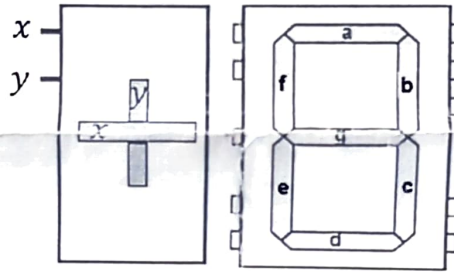


CALCULATORS ARE NOT ALLOWED

Numbers in square brackets [x] after a statement show the marks for that question.

Numbers in {} brackets are for administrative use. Please ignore.

Q1. 7-segment LED displays are a very important part of any digital system. For example, for showing which floor a lift is currently present. Imagine a building with 7 floors, 1 ground floor and 8 basement floors. To keep the arithmetic simple, the building management system uses 2's complement binary system of appropriate bit length to store the floor number of the lift (positive for floors above ground, 0 for ground floor and negative for basement floors). For the lift display, we decide to denote the ground floor as "0" on the 7-segment display. For floors above the ground floor, we display the floor number with a "+" sign, and in case of basement floors, we display the floor number with a "-" sign. We are able to do this using a 2-segment LED display in front of the 7-segment display, as shown in the figure below. Also, just to keep the aesthetics, we decide that when "0" is displayed, we will keep the 2-segment display off, because "+0" or "-0" looks weird! Your job is to design a circuit that takes input from the building management system about the floor number of the lift, and displays it using the lift display. You can assume that you already have 7447 decoder ICs (BCD to 7-segment decoder) with you, so you can denote it as a black box. Don't worry about optimizing the transistor count or the timing.



[12 marks]{CO-2}

Q2. Let us define a set $B = \{1, p, q, pq\}$, where p and q are distinct prime natural numbers. Let us define two operators on the elements of this set as Least Common Multiple denoted by $(a \# b)$, and Greatest Common Divisor (also called Greatest Common Factor or Highest Common Factor) denoted by $(a * b)$. Are the operations $\#$ and $*$ closed on the set B [4]? Do p and q need to be prime for the closure to work [2]? Do the operations $\#$ and $*$ have an identity element, if so, what are they [4]?

[10 marks]{CO-1}

Q3. MUXes are implemented using the sum of products form such that each minterm of the select lines is calculated and ANDed with each input line, and the result is ORed. How many transistors are required to create a 2^n -to-1 MUX in such a manner [2]? For a large n , we can create two minterms by first ANDing $(n - 1)$ terms, then ANDing the remaining term (and its complement) with two separate 2-input AND gates. If this is done, what is the transistor count for our 2^n -to-1 MUX [4]? Above what value of n is this trick advisable [2]?

[8 marks]{CO-2}

Q4. We all love the 2's complement representation because it makes arithmetic very easy. Numbers stored in 2's complement form can simply be read from memory, added and subtracted and the result is also correct in 2's complement form. Prove this mathematically. Ignore cases of overflow.

[10 marks]{CO-1}