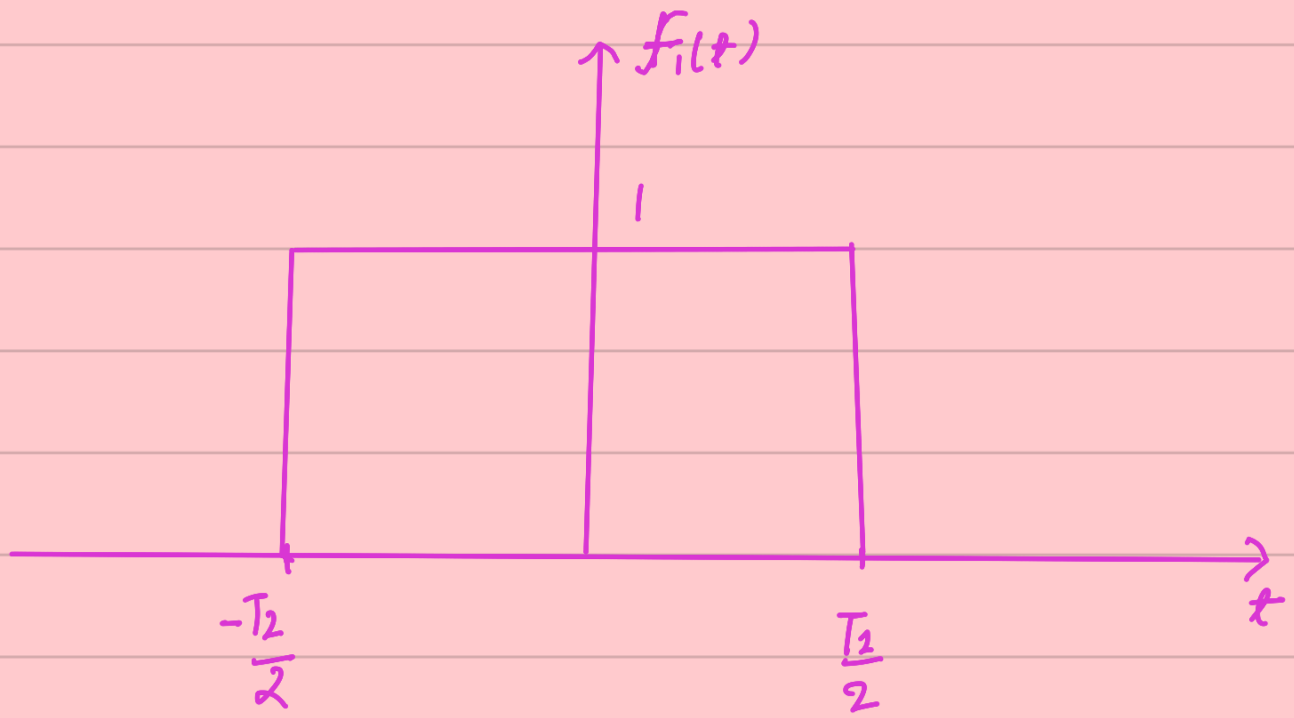
Q.1

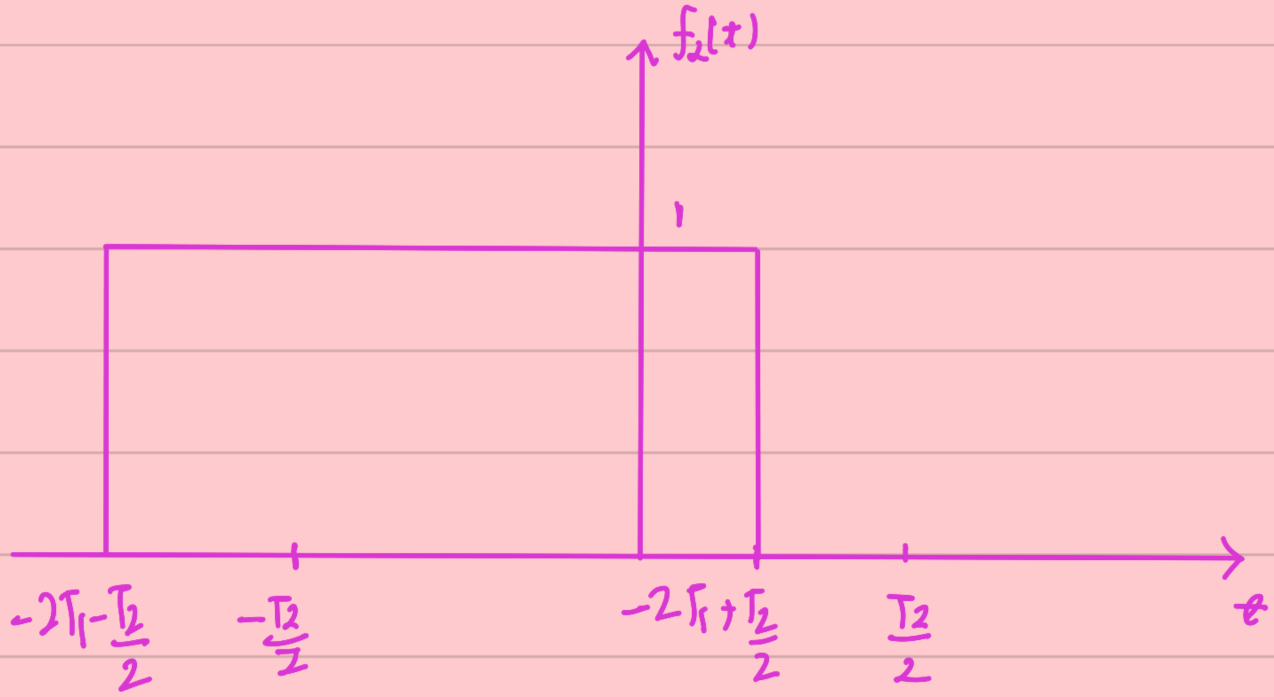
$$\text{rect}(t) \triangleq \begin{cases} 1 & \text{if } |t| \leq 0.5 \\ 0 & \text{otherwise} \end{cases}$$

$$f_1(t) = \text{rect}\left(\frac{t}{T_2}\right) = \begin{cases} 1 & \text{if } |t| \leqslant \frac{T_2}{2} \\ 0 & \text{otherwise} \end{cases}$$

$$f_2(t) = \text{rect}\left(\frac{t - KT_1}{T_2}\right) = \begin{cases} 1 & \left|\frac{t - KT_1}{T_2}\right| \leqslant 0.5 \\ 0 & \text{otherwise} \end{cases}$$

$$f_2(t) = \begin{cases} 1 & KT_1 - \frac{T_2}{2} \leqslant t \leqslant KT_1 + \frac{T_2}{2} \quad (K = -2) \\ 0 & \text{otherwise} \end{cases}$$





First, we find the FT of rect(t)

$$\because \quad FT\{rect(t)\} = \int_{-\infty}^{\infty} rect(t)\, e^{-j2\pi ft}\, dt$$

$$= \int_{-0.5}^{0.5} e^{-j2\pi ft}\, dt$$

$$= \frac{-1}{j2\pi f}\left(e^{\frac{-j2\pi f}{2}} - e^{+\frac{j2\pi f}{2}}\right)$$

$$= \frac{\cancel{2}j\sin(\pi f)}{\cancel{2}j\pi f}$$

$$= \frac{\sin(\pi f)}{\pi f} = \text{sinc}(f)$$

Using time-scaling property of Fourier Transform.

$$F_1(f) = FT\{f_1(t)\} = \underline{T_2 \operatorname{Sinc}(fT_2)}$$

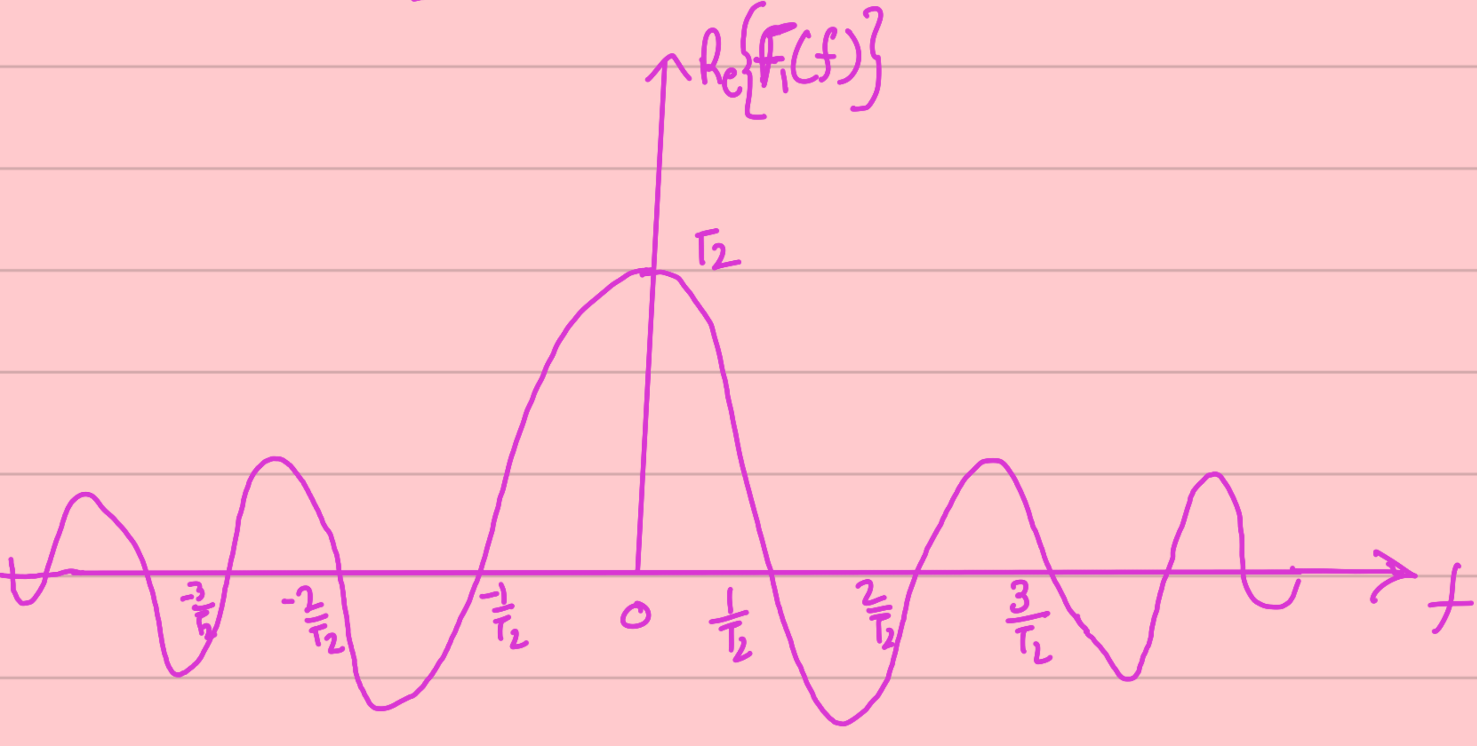
To find FT of $f_2(t)$, we use scaling and then shifting properties.

$$F_2(f) = FT\{f_2(t)\} = T_2 \operatorname{sinc}(fT_2)\, e^{-j2\pi f \cdot KT_1}$$

$$= \underline{T_2 \operatorname{sinc}(fT_2)\, e^{j4\pi f T_1}}$$

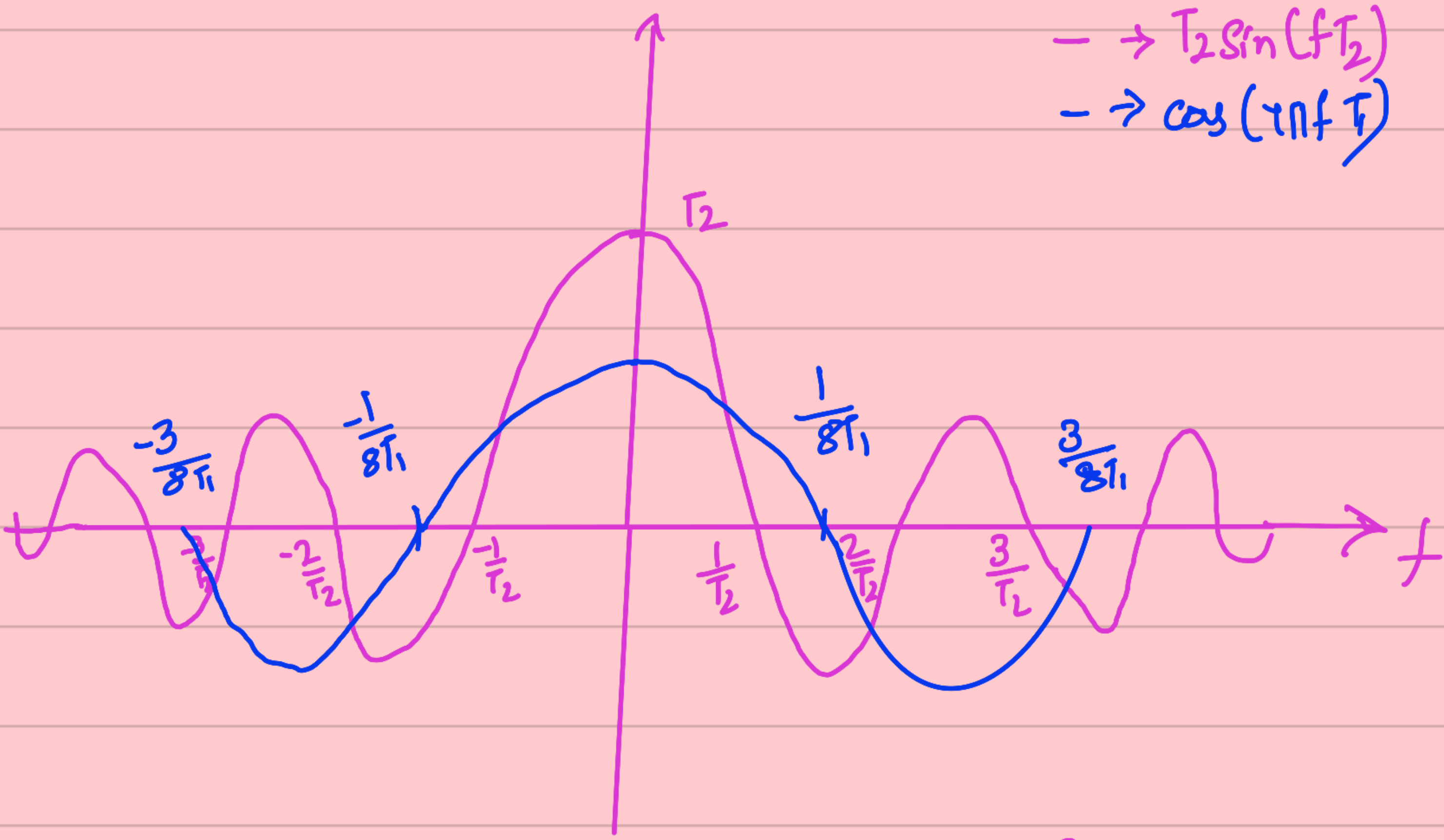
To sketch the real part, we do

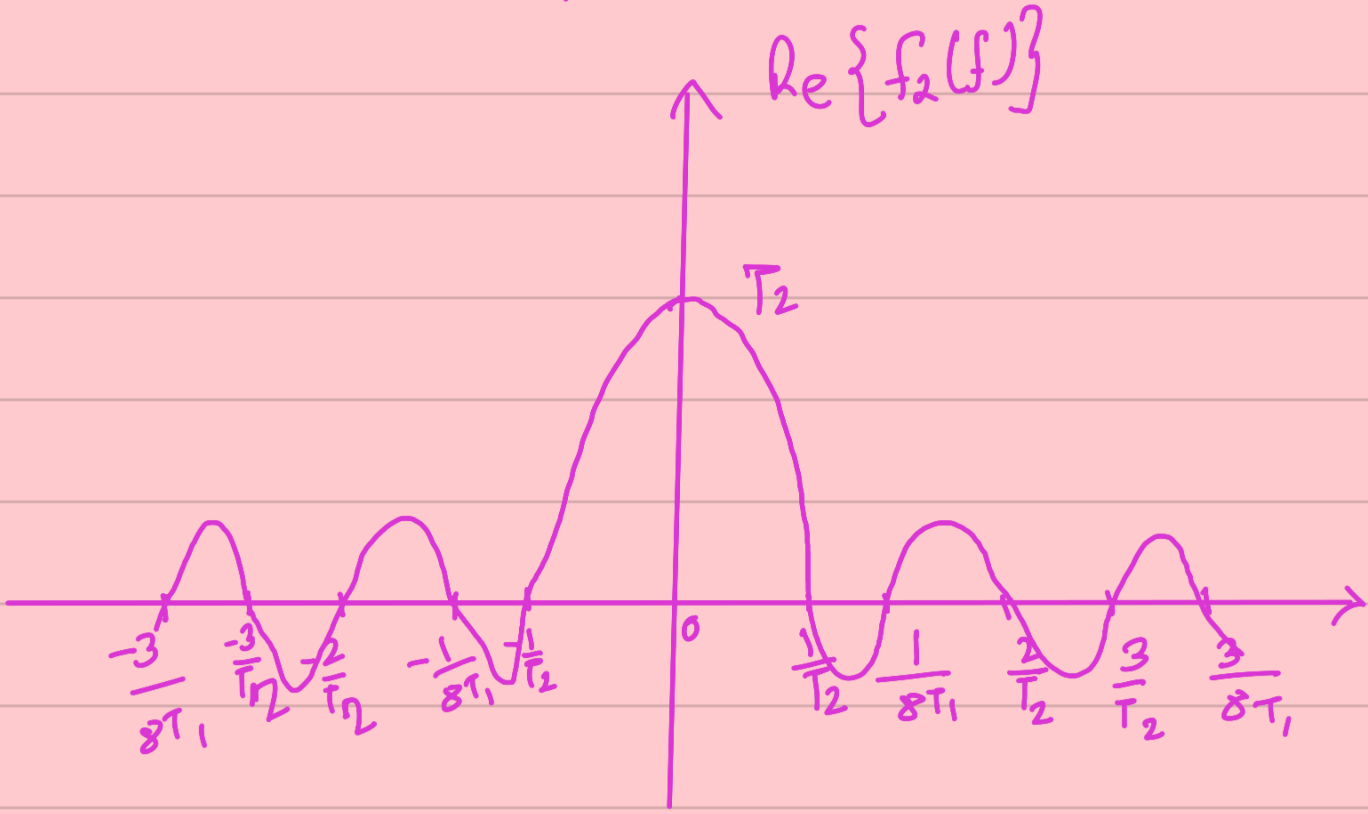
$$Re\{F_1(f)\} = T_2 \operatorname{sinc}(fT_2)$$



$$Re\left\{F_2(f)\right\} = T_2 \,\text{sinc}(fT_2)\cos\left(4\pi f \frac{T_1}{T_2}\right)$$

Assuming $T_1 > T_2$,





Q.2

$$m(t) = \sum_{k \in \mathbb{Z}} \text{rect}\left(\frac{t - kT_1}{T_2}\right) \qquad T_2 < T_1$$

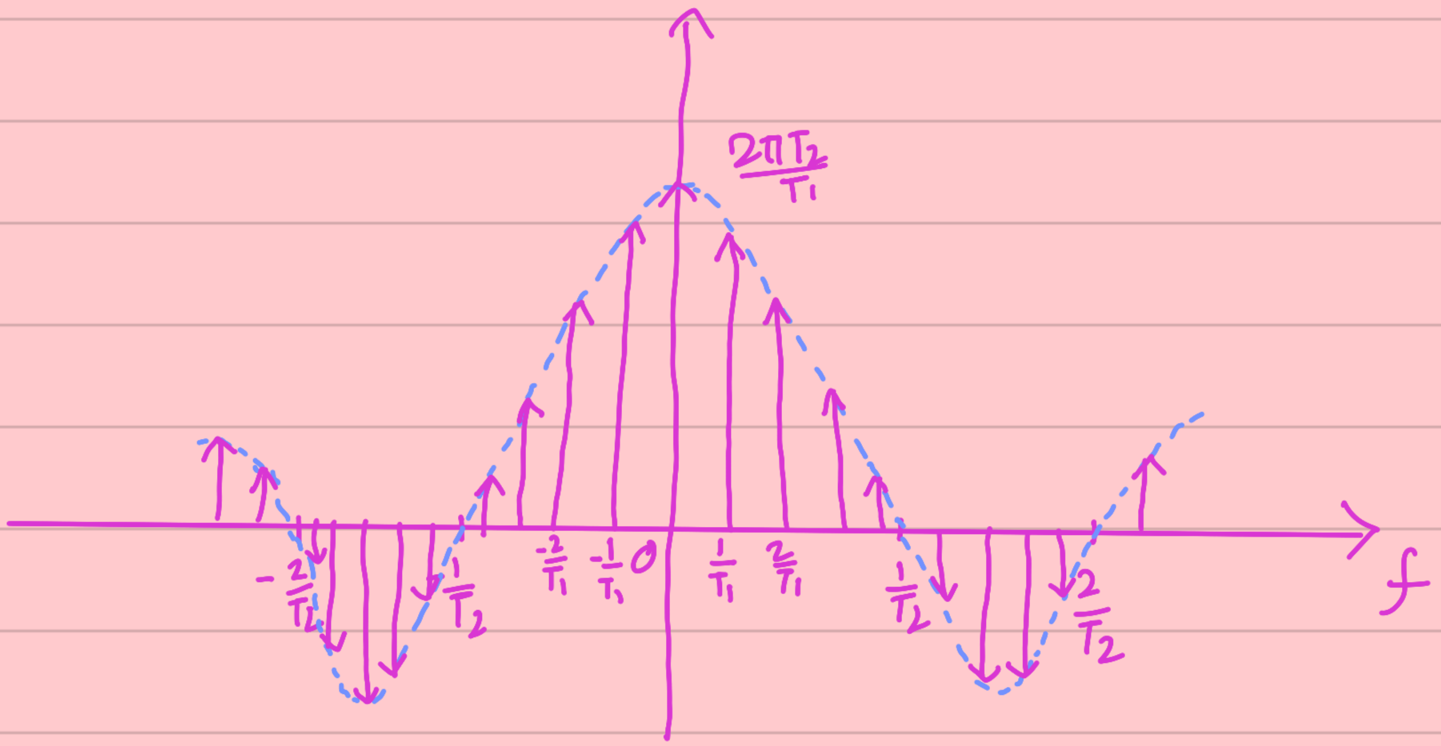
$$= \left[\sum_{k \in \mathbb{Z}} \delta(t - kT_1)\right] * \text{rect}\left(\frac{t}{T_2}\right)$$

Using: $f\left(\frac{t}{T_2}\right) * \delta(t - kT_1) = \int_{-\infty}^{\infty} \delta(\tau - kT_1)\, f\left(\frac{t-\tau}{T_2}\right) d\tau$

$$= f\left(\frac{t - kT_1}{T_2}\right)$$

To find FT of $m(t)$, use convolution in time domain

$$\therefore\ FT\{m(t)\} = \frac{2\pi}{T_1} \sum_{k \in \mathbb{Z}} \delta\left(f - \frac{k}{T_1}\right) T_2 \,\text{Sinc}(fT_2)$$

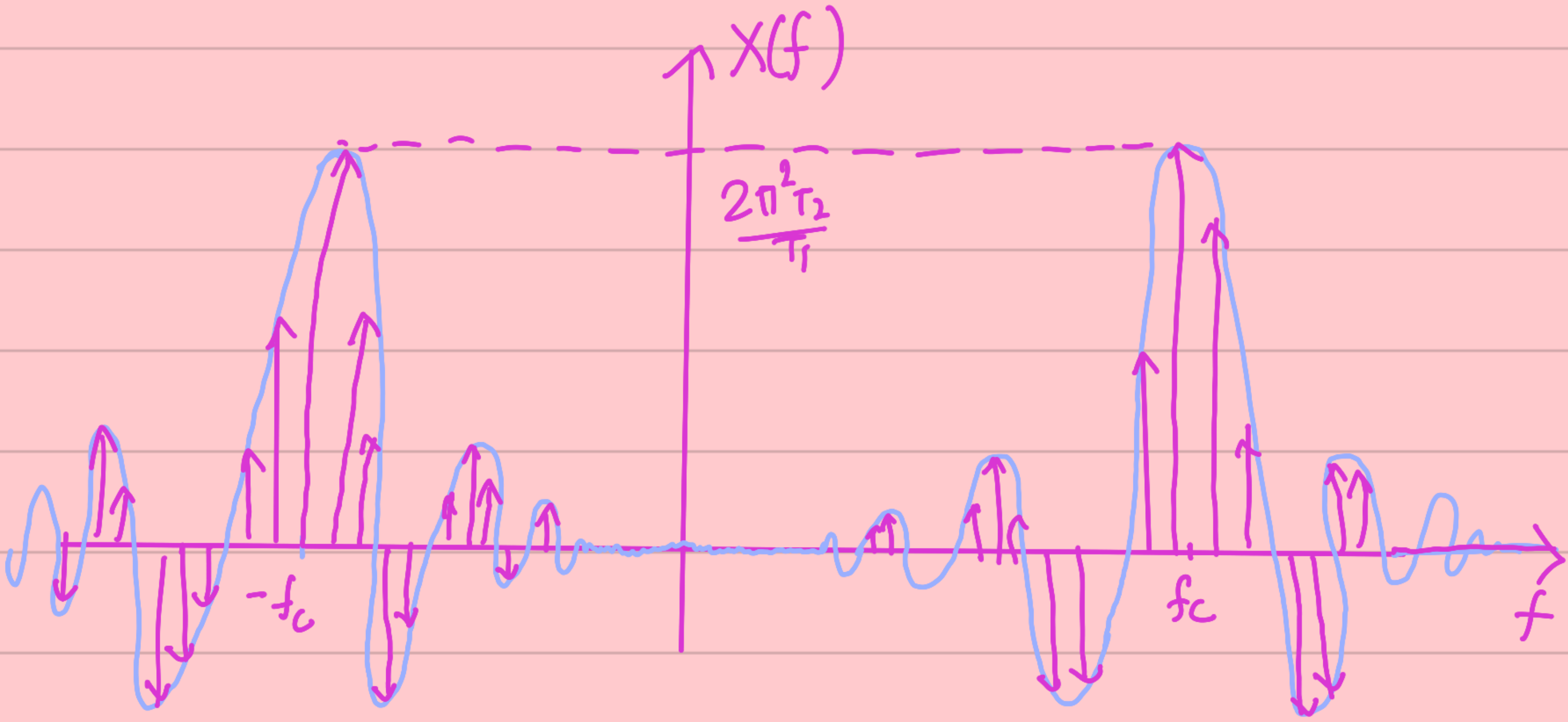


Q.3 $\quad x(t) = m(t)\cos(2\pi f_c t) \qquad f_c \gg t$

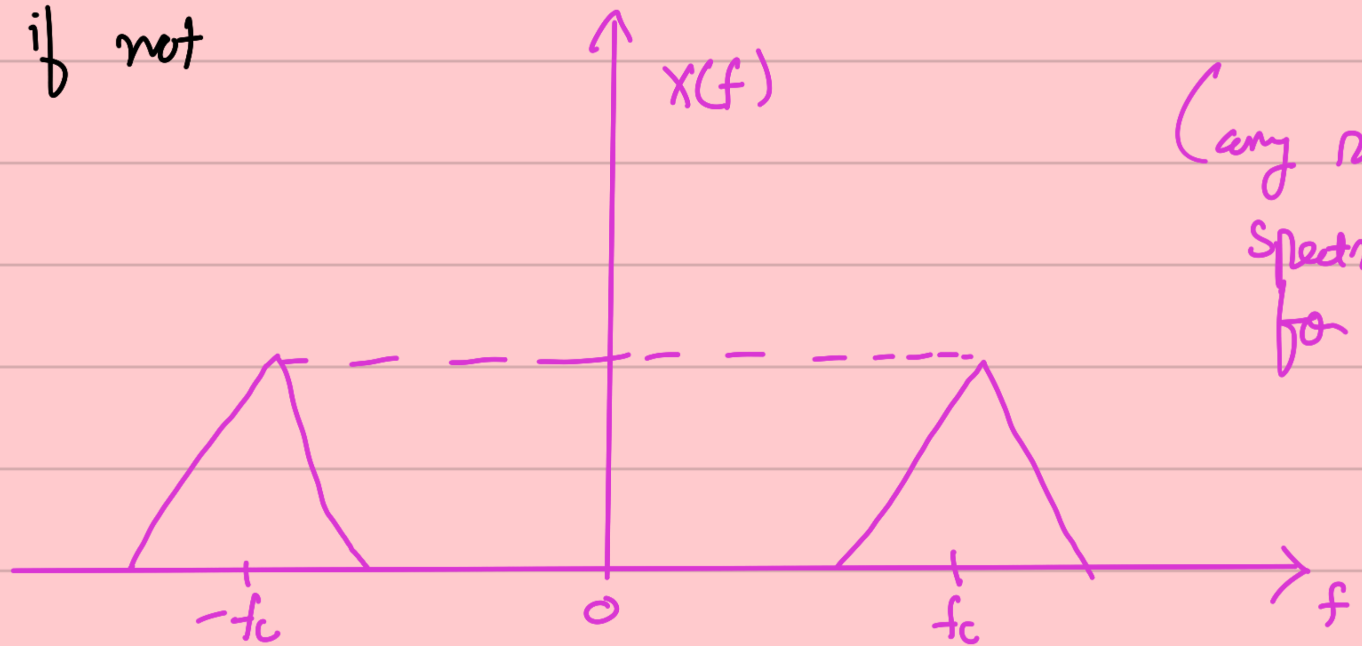
$$\mathcal{FT}\{x(t)\} = M(f) * \pi\left(\delta(f-f_c) + \delta(f+f_c)\right)$$

$$= \pi\left(M(f-f_c) + M(f+f_c)\right)$$

If $m(t)$ is from q2.



if not



(any random spectrum for m(t))