COUPLES:

(I) Base ~~difference~~ SUM constant: ${}^mC_0\,{}^nC_r + {}^mC_1\,{}^nC_{r-1} + {}^mC_2\,{}^nC_{r-2} + \dots + {}^mC_r\,{}^nC_0 = {}^{m+n}C_r$: Team of r from m boys & n girls

$\Rightarrow {}^mC_0\,{}^nC_r - {}^mC_1\,{}^nC_{r-1} + \dots + (-1)^r\,{}^mC_r\,{}^nC_0$ = Coeff of x^r in $(1-x)^m(1+x)^n$

(II) Base difference constant: ${}^mC_0\,{}^nC_r + {}^mC_1\,{}^nC_{r+1} + {}^mC_2\,{}^nC_{r+2} + \dots + {}^mC_{n-r}\,{}^nC_n$

$$(1+x)^m\left(1+\frac{1}{x}\right)^n = \left({}^mC_0 + {}^mC_1 x + {}^mC_2 x^2 + \dots + {}^mC_m x^m\right)\left({}^nC_0 + {}^nC_1\frac{1}{x} + {}^nC_2\frac{1}{x^2} + \dots + {}^nC_n\frac{1}{x^n}\right)$$

Σ = coeff of x^{-r} in $(1+x)^m\left(1+\frac{1}{x}\right)^n$ = coeff of x^{n-r} in $(1+x)^m(1+x)^n = (1+x)^{m+n}$

$= {}^{m+n}C_{n-r}$

* $$\sum_{r=1}^{n} {}^{m+r}C_r = {}^{m+n+1}C_{n+1}$$

BINOMIAL : SUM OF SERIES : Prop. → ${}^nC_r = \frac{n}{r}\,{}^{n-1}C_{r-1}$ → ${}^nC_r = \frac{n-(r-1)}{r}\,{}^nC_{r-1}$

DENTIST Prop. degradation

$(1+x)^n = {}^nC_0 + {}^nC_1 x^1 + \dots + {}^nC_r x^r + \dots + {}^nC_n x^n$ → ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$ upgradation

(I) $\sum_{r=0}^{n} {}^nC_r = 2^n \; (=(1+1)^n)$

$\sum_{r=0}^{n} r\,{}^nC_r = \sum_{r=0}^{n} r\cdot\frac{n}{r}\cdot{}^{n-1}C_{r-1} = n\sum_{r=0}^{n} {}^{n-1}C_{r-1} = n\cdot 2^{n-1}$

Similarly, $\sum r(r-1)\,{}^nC_r = n(n-1)2^{n-2}$ OR $\sum r^2\,{}^nC_r = n(n-1)2^{n-2} + \sum r\,{}^nC_r = n(n+1)2^{n-2}$

(II) $\sum_{r=0}^{n} (-1)^r\,{}^nC_r = (1+(-1))^n = 0 \,;\, n\geq 1$

$\Rightarrow n(1+x)^{n-1} = {}^nC_1 + 2\,{}^nC_2 x + \dots + r\,{}^nC_r x^{r-1} + \dots + n\,{}^nC_n x^{n-1}$

$\sum_{r=0}^{n} (-1)^r\cdot r\cdot{}^nC_r = n(1+(-1))^n = 0 \,;\, n\geq 2$

Similarly, $\sum r^2 (-1)^r\,{}^nC_r = 0 \,;\, n\geq 3$

(III) $C_0 + 3C_1 + 5C_2 + \dots + (2n+1)C_n = \sum_{r=0}^{n} (2r+1)\,{}^nC_r = 2\cdot n2^{n-1} + 2^n = (n+1)2^n$

$\Rightarrow \int (1+x)^n dx = {}^nC_0 x + \frac{{}^nC_1 x^2}{2} + \frac{{}^nC_2 x^3}{3} + \dots + \frac{{}^nC_r x^{r+1}}{r+1} + \dots + \frac{{}^nC_n x^{n+1}}{x+1}$

$\sum_{r=0}^{n} \frac{{}^nC_r}{r+1} = \frac{2^{n+1}-1}{n+1}$ OR $2C_0 + 2^2\frac{C_1}{2} + 2^3\frac{C_2}{3} + \dots + 2^{n+1}\frac{C_n}{n+1} = \frac{3^{n+1}-1}{n+1}$

$\Rightarrow \frac{C_1}{2} + \frac{C_3}{4} + \frac{C_5}{6} + \dots + \frac{C_{2r-1}}{2r} + \dots + \frac{C_{2n-1}}{2n} = \sum_{r=1}^{n} \frac{{}^nC_{2r-1}}{2r} = \sum_{r=1}^{n} \frac{{}^{n+1}C_{2r}}{n+1} = \frac{1}{n+1}\sum_{r=1}^{n} {}^{n+1}C_{2r} = \frac{1}{n+1}(2^{n+1}-1)$

(IV) $\sum_{r=0}^{n} {}^rC_k = \underbrace{{}^0C_k + {}^1C_k + \dots + {}^{k-1}C_k}_{\text{zero}} + {}^kC_k + {}^{k+1}C_k + \dots + {}^nC_k = {}^{k-1}C_k + {}^{k-1}C_{k+1} + {}^kC_k + \dots + {}^nC_k \underbrace{- {}^{k-1}C_{k+1}}_{\text{zero}}$

$= {}^kC_{k+1} + {}^kC_k + \dots = {}^{k+1}C_{k+1} + {}^{k+1}C_k + {}^{k+2}C_k + \dots + {}^nC_k = {}^{n+1}C_{k+1}$

DEPENDENT & INDEPENDENT SUMMATIONS:

$$\begin{array}{c|c}
 & i \\
\hline
j & \begin{bmatrix} {}^nC_0\,{}^nC_0 & {}^nC_0\,{}^nC_1 & \cdots & {}^nC_0\,{}^nC_j & \cdots & {}^nC_0\,{}^nC_n \\ {}^nC_1\,{}^nC_0 & {}^nC_1\,{}^nC_1 & \cdots & {}^nC_1\,{}^nC_j & \cdots & {}^nC_1\,{}^nC_n \\ \vdots & \vdots & & \vdots & & \vdots \\ {}^nC_i\,{}^nC_0 & {}^nC_i\,{}^nC_1 & \cdots & {}^nC_i\,{}^nC_j & \cdots & {}^nC_i\,{}^nC_n \\ \vdots & \vdots & & \vdots & & \vdots \\ {}^nC_n\,{}^nC_0 & {}^nC_n\,{}^nC_1 & \cdots & {}^nC_n\,{}^nC_j & \cdots & {}^nC_n\,{}^nC_n \end{bmatrix} \begin{array}{l} T \\ \\ \\ \\ \\ D \end{array}
\end{array}$$

$$\sum_{i=0}^{n}\sum_{j=0}^{n} {}^nC_i\,{}^nC_j = \sum_{i=0}^{n}\left({}^nC_i\sum_{j=0}^{n} {}^nC_j\right) = \sum_{i=0}^{n} 2^n\cdot {}^nC_i = 2^{2n}$$

Also, $\displaystyle\sum_{i=0}^{n}\sum_{j=0}^{n} {}^nC_i\,{}^nC_j = 2\sum_{0\le i<j\le n} {}^nC_i\,{}^nC_j + \sum_{i=0}^{n}\left({}^nC_i\right)^2$

$$\Rightarrow 2^{n} = 2\left(\sum_{0\le i<j\le n} {}^nC_i\,{}^nC_j\right) + {}^{2n}C_n \Rightarrow \sum_{0\le i<j\le n} {}^nC_i\,{}^nC_j = \frac{2^{2n} - {}^{2n}C_n}{2}$$

Similarly, $\displaystyle\sum\sum_{0\le i<j\le n}(C_i-C_j)^2 = \frac{\sum_{i=0}^{n}\sum_{j=0}^{n}(C_i-C_j)^2 - 0}{2} = \frac{\sum_{j=0}^{n}\sum_{i=0}^{n}\left(C_i^2 - 2C_iC_j + C_j^2\right)}{2} = \frac{\sum_{i=0}^{n}\left((n+1)C_i^2 - 2^{n+1}C_i + {}^{2n}C_n\right)}{2}$

$$= \frac{(n+1)\,{}^{2n}C_n - 2^{2n+1} + (n+1)\,{}^{2n}C_n}{2} = (n+1)\,{}^{2n}C_n - 2^{2n}$$

$$\sum\sum_{0\le i<j\le n}(i+j)(C_i+C_j+C_iC_j) = S = \sum\sum_{0\le i<j\le n}(n-i+n-j)(C_i+C_j+C_iC_j) = 2n\sum\sum_{0\le i<j\le n}(C_i+C_j+C_iC_j) - S$$

$$\Rightarrow S = n\,\frac{\sum_{i=0}^{n}\sum_{j=0}^{n}(C_i+C_j+C_iC_j) - \sum_{i=0}^{n}(2C_i+C_i^2)}{2} = \frac{n}{2}\left(\sum_{i=0}^{n}\left((n+1)C_i + 2^n + 2^nC_i\right) - \left(2^{n+1} + {}^{2n}C_n\right)\right)$$

$$= \frac{n}{2}\left((n+1)2^n + (n+1)2^n + 2^{2n} - 2^{n+1} - {}^{2n}C_n\right) = \frac{n}{2}\left(n2^{n+1} + 2^{2n} - {}^{2n}C_n\right)$$

$$\sum\sum_{0\le i<j\le n}(i\cdot j)\,C_i\cdot C_j = \sum\sum_{0\le i<j\le n}\left(n^2 - (i+j)n + i\cdot j\right)C_iC_j$$

$$\sum\sum_{0\le r<s\le n} {}^nC_s\,{}^sC_r = \sum_{r=0}^{n}\sum_{s=0}^{n} {}^nC_s\,{}^sC_r - \sum_{s=0}^{n} {}^nC_s\,{}^sC_s$$

$$= \sum_{s=0}^{n} 2^s\cdot {}^nC_s - 2^n$$

$$= \underline{\underline{3^n - 2^n}}$$

upper triangle is zero

$$\begin{array}{c|c}
 & s \\
\hline
r & \begin{bmatrix} {}^nC_0\,{}^0C_0 & {}^nC_0\,{}^0C_1 & \cdots & {}^nC_0\,{}^0C_r & \cdots & {}^nC_0\,{}^0C_n \\ {}^nC_1\,{}^1C_0 & {}^nC_1\,{}^1C_1 & \cdots & {}^nC_1\,{}^1C_r & \cdots & {}^nC_1\,{}^1C_n \\ \vdots & \vdots & & \vdots & & \vdots \\ {}^nC_s\,{}^sC_0 & {}^nC_s\,{}^sC_1 & \cdots & {}^nC_s\,{}^sC_r & \cdots & {}^nC_s\,{}^sC_n \\ \vdots & \vdots & & \vdots & & \vdots \\ {}^nC_n\,{}^nC_0 & {}^nC_n\,{}^nC_1 & \cdots & {}^nC_n\,{}^nC_r & \cdots & {}^nC_n\,{}^nC_n \end{bmatrix}
\end{array}$$

$$e^{i\theta} = \cos\theta + i\sin\theta \Rightarrow (e^{i\theta})^n = (\cos\theta + i\sin\theta)^n = e^{in\theta}$$

$$\Rightarrow {}^nC_0\cos^n\theta + {}^nC_1\, i\sin\theta\cos^{n-1}\theta + {}^nC_2\, i^2\sin^2\theta\cos^{n-2}\theta + \dots + {}^nC_n\, i^n\sin^n\theta = \cos n\theta + \sin n\theta$$

EXPANSIONS OF $\cos n\theta$ & $\sin n\theta$

$$\cos n\theta = {}^nC_0\cos^n\theta - {}^nC_2\cos^{n-2}\theta\sin^2\theta + \dots$$

$$\sin n\theta = {}^nC_1\sin\theta\cos^{n-1}\theta - {}^nC_3\sin^3\theta\cos^{n-3}\theta + \dots$$

OR

$$\cos n\theta = \cos^n\theta\,(1 - {}^nC_2\tan^2\theta + {}^nC_4\tan^4\theta - \dots)$$

$$\cancel{\sin = \sin^n\theta}$$

$$\sin n\theta = \cos^n\theta\,({}^nC_1\tan\theta - {}^nC_3\tan^3\theta + {}^nC_5\tan^5\theta - \dots)$$

$$\therefore \tan n\theta = \frac{{}^nC_1\tan\theta - {}^nC_3\tan^3\theta + {}^nC_5\tan^5\theta - \dots}{1 - {}^nC_2\tan^2\theta + {}^nC_4\tan^4\theta - \dots} \text{ aka } \frac{S_1 - S_3 + S_5 - \dots}{1 - S_2 + S_4 - \dots}$$

Generalisation,

$$\cos(\theta_1 + \theta_2 + \dots + \theta_n) = \prod_{i=1}^{n}\cos\theta_i \cdot (1 - S_2 + S_4 - \dots)$$

$$\sin(\theta_1 + \theta_2 + \dots + \theta_n) = \prod_{i=1}^{n}\cos\theta_i \cdot (S_1 - S_3 + S_5 - S_7 + \dots)$$

$$\tan(\theta_1 + \theta_2 + \dots + \theta_n) = \frac{S_1 - S_3 + S_5 - S_7 + \dots}{1 - S_2 + S_4 - S_6 + \dots}$$

$$\int \frac{dx}{x^2+a^2} = \int \frac{a\sec^2\theta\, d\theta}{a^2\tan^2\theta + a^2} = \frac{1}{a}\int \frac{\sec^2\theta\, d\theta}{\sec^2\theta} = \frac{1}{a}\cdot\theta + c$$

$x = a\tan\theta$

$dx = a\sec^2\theta\, d\theta$

$$= \frac{1}{a}\tan^{-1}\frac{x}{a} + C$$

$$\int \frac{dx}{x^2-a^2} = \int \frac{a\cos\theta\, d\theta}{a^2\sin^2\theta - a^2} = -\frac{1}{a}\int \frac{d\theta}{\cos\theta} = -\frac{1}{a}\ln|\sec\theta+\tan\theta| + c$$

$x = a\sin\theta$

$dx = a\cos\theta\, d\theta$

$$= -\frac{1}{a}\ln\left|\frac{1+\sin\theta}{\cos\theta}\right| + c = -\frac{1}{a}\ln\left|\frac{1+\frac{x}{a}}{\sqrt{1-\frac{x^2}{a^2}}}\right| = -\frac{1}{2a}\left|\frac{(1+\frac{x}{a})^2}{(1-\frac{x}{a})(1+\frac{x}{a})}\right| + c$$

$$= \frac{1}{2a}\ln\left|\frac{a-x}{a+x}\right| + c$$

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \int \frac{a\sec\theta\tan\theta\, d\theta}{a\sqrt{\sec^2\theta-1}} = \ln|\sec\theta+\tan\theta| + c = \ln\left|\frac{x}{a} + \sqrt{\frac{x^2}{a^2}-1}\right| + c$$

$x = a\sec\theta$

$dx = a\sec\theta\tan\theta\, d\theta$

$$= \ln\left|\frac{x+\sqrt{x^2-a^2}}{a}\right| + c = \ln|x+\sqrt{x^2-a^2}| - \ln|a| + c$$

$$\int \frac{dx}{\sqrt{a^2+x^2}} = \int \frac{a\sec^2\theta\, d\theta}{a\sqrt{1+\tan^2\theta}} = \int \sec\theta\, d\theta = \ln|\sec\theta+\tan\theta| + c$$

$x = a\tan\theta$

$dx = a\sec^2\theta\, d\theta$

$$= \ln\left|\sqrt{\frac{x^2}{a^2}+1} + \frac{x}{a}\right| + c = \ln\left|\sqrt{x^2+a^2} + x\right| + c$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \int \frac{a\cos\theta\, d\theta}{a\sqrt{1-\sin^2\theta}} = \int d\theta = \sin^{-1}\frac{x}{a} + c = -\cos^{-1}\frac{x}{a} + c$$

$x = a\sin\theta$

$dx = a\cos\theta\, d\theta$

Classical theory of Probability: All outcomes are assumed to be 'equally-likely'

$$\text{Probability of an event} = \frac{\text{Total no. of outcomes favourable to the event}}{\text{Total no. of equally likely outcomes}}$$ aka "Theoretical Probability"

Empirical Probability / Statistical Approach of Probability:
When the prob. of the occurence of event A can't be worked out exactly (due to practical limitations & finite no. of trials), an empirical value is worked out.

Probability of occurence of event A, $P(A) = \lim_{n \to \infty} \left(\frac{r}{n}\right)$

where r = "number of successes", n = "no. of trials" i.e. (successes + failures)

This is aka "Experimental probability"

Axiomatic Approach to Probability: Let S be the sample space of a random experiment
The probability P is a real valued function whose domain is the power set of S and range is the interval [0, 1] satisfying the following axioms:

(1) For any event $P(E)$, $P(E) \geq 0$
(2) $P(S) = 1$
(3) If E and F are mutually exclusive events, then $P(E \cup F) = P(E) + P(F)$

* Let S be a sample space containing outcomes $\omega_1, \omega_2, \ldots \omega_n$, i.e. $S = \{\omega_1, \omega_2, \ldots \omega_n\}$

It follows that: (1) $0 \leq P(\omega_i) \leq 1$ for each $\omega_i \in S$
(2) $P(\omega_1) + P(\omega_2) + \ldots + P(\omega_n) = 1$
(3) For any event A, $P(A) = \sum P(\omega_i)$, $\omega_i \in A$

BINOMIAL THEOREM FOR ANY INDEX

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \frac{n(n-1)(n-2)(n-3)}{4!}x^4 + \cdots \quad , \text{ if } |x|<1$$

$$(1-x)^{-1} = \frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

$$\left.\begin{array}{l} (1+x)^{-n} = 1 - {}^{n}C_1 x + {}^{n+1}C_2 x^2 - {}^{n+2}C_3 x^3 + \cdots + {}^{n+k-1}C_k (-1)^k x^k + \cdots \\ (1-x)^{-n} = 1 + {}^{n}C_1 x + {}^{n+1}C_2 x^2 + {}^{n+2}C_3 x^3 + \cdots + {}^{n+k-1}C_k x^k + \cdots \end{array}\right\} n \in \mathbb{Z}$$

= no. of such distinct arrangements $(n_1, n_2, n_3, n_4 \ldots$

probability distribution

X :	x_1	x_2	...	x_n
$P(X)$:	p_1	p_2	...	p_n

$p_i > 0,\ i = 1, 2, \ldots, n,\ \sum_{i=1}^{n} p_i = 1$

RANDOM VARIABLE: A random variable is a real valued function whose domain is the sample space of a random experiment.

MEAN of X / expected value of X denoted by $E(X)$

$$\boxed{= \mu = E(X) = \sum_{i=1}^{n} x_i p_i}$$

$$\boxed{\text{variance},\ \sigma^2 = \sum_{i=1}^{n} (x_i - \mu)^2 p_i = \sum_{i=1}^{n} x_i^2 p_i - \mu^2 = E(X-\mu)^2}$$

$$\boxed{\text{standard deviation},\ \sigma = \sqrt{\sum_{i=1}^{n} (x_i - \mu)^2 p_i}}$$

Conditional probability: $P(E|F) = \dfrac{P(E \cap F)}{P(F)}$ provided $P(F) \neq 0$

For random variable: mean $(\mu) = np$; $n \to$ no. of trials in Bernoulli

$p \to$ prob. of success in each trial

variance $(\sigma) = npq$; $q \to$ prob. of failure

#Wikipedia: If the random variable X follows the binomial distribution with parameters $n \in \mathbb{N}$ and $p \in [0, 1]$, we write $X \sim B(n, p)$. The probability of getting exactly k successes in n independent Bernoulli trials is given by the probability mass function:

$$f(k, n, p) = Pr(k; n, p) = Pr(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

The 'Expected value', $E[X] = E[\underbrace{X_1 + X_2 + \cdots + X_n}] = E[X_1] + E[X_2] + E[X_3] + \cdots + E[X_n] = p + p + \cdots + p\ \boxed{= np}$

X is sum of n identical Bernoulli random variables, each with expected value p, E[X] is linear no. of such distinct

Also, $E[X] = \sum_{r=0}^{n} r\binom{n}{r} p^r (1-p)^{n-r} = \sum_{r=0}^{n} n\binom{n-1}{r-1} p^{r-1}(1-p)^{n-r}\cdot p = np\sum_{r=0}^{n}\binom{n-1}{r-1}p^{r-1}(1-p)^{(n-1)-(r-1)} = np(p+1-p)^{n-1}$

$\Rightarrow E[X] = np$

$Var[X] = E[X^2] - (E[X])^2 = \left[\sum_{r=0}^{n} r^2\binom{n}{r}p^r(1-p)^{n-r}\right] - (np)^2 = -n^2p^2 + \sum_{r=0}^{n} r(r-1)\binom{n}{r}p^r(1-p)^{n-r} + \sum_{r=0}^{n} r\binom{n}{r}p^r(1-p)^{n-r}$

$= -n^2p^2 + n(n-1)p^2\sum_{r=0}^{n}\binom{n-2}{r-2}p^{r-2}(1-p)^{(n-2)-(r-2)} + \sum_{r=0}^{n} r\binom{n}{r}p^r(1-p)^{n-r}$

$= -n^2p^2 + n(n-1)p^2 + np = np(-np + pn - p + 1) = np(1-p) = \boxed{npq}$

Q. Let $n_1 < n_2 < n_3 < n_4 < n_5$ be +ve int. such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. Then the no. of such distinct arrangements $(n_1, n_2, n_3, n_4, n_5)$ is

Let $n_2 = n_1 + a$, $n_3 = n_2 + b = n_1 + a + b$, $n_4 = n_3 + c = n_1 + a + b + c$, $n_5 = n_4 + d = n_1 + a + b + c + d$

where $n_1, a, b, c, d > 0$

$\therefore n_1 + n_2 + n_3 + n_4 + n_5 = 20$

$\Rightarrow 5n_1 + 4a + 3b + 2c + d = 20$

No. of ways = coeff. of x^{20} in $(\cancel{1}+x+x^2+\dots)^5 (\cancel{1}+x+x^2+\dots)^4 (\cancel{1}+x+x^2+\dots)^3 (\cancel{1}+x+x^2+\dots)^2 (\cancel{1}+x+x^2+\dots)$

$=$ " x^{20} in $(\cancel{1}+x+x^2+\dots)^{15}$

$=$ " x^5 in $(1+x+x^2+\dots)^{15}$

$=$ " x^5 in $(1-x)^{-15}$

$= {}^{15+5-1}C_5 = {}^{19}C_5$

No. of ways = coeff of x^{20} in $(x^5 + x^{10} + \dots)(x^4 + x^8 + \dots)(x^3 + x^6 + \dots)(x^2 + x^4 + \dots)(x + x^2 + \dots)$

$=$ " x^{20} in $\dfrac{x^5}{(1-x^5)} \dfrac{x^4}{(1-x^4)} \dfrac{x^3}{(1-x^3)} \dfrac{x^2}{(1-x^2)} \dfrac{x}{(1-x)}$

$=$ " x^5 in $(1-x^5)^{-1}(1-x^4)^{-1}(1-x^3)^{-1}(1-x^2)^{-1}(1-x)^{-1}$

$=$ " x^5 in $(1 + x^5 + x^{10} + \dots)(1 + x^4 + x^8 + \dots)(1 + x^3 + x^6 + \dots)(1 + x^2 + x^4 + x^6 + \dots)$
$(1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + \dots)$

$= (0,0,0,0,5)\ (00023)\ (00102)\ (04001)\ (\cancel{00321})\ (00041)\ (50000)$

Q. Two friends visit a restaurant randomly during 5pm to 6pm. ~~Any~~ Among the two, whoever comes first waits for 15 min & then leaves. The probability that they meet is?

M-1: If first comes in (0-45) min, second must come within next 15 min

$$\text{Prob.} = \left(\frac{3}{4}\right) \times \left(\frac{1}{4}\right)$$

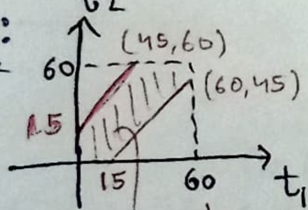
$$\text{i.e. } \left(\frac{45}{60}\right) \quad \left(\frac{n+15-n}{60}\right)$$

If first comes in (45-60) min, whenever next comes, they'll meet

$$\text{Prob} = \left(\frac{60-45}{60}\right) = \frac{1}{4}$$

$$\therefore \text{Req. prob} = \frac{3}{4}\times\frac{1}{4}+\frac{1}{4} = \frac{7}{16}$$

M-2:



↳ when they meet

A arrives at 5pm t_1 min
B arrives at 5pm t_2 min
They meet if $|t_1 - t_2| \le 15$ (shaded region)

$$\therefore \text{Prob.} = \frac{3600 - 2\times\frac{1}{2}\times 45^2}{3600} = \frac{7}{16}$$

$$\sum_{k=2}^{\infty} \frac{k^4+3k^2+10k+10}{2^k(k^4+4)} = \sum_{k=2}^{\infty}\left(\frac{1}{2^k} + \frac{3k^2+10k+6}{2^k(k^4+4k^2+4-4k^2)}\right) = \frac{1/4}{1-1/2} + \sum \frac{3k^2+10k+6}{2^k((k^2+2)^2-(2k)^2)}$$

$$= \frac{1}{2} + \sum \frac{3k^2+10k+6}{2^k(k^2+2k+2)(k^2-2k+2)}$$

(due to 2^k in denominator, multiples of 2 are allowed to be in the repeating term)

$$= \frac{1}{2} + \sum \frac{4(k^2+2k+2)-(k^2-2k+2)}{2^k(k^2+2k+2)(k^2-2k+2)}$$

$$= \frac{1}{2} + \sum_{k=2}^{\infty}\left(\frac{1}{2^{k-2}(k^2-2k+2)} - \frac{1}{2^k(k^2+2k+2)}\right) = \frac{1}{2} + \frac{1}{2^0\cdot 2} - \frac{1}{4\cdot 10} + \frac{1}{2\cdot 5} - \frac{1}{8\cdot 17} + \frac{1}{4\cdot 10} - \frac{1}{16\cdot 26} + \frac{1}{8\cdot 17} + \cdots$$

$\hookrightarrow ((k-2)^2+2(k-2)+2)$

☆ Terms REPEAT

$$= \frac{1}{2} + \frac{1}{2} + \frac{1}{10} = \boxed{\frac{11}{10}}$$

Q. The value of $\int_0^{\pi/4} e^{x\sec^2 x - \tan x}\left(x\tan x - \frac{\sin 2x}{2}\right)dx$ is

$\frac{d}{dx}\left(e^{x\sec^2 x - \tan x} f(x)\right) = e^{x\sec^2 x - \tan x}\left(f'(x) + (\cancel{\sec^2 x} + x\cdot 2\sec^2 x\tan x - \cancel{\sec^2 x}) f(x)\right)$

$I = \int_0^{\pi/4} e^{x\sec^2 x - \tan x}\left(x\tan x\cdot 2\sec^2 x\cdot \frac{1}{2}\cos^2 x + \cos x(-\sin x)\right)dx$

$= \int_0^{\pi/4} d\left(e^{x\sec^2 x - \tan x}\cdot \frac{1}{2}\cos^2 x\right)dx = \left[e^{x\sec^2 x - \tan x}\cdot \frac{1}{2}\cos^2 x\right]_0^{\pi/4}$

$= \frac{1}{2}\left[e^{\frac{\pi}{4}\times 2 - 1}\cdot\frac{1}{2} - 1\right] = \frac{1}{4}e^{\frac{\pi}{2}-1} - \frac{1}{2}$

Q. If x and y are non-zero numbers satisfying $x^2+y^2 = xy(x^2-y^2)$. Then the min value of x^2+y^2 is

Q. 5 girls and 10 boys sit at random in a row having 15 chairs numbered 1 to 15. What is the probability that the end seats are occupied by girls and b/w any two girls an odd no. of boys take their seats?

G a G b G c G d G $a+b+c+d=10 \Rightarrow$ (All odd) $\Rightarrow 2a_0+2b_0+2c_0+2d_0=6$

$\Rightarrow a_0+b_0+c_0+d_0=3 \rightarrow$ no. of non-negative integral sol^{n}s $\equiv$ answer

no. of ways $= 5!\cdot 10!\cdot {}^6C_3$

Q. Let a_r denotes the number of distributing r objects into 4 distinct boxes such that boxes 1 and 2 must hold an even no. of object and box 3 must hold an odd no. of objects, then $a_r = ?$

Sol^n: Suppose a case, $\boxed{2}\ \boxed{4}\ \boxed{3}\ \boxed{2} \rightarrow$ no. of ways $= \dfrac{11!}{2!\,4!\,3!\,2!}$

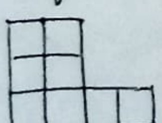
$\therefore$ Total no. of ways $=$ coeff of x^r in $r!\left(1+\frac{x^2}{2!}+\frac{x^4}{4!}+\cdots\right)^2\left(\frac{x}{1!}+\frac{x^3}{3!}+\cdots\right)\left(1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots\right)$

$=$ coeff of x^r in $r!\left(\frac{e^x+e^{-x}}{2}\right)^2\left(\frac{e^x-e^{-x}}{2}\right)e^x$

$=$ " $\frac{r!}{8}\left(e^{2x}+e^{-2x}+2\right)\left(e^{2x}-1\right) =$ coeff of x^r in $\frac{r!}{8}\left(e^{4x}+1+2e^{2x}-e^{2x}-e^{-2x}-2\right)$

$=$ " $\frac{r!}{8}\left(e^{4x}+e^{2x}-e^{-2x}-1\right) = \frac{r!}{8}\left(\frac{4^r}{r!}+\frac{2^r}{r!}-\frac{(-2)^r}{r!}\right) = \boxed{\frac{1}{8}\left(4^r+2^r-(-2)^r\right)}$

Q. The no. of ways in which the letters of the word RAMESH can be placed in the squares of the given figure so that no row remains empty, is



(1) 17280 (3) 15840
(2) 18720 (4) 14400

Sol: Total − cases when atleast a row empty

$= {}^8C_6\times 6! - {}^6C_6\times 6!\times 2 = \underline{18720}$

PROBABILITY : A, B and C throw a dice one by one. Whomesoever throws a six wins the game. If it starts with A, then B and then C, find the respective chances of their winning.

$$P(A) = A + A'B'C'A + A'B'C'A'B'C'A + \dots \infty$$
$$= \frac{1}{6} + \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{1}{6}\right) + \dots = \frac{\frac{1}{6}}{1-\left(\frac{5}{6}\right)^3} = \frac{36}{91}$$

$$P(B) = A'B + A'B'C'A'B + \dots \infty = \frac{\frac{5}{6}\times\frac{1}{6}}{1-\left(\frac{5}{6}\right)^3} = \frac{30}{91}$$

$$P(C) = A'B'C + A'B'C'A'B'C + \dots \infty = \frac{\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}}{1-\left(\frac{5}{6}\right)^3} = \frac{25}{91}$$

Q. A newly opened fast food restaurant, has 5 menu items. If the first 4 customers choose one menu item at random ~~and the~~, what is the probability that the 4^{th} customer orders a previously unordered item?

Sol^n: Total no. of ways to order $= 5\times5\times5\times5 = 5^4$

4^{th} one orders previously unordered item, choose one & reserve for him

no. of ways $= {}^5C_1 \times 4\times4\times4\times1 = 5\times4^3$

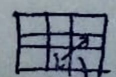
$\therefore P = \frac{5\cdot4^3}{5^4} = \frac{64}{125}$

Q. Three critics review a book. Odds in favour of the book are 5:2, 4:3 & 3:4 resp. for the three critics. The probability that majority are in favour of the book is?

Sol^n: favour: $\frac{5}{7}, \frac{4}{7}, \frac{3}{7}$; $P_{required} = \frac{5}{7}\times\frac{4}{7}\times\frac{3}{7} + \frac{5}{7}\times\frac{4}{7}\times\frac{4}{7} + \frac{5}{7}\times\frac{3}{7}\times\frac{3}{7} + \frac{2}{7}\times\frac{4}{7}\times\frac{3}{7} = \frac{209}{343}$

Q. If four squares are chosen at random on a chessboard and the probability that they lie on a diagonal line is $\frac{\lambda}{{}^{64}C_4}$, then sum of digits of λ = ?

$$\lambda = \left(\left({}^4C_4 + {}^5C_4 + {}^6C_4 + {}^7C_4\right)\times2 + {}^8C_4\right)\times2 = \left({}^8C_5\times2 + {}^8C_4\right)\times2 = \underline{\underline{364}}$$



this is also a diagonal line

DEFINITIONS:

→ An experiment is called random experiment if (i) It has more than one possible outcome and (ii) It is not possible to predict the outcome in advance

→ A possible result of a random experiment is called its **outcome**.

→ The set of all possible outcomes of a random experiment is called the sample space (S) associated with the experiment.

→ Each element of the sample space is called a sample point.

→ Any subset E of a sample space S is called an event.

→ The event E is of a sample space S is said to have occured if the outcome ω of the experiment is such that $\omega \in E$. If the outcome ω is such that $\omega \notin E$, we say that the event E has not occured.

→ ϕ → impossible event & S → sure event

→ Simple (or elementary) event: event E has only one sample point

→ Compound event: event E has more than one sample point

→ In general, two events A and B are called mutually exclusive events if the occurence of any one of them excludes the occurence of the other event, i.e., if they can not occur simultaneously. Generally $A \cap B = \phi$.

→ If $E_1, E_2, \ldots, E_n$ are n events of a sample space S and if $E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n = \bigcup_{i=1}^{n} E_i = S$ then $E_1, E_2, \ldots, E_n$ are called exhaustive events.