Mid-term Examination

Information and Communication (Spring 2023)

Time : 1hr 30 mins, Total Marks: **50**

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Instructions:

- Reasons for all steps should be given, in general.
- This is a closed book, traditional, exam.
- Malpractice will directly result in 0 and further academic action will be initiated.

Questions:

1. ($5 \times 3 = 15$ marks) Answer whether the following statements are true or false (T/F), giving appropriate brief reasons for the same (2-3 lines max).

   (a) T/F? : Reconstructing an analog signal from its sampled version is impossible for most signals of importance in engineering.

   (b) T/F? : The bandwidth of a baseband signal with highest non-zero frequency component at 40 KHz and lowest non-zero frequency component at 20 KHz is 20 KHz.

   (c) It is known that all stars with mass greater than C (the so-called Chandrashekar Limit) could possibly collapse to form a black-hole. The statement you have to check whether true or false is given in **bold** below.

   event space.

   - Consider the set Ω as the set of all stars. Let $\mathcal{F}$ denote the collection $\{A \subseteq \Omega :$ each star in A has mass greater than $C\}$. Let $P : \mathcal{F} \to \mathbb{R}$ be a function, such that for each $A \in \mathcal{F}$, $P(A)$ denotes the probability that at least one star in the subset A ends up as a black-hole. **T/F?: The triple $(\Omega, \mathcal{F}, P)$ form a valid probability space.**

   (d) T/F?: The function G defined below represents the CDF of a Discrete Random Variable.

$$G(x) = \begin{cases} 0, & \forall x < 1 \\ 1 - 1/a, & \forall x \in [a, a+1), \forall a \in \{1, 2, \ldots, 100\} \\ 99/100 & \forall x \in [101, 101 + 10^{-100}) \\ 1, & \forall x \geq 100 + 10^{-100}, \end{cases}$$

   (e) T/F? : In any probability space, two events that are independent can never be mutually exclusive.

2. (5 marks) Write clearly and completely the probability density function (p.d.f) of a Gaussian Random variable whose mean is 15, and whose second moment is 10 times its variance.

3. (5+3+8=16 marks)

(a) For any signal $x(t)$, let $x_s(t)$ denote the signal obtained by sampling $x(t)$ with sampling period T. Show that, the signal obtained by passing $x_s(t)$ through an Ideal Low Pass Filter with cutoff frequency $1/2T$ is $\sum_{k \in \mathbb{Z}} x(kT) sinc(t/T - k)$. (Hint: Think what happens in the frequency domain when passing $x_s(t)$ through the ideal LPF. Then map it back to time domain. Standard FT pairs can be used directly to answer this question).

(b) Assume that $x(t) = cos(400\pi t)$. What is the Nyquist sampling rate for this signal? Will sampling at exactly the Nyquist sampling rate enable reconstruction? Argue with reasons.

(c) Suppose the above signal $x(t)$ is sample at a rate of 800 samples per second. Describe precisely (using mathematical equations) the sampled signal $x_s(t)$. Find the spectrum of the sampled signal. Show (perhaps, by using this spectrum) that, by passing the sampled signal through an ideal LPF with cut-off at 400 Hz, we get the original signal $x(t)$ (you can use some of the observations/results done in the class for answering the last part of this bit).

4. (14 marks) A string $\boldsymbol{x}$ of length 100 bits (that is, a vector $\boldsymbol{x} \in \{0,1\}^{100}$) is passed through a channel. This channels flips each transmitted bit (i.e., changes the bit from 0 to 1 or vice-versa) independently, with probability $p = 0.2$. The receiver gets the resulting vector.

(a) (6 marks) Describe an appropriate (one that fits the above channel description) probability distribution (PMF) for the number of flips that the channel imposes on $\boldsymbol{x}$. Prove that it is a valid distribution.

(b) (5 marks) Find the expected value of the number of flips. Do not use a formula directly. You may prove the formula first, or calculate the mean directly from first principles.

(c) (3 marks) Give a computable mathematical expression that captures the probability that the number of flips is more than 70.