EC5.101 - Network, Signals and Systems
Quiz 1

Date: 25th November, 2022
Exam duration: 45 minutes

Maximum marks: 24

Instructions:

a) There are *6 questions* for a total of *24 marks*.

b) Mention any additional assumptions you make that is not given in the question.

c) Clearly show the steps used to arrive at the solutions.

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1. [4 marks] Consider the periodic signal given below:

$$x(t) = \sin(2t) + \sin^2(t).$$

   (a) Find the trigonometric Fourier series (FS) coefficients of $x(t)$.
   (b) Find the complex FS coefficients of $x(t)$.

2. [6 marks] The complex FS coefficients of a periodic signal $x(t)$ are $\{j, -1, 2, \underset{\uparrow}{0}, 2, -1, -j\}$ where $j = \sqrt{-1}$. Find the FS coefficients of the following signals:

   (a) $2x(2t)$
   (b) $x(2-t)$
   (c) $x(t) - x(-t)$
   (d) $2 - x(t)$

3. [2 marks] Identify whether the given signals are odd, even, or neither.

   (a) $x(t) = t^2\cos(3t)$
   (b) $x(t) = t\cos(70t) + t\sqrt{1+t^2}$
   (c) $x(t) = \sin^2(7t) + \sin(t)$
   (d) $x(t) = \log(\cos(t))$

4. [4 marks] Consider the operator representation of a system given by

$$Y = X + \mathcal{A}(X + \alpha Y)$$

(a) Sketch the block diagram representation of this sytem.

(b) Write down the differential equation representation of this sytem.

5. [4 marks] Let the trigonometric FS coefficients of a periodic signal $x(t)$ be written in the format $\{a_0, [a_1, a_2, \ldots], [b_1, b_2, \ldots]\}$. FS coefficients of two periodic signals $x_1(t)$ and $x_2(t)$ having the same period are given below:

(a) $x_1(t) \longleftrightarrow \{1, [1, 0, 1], [2]\}$ and $x_2(t) \longleftrightarrow \{0, [-1, 0, -1], [-2]\}$

(b) $x_1(t) \longleftrightarrow \{0, [1, 0, 1], [0, 2]\}$ and $x_2(t) \longleftrightarrow \{1, [0, 1, 0], [2, 0]\}$

Assume that all other coefficients are zero. In each case, identify whether the pair of signals $x_1(t)$ and $x_2(t)$ are orthogonal or not. Justify your answer without performing explicit computations.

6. [4 marks] For a periodic signal with period $T = 1$ it is given that,

$$\int_0^1 |x(t)|^2 dt = 4 = |a_{-1}|^2 + |a_0|^2 + |a_1|^2$$

where a_k denote the complex FS coefficients.

(a) If it is known that the signal is even and $a_{-1} = j$, then compute a_0.

(b) If it is known that the signal is even and has half-wave symmetry, compute a_{-1}.