2022101022

Linear Algebra Quiz : 2
Total Marks:20

Question 1 [2 Marks]

Let A be an $m \times n$ matrix. Then prove that the matrix transformation T_A : $\mathbb{R}^n \to \mathbb{R}^m$ defined by :

$$T_A = Ax \text{ (for } x \text{ in } \mathbb{R}^n)$$

is a linear transformation.

Question 2 [3+3=6 Marks]

Show that the rotation about the origin through an angle θ defines a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ and find its standard matrix.

Question 3 [3+2=5 Marks]

A market research team is conducting a controlled survey to determine people's preferences in toothpaste. The sample consists of 200 people, each of whom is asked to try two brands of toothpaste over a period of several months. There are just two states using Brand A and using Brand B, and the transition probabilities are those indicated in Fig. 1. Suppose that, when the survey begins, 120 people are using Brand A and 80 people are using Brand B.

- Write the Markov chain equation at $k-th$ month. How many people will be using each brand 1 month later? 2 months later?
- Suppose, we wanted to keep track of not the actual numbers of toothpaste users but, rather, the relative numbers using each brand. Can you guess (approximately) the steady state vector (the fraction of user A and User B with respect to the total users) for a long term limit? Does it depend on the initial fraction?

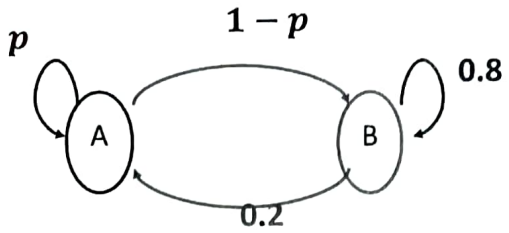

Figure 1: Schematic diagram of transition matrix. Here p is the last digit of your roll number $\times 0.1$.

Question 4 [3+2+2=7 Marks]

Five people are all connected by e-mail. Whenever one of them hears a juicy piece of gossip, he or she passes it along by e-mailing it to someone else in the group according to Table

Sender	Recipients
Ann	Carla, "YOU"
Bert	Carla, Dana
Carla	"YOU"
Dana	Carla, Ann
"YOU"	Bert, Dana

- Draw the digraph that models this "gossip network" and find its adjacency matrix A.
- If "YOU" hears a rumor, how many steps will it take for everyone else to hear the rumor? What matrix calculation reveals this?
- How many directed 2- paths exist from Dana to "YOU" ?

Definition: If G is a diagraph with n vertices, then its adjacency matrix is the $n \times n$ matrix A defined by

$$A_{ij} = \begin{cases} 1 & \text{if there is an edge from vertex i to vertex j} \\ 0 & \text{otherwise} \end{cases}$$

Note : Replace "YOU" by your name.