LOYOLA ACADEMY

DEGREE & PG COLLEGE

OLD ALWAL, SECUNDERABAD-500 010, TELANGANA, INDIA

(Autonomous and Affiliated to Osmania University)

II-SEMESTER-END THEORY EXAMINATIONS, APRIL–2023

Course : B.Sc. Computer Data Science & Data Analytics Engg. UID No.:

Subject : Abstract Algebra Max. Duration : 3 Hours.

Code : DS18201 Max. Marks : 60 Marks

SECTION – A

Answer ALL the following short questions: [10 x 2 = 20 M]

1. Prove that $\{1\ ,-1\}$ form an abelion group under multiplication.
2. Define Cosets and Formalizer of an element.
3. Show that the intersection of any two normal subgroups of a group is normal subgroup.
4. Define Homomorphism and Kernel of a homomorphism.
5. Define Even and Odd permutations.
6. Find the number of generators of cyclic group of order 60.
7. Define Echelon form of a matrix.
8. State the condition under which a system of Non Homogeneous equations will have a Unique solution.
9. Prove that ± 1 can be the only real characteristic roots of an orthogonal matrix.
10. Obtain the matrix corresponding to the quadratic form $x^2+2y^2+3z^2+4xy+5yz+6zx$.

SECTION – B

Answer any ONE of the two essay questions from each of the following units: [5 x 8 = 40 M]

UNIT – I

1. Define Binary operation and Prove that the set of n^{th}roots of unity under multiplication forms a Finite group.

2. (a) Define sub group and Abelion Group.

   (b) If H and K are two subgroups of a G , then HK is subgroup of G iff HK=KH

UNIT – II

3. Let H is a normal subgroup of G, then show that the set $\frac{G}{H}$ of all cosets of H in G forms a Group With respect to coset multiplication.

4. Let S = $\{\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ | a, b ∈ R }.Show that $\phi : (C ,+)\rightarrow(S,+)$ given by

   $\Phi(a+bi) = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ is a Group isomorphism.

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II– SEMESTER-SUPPLEMENTARY THEORY EXAMINATIONS, NOV–2022

ourse : B.Sc. (Computer Data Science & Analytics Engineering) UID No.:

ubject : Abstract Algebra Max. Duration : 3 Hours

ode : DS18201 Max. Marks : 60 Mark

SECTION – A

Answer ALL the following short questions: [10 x 2 = 20 M]

Define Group and Subgroup.

Define order of an element.

Show that every subgroup of an abelion group is normal.

Define Automorphism.

Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 5 & 1 & 4 & 6 & 8 & 7 \end{bmatrix}$, Find the order of permutation α .

Show that every group of prime order is cyclic.

Define Rank of a matrix.

If **A** be an n – rowed non singualar matrix , **X** be an $n \times 1$ matrix , **B** be an $n \times 1$ matrix , the syste of equations **AX**= **B** has a unique solution

Find the Eigen values of the identity matrix.

Write down the quadratic form corresponding to the matrix A= $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & 3 \\ 3 & 3 & 1 \end{pmatrix}$.

SECTION – B

Answer any ONE of the two essay questions from each of the following units: [5 x 8 = 40 M

UNIT – I

Prove that a non – empty finite subset of a group which is closed under multiplication is a Sub gr of G.

(a) State and Prove Lagrange's theorem.

(b) If $H = \{1, -1\}$ and $G = \{1, -1, i, -i\}$, Find all the Right cosets of H in G.

UNIT – II

(a) Define Centre Z of G and Normalizer N(a).

(b) If G is a group then the centre Z of G is a normal subgroup of G.

State and prove fundamental theorem on Homomorphism of groups.

UNIT – III

Define Symmetric Group and Show that the set A_n of all even permutations of degree n forms a Group of order $\frac{n!}{2}$ w.r.t permutation multiplication.

Show that any infinite cyclic group is isomorphic to the additive group of integers (Z , +) and ev finite cyclic group G of order n is isomorphic to the group of integer's addition modulo n $i.e., (Z_$

UNIT – IV

Find the ranks of A and B , where A= $\begin{pmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$, B= $\begin{pmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{pmatrix}$.

Show that the equations

$$x + y + z = -3$$
$$3x + y - 2z = -2$$
$$2x + 4y + 7z = 7$$

are not Consistent.

UNIT – V

Find the Eigen values and Eigen vectors of the matrix A= $\begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}$

Reduce the quadratic form $x^2 + 4y^2 + 9z^2 + t^2 - 4xy - 12yz + 6zx - 2xt - 6zt$ to canonical

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II–SEMESTER-END THEORY EXAMINATIONS, APRIL–2024

ourse : B.Sc. Computer Data Science & Data Analytics
bject : Abstract Algebra
de : DS18201

UID No.:
Max. Duration : 3 Hours.
Max. Marks : 60 Marks

SECTION – A

Answer ALL the following short questions: (10 x 2 = 20 M)

	M	CO	BT
Prove that If G is a finite group, then there exists a positive integer k such that $x^k = e, \forall x \in G$	(2)	1	L5
Write the elementary properties of groups	(2)	1	L1
Define homomorphism	(2)	2	L1
Prove that all abelian groups have normal subgroups.	(2)	2	L5
Define cyclic group	(2)	3	L1
What is permutation group	(2)	3	L2
Define Rank of a matrix	(2)	4	L1
Find the value of 'k' such that the rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & k & 7 \\ 3 & 6 & 10 \end{bmatrix}$ is 2	(2)	4	L6
What is a quadratic form.	(2)	5	L2
Define eigen values and eigen vectors of the matrix	(2)	5	L1

SECTION – B

Answer any ONE of the two essay questions from each of the following units: (5 x8 = 40 M)

UNIT – I	M	CO	BT
f G is a group such that $(ab)^n = a^n b^n$ for three consecutive integers, then prove that $b = ba$	8	1	5
rove that the union of two subgroups of a group is a subgroup of the group iff one is ontained in the other.	8	1	2
UNIT – II			
homomorphism $\emptyset: G \to H$ is injective if and only if $ker\emptyset = \{e\}$	8	2	4
ove that Every cyclic group is isomorphic to Z or to $Z/(n)$ for some $n \in N$	8	2	4
UNIT – III			
ove that every subgroup of a cyclic group is cyclic	8	3	3
te and prove Cayley's theorem.	8	3	5

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II – SEMESTER SUPPLEMENTARY EXAMINATIONS, NOVEMBER – 2018

rse : B.Sc. Computer Data Science & Analytics Engineering UID No. :
ject : Abstract Algebra Max. Duration : 3 hrs.
e : 311203 Max. Marks : 60 M

SECTION – A

Answer ALL the following short questions: [10 x 2 = 20 M]

Define Group with example.
State Lagrange's Theorem.
Define Normal Subgroup.
Define Homomorphism of groups.
State Cayley's Theorem.
Define cyclic group with an example.
Define Echelon form of a matrix.
Give one example of Rank of Matrix.
State Cayley-Hamilton Theorem.
). Define Eigen value of a matrix.

SECTION – B

Answer any ONE of the two essay questions from each of the following units: [5 x 8 = 40 M]

UNIT – I

[a] If a, b are any two elements of a group G, then the equations $ax = b$ and $ya = b$ have unique Solutions in G.

[b] If $G = \left\{ \begin{bmatrix} a & o \\ o & o \end{bmatrix} : a \text{ is any non - zero real number} \right\}$, Show that G is a Commutative group under Matrix Multiplication.

[a] If a, b are any two elements of a group G and H is any subgroup of G, then prove that $Ha = Hb \Leftrightarrow ab^{-1} \in H$ and $aH = bH \Leftrightarrow a^{-1}b \in H$.

[b] If H is any Subgroup of a group G and $h \in H$, then prove that $Hh = H = hH$.

UNIT – II

[a] Prove that subgroup H of a group G is normal $\Leftrightarrow xHx^{-1} = H \ \forall x \in G$.

[b] Prove that the intersection of any two normal subgroups of a group is a normal subgroup.

Suppose G is a group and N is a normal Subgroup of G. Let f be a mapping from G to $\frac{G}{N}$ defined by $f(x) = Nx \ \forall x \in G$. Then show that f is a homomorphism of G onto $\frac{G}{N}$ and Kernel f =N.

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II – SEMESTER END THEORY EXAMINATIONS JULY - 2021

urse : B.Sc. [Computer Data Science & Data Analytics] UID No. :

bject : Abstract Algebra Max. Duration : 2 hrs.

de : DS18201 / 311203 Max. Marks : 60 M

SECTION – A

Answer any FIVE of the following short Answer Questions: [5 x 2 = 10M]

1. Define a monoid with example.
2. Show that the composition * defined by $a * b = ab^2$ is not associative.
3. Show that in a group G , $(a^{-1})^{-1} = a\ \forall a \in G$.
4. If H is a subgroup of group G prove that HH = H.
5. If Ha and Hb are any two right cosets of H in G then prove that Ha = Hb if and only if $ab^{-1} \in H$.
6. Find the inverse of permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 4 & 5 & 1 & 6 & 7 & 9 & 8 \end{pmatrix}$
7. Define the rank of a matrix and find the rank of the Matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$
8. Define the characteristic Equation of a matrix A and State Cayley-Hamilton Theorem.
9. Find the Characteristic roots of the matrix $A = \begin{bmatrix} 2 & -1 \\ 6 & 5 \end{bmatrix}$
10. Find the quadratic form of the matrix $A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & -1 & -2 \\ 3 & -2 & 6 \end{bmatrix}$

SECTION – B

Answer any FOUR units, choosing one question from each unit: [4 x 12 ½ =50M]

UNIT – I

1. (a) Define a Group. Show that the set G = { 1, 2, 3, 4, 5, 6 } form an abelian group w.r.t $\times_7$. Also find order of each element in G.

   (b) Show that the set a group G is abelian if and only if $(a\,b)^2 = a^2.b^2.\forall a, b \in G$.

2. (a) Show that the product of two subgroups H & K is subgroup of group G iff HK =KH.

   (b) State and prove Lagrange's theorem.

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