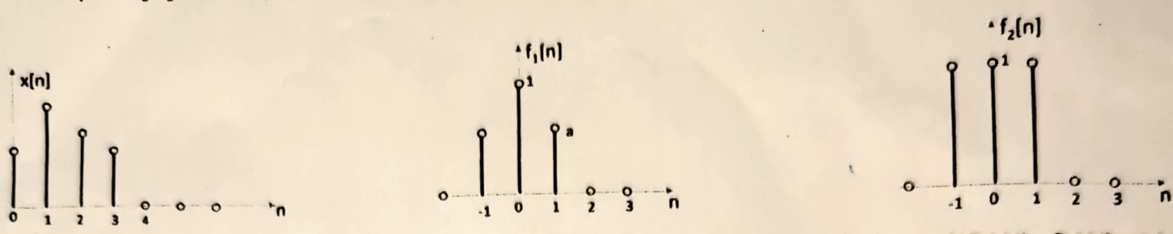
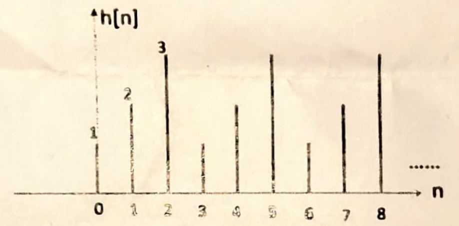
1. Upsampling a sequence x[n] by a factor of M is desired. A proposed method for this is :
   (i) create a sequence $x_e[n]$ by introducing M-1 zeros between successive samples of x[n].
   (ii) convolve $x_e[n]$ with a suitable sequence f[n] to obtain the final result.

   A sample x[n] and 2 possible candidates for f[n] are shown below. [2x5 +2 marks]



   a. Which of these sequences (f_1 or f_2) will give the best upsampled result? Why? What is the length of the final result with either of these sequences? Answer *without* doing any convolution.
   b. Sketch the appropriate f[n] for upsampling with M = 4.
   c. Identify the purpose of each of the steps in the proposed method.

2. A causal system with input x[n] and output y[n] has impulse response h[n] as given below. [6+6 marks]

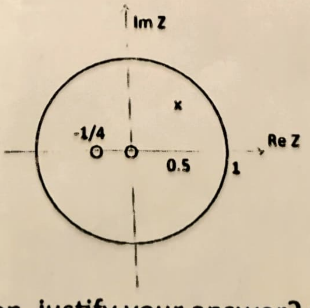


   a. Write h[n] in terms of δ[n] and use it to write the difference equation (part of which is given below) of this system.

   $$y[n] - y[n-3] = \_\_\_\_\_\_\_\_.$$

   b. Implement this equation using least number of these units: adders, delays and gains.

3. An *even* sequence is one which satisfies x[n] = x[-n]. Assuming it has a *rational* z-transform X(z) answer the following. [3+4+3 marks]
   a. What is the consequence of the *even* nature of the sequence for X(z)?
   b. Complete the partial pole-zero plot shown below for a *real, even* sequence and find X(z). What could be its ROC?



   c. Is the DTFT of the x[n] real and even, justify your answer? Can you find another signal as a function of x[n] such the that resultant signal DTFT is complex and even?

[PTO]

4. Consider the $x_e[n]$, $f[n]$ and $x[n]$ defined in Question 1 and identify the following. [3+5+2 marks]

   a. Relate the DTFT of $x_e[n]$ with DTFT of $x[n]$.

   b. Let the $y[n]= x_e*f[n]$ and $y_1[n]= y[2n]$ then

      i. Express the DTFT of $y[n]$ in terms of DTFTs of $x[n]$ and $f[n]$.

      ii. Express the DTFT of $y_1[n]$ in terms of DTFTs of $x[n]$ and $f[n]$

      iii. Express the $y_1[n]$ in terms of $x[n]$ and $f[n]$

   c. Consider the relation obtained in b.iii and show that the relation holds for the sequence of $x[n]$ and $f[n]$ provided in Question 1.