $H[k]$ is the values obtained by sampling $H(e^{j\omega})$ at four equidistance locations of ω between -π to π or Ω between $-\Omega_s/2$ to $\Omega_s/2$, then compute $h[n]$. [2 Marks]

(iii) Let $H_1(z)$ and $H_2(z)$ are two z-transforms which results the same amplitude spectrum of $H(e^{j\omega})$ but with two different non-zero phase spectrums, then draw the pole-zero plots of $H_1(z)$ and $H_2(z)$? [3 Marks]

Q4.b: Let $H(s) = \frac{1000\pi}{s+1000\pi}$ then

(i) Find continuous-time angular frequencies Ω_c, Ω_p and Ω_s, where gains δ_c at Ω_c is -3dB, δ_p at Ω_p is -1dB and δ_s at Ω_s is -40dB. [3 Marks]

(ii) Considering Ω_p, δ_p, Ω_s and δ_s, deduce $H_1(s)$ using Butterworth filter design approach. [4 Marks]

(iii) Is there any mismatch between $H(s)$ and $H_1(s)$ and justify the answer? [1 Mark]

Q5.a: Let a linear convolution of $x[n]$ and $h[n]$ is $\{1, 1, 1, 1, 1, 1, 1\}$ and three different lengths of $x[n]$ results the same linear convolution with three different $h[n]$ when the lengths of $x[n]$ is 3, 4 and 5 then

(i) Is the length of the circular convolution of $x[n]$ and $h[n]$ the same? Justify your answer.

[2 Marks]

(ii) Compute the circular convolution values under all three cases of length of $x[n]$?

Length of Linear Conv $= 7 = \boxed{l_1 + l_2 - 1}$

[3 Marks]

* ∵ Circular Conv. & Linear Convolution have same values but only different lengths.

Q5.b: Let $x[n]$ is even and complex then prove that its DFT $X[k]$ is real without using any properties?

[5 Marks]

Q5.c: There is an 8-point DIF FFT chip, which takes the arbitrary signal $x[n] = \{x[0], x[1], x[2], x[3], x[4], x[5], x[6], x[7]\}$ in normal sequence order and produce the DFT in bit reversal order $\{X[0], X[4], X[2], X[6], X[1], X[3], X[5], X[7]\}$. Unfortunately, in this chips 5th, 6th, 7th and 8th input locations are grounded. Without knowing this, an input sequence of length 8 is given then what is the output from the FFT chip? Clearly justify the answer with necessary computations.

[5 Marks]

Q6.a: Considering DTFT analysis equation $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$, prove the synthesis equation $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$? [3 Marks]

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}\,d\omega \cancel{= \frac{1}{2\pi}\int_{-\pi}^{\pi}\sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}\cdot e^{j\omega n}\,d\omega}$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi}\sum_{m=-\infty}^{\infty} x[m]e^{-j\omega m}e^{j\omega n}\,d\omega$$

$$= \frac{1}{2\pi}\left(\sum_{m=-\infty}^{\infty} x[m]\right)\int_{-\pi}^{\pi} e^{-j\omega(m-n)}\,d\omega = \frac{1}{2\pi}\left(\sum_{m=-\infty}^{\infty} x[m]\right)\left[\frac{e^{-j\omega(m-n)}}{-j(m-n)}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{2\pi}\left(\sum_{m=-\infty}^{\infty} x[m]\right)\left[e^{-j\pi(m-n)} - e^{+j\pi(m-n)}\right]\frac{1}{(-j(m-n))}$$

$$= \frac{1}{2\pi}\left(\sum_{m=-\infty}^{\infty} x[m]\right)\left[\cos(\pi(m-n)) - j\sin(\pi(m-n)) - \cos(\pi(m-n)) - j\sin(\pi(m-n))\right]\frac{1}{-j(m-n)}$$

$$= \frac{1}{2\pi}\left(\sum_{m=-\infty}^{\infty} x[m]\right)\left[-2j\sin(\pi(m-n))\right]\frac{1}{-j(m-n)}$$

$$= \frac{1}{2\pi}\left(\sum_{m=-\infty}^{\infty} x[m]\right)\left(\frac{2\sin(\pi(m-n))}{m-n}\right).$$

Contd.

Q6.b: Compute the DTFT of unit step signal $u[n]$? [4 Marks]

Q6.c: Let the pole-zero plot of a systems is shown below then compute and precisely plot the phase spectrum of the system? [3 Marks]

$H(z) = \left(z - e^{j\frac{2\pi}{3}}\right)\left(z - e^{-j\frac{2\pi}{3}}\right)$

Q6.d: Let the impulse response of a system is $h[n] = \{1, 1, 1, 1, 1\}$ then

(i) Compute and plot the DTFT of $h[n]$? [3 Marks]

(ii) Compute the output $y[n]$ when the input to the system is $\cos\left(\frac{\pi}{3}n\right)$? [2 Marks]

Consider $h[n] = \{1, 1, 1, 1, 1\}$