(CS1.301) Algorithm Analysis and Design (Monsoon 2023) Date: 20.10.2023

Deep Quiz 2

Alloted time: 45 minutes Total marks: 15

Instructions:

- There are a total of 3 questions with varying credit. Partial credit exists for all questions.
- Discussions amongst the students are not allowed. No electronic devices nor notes/books of any kind are allowed.
- Any dishonesty shall be penalized heavily.
- Place your identity cards on the table for verification.
- Be clear in your arguments. Vague arguments shall not be given any credit.

Question 1

[4 marks]

As input we are given n items such that item j has profit p_j and weight w_j and we are also given a knapsack of capacity B. All of these are positive integers. We need to find the subset $S \subseteq 1, \ldots, n$ which maximizes $\sum_{j\in S} p_j$ and "fits" in the knapsack; that is, $\sum_{j\in S} w_j \leqslant B$.

Question 2

[5.5 marks]

Let the matrices A, B and C be of dimensions $10 \times 5, 5 \times 30$ and 30×15. The task at hand is to compute the product of the matrices as ABC (note that the ordering matters). You could possibly multiply AB first (with $10 \times 5 \times 30$ operations) and multiply the result with C (with $10 \times 30 \times 15$ operations). This takes a total of 6000 operations. You could also possibly multiply A with the product BC which is precomputed. This takes $10 \times 5 \times 15 + 5 \times 30 \times 15 = 3000$ operations. Clearly, the second approach is more optimal.

Let $A_1, A_2, \ldots, A_n$ be n matrices such that for all integers $1 \leqslant i \leqslant n$ matrix A_i has dimension $d_{(i-1)} \times d_i$. We would like to find an optimal way to multiply these matrices $A_1, A_2, \ldots, A_n$.

Let us say we get the optimal way of multiplying matrices $A_i, \ldots, A_j$ is the minimum over all k, the product of $A[i..k]$ and $A[k+1..j]$ where $A[i..k]$ is the product of the matrices $A_i, \ldots, A_k$ and $A[k+1..j]$ is the product of matrices $A_{k+1}, \ldots, A_j$.

For $i < j$, let $\mathrm{Opt}(i, j)$ be the optimal way of multiplying matrices $A_i, \ldots, A_j$. Please write a mathematical expression (recurrance relation) for $\mathrm{Opt}(i, j)$ using the above strategy, construct the memoization matrix, and explain how this helps us arrive at the optimal computation of matrices A, B and C as given in the example above.

Question 3 [5.5 marks]

Suppose you are given three strings, S_1, S_2, and S_3, where $|S_1| = n$, $|S_2| = m$, and $|S_3| = m+n$. We say that S_3 is an interleaf of S_1 and S_2 if and only if S_3 can be formed by interleaving sequences of characters from S_1 and S_2 in a way that maintains the left-to-right ordering of S_1 and S_2. For example, "split" is an interleaving of "spit" and "l", but "splti" is not, and "cchocohilaptes" is an interleaf of "chocolate" and "chips".

Give an efficient dynamic programing algorithm that takes S_1, S_2, and S_3 as parameters and determines whether S_3 is an in interleaf of S_1 and S_2.

Here memoization matrix could take True or False values in each entry where True in entry $M_{i,j}$ could represent if the first $i+j$ letters of S_3 are formed by interleaving of first i letters of S_1 and first j letters of S_2.