

To minimize quantization error we must reduce Δ .

↳ noise margin ↓

NUMBER SYSTEM.

A number system with base 'b' will have b different digits, ranging from 0 to $b-1$.

NOTE:-

- b must not equal to 0
- b must be positive integer.

① Other base system to Non-decimal system.

Steps:-

- Convert original number to the decimal.
- Convert the decimal to the required base.

Example: $(101101)_2 \rightarrow (?)_3$

$$1 \times 2^0 + 1 \times 2^2 + 1 \times 2^3 + 1 \times 2^3 = (45)_{10}$$

$$\begin{array}{r} 3 \\ | \\ 450 \\ -3 \\ \hline 150 \\ -3 \\ \hline 52 \\ -3 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 3 \\ | \\ 150 \\ -3 \\ \hline 50 \\ -3 \\ \hline 2 \\ \times 3 \\ \hline 6 \\ -3 \\ \hline 3 \\ \times 3 \\ \hline 9 \\ -9 \\ \hline 0 \end{array}$$

No of bits required formula by rounding off the numbers is given by

$$\log(n)$$

$$\text{No. of } n = \lceil \log_2(n) \rceil = \text{no. of coefficient}$$

Group of 4 bit = nibble

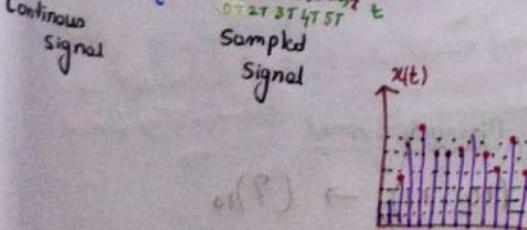
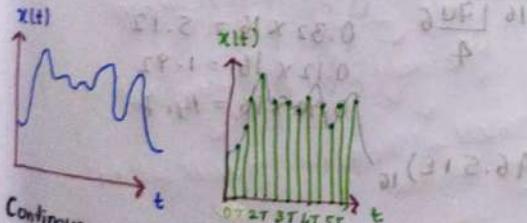
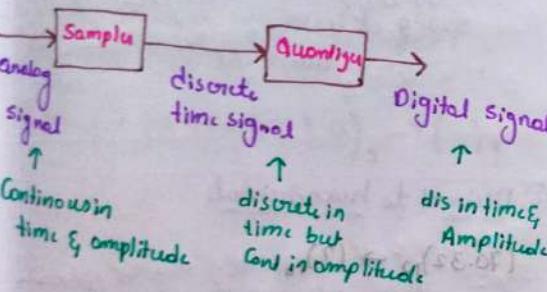
8-bit = Byte

16-bit = Word

32-bit = double Word

- Digital has been derived from the word digit, represents the system with the finite number of possibilities.
- Analog is similar to real world signal and infinite number of possibility.
- Analog is more natural, so it need to convert by data converters.
- Conversion by digital is more powerful and requires less storage, accuracy, noise distortion.
- High speed, Easy measurement, Security

How digital Signal obtained?



We approximate value of each sample to the nearest allowed level
↳ introduce an error called as Quantization.
 Δ = Step size
 $\max \text{ error} = \Delta/2$.

① Decimal to binary

$$(70.32)_{10} \rightarrow (?)_2$$

$$\begin{array}{r} 700 \\ 2 \overline{)351} \\ 2 \overline{)171} \\ 2 \overline{)80} \\ 2 \overline{)40} \\ 2 \overline{)20} \\ \hline \end{array}$$

$$\begin{array}{l} 0.32 \times 2 = 0.64 \\ 0.64 \times 2 = 1.28 \\ 0.28 \times 2 = 0.56 \\ 0.56 \times 2 = 1.12 \end{array}$$

$$(1000110.0101)_2$$

② Decimal to octal

$$(70.32)_{10} \rightarrow (?)_8$$

$$\begin{array}{r} 706 \\ 8 \overline{)80} \\ 8 \overline{)0} \\ \hline \end{array}$$

$$\begin{array}{l} 0.32 \times 8 = 0.56 \\ 0.56 \times 8 = 4.48 \end{array}$$

$$(106.24)_8$$

③ Decimal to hexadecimal.

$$(70.32)_{10} \rightarrow (?)_{16}$$

$$\begin{array}{r} 706 \\ 16 \overline{)4} \\ 4 \overline{)0} \\ \hline \end{array}$$

$$(46.51E)_{16}$$

④ Binary to decimal

$$(101.11)_2 \rightarrow (?)_{10}$$

$$\begin{aligned} &= 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + \frac{1}{2} + \frac{1}{4} \\ &= 4 + 0 + 1 + 0.5 + 0.25 \\ &= (5.75)_{10} \end{aligned}$$

⑤ Binary to Octal

Group 3 bit and write convenient number

⑥ Binary to hexa

Group four four bit and then write its equivalent

NOTE-3: From higher to lower base

$$()_9 \rightarrow ()_3$$

$$(783.437) \rightarrow (?)_3$$

$$(31\ 22\ 10.\ 1\ 10\ 21)_3$$

NOTE: While grouping to convert the number from binary to hexadecimal or to octal.

Real \rightarrow left to right

Fraction \rightarrow Right to left

Problem-1: $(123)_5 = (xy)_y$

$$35 + 10 + 3 = xy + 8$$

$$38 = xy + 8$$

$$xy = 30$$

$$y > x \& y > 8$$

hence $y = 6 \& x = 5$

$$1 \times 30 = 30$$

$$2 \times 15 = 30$$

$$3 \times 10 = 30$$

Problem-2: $\frac{312}{30} = 13.1$ find no. x

$$\frac{(312)x}{(30)x} = (13.1)_x$$

$$= \frac{3xz^2 + 1xx + 2}{3x} = 1x + 3 + x^{-1}$$

$$= 3x^2 + 6x + 2 \quad (\text{RHS})$$

$$x^2 - 5x = 0$$

$$x(x-5) = 0$$

$$x = 0 \text{ or } x = 5$$

$KB = 2^{10} \text{ Bit}$

$MB = 2^{20} \text{ Bit}$

$GB = 2^{30} \text{ Bit}$

L-2

Negative Number representation

24/10/2025

Object	Range
Signed	
$(n-1)$'s Comp	$0's$ Complement
$n's$ Complement	$n's$ Complement
1) Unsigned number	$0 - 2^n - 1$
2) Signed	$-(2^{N-1})$ to $(2^{N-1} - 1)$
3) Unsigned no. of bits	$\log_2 N$
4) 1's Complement	$-(2^{N-1} - 1)$ to $(2^{N-1} - 1)$
5) 2's Complement	$-(2^{N-1})$ to $(2^{N-1} - 1)$

1 → represents -ve
0 → represents +ve

Limitation of Sign magnitude

$$+0 \rightarrow 0000$$

$$-0 \rightarrow 1000$$

Since both should have same binary value but here it is different, this can be solved by signed mag by 2's complement.

NOTE : ① 2's Complement = 1's Complement + 1.

$$\text{② Scale bit } 10010 \xrightarrow{2^4} 01110.$$

Advantages of 2's Complement

- ① 0 is uniquely represented.
- ② Extended range.
- ③ No need additional hardware while performing $-A - B$ operation.

8's Complement & 7's Complement

$$of (3674)_8$$

$$\begin{array}{r}
 7777 \\
 3674 \\
 \hline
 4103 \rightarrow 1's \text{ Complement} \\
 4104 \rightarrow 8's \text{ Complement}
 \end{array}$$

NOTE: Every computer uses 2's Complement representation.

While performing addition operation try to choose the N value which has higher among them.

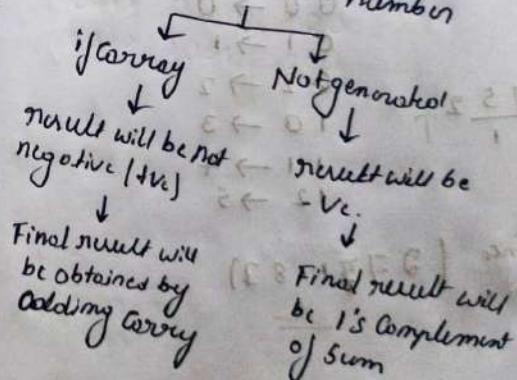
Trick

$$\begin{array}{r}
 1111 \\
 1001 \\
 11010 \\
 11010 \\
 11011 \\
 \hline
 00010
 \end{array}$$

• if even no. of 1's = 0
Odd no. of 1's = 1
• no. of pairs = Carry

Addition of the form $A - B$

Step 1:- Take 1's Complement of negative number
Step 2:- Add both the numbers



$$\begin{array}{r}
 5 \\
 -4 \\
 \hline
 1
 \end{array} \quad N=4$$

$$\begin{array}{r}
 0101 \\
 0100 \\
 \hline
 1011 \\
 0000
 \end{array}$$

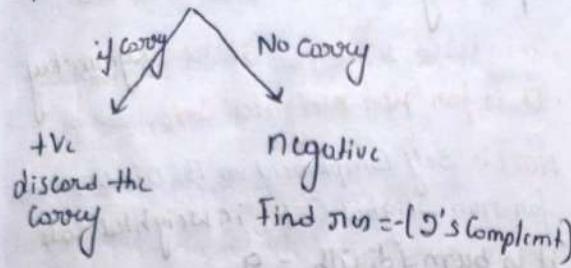
$$\begin{array}{r}
 0000 \\
 \hline
 0001
 \end{array}$$

$$\begin{array}{r}
 4 \\
 -5 \\
 \hline
 -1
 \end{array} \quad \begin{array}{r}
 0100 \\
 0100 \\
 \hline
 10100
 \end{array} \quad \begin{array}{r}
 10100 \\
 \hline
 1110
 \end{array}$$

$$\begin{array}{r}
 \text{No carry} \\
 -(0001) \\
 = -1 \\
 \hline
 \frac{1}{2}
 \end{array}$$

Subtraction by 2's Complement

Step 1: Take 2's of neg number
Step 2: Add both numbers



Subtraction is of the form :-

$$\begin{array}{r} -A \\ -B \\ \hline \end{array}$$

- Step 1:- Choose correct No. of bit
- Step 2:- Take 2's complement of both numbers
- Step 3:- It will always generate a carry
it should be discarded
- Step 4:- Result is equal to (2's complement of final)

Addition (\gg)

$$\begin{array}{r} (4\ 3\ 2\ 7)_8 \\ (3\ 7\ 4\ 5)_8 \\ \hline (1\ 0\ 2\ 7\ 4)_8 \end{array}$$

$$\begin{array}{r} (11)_8 \\ (7)_8 \\ \hline ? \end{array} \rightarrow \begin{array}{r} (9)_8 \\ (7)_8 \\ \hline \end{array} \quad \begin{array}{l} \text{Every carry no also} \\ \text{need to convert into} \\ \text{Octal} \end{array}$$

3's Compliment Advantage

- ① Range is Expanded
 - ② No need Extra adder.
 - ③ 0/±0 ambiguity has been resolved.

100
120
140
160

$$1KB = 2^{10} \text{ bytes}$$

$$2^{10} \text{ bits}$$

$$1MB = 2^{10} Kbytes$$

$$2^{20} \text{ bytes}$$

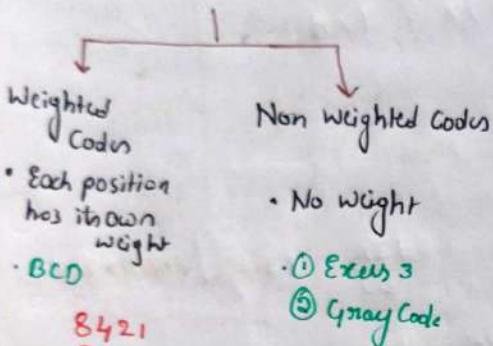
$$2^{23} \text{ bits}$$

$$1GB = 2^{10} Mbytes$$

$$2^{30} \text{ bytes}$$

$$2^{33} \text{ bits}$$

Binary Coders



default Value

ASCII \rightarrow 8bit
EBCD \rightarrow 9bit representation

BCD :-

- ① Very popular and easy to represent
- ② Varies from 0-9
- ③ Each digit is group of 4 bit

Excess -3.

0	0011
1	0100
2	0101
3	0110

Additional feature of Excess 3 is
Self Complement.

Self Complement: Code whose 9's complement can be obtained by replacing 0 with 1 and 1 with 0

- It is used because it has Symmetry
- It is for Non-Weighted Code

NOTE:- Self Complement in BCD is applicable for non weighted code, in weighted code it is sum of digits = 9.

$$\text{Ex:- } 0421 = 0+4+2+1 = 9$$

$$5211 = 9$$

$$84-2-1 = 9$$

①

$$\begin{array}{r}
 4 \xrightarrow{\text{Excess-3}} 0111 \\
 \downarrow \\
 9' \text{Complement} \quad 1000
 \end{array}$$

↑ Excess 3

② 435 Excess 3 9' Complement

$$\begin{array}{r}
 0111 \quad 0110 \quad 100 \\
 1000 \quad 1001 \quad 0111 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 9 \quad 9 \quad 9 \\
 4 \quad 3 \quad 5 \\
 \hline
 5 \quad 6 \quad 4 \\
 + \quad 3 \quad 3 \quad 3 \\
 \hline
 8 \quad 9 \quad 7
 \end{array}$$

123 in binary

$$\begin{array}{r}
 2 | 103 \\
 2 | 61 \\
 2 | 30 \\
 2 | 15 \\
 2 | 7 \\
 2 | 3
 \end{array}$$

Ex-3 in self Complementing code

	8421	2421	5211	8421
0	0000	0000	0000	0000
1		0001	0001	
2	1000	0100	0100	1000
3	0010	0011	0011	0010
4	1001	0110	0110	1001
5	0010	0101	0101	0010
6	1010	0111	0111	1010
7	0100			0100
8	0101	1000	1000	0101
9	1011			1011

Question: What are the different ways which 5 can be represented in 2421.

Ans 2 ways $\{1011\}$ or $\{0101\}$

Question: No of unique representation of 2421

$\{9, 0, 1, 8\}$

and it is also self complement.

Question: Unique representation of 5211

$\{0, 4, 5, 9\}$

Gray Code: A code having a hamming distance of 1, is called gray code.

Application:
 1. used in lower power device
 2. used in FIFO and K-Map

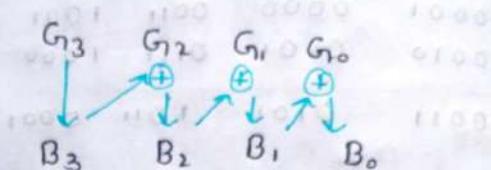
Binary to gray:

$$(B_2 \oplus B_1 \oplus B_0)$$

$$(G_2, G_1, G_0)$$

$$\begin{aligned} G_2 &= B_2 \\ G_1 &= B_2 \oplus B_1 \\ G_0 &= B_1 \oplus B_0 \end{aligned}$$

Gray to Binary



BCD Addition:

Step 1:- Replace each BCD digit by its 4 bit binary number

Step 2:- Perform binary addition on each nibble

Step 3:- If sum of each nibble is either > 1001 (6) or carry is generated

Step 4:- Correct the nibble by adding 0110.

Example 1:-

$$\begin{array}{r} 432 \\ 739 \\ \hline 1171 \end{array}$$

Two corrections are needed. > 6

$$\begin{array}{r} 0100 \quad 0011 \quad 0010 \\ 0111 \quad 0011 \quad 1001 \\ \hline 1011 \quad 0110 \quad 1011 \end{array}$$

$$\begin{array}{r} 0110 \quad 0110 \\ 10001 \quad 0111 \quad 0001 \end{array}$$

Hence the results are matching

$$\begin{array}{r} 1039 \\ 2578 \\ \hline 3617 \end{array}$$

2. Connection required

$$\begin{array}{cccc} 0001 & 0000 & 0011 & 1001 \\ 0010 & 0101 & 0111 & 1000 \\ \hline 0011 & 0101 & 1011 & 0001 \\ & & 0110 & 0110 \end{array}$$

$$0011 \quad 0110 \quad 0001 \quad 0111$$

hence the result is matching

Why we are adding 6 only?

①

BCD	$\left\{ \begin{array}{l} 0 \\ \vdots \\ 9 \end{array} \right.$
4-bit position	$\left\{ \begin{array}{l} \vdots \\ 16 \end{array} \right.$

6 remaining

② $2^4 = 16$

$$\begin{array}{r} 10 \\ \hline 6 \\ \hline = \end{array}$$

1100 0010
1100 1110

Logic Gates:

Positive logic
 $V_{cc} = 1$
 $Gnd = 0$

Negative logic
 $V_{cc} = 0$
 $Gnd = 1$

NOTE: By using duality we can convert +ve logic to negative logic

Logic gates

Basic

AND
OR
NOR

Universal

NAND
NOR

Special Purpose
XOR
XNOR

AND

OR

NOT



A
B



A
B

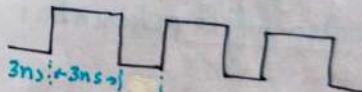
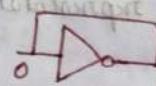


A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

A	B	Y
0	1	1
1	0	1
1	1	0
0	0	1

A	Y
0	1
1	0
0	0
1	1

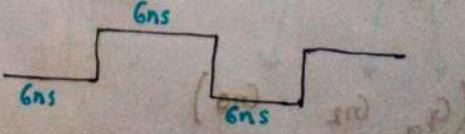
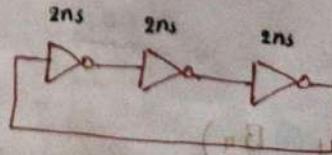
X = Unknown \rightarrow (When proper inputs are not defined)
Z = High impedance \rightarrow (When input is not connected to output logic)



D T_p (Time period) = 6ns

② Frequency = $\frac{1}{T_p} = \frac{1}{6\text{ns}} = 166\text{MHz}$

③ Duty cycle = $\frac{T_{on}}{T_p} = \frac{3\text{ns}}{6\text{ns}} = 0.5$ or 50%

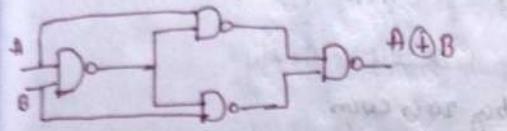


$$T_p = 12 \text{ ns}$$

$$f_{req} = \frac{1}{12 \text{ ns}} = 83 \text{ MHz}$$

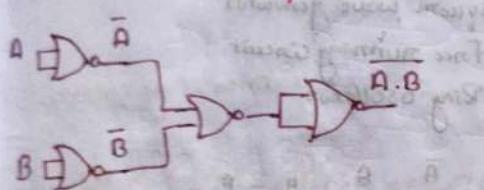
$$\text{Duty cycle} = \frac{6 \text{ ns}}{12 \text{ ns}} = 0.5 \text{ or } 50\%$$

XOR Gate using NAND :-

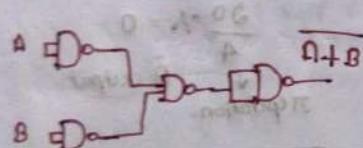


NAND Gate by NOR Gate :-

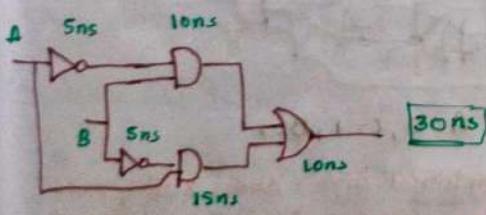
- Symbol need to be used
- Equation Also.



NOR using NAND :-

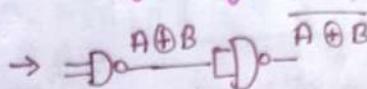


Find the total delay in the circuit

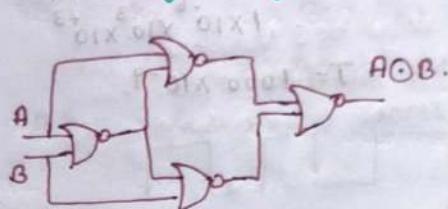


It will try to take the largest delay.

XNOR by using NAND :-



XNOR by using NOR :-



followed by another Not gate
will give me XOR gate

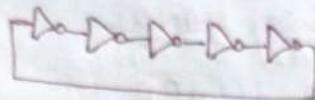
The Time period of Square Wave Generated by AMW = $T = 2nT_p$

$$f = \frac{1}{T}$$

* * * NOR Gate is used in low power devices.

27/09/2025

Ring Oscillator with 5 inverters generate a waveform 1MHz. What is the delay? What is the delay of each inverter?



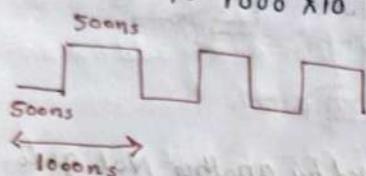
Given

$$f = 1 \text{ MHz}$$

$$T = \frac{1}{f} = \frac{1}{1 \times 10^6} = 1 \times 10^{-6} = 1 \mu\text{s}$$

$$1 \times 10^{-6} \times 10^3 \times 10^3$$

$$T = 1000 \times 10^{-9}$$

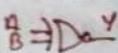


$$\text{Delay} = \frac{T_{on}}{T_p} = \frac{500 \text{ ns}}{5 \text{ ns}} = 100$$

Exon



EXNOR



$$Y = A \oplus B$$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

$$Y = A \odot B$$

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

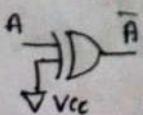
Inequality

- Odd one detector
- Adder/Subtractor
- Parity generator

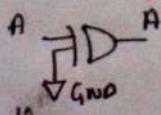
$$Y = A\bar{B} + \bar{A}B$$

$$Y = \bar{A}\bar{B} + AB$$

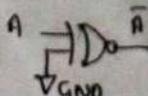
XOR as Inverter



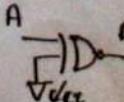
XOR as Buffer



Xnor as Inverter

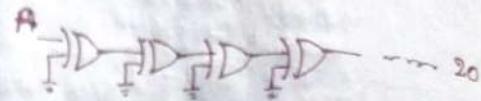


Xnor as Buffer



If even number of inverters are connected it will be

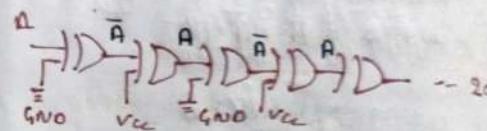
- Bi-stable multivibrator
- Basic memory element
- Used in FF/latch.



Output will be A only
bcoz 20 is even.

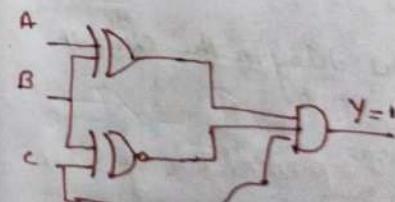
If odd number of inverters are connected it will be

- Astable multivibrator
- Square wave generator
- Free running circuit
- Ring oscillator



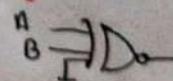
$$\frac{30}{4} \cdot 1 = 0$$

↓ Negation.
 \bar{A} output



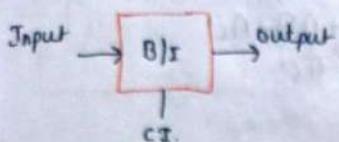
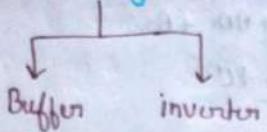
$$A = 0, C = 1, B = 1$$

Implement higher order XOR & Xnor by lower Exon gate design XOR gate using 2-input XNOR gate.

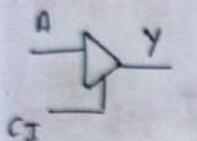


Hint whenever we are implementing higher by lower we can use cascading technique.

Tristate logic

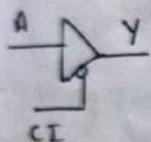


① Active high tristate Buffer.



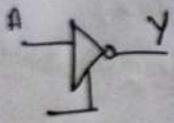
A	C2	Y
0	X	Z (inactive)
1	0	0
1	1	1 } active

② Active Low tristate Buffer.



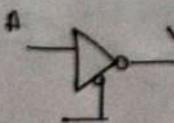
A	C2	Y
0	0	0 } active
0	1	1 } inactive
1	0	1 } active
1	1	1 } inactive

③ Tristate Active high inverter



A	C2	Y
0	X	Z
1	0	1
1	1	0
	0	1

④ Active low tristate inverter.



A	C2	Y
0	0	1
0	1	0
1	0	1 } active
1	1	1 } inactive

Logic Minimization:-

- Boolean algebra invented by George Boole in 1854.
- Used to simplify logical selection.

Properties:

1) $A + A + A = A$	$A \cdot A = A$
2) $A + 0 = A$	$A \cdot 0 = 0$
3) $A + 1 = 1$	$A \cdot 1 = A$
	$A \cdot A' = 0$

Dual ↗

Boolean Law /

① Dominance Law / Annulation Law.

- 1) $A \cdot 0 = 0$
- 2) $A + 1 = 1$.

② Identity law

- 1) $A \cdot 1 = A$
- 2) $A + 0 = A$.

④ Idempotent Law

- 1) $A \cdot A = A$
- 2) $A + A = A$

⑤ Complement Law.

- 1) $A \cdot \bar{A} = 0$
- 2) $A + \bar{A} = 1$.

⑥ Commutative Law.

- 1) $A \cdot B = B \cdot A$
- 2) $A + B = B + A$.

⑦ Double negation Law.

$$\bar{\bar{A}} = A.$$

⑧ DeMorgan's Law.

- 1) $\overline{A \cdot B} = \bar{A} + \bar{B}$
- 2) $\overline{A + B} = \bar{A} \cdot \bar{B}$.

⑨ Distributive Law.

$$A \cdot (B + C) = AB + AC$$

$$A + (B \cdot C) = (A + B) (A + C)$$

Absorption Law

$$A + (AB) = A$$

$$A \cdot (A+B) = A$$

Associative Law

$$A + (B+C) = (A+B)+C$$

$$A \cdot (B \cdot C) = (A \cdot B) \cdot C$$

Consensus Theorem / Redundancy Theorem

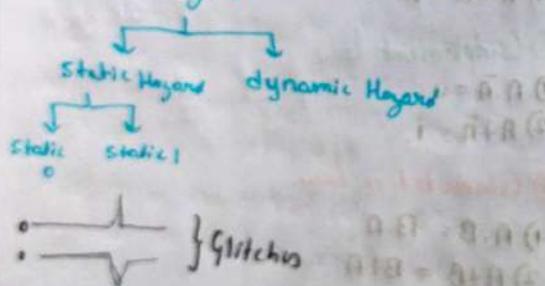
- ① There must be 3 Variables
- ② Each Variable must be repeated twice
- ③ One Variable must be Complemented

$$Y = AB + A'B + BC$$

$$Y = AB + A'C$$

NOTE: Significance when ever we come across Hazard problem it can be solved by adding some redundant term to it.

Hazard



Cost will be increase but it will be getting correct output.

$$\textcircled{1} \quad AB + A(B+C) + B(B+C)$$

$$= AB + AB + AC + BB + BC$$

$$AB + AC + BB + BC$$

$$AB + AC \quad \text{B is redundant}$$

$$\underline{AB}$$

$$\textcircled{5} \quad A(A+B) + (B+A)(A+B)$$

$$= A$$

$$\textcircled{3} \quad (A+C)(AD+AC) + AC + C$$

$$AD+AC + ADC + ADC + AC'$$

$$\underline{AD+AC+AC'}$$

$$\textcircled{4} \quad A(A+BC) + A(BC+C')$$

$$A+ABC + AB + AC'$$

$$A + A + AC'$$

$$\underline{A}$$

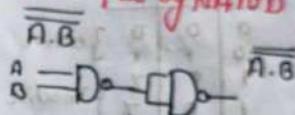
$$\textcircled{5} \quad \underline{A \cdot \overline{AB}} \cdot \underline{B \cdot \overline{AB}}$$

$$\overline{AB} + \underline{AB}$$

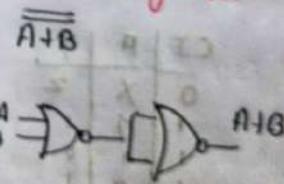
Implementation of Logic Gate using NANO & NOR

SOP \rightarrow Directly implement by NANO gate
POS \rightarrow NOR Gate

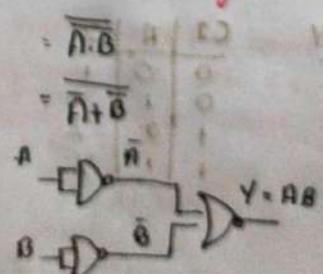
AND Gate by NANO



OR Gate by NOR

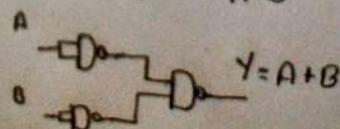


AND Gate by NOR

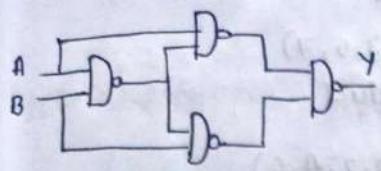


OR Gate by NANO

$$(A+B) = \overline{\overline{A} \cdot \overline{B}}$$

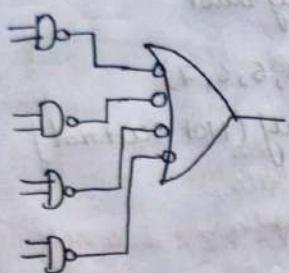


Implementation of XOR gate using min number of NAND Gates



by NOR just replace by NOR.

$$\textcircled{1} \quad Y = AB + BC + CA.$$



By using 2 input we can use this technique

try to implement by 2 input NAND gate.

Alternate Symbols :-

$$\textcircled{1} \quad \begin{array}{c} A \\ \oplus \\ B \end{array} \equiv \begin{array}{c} \overline{A \cdot B} \\ \oplus \\ \overline{A + B} \end{array}$$

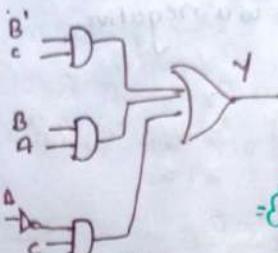
$$\textcircled{2} \quad \begin{array}{c} \overline{A + B} \\ \oplus \\ \overline{A \cdot B} \end{array} \equiv \begin{array}{c} A \\ \oplus \\ B \end{array}$$

$$\textcircled{3} \quad \begin{array}{c} A \oplus B \\ \oplus \\ A \oplus B \end{array} \equiv \begin{array}{c} \overline{A \cdot B} \\ \oplus \\ \overline{A \cdot B} \end{array}$$

	NAND	NOR
NOT	1	1
AN	2	3
OR	3	2
XOR	4	5
XNOR	5	4
NAND	1	4
NOR	4	1

Finding circuit efficiency after minimization :-

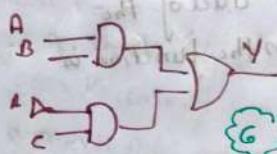
$$Y = AB + BC + \overline{AC}$$



NOTE : Cost is Equal to no of inputs.
Each input has cost of 1.

After Simplification

$$Y = AB + A'C.$$



Circuit efficiency :-

$$\frac{9 - 6}{9} = \frac{3}{9} = \frac{1}{3} \times 100$$

33% Efficient

Note :- Fan 3 input EXOR gate to implement in 2 input EXNOR gate we required 3 EXNOR gate

$$\begin{array}{c} A \\ \oplus \\ B \\ \oplus \\ C \end{array} \Rightarrow D \rightarrow D \rightarrow D$$

For EXOR gate no of inputs - 1.

$$\begin{array}{c} A \\ \oplus \\ B \\ \oplus \\ C \end{array} \Rightarrow D \rightarrow D$$

Duality Principle:

3/03/25

Check whether it is self dual or not

- + logic \rightarrow Active high buffer
- logic \rightarrow Active low buffer

- Bubble indicate it is a negative logic

From	To
+	.
.	+
0	1
1	0

Ex: $AB + BC + CA$

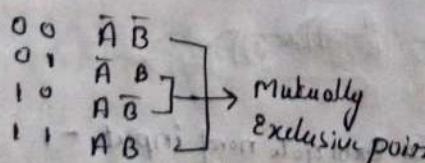
$$(A+B)(B+C)(C+A)$$

$A \cdot 0 \rightarrow n+1$

Self dual function: dual of the function is same as the function is called self dual.

Testing of self dual function:

- Function must be "neutral"
 - The function which has equal no of minterm and maxterm then it is called neutral.
- If n is no of variables 2^n minterms and maxterms are possible.
- It must not contain mutually exclusive terms.



(a) $f = \Sigma(1, 2, 3, 4)$

= Non self

(b) $f = \Sigma(1, 3, 5, 7)$

= self dual

(c) $f = \Sigma(0, 2, 4, 6)$

= self dual

(d) $f = \Sigma(0, 1, 6, 7)$

= Non self dual

(e) $f = \Sigma(1, 2, 5, 6, 7)$

= Non self (Not neutral)

Prove $XY + YZ + ZX$ is a self dual.

$XY + YZ + ZX$ is self dual

$$(X+Y)(Y+Z)(Z+X)$$

$$[XY + XZ + Y + YZ] [Z + X]$$

$$[Y(X+1+Z) + XZ] [Z+X]$$

$$(Y+XZ)[Z+X]$$

$$= YZ + YX + XZ + XZX$$

$$= YZ + YX + XZ$$

\rightarrow dual of the function is same

hence it is self dual

NOTE: If n variable function total how many self dual function

$$2^{n-1}$$

Trick for demorgan's law.

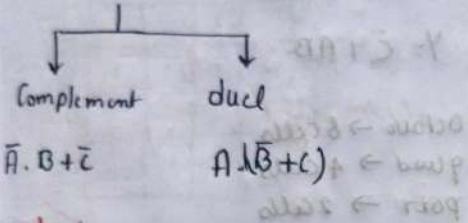
$$\begin{array}{l} \text{Replace} \\ + \rightarrow \cdot \\ \cdot \rightarrow + \\ 0 \rightarrow 1 \\ 1 \rightarrow 0 \end{array}$$

Example :- $(A+B)(B+C)(C+A)$

$(\bar{A} \cdot B) + (\bar{B} \cdot \bar{C}) + (\bar{C} \cdot A)$

Complement vs dual

$f = f_1(A + \bar{B}C)$



dual:

$$\begin{array}{l} + \rightarrow \cdot \\ \cdot \rightarrow + \end{array}$$

Simplify

$$\begin{aligned} & \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + AB\bar{C} \\ &= \bar{A}\bar{C} + B\bar{C} + \bar{A}B. \end{aligned}$$

K-MAP :-

Minterm

* SOP

* It is a product term.
Produces output always one

* All variables/literals are present in True/False

* Total no. of minterms
 $= 2^n$: n no. of variable

Minterm is rep by
 $m_{(sub)}$
 \hookrightarrow rep decimal value.

A	B	Min
0	0	$\bar{A}\bar{B}$
1	0	$A\bar{B}$
0	1	$\bar{A}B$
1	1	AB

Rules of K-Map :-

- ① Make a longest group.
- ② Not in the diagonal form.
- ③ Grouping will have 1 Variable change.
- ④ We can have overlapping allowed but no Redundant group.

NOTE:- K-map reduces the bool exp/eqn in SOP/POLY

- We have grouping of 1, 2, 4, 8 ✓
- 3, 5, 6, 7, 9 X.

Grouping order :- 32 → 16 → 8 → 4 → 2

Rules:-

- group should only include cells containing ones in case of SOP.
- group should only include zeros in POS.
- group should be only horizontal or vertical.
- group must contain 2^n cells
- Overlapping of groups is allowed.

Don't care :-

helps to reduce the ckt but we try to use less.

Maxterm

* POS

* output = 0

$\Rightarrow A+B$

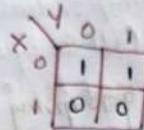
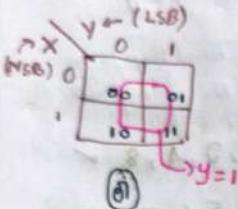
* represented by Π

A	B	Max
0	0	$A+B$
0	1	$A+\bar{B}$
1	0	$\bar{A}+B$
1	1	$\bar{A}+\bar{B}$

2-Variable - K-MAP

Total number of min = 4
max = 4

x	y	Min	Max
0	0	$\bar{x}\bar{y}$	$x+y$
0	1	$\bar{x}y$	$x+\bar{y}$
1	0	$x\bar{y}$	$\bar{x}+y$
1	1	xy	$\bar{x}+\bar{y}$



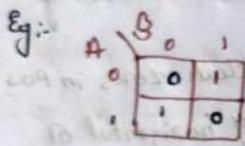
$$Y = \sum m(0, 1)$$

$$Y = \pi(2, 3)$$

y	x	0	1
0	0	00	10
1	0	01	11

$$\bar{X} = SOP$$

$$X = POS$$



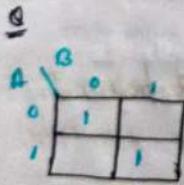
$$Y = \bar{A}B + A\bar{B}$$

(SOP)

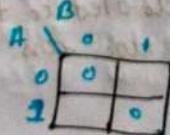
$$Y = (A+B)(\bar{A}+\bar{B})$$

(POS)

Focus on 0 term only



$$Y = \bar{A}\bar{B} + AB$$



$$Y = (A+B)(\bar{A}+\bar{B})$$

3-Variable - K Map

$$2^3 = 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$$

4

BC		AC	
00	01	11	10
m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6

AC	
0	2
2	3

C

AB	
0	2
1	3

Eg

BC	
00	01
1	1

$$Y = \bar{C} + AB$$

Octave \rightarrow 8 cells

quad \rightarrow 4 cells

pair \rightarrow 2 cells

Reduce $Y = \sum m(1, 2, 5, 6, 7)$ in SOP from using K-map

4

BC	
0	1
1	0

$$Y = \bar{B}C + B\bar{C} + AC$$

$$Y = \bar{B}C + B\bar{C} + AB$$

4-Variable - K-map

$$2^4 = 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1$$

AB

CD		AC	
00	01	11	10
0	1	3	2
4	5	7	6
12	13	15	14
8	9	11	10

CD

00	01	11	10
0	0	0	0
1	0	0	1
0	1	1	1
1	1	1	1

Reduced $Y(A, B, C, D) = \Sigma m(0, 1, 2, 3, 4, 5, 6, 7, 13, 15)$ in SOP from using K-map

	C\B		D	
A\B	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$$Y = (\bar{A} + B)(\bar{A} + D)$$

Note:-

	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

In this above there is no need of quad because it creates an extra problem.

5-Variable K-MAP

Two ways:-

① Two K map separately for MSB and MSB(1).

(i)

$$A=0$$

	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$$A=1$$

	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

Try to form a group by combining them

Reduce by using K-map.

$$\Sigma(0, 2, 3, 10, 11, 12, 13, 17, 19, 20, 21, 26, 27)$$

$$A=0$$

	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$$BCD + \bar{C}D + \bar{B}\bar{C}E + B\bar{C}\bar{D} + \bar{B}CD$$

NOTE: Try to form a group with less number of redundant term.

	00	01	11	10
00	(X)	1	1	X
01	X	1		
11		1		
10		1		

→ don't do this grouping.

Canonical SOP :- Each literal a contains all variables in it.

$$\text{in SOP } Y = AB + BC + CA$$

Step 1 :- Observe the missing literals and add $C\bar{C}$ and apply distribution.

Step 2 :- do for all the product term.

Step 3 :- Remove all the replicated term
Keep only one copy

Step 4 :- Represent in sum of minterm.

$$\begin{aligned} & AB(C\bar{C}) + BC(A+\bar{A}) + CA(B+\bar{B}) \\ & = ABC + A\bar{B}C + BCA + BCA\bar{A} + CAB + CAB\bar{A} \\ & = ABC + \bar{A}BC + ABC + \bar{A}BC + CAB + \bar{A}BC \end{aligned}$$

$$Y = \Sigma_m(3, 4, 5, 6, 7)$$

$$\begin{aligned} Y &= A(B+\bar{B})(C+\bar{C}) + BC(A+\bar{A}) \\ &= (AB+AB)(C+\bar{C}) + ABC + \bar{A}BC \\ &= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + ABC \\ &= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC \end{aligned}$$

5-variable K-map

	000	001	011	010	110	111	101	100
00								
01								
11								
10								

\bar{A} A

0	1	3	2
4	5	7	6
12	13	15	14
8	9	11	10

16	17	19	18
20	21	23	22
28	29	31	30
24	25	27	26

While grouping consider \bar{A} and A also and try to make common grouping.

NOTE: Standard derived results :-

- With n variables possible min or maximum terms $= 2^n$

- With n Variables Maximum possible logical expression $= 2^{2^n}$

- With n Variable Maximum possible self dual $= 2^{2^{(n-1)}}(2^0 + 2^1 + 2^3 + 2^7 + 2^{15} + 2^{31} + 2^{63} + 2^{127} + 2^{255} + 2^{511} + 2^{1023} + 2^{2047} + 2^{4095} + 2^{8191} + 2^{16383} + 2^{32767} + 2^{65535} + 2^{131071} + 2^{262143} + 2^{524287} + 2^{1048575} + 2^{2097151} + 2^{4194303} + 2^{8388607} + 2^{16777215} + 2^{33554431} + 2^{67108863} + 2^{134217727} + 2^{268435455} + 2^{536870911} + 2^{1073741823} + 2^{2147483647} + 2^{4294967295} + 2^{8589934591} + 2^{17179869183} + 2^{34359738367} + 2^{68719476735} + 2^{137438953471} + 2^{274877906943} + 2^{549755813887} + 2^{1099511627775} + 2^{2199023255551} + 2^{4398046511102} + 2^{8796093022204} + 2^{17592186044408} + 2^{35184372088816} + 2^{70368744177632} + 2^{140737488355264} + 2^{281474976710528} + 2^{562949953421056} + 2^{1125899906842112} + 2^{2251799813684224} + 2^{4503599627368448} + 2^{9007199254736896} + 2^{18014398509473792} + 2^{36028797018947584} + 2^{72057594037895168} + 2^{144115188075790336} + 2^{288230376151580672} + 2^{576460752303161344} + 2^{1152921504606322688} + 2^{2305843009212645376} + 2^{4611686018425290752} + 2^{9223372036850581504} + 2^{18446744073701163008} + 2^{36893488147402326016} + 2^{73786976294804652032} + 2^{147573952589609304064} + 2^{295147905179218608128} + 2^{590295810358437216256} + 2^{1180591620716874432512} + 2^{2361183241433748865024} + 2^{4722366482867497730048} + 2^{9444732965734995460096} + 2^{18889465931469990920192} + 2^{37778931862939981840384} + 2^{75557863725879963680768} + 2^{151115727451759927361536} + 2^{302231454903519854723072} + 2^{604462909807039709446144} + 2^{1208925819614079418892288} + 2^{2417851639228158837784576} + 2^{4835703278456317675569152} + 2^{9671406556912635351138304} + 2^{19342813113825270702276608} + 2^{38685626227650541404553216} + 2^{77371252455301082808556432} + 2^{154742504910602165617112864} + 2^{309485009821204331234225728} + 2^{618970019642408662468451456} + 2^{1237940039284817324936902912} + 2^{2475880078569634649873805824} + 2^{4951760157139269299747611648} + 2^{9903520314278538599495223296} + 2^{19807040628557077198985446592} + 2^{39614081257114154397970893184} + 2^{79228162514228308795941786368} + 2^{158456325228456617591883572736} + 2^{316912650456913235183767145472} + 2^{633825300913826470367534290944} + 2^{1267650601827652840735068581888} + 2^{2535301203655305681470137163776} + 2^{5070602407310611362940274327552} + 2^{10141204814621222725880548655104} + 2^{20282409629242445451761097310208} + 2^{40564819258484890903522194620416} + 2^{81129638516969781807044389240832} + 2^{162259277033939563614088778481664} + 2^{324518554067879127228177556963328} + 2^{649037108135758254456355113926656} + 2^{1298074216271516508912710227853312} + 2^{2596148432543033017825420455706624} + 2^{5192296865086066035650840911413248} + 2^{10384593730172132071301681822826496} + 2^{20769187460344264142603363645652992} + 2^{41538374920688528285206727291305984} + 2^{83076749841377056570413454582611968} + 2^{166153499682754113140826909165223936} + 2^{332306999365508226281653818330447872} + 2^{664613998731016452563267636660895744} + 2^{1329227997462032905126535273321791488} + 2^{2658455994924065810253070546643582976} + 2^{5316911989848131620506141093287165952} + 2^{10633823979696263241012282186574331904} + 2^{21267647959392526482024564373148663808} + 2^{42535295918785052964049128746297327616} + 2^{85070591837570105928098257492594655232} + 2^{170141183675140201856196514985189305464} + 2^{340282367350280403712393029970378610928} + 2^{680564734700560807424786059940757221856} + 2^{1361129469401121614849572119881514443712} + 2^{2722258938802243229699144239763028887424} + 2^{5444517877604486459398288479526057774848} + 2^{10889035755208972918796578990532155548896} + 2^{21778071510417945837593157981064311097792} + 2^{43556143020835891675186315962128622195584} + 2^{87112286041671783350372631924257244391168} + 2^{174224572083343566700745263848514488782336} + 2^{348449144166687133401490527697028977646672} + 2^{696898288333374266802981055394057955293344} + 2^{1393796576666744533605962110788115910586688} + 2^{2787593153333489067211924221576238221173376} + 2^{5575186306666978134423848443152476442346752} + 2^{1115037261333395626884769688630493284683304} + 2^{2230074522666791253769539377260986569366608} + 2^{4460149045333582507539078754521973138733216} + 2^{8920298090667165015078157509043946277466432} + 2^{1784059618133433003015635501808789255493264} + 2^{3568119236266866006031271003617578510986528} + 2^{7136238472533732012062542007235157021973056} + 2^{14272476945067464024125844014470340443946112} + 2^{28544953890134928048251688028940680887892224} + 2^{57089907780269856096503376057881361775784448} + 2^{11417981556053971219300675211576273555156896} + 2^{22835963112107942438601350423152547110313792} + 2^{45671926224215884877202700846305094220627584} + 2^{91343852448431769754405401692610188441255168} + 2^{182687704896863539508810803385220376882503336} + 2^{365375409793727079017621606770440753765006672} + 2^{730750819587454158035243213540881507530013344} + 2^{1461501639174908316070486427081763015060026688} + 2^{2923003278349816632140972854163526030120053376} + 2^{5846006556699633264281945708327052060240106752} + 2^{11692013113399266528563891416654104120480213504} + 2^{23384026226798533057127782833308208240960427008} + 2^{46768052453597066114255565666616416481920854016} + 2^{93536104907194132228511131333232832963841708032} + 2^{187072209814388264457022262666465665927683416064} + 2^{374144419628776528914044525332931331855366832128} + 2^{748288839257553057828089050665862663710733664256} + 2^{1496577678515106115656178101331725327421473328512} + 2^{2993155357030212231312356202663450654842946657024} + 2^{5986310714060424462624712405326901309685893314048} + 2^{11972621428120848925249424810653802619371786628096} + 2^{23945242856241697850498849621307605238743573256192} + 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AB	CD	Q1	Q2
00	1 1	1	
01	X		
11	X		
10	1 1		X

$$Y = \overline{BC} + \overline{BD}$$

$$Y = \sum_m (1, 2, 3, 4, 5) + d(10, 11, 12, 13, 14, 15)$$

AB	CD	Q1	Q2	Q3
00	1 1	1	1	1
01	1 1			
11	X X	X	X	X
10		X X		

$$\overline{BC} + \overline{BD} + \overline{AC}D$$

Variable Entered Mapping (VEM)

i) Map the larger no. of Variable to lower variables

$$\Sigma (0, 2, 3, 5, 6, 7, 8, 10, 11, 13, 14, 15)$$

Step-1

A	B	C	D	F
0	0	0	0	1
0	0	0	1	0 } D
0	0	1	0	1 } 1
0	0	1	1	1 } 1
0	1	0	0	0 } D
0	1	0	1	1 } D
0	1	1	0	1 } 1
1	0	0	0	1 } D
1	0	0	1	0 } D
1	0	1	0	1 } 1
1	0	1	1	1 } 1
1	1	0	0	0 } D
1	1	0	1	1 }

$$Q_1 = (0, 1, 8, 9)$$

$$Q_2 = (0, 2, 8, 10)$$

Hints: Take LSB bit as reference of all the conditions.

Step-2

A	B	C	F
0	0	0	D
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	D
1	0	1	1
1	1	0	0
1	1	1	1

In this process we are reducing the 4 Variables into 3 Variables.

From above Truth Table

A	BC	00	01	11	10
0	\overline{D}	1	1	D	
1	\overline{D}	1	1	D	

Steps involved in Solving this :-

- ① Treat all the Variables as zero and get SOP (D)
- ② Marks one Variable of all as 1 with previous 1's. & don't care Σ get SOP
- ③ Multiply SOP with Variable
- ④ Repeat step-2 & 3 until all Variables are covered
- ⑤ SOP of VEM is sum of all VEM obtained earlier.

⑥ Make $D \bar{Q} \bar{D} = 0$

Keep 1, 0, X = As it is

A	BC	00	01	11	10
0	\overline{D}	1	1	D	
0	\overline{D}	1	1	D	

C

- ⑤ $D=0 \quad \bar{D}=1$
 Replace 1 with X
 0/x unchanged.

	00	01	11	10
0	1	X	X	0
1	1	X	X	0

\downarrow
 $B\bar{D}$

③ $D=1 \quad \bar{D}=0$

Replace 1 with X

0/x → unchanged

	BC	00	01	11	10
0	00	0	X	X	1
1	01	0	X	X	1

\downarrow
 $B\bar{D}$

$$Y = C + \bar{B}\bar{D} + B\bar{D}$$

$\Sigma(0,1,2,3,4,5) + d(10,11,12,13,14,15)$

	00	01	11	10		1111	X	X
00	1	1	1	1				
01	1	1						
11								
10								

$\rightarrow \bar{A}\bar{B} + A\bar{C}$

A B C D Y

0 0 0 0 1

0 0 0 0 1 } 1

0 0 0 1 1 }

0 0 1 0 1 }

0 0 1 1 1 }

0 1 0 0 0 }

0 1 0 1 0 }

0 1 1 0 0 }

0 1 1 1 0 }

1 0 0 0 0 }

1 0 0 1 0 }

1 0 1 0 X }

1 0 1 1 X }

1 1 0 0 X }

1 1 0 1 X }

1 1 1 0 X }

A	B	C	f
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	X
1	1	0	X
1	1	1	X

① Since there is no Variable it is similar to normal K-Map

A	B	C	00	01	11	10
0	1	1	0	0	0	1
1	0	X	X	X	X	X

$A\bar{B} + B\bar{C}$

LSB on mapping Variable:-

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

Now doing Mapping Variable on A :-Trick: Mapping Variable on LSB

B	C	A	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

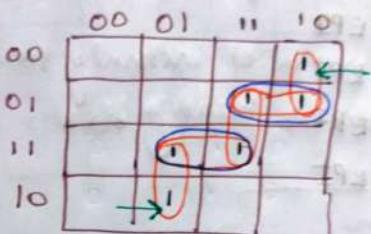
Given the minimized expression and write the output. By comparing the sequence by dividing the groups we can easily see the variables.

Implicants: All terms which are mentioned in K-map are also known as implicants.

Prime implicants: All possible pairs in a K-map are known as prime implicants.

Essential prime implicants: All possible pairs in which at least one min term which can't be pair with any other way considered as EPI.

Redundant prime implicant: The prime implicants whose each '1' is covered by atleast one important / essential prime implicants.



$$\text{Implicants} = 6 \{2, 6, 7, 9, 13, 15\}$$

$$\text{Prime implicants} = 5$$

$$\text{Essential implicants} = 2$$

$$\text{Redundant prime implicants} = 2$$

$$\text{Prime implicants} =$$

$$\bar{A}CD + \bar{A}BC + BCD + ABD, AC'D$$

$$\text{Essential prime} =$$

$$AC'D, ACD$$

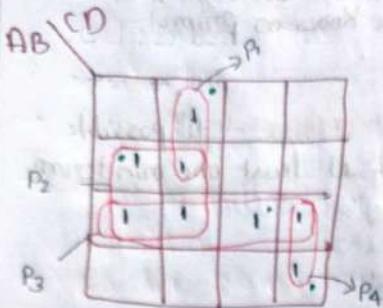
$$\text{Redundant prime} =$$

$$A'BC, ABD$$

Selective prime implicant: Selecting one more extra term.

Q. Which of the following is essential

$$\text{Prime implicant } Y(ABCD) = \{1, 4, 5, 10, 12, 13, 14, 15\}$$



P₁ EPI (allow one minterm which is not grouped with any other)

$$P_1 = \text{EPI}$$

$$P_2 = \text{EPI}$$

$$P_3 = \text{EPI}$$

$$P_4 = \text{EPI}$$

1	d	1
	1	1
d		
d	1	1

$$\text{EPI} = 3$$

In all the group
there will be many

Unpaired terms

NOTE:- Once the don't care is used you can use to minimize them.

Tabular Method

- Used to minimize Variable more than 4

Simplify $Y(A,B,C,D) = \Sigma(0,1,3,7,8,9,11,13)$
using Tabular Method.

AB	CD	00	01	11	10
00	1	1	1	1	1
01	1	1	1	1	1
11		1	1	1	1
10	1	1	1	1	1

$$Y = \overline{B}\overline{C} + CD$$

Table - 1

Minterms	Variables	A	B	C	D
0		0	0	0	0
1		0	0	0	1
3		0	0	1	1
7		0	1	1	1
8		1	0	0	0
9		1	0	0	1
11		1	0	1	1
15		1	1	1	1

Table - 2 : Based on number of ones.

No of 1's	Minterm	Variables			
		A	B	C	D
0	0	0	0	0	0
1	1	0	0	0	1
2	3	0	0	1	1
3	7	0	1	1	1
4	15	1	1	1	1

Table 3:- Make a group by one bit change
and change by -

Group	Match Pair	Variable			
		A	B	C	D
Group 0	(0,1)	0	0	0	-
	(0,8)	-	0	0	0
Group 1	(1,3)	0	0	-	1
	(1,9)	-	0	0	1
	(8,9)	1	0	0	-
Group 2	(3,7)	0	-	1	1
	(3,11)	-	0	1	1
	(9,11)	1	0	-	1
Group 3	(7,15)	-	1	1	1
	(11,15)	1	-	1	1

Note:- After striking put check mark

Table 4:- Combining 4 Matching pair.

Group	Match Pair	Variable			
		A	B	C	D
Group 0	(0,1,8,9)	-	0	0	-
	(0,8,1,9)	-	0	0	-
Group 1	(1,3,9,11)	-	0	-	1
	(1,9,3,11)	-	0	-	1
Group 2	(3,7,11,15)	-	-	1	1
	(3,11,7,15)	-	-	1	1

Group	Matched Variable	Variable			
		A	B	C	D
Group 0	(0,1,8,9)	-	0	0	-
Group 1	(1,3,9,11)	-	0	-	1
Group 2	(3,7,11,15)	-	-	1	1

Pomme implicant Table :-

Prime Implicant	Minterms involved	0	1	3	7	8	9	11	15
$B'C'$	(0,1,8,9)	X	X					X X	
$B'D$	(1,3,9,11)		X	X				X X	
CD	(3,7,11,15)			X	X			X X	

Choose column with single cross, write associated minterm.

$$Y = B'C' + CD$$

EPI Verification:-

EPI	Mint	0	1	3	7	8	9	11	15
$\bar{B}\bar{C}$	(0,1,8,9)	✓	✓					✓	✓
CD	(3,7,11,15)			✓	✓			✓	✓

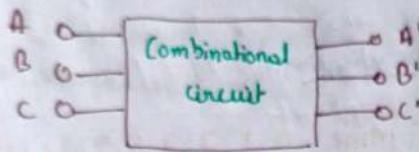
NOTE:- If you have don't care form
don't write in the PI table



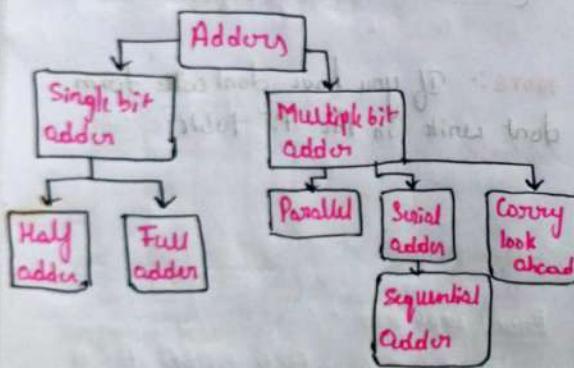
Don't care terms are marked with question marks

Combinational circuits:-

- output depends only on present input
- Add, Mul, demux, Encoder, decoder
- Parity encoder, MUX
- Speed is fast
- Its independent in time
- Used for arithmetic & Boolean operations.
- No clock, No memory, No feedback

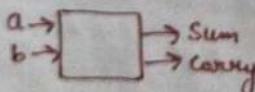


- Output will be at instant of time
- Basic gates are building blocks



• Parallel adder = Carry ripple adder

Half adder:



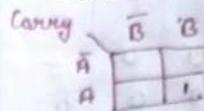
A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$\text{Sum} = \Sigma m(1, 2)$$

$$\text{Carry} = \Sigma m(3)$$

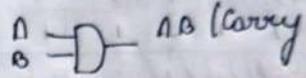
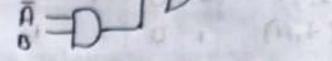
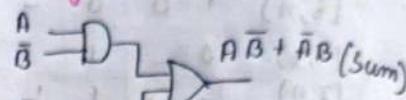


$$\text{Sum} = A\bar{B} + \bar{A}B = A \oplus B$$

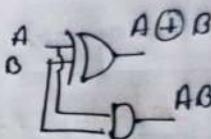


$$\text{Carry} = AB$$

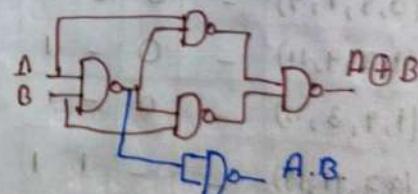
Basic gate:-



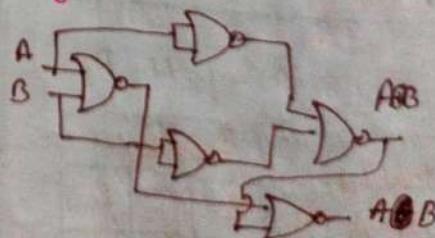
EXOR:-



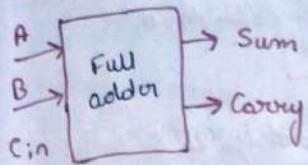
By NAND Gate:-



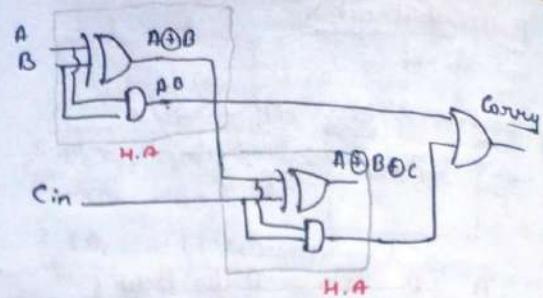
By NOR Gate:-



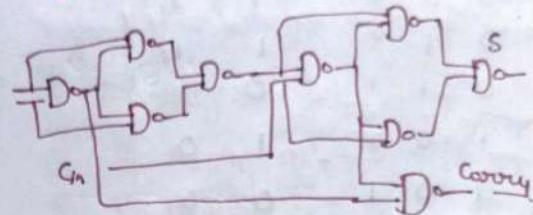
Full adder:



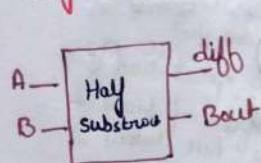
A	B	C	Sum	Carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1



By Nand Gate:



Half subtractor:



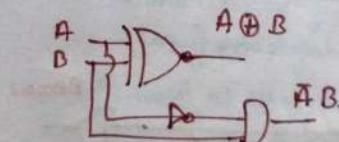
A	B	D	Bout
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

$$D \Sigma (1, 2)$$

$$B_{out} (1)$$

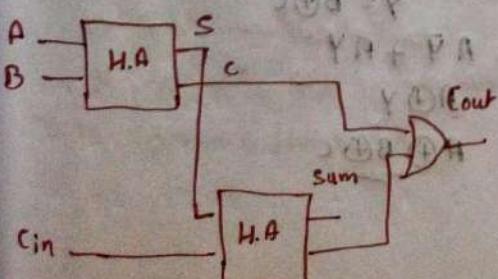
$$\begin{aligned} D &= \bar{A}\bar{B} + A\bar{B} \\ &= A \oplus B \end{aligned}$$

$$B_{out} = \bar{A}B$$

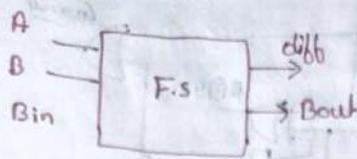


Steps to design Combinational circuit.

1. Analyse the statement and draw a block diagram.
2. Tabulate the truth-table.
3. Get reduced expression from K-Map.
4. Design internal like by specified logic gates.



Full Subtractor:



A	B	Bin	Diff	Bout
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

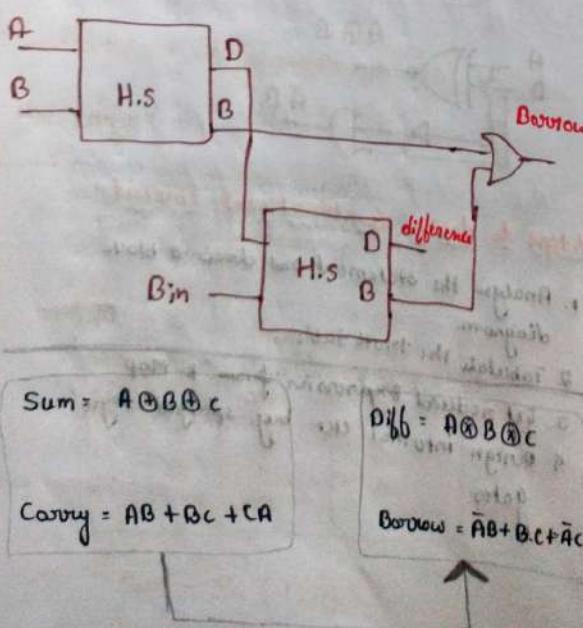
$$\text{Diff} = \sum m(1, 2, 4, 7)$$

$$\text{Bout} = \sum m(1, 2, 3, 7)$$

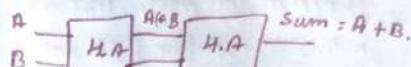
$$\text{Diff} = A \oplus B \oplus C$$

$$\begin{aligned} \text{Bout} &= \bar{A}B + B\bar{B}_{\text{in}} + \bar{A}\bar{B}_{\text{in}} \\ &= \bar{A}B + (A \otimes B) \text{ Bin} \\ &\quad AB + BC + AC \end{aligned}$$

Full Subtractor by using Half Subtractor:



Implement of A+B by half adder



$$= (A \oplus B) \oplus AB$$

$$= A \oplus B = \bar{A}B + A\bar{B}$$

$$= (\overline{A \oplus B}) AB + (A \oplus B) \bar{A}\bar{B}$$

$$= (A \oplus B) AB + (A \oplus B) \bar{A}\bar{B}$$

$$= (\bar{A}\bar{B} + AB) AB + (\bar{A}\bar{B} + A\bar{B}) \bar{A}\bar{B}$$

$$\bar{A}\bar{B} \bar{A}' \bar{B}' + AB$$

①

$$(\bar{A}B)(\bar{A}\bar{B}) + (A\bar{B})(\bar{A}\bar{B})$$

$$(\bar{A}B)(\bar{A} + \bar{B}) \quad \left| \begin{array}{l} (\bar{A}\bar{B})(\bar{A} + \bar{B}) \\ = \bar{A}B\bar{A} + \bar{A}B\bar{B} \end{array} \right.$$

$$= \bar{A}B \quad \left| \begin{array}{l} \bar{A}B\bar{A}' + A\bar{B}\bar{B} \\ = A\bar{B} \end{array} \right.$$

$$= AB + \bar{A}B + A\bar{B}$$

$$= B(A + \bar{A}) + A\bar{B}$$

$$B + A\bar{B}$$

$$(A + B)(B + \bar{B})$$

$$\underline{\underline{A + B}}$$

Bout Equation -

$$A\bar{B}\bar{C} + \bar{A}\bar{B}C + ABC + \bar{A}B\bar{C}$$

$$A(\bar{B}\bar{C} + BC) + \bar{A}(\bar{B}C + B\bar{C})$$

$$A(B \oplus C) + \bar{A}(B \oplus C)$$

$$A(\bar{B} \oplus C) + \bar{A}(B \oplus C)$$

$$Y = B \oplus C$$

$$A \bar{Y} + \bar{A} Y$$

$$A \oplus Y$$

$$\underline{\underline{A \oplus B \oplus C}}$$

10/03/2025

Note: Why we are implementing the full adder by using 2 half adder.

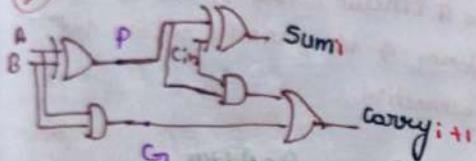
→ Based on requirement we need to implement them.

Distributive Law :-

$$\begin{aligned} B + \bar{B} C & \quad (\text{SOP}) \\ = (B + \bar{B}) (B + C) & \quad (\text{POS}) \end{aligned}$$

Full Subtraction circuit can be used for carry full adder circuit to implement carry lookahead adder for carry propagation and carry generation.

NOTE

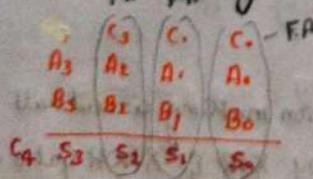


$$\begin{aligned} P &= \text{Propagation} = A \oplus B \\ G &= \text{Generation} = A \cdot B. \end{aligned}$$

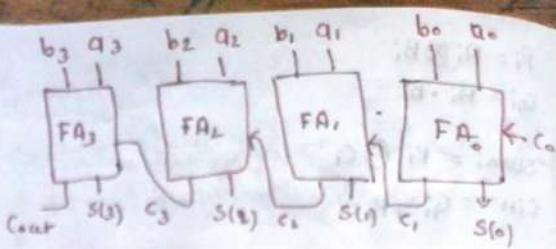
NOTE: Now with full and half adders, we can add 2 bit or 3 not more than those this can be implemented by

- ① Parallel adder (Ripple Carry)
- ② Carry look ahead adder.

→ When we compare both carry look adder has less propagation delay



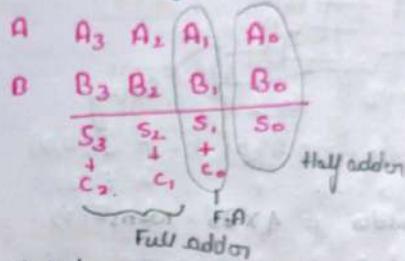
→ So it required 4 full adders



→ FA_i will be activated when C_i will be propagated.

→ If every FA has t_i delay total delay will be 4t_i.

No input carry:



→ It has large propagation delay, it need to wait for before carry mean it need to wait not working in full parallel mode.

Carry Look Ahead adder:

→ It include carry generation & carry propagation.

Involved 3 steps:

① Obtain carry generation (G_i) & carry propagation (P_i)

② Sum at every stage

$$\text{Sum} = P_i \oplus C_i$$

$$S_0 = P_0 \oplus C_0$$

$$S_1 = P_1 \oplus C_1$$

$$S_2 = P_2 \oplus C_2$$

$$S_3 = P_3 \oplus C_3$$

③ Next stage carry

$$C_{i+1} = G_i + P_i C_i$$

$$C_1 = G_0 + P_0 C_0$$

$$C_2 = G_1 + P_1 C_1 = G_1 + P_1 (G_0 + P_0 C_0)$$

$$C_3 = G_2 + P_2 C_2.$$

$$C_4 = G_3 + P_3 C_3.$$

$$P_i = A_i \otimes B_i$$

$$G_{ii} = A_i + B_i$$

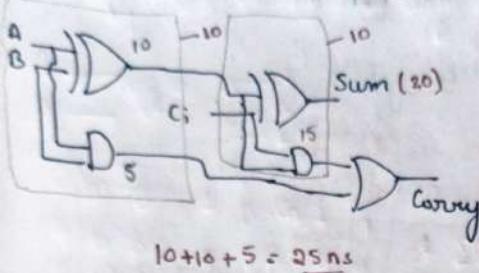
$$\begin{aligned} S_{ii} &= P_i \otimes C_i \\ C_{ii} &= G_{ii} + P_i C_i \end{aligned}$$

④

If we consider a Full adder circuit with half adder. Each $XOR = 10\text{ns}$
 $AND = 5\text{ns}$
 $OR = 5\text{ns}$

delay

For 4 bit adder total latency.



For 4 Full adder $= 4 \times 25 = 100\text{ns}$

NOTE: By these 4 Equation we can reduce the carry propagation delay

$$\begin{array}{cc} AB & A \oplus B \\ \downarrow & \downarrow \\ 5 & 10 \end{array}$$

$$\begin{array}{l} \text{Step 1} \rightarrow 10 \text{ (delay)} \\ \text{Step 2} \rightarrow 10 \end{array} \left. \begin{array}{l} \text{Sum} = P_i \oplus C_i = 20 \\ \uparrow 10 \end{array} \right.$$

$$\begin{array}{l} \text{Step 3} \rightarrow G_{ii} + P_i C_i \\ \downarrow \quad \downarrow \\ 5 \quad 10 \end{array}$$

→ Carry Ripple Adder:

- Parallel adder (cascaded)
- Propagation delay is added to each circuitry

→ CLA:

- Additional circuitry for C_J & C_P but latency is reduced.

A n-bit parallel adder requires

- n Full adder (8) ($n-1$) Full adder and Half adder.

- by half adder we required 2n half adder and n -gate.

→ In n-bit parallel adder, the minimum delay to produce the final result is equal to $2n \times tpd$ [where tpd is the propagation delay of each]

→ To speed up the process look Ahead Carry Adder is used, which is based on carry generation & propagation Fully parallel circuit. Cost extra circuit

Design a circuit which is able to perform 4-bit parallel addition and subtraction.

$$\begin{array}{c} A \\ + B \\ \hline \end{array} \quad \begin{array}{c} A \\ - B \\ \hline \end{array}$$

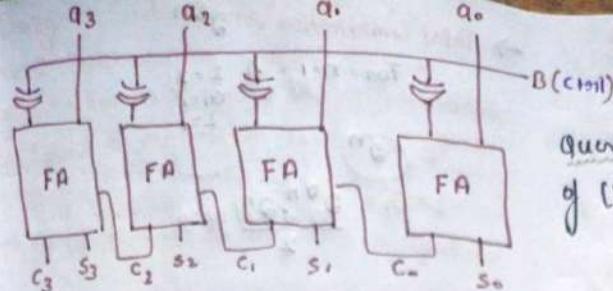
$0 = \text{Adder}$
 $1 = \text{Subtractor}$
ctrl. logic

$$\begin{array}{c} A \\ + B \\ \hline \text{no change} \end{array} \quad \begin{array}{c} A \\ - B \\ \hline \text{B's complement} \end{array}$$

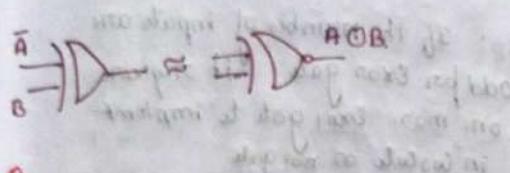
A	B	y
0	0	0
0	1	1
1	0	1
1	1	0

Buffer
Complement

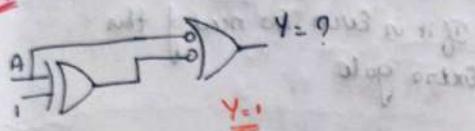
NOTE: decision making circuit will be designed by XOR & $XNOR$ Gates.



NOTE: In layout equation of full subtractor one need to be Complemented.



9



$$\text{Bubble } \oplus \approx \text{NAND}$$

How many 4-bit parallel adder would be required to add two binary numbers each representing decimal number up through 250?

2	<u>360</u>	0			
2	<u>140</u>	0			
2	<u>70</u>	0			
2	<u>360</u>	0			
2	<u>18</u>	0			
2	<u>9</u>	1			
2	<u>40</u>	0			
2	<u>20</u>	0			

1001 0000 0

For 1024 (4,000)

→ Not to big

$$= 2^{10} - 1$$

= 1024-1

- 1023 -

$$10+1 = 11 \text{ bits}$$

3-4 bit adder required

Question: $(36x70)_{10}$ is 10's Complement of $(4240)_{10}$ then find x,y,z

$$\begin{array}{r} 999 \\ \times 420 \\ \hline 36 \end{array}$$

$$\begin{array}{r} 9 - 7 = 6 \\ 9 - 6 = \boxed{3} \\ \boxed{7 = 3} \end{array} \quad \begin{array}{r} 9 - 4 = 3 \\ 9 - 3 = 4 \\ \boxed{4 = 6} \end{array}$$

$$\begin{array}{r} 10 - 2 = 1 \\ 10 - 7 = 2 \\ \hline (6,0) 3 \quad | 2 - 3 \\ (6,3) 3 \quad | \end{array} \quad \begin{array}{r} 9 - 4 = x \\ 9 - 6 = x \\ \hline 3 = x \end{array}$$

Q. Design a circuit with 3 inputs to Count 1's present in input combination using full adder.

Q 1's Compliment addition and Subtraction using parallel adder. & 2's Compliment also.

$G \approx 3^{\circ}$ not enough to

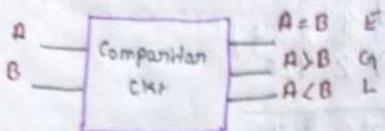
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$\rightarrow \text{Total Combination} = 2^n$

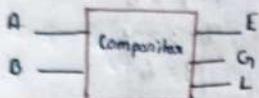
For $n=1 = 4$ E=2
 $G_1=1$ L=1

Comparator Circuit:

- D performs 3 different operation
 - greater, less than, equal.



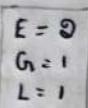
→ n bit Comparator required only 1 output

1-bit Comp.

A	B	E	G ₁	L
0 0	1	0	0	
0 1	0	0	1	
1 0	0	1	0	
1 1	1	0	0	

$E = \bar{A}\bar{B} + AB = A \oplus B$
 $G_1 = A\bar{B}$
 $L = \bar{A}B$.

Note: 1 bit Comparator

2-bit Comparator:

A, A ₀	B, B ₀	E	G ₁	L
0 0	0 0	1	0	0
0 0	0 1	0	0	1
0 0	1 0	0	0	1
0 0	1 1	0	0	1
0 1	0 0	0	1	0
0 1	0 1	1	0	0
0 1	1 0	0	0	0
0 1	1 1	0	0	1
1 0	0 0	0	0	1
1 0	0 1	0	1	0
1 0	1 0	0	1	0
1 1	0 0	0	1	0
1 1	0 1	0	1	0
1 1	1 0	0	1	0
1 1	1 1	1	0	0

$$\Sigma E = (0, 5, 10, 15)$$

$$\Sigma L = (1, 3, 6, 7, 11)$$

$$\Sigma G_1 = (4, 8, 9, 12, 13, 14)$$

$$E = 2^n$$

$$L = G_1 = \frac{2^{0.5} - 2^n}{2}$$

Note: Exnor gate is used for Comparator is Equality.

Tip: If the number of inputs are odd for Exor gate it require one more Exor gate to implement in cascade or not gate.

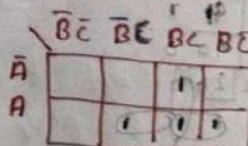
- If it is Even no need of this Extra gate

Non Conventional Combinational logic circuit.

Design the Combinational CKT with 3 input to detect the majority function



A	B	C	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1



$$Y = AB + BC + CA$$

$$f = (3, 6, 5, 7)$$

Output is Exactly Equal to fulladder

Design a Combinational CKT which has 3 input and one output and is equal 1 when no of ones is entire combination is equal or more than two the output is one.

→ It is similar to majority output.

Q. Palindromic Sequence:-

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

$Y = \sum(0, 2, 5, 7)$

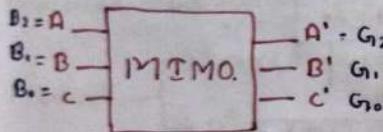
$\bar{B}_2 \quad \bar{B}_1 \quad B_2 \quad B_1$

\bar{B}_2	\bar{B}_1	B_2	B_1
1	1	1	1
1	0	0	1
0	1	1	0
0	0	0	0

$Y = AB + \bar{A}\bar{C}$

$Y = A \oplus C$

Q. Design a circuit which is able to convert 3 bit binary to gray number.



B_2	B_1	B_0	G_{12}	G_{11}	G_{10}
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	1
0	1	1	0	1	0
1	0	0	1	1	0
1	0	1			
1	1	0	1	1	1
1	1	1	1	0	1
1	1	1	1	0	0

$$G_{12} = (4, 5, 6, 7)$$

$$G_{11} = (2, 3, 4, 5)$$

$$G_{10} = (1, 2, 5, 6)$$

$\bar{B}_2 \quad \bar{B}_1 \quad B_2 \quad B_1$

\bar{B}_2	\bar{B}_1	B_2	B_1
1	1	1	1

$G_{12} = B_2$

$\bar{B}_2 \bar{B}_1$	$\bar{B}_2 B_1$	$B_2 \bar{B}_1$	$B_2 B_1$
1	1	1	1
1	1	1	1

$$G_{12} = \bar{B}_2 B_1 + B_2 \bar{B}_1$$

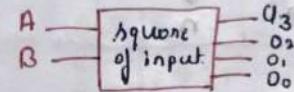
$$G_{12} = B_2 \oplus B_1$$

$\bar{B}_2 \bar{B}_1$	$\bar{B}_2 B_1$	$B_2 \bar{B}_1$	$B_2 B_1$
1	1	1	1
1	1	1	1

$$G_{11} = B_2 \oplus B_1$$

NOTE: Gray Code low power division is used.

Design a Combinational circuit which produces square of 2 bit numbers



A	B	O_3	O_2	O_1	O_0
0	0	0	0	0	0
0	1	0	0	0	1
1	0	0	1	0	0
1	1	1	0	0	0

Multiplication :-

i)

$$\begin{array}{r} A_1 \quad A_0 \\ B_1 \quad B_0 \\ \hline A_1 B_0 & A_0 B_0 \\ A_1 B_1 & B_1 A_0 \\ \hline \end{array} \times$$

Resource requirement :-

→ 4 (2 input AND Gate)

→ 2 Half adder

ii)

$$\begin{array}{r} A_2 \quad A_1 \quad A_0 \\ B_2 \quad B_1 \quad B_0 \\ \hline A_2 B_0 & A_2 B_1 & A_2 B_2 \\ A_1 B_0 & A_1 B_1 & A_1 B_2 \\ A_0 B_0 & A_0 B_1 & A_0 B_2 \\ \hline \end{array} \times \quad \times$$

1 F 1FH
H 1FH
H H O

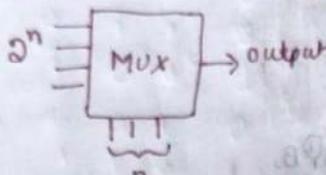
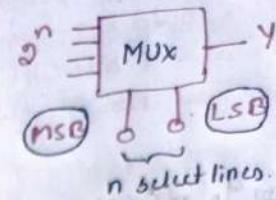
→ 9 (2 input AND Gate)

3 (Half adder)

3 (Full adder)

MULTIPLEXER :

- Multiple inputs single output
- Parallel in Serial out
- Arithmetic operator
- Universal Ckt
- Known as MUX



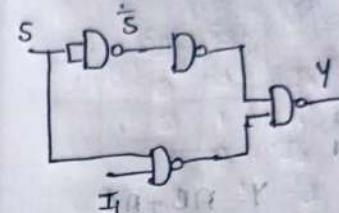
$$\rightarrow S = \log_2 N. \quad (N = \text{No. of inputs})$$

	inputs	selectline
2:1 Mux	2	1
4:1 Mux	4	2
8:1 Mux	8	3
2^n Mux	2^n	n

Q:1 MUX using NAND

$$Y = \overline{\overline{S} I_0 + S I_1}$$

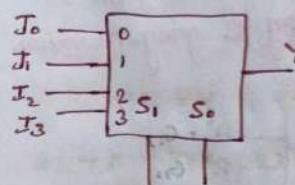
$$Y = \overline{\overline{S} I_0} \cdot \overline{S I_1}$$



Q:1 MUX

Non linear design :-

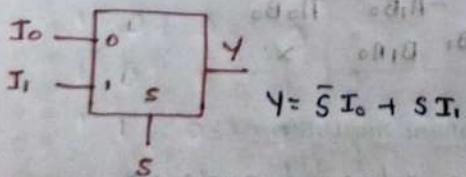
	input	selectline
3:1 Mux	3	2
5:1 Mux	5	1, 3
10:1 Mux	10	4
100	100	7



S_1	S_0	Y
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3

$$Y = I_0 \bar{S}_0 \bar{S}_1 + I_1 \bar{S}_0 S_1 + I_2 S_0 \bar{S}_1 + I_3 S_0 S_1$$

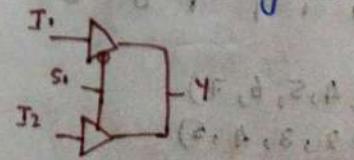
Q:1 MUX



Proof of property

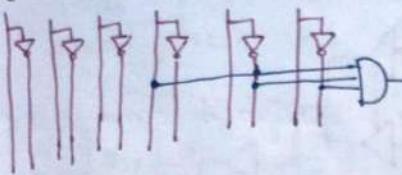
I_0	I_1	S	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

design Q:1 MUX using tristate buffer



Reconfigurable Architecture :-

I₀ I₁ I₂ I₃ I₄ S₁ S₂



PLA	AND		OR	
	ON	ON	ON	X
PAL	ON	X	X	
ROM	X			+

Types of questions they may ask.

- ① Implement any boolean function circuit using Mux.
- ② Implement higher order mux using lower order mux.
- ③ Implementation of Analysis of Network

④ Implementation of boolean expression by Mux

Find which mux have to use

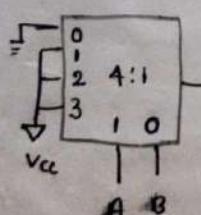
n Variable → 2^n x 1 mux (all n inputs are select lines)

→ 2^{n-1} x 1 (n-1 input → select line, 1 input Data line)

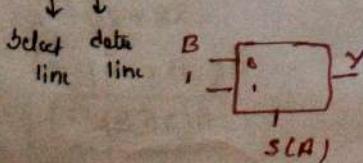
→ 2 x 1 (n-1 → Data line, 1 input select line).

a) $y = \Sigma(1, 2, 3)$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

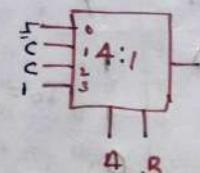
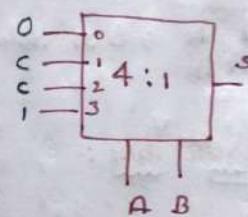


A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

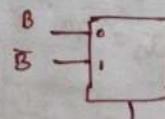


Implementation of 4x1 Mux.

A	B	C	S	C
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
			<u>s</u>	<u>d</u>



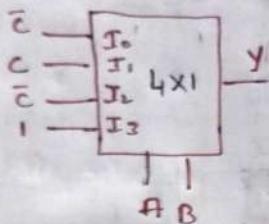
XOR Gate by 2:1 Mux



A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

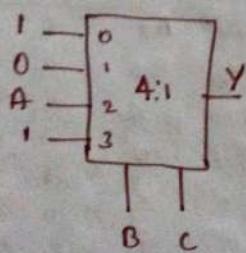
Implement $y = \Sigma(0, 3, 4, 6, 7)$ using
4:1 Mux AB(selectline)

A	B	C	Y
0	0	0	1 } $I_0 = 1$
0	0	1	0 } $I_0 = 0$
0	1	0	0 } $I_1 = 0$
0	1	1	3 } $I_1 = 3$
1	0	0	1 } $I_2 = 1$
1	0	1	0 } $I_2 = 0$
1	1	0	0 } $I_3 = 0$
1	1	1	1 } $I_3 = 1$

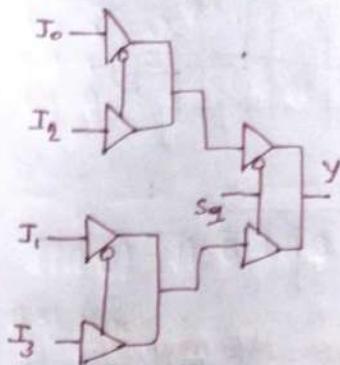


Now BC as select line.

B	C	A	Y
0	0	0	1 } $I_0 = 1$
0	0	1	1 } $I_0 = 1$
0	1	0	0 } $I_1 = 0$
0	1	1	0 } $I_1 = 0$
1	0	0	0 } $I_2 = 0$
1	0	1	1 } $I_2 = A$
1	1	0	1 } $I_3 = 1$
1	1	1	1 } $I_3 = 1$



Design 4x1 Mux using tristate buffer.



Sg	Sg0	Y
0	0	I0
0	1	I1
1	0	I2
1	1	I3

Now AB as select line.

(initial state) when ABC = 000 → initial state

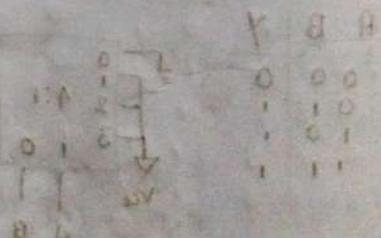
(initial state) when ABC = 001 → initial state

(initial state) when ABC = 010 → initial state

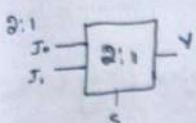
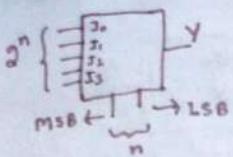
(initial state) when ABC = 011 → initial state

(initial state) when ABC = 100 → initial state

(initial state) when ABC = 101 → initial state



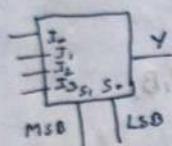
Y	A	B
0	0	0
1	0	0
0	1	0
1	1	0
0	0	1
1	0	1
0	1	1
1	1	1



S	Y
0	J_0
1	J_1

$$Y = I_0 \bar{S}_0 + I_1 S_0$$

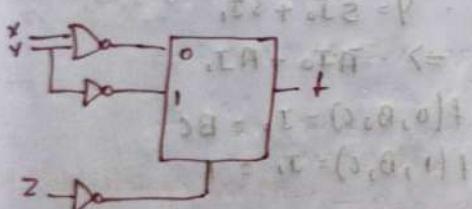
→ By using Nand gate we require 4 Nand gates.



S_0, S_1	Y
0, 0	J_0
0, 1	J_1
1, 0	J_2
1, 1	J_3

$$Y = J_0 \bar{S}_0 \bar{S}_1 + I_1 S_0 \bar{S}_1 + I_2 \bar{S}_0 S_1 + I_3 S_0 S_1$$

Ex:



$$f = \bar{z}\bar{y} + (\bar{x}+y)z$$

$$= \bar{z}\bar{y} + (\bar{x} \cdot \bar{y})z$$

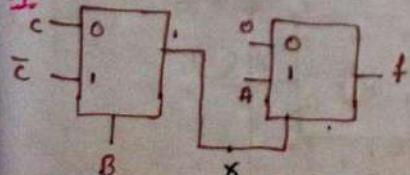
$$= \bar{z}\bar{y} + \bar{x}\bar{y}z$$

$$= \bar{y}(\bar{z} + \bar{x}z)$$

$$= \bar{y}(\bar{x} + \bar{z})$$

$$f = \bar{y}\bar{x} + \bar{y}\bar{z}$$

Ex:

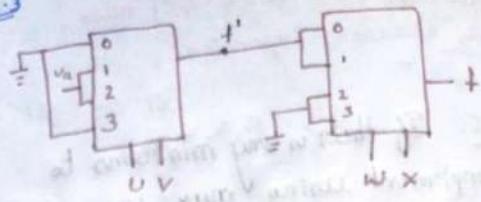


$$X = \bar{B}C + B\bar{C}$$

$$f = (\bar{B}C + B\bar{C})A$$

$$f = (B \oplus C)A$$

Ex:



$$f = \bar{U}V + U\bar{V}$$

$$f' = U \oplus V$$

$$= \bar{W}\bar{X}(U \oplus V) + \bar{W}X(U \oplus V)$$

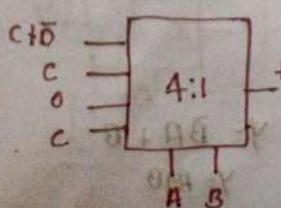
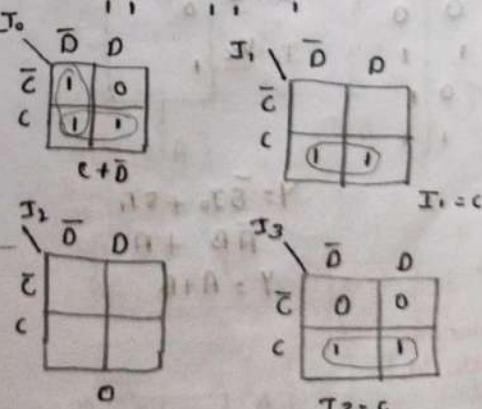
$$= U \oplus V(\bar{W}\bar{X} + \bar{W}X)$$

$$= U \oplus V(\bar{W})$$

$$= (U \oplus V)\bar{W}$$

Q. 4: $\Sigma(0, 2, 3, 6, 7, 14, 15)$ using 4:1 MUX.

A	B	C	D	f
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

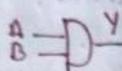


$Y = AB + BC + CA$ using 4×1 MUX.
use Shannon's Expansion.

NOTE: If there is any minterms to implement using MUX then only Tabular Method.

→ Shannon method only for $2:1$ MUX only.

2-input AND Gate using $2:1$ MUX



A	B	Y
0	0	0 } $J_0 = 0$
0	1	0 } $J_1 = 0$
1	0	0 } $J_0 = 0$
1	1	1 } $J_1 = 1$

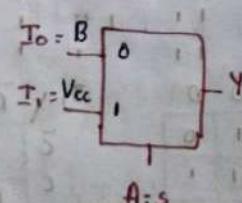
Here A = Select line & B on data line



$$Y = \bar{S}J_0 + SJ_1 \\ = \bar{A}0 + AB \\ Y = \underline{\underline{AB}}$$

$2:1$ MUX using OR Gate.

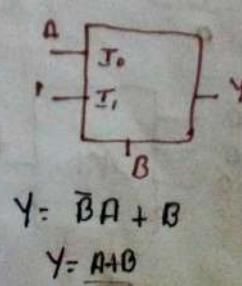
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1



$$Y = \bar{S}J_0 + SJ_1 \\ = \bar{A}B + A \\ Y = \underline{\underline{A+B}}$$

B → Select line
A → Data line

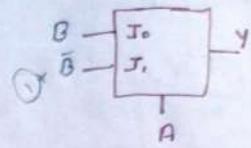
B	A	Y
0	0	0
0	1	1
1	0	1
1	1	1



$$Y = \bar{B}A + B \\ Y = \underline{\underline{AB}}$$

3-input X-OR Gate using $2:1$ MUX

A	B	C	Y
0	0	0	0
0	1	0	1
1	0	0	1
1	1	0	0



Using Shannon Expansion (Only 3)

$$f(A, B) = A \cdot \bar{B} + \bar{A}B$$

A sketchline:

$$f(0, B) = 0\bar{B} + 1B = B$$

$$f(1, B) \cdot 1\bar{B} + 0 = \bar{B}$$

$$Y = \bar{A}f(0, B) + Af(1, B)$$

$$Y = \bar{A}B + AB$$

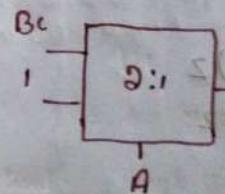
By using $\bar{S}J_0 + SJ_1$,

$Y = \bar{S}J_0 + SJ_1$

$$\Rightarrow \bar{A}J_0 + AJ_1$$

$$f(0, B, C) = J_0 = BC$$

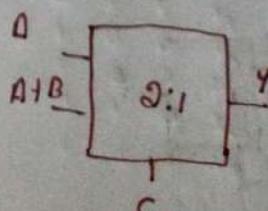
$$f(1, B, C) = J_1 = 1$$



as select line :-

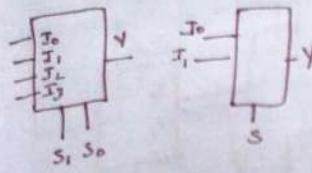
$$f(A, B, 0) = J_0 = A$$

$$f(A, B, 1) = A + B$$



Implementation of higher order MUX using lower order MUX.

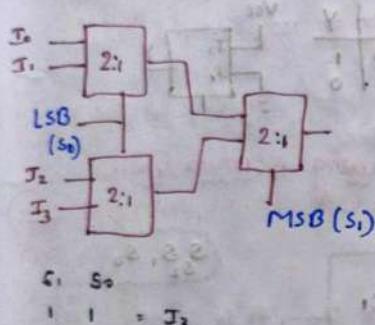
Q Implement 4:1 Mux using 2:1 Mux



$$\frac{4^2}{2} = \frac{2^1}{2}$$

level-1 level-2.

Note: Selection line will be same for each level.

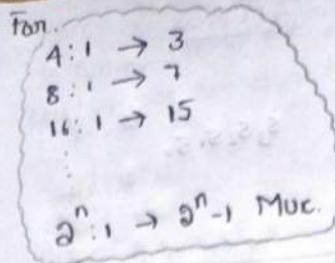
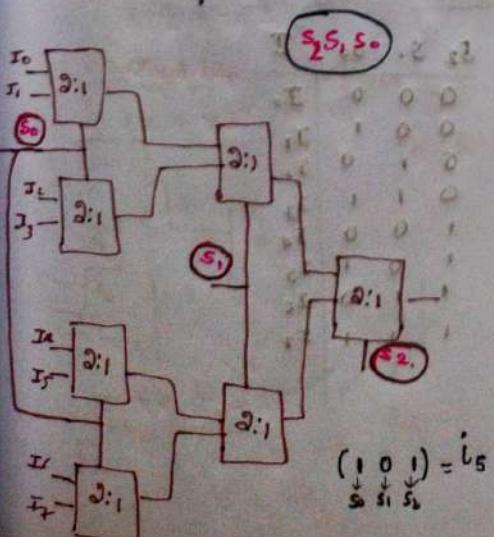


8:1 Mux using 2:1 Mux

$$\frac{8}{2} = \frac{4}{2} \quad \frac{4}{2} = \frac{2}{2}$$

level-1 level-2 level-1

7 Mux using 2:1 Mux



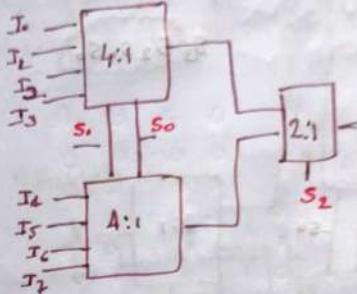
Q 8:1 Mux using 4:1 Mux & 2:1 Mux

$$\frac{8}{4} = 2 \text{ (4:1 Mux)}$$

$$\frac{2}{2} = 1 \text{ (2:1 Mux)}$$

(S₂, S₁, S₀)

O



16:1 Mux using 4:1 Mux

$$\frac{16}{4} = \frac{4}{4} = 1$$

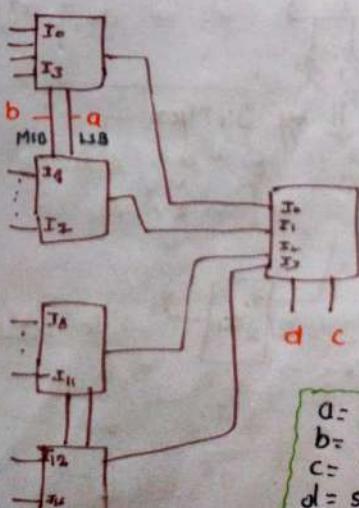
Selection lines to req = 16.

(S₃, S₂, S₁, S₀)

level-4 (4x1)

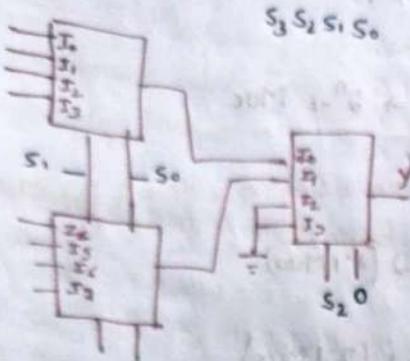
(4:1)

level-8 (4x1)

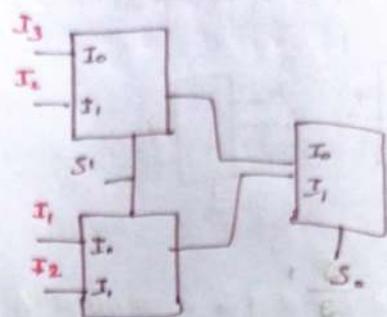


a = S₀
b = S₁
c = S₂
d = S₃

8:1 MUX using 4:1 MUX

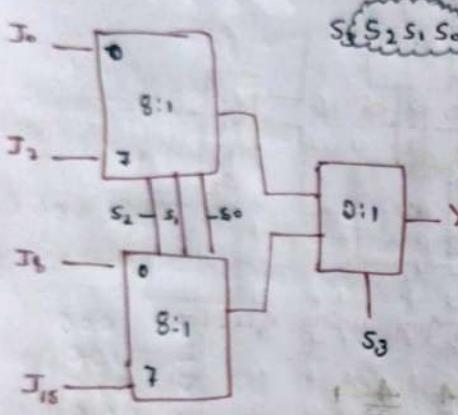


What if I change the Order of the Select line.

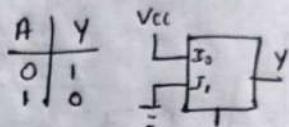


due to change in the Select line

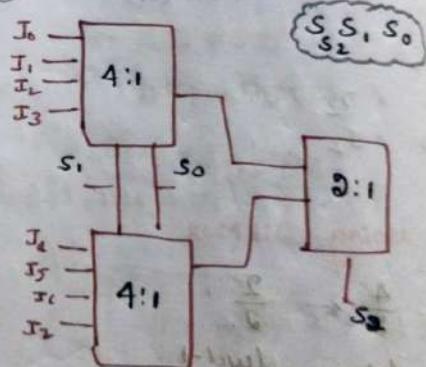
16:1 Mux using 8:1 Mux & 2:1 Mux



Inverters using 3:1 Mux

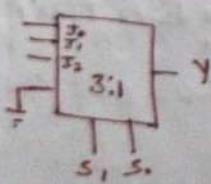


8:1 Mux using 4:1 mux & 2:1

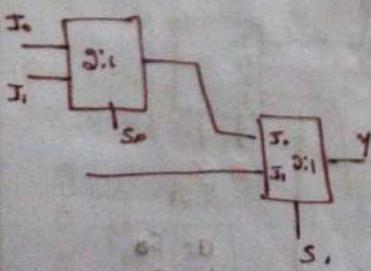


Non Conventional Multiplexer:

3:1 Mux using 2:1 Mux



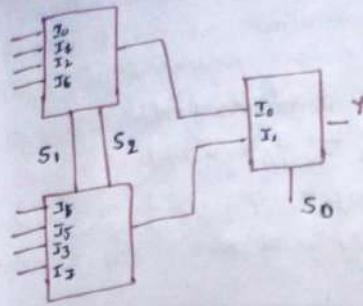
|| by 2:1 Mux



Actual

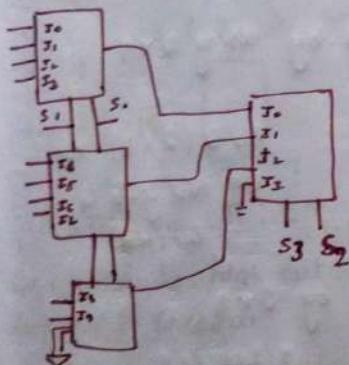
S ₂	S ₁	S ₀	J
0	0	0	I ₀
0	0	1	I ₁
0	1	0	I ₂
0	1	1	I ₃
1	0	0	I ₄
1	0	1	I ₅
1	1	0	I ₆
1	1	1	I ₇

Note: In this below Example No need to connect the 3rd mux in the Level-1 we can directly connect.

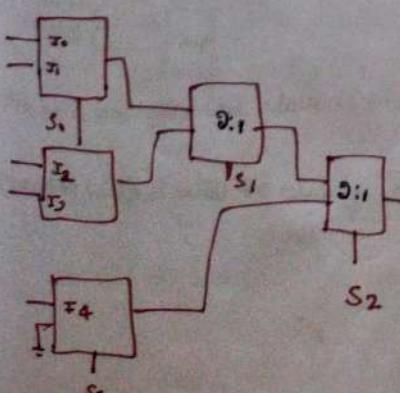


S ₀	S ₁	S ₂	Y
0	0	0	J ₀
0	0	1	J ₄
0	1	0	J ₂
0	1	1	J ₆
1	0	0	J ₁
1	0	1	J ₅
1	1	0	J ₃
1	1	1	J ₇

Ex: 10:1 Mux using 4:1 Mux

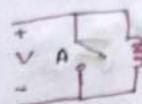


5:1 Mux using 2:1 Mux.



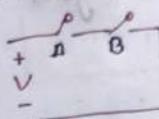
Switching circuit.

NOT Gate :-



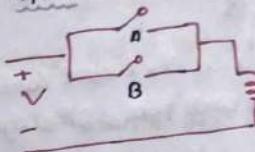
A	Y
0	1
1	0

AND Gate :-



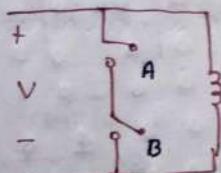
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

OR Gate :-



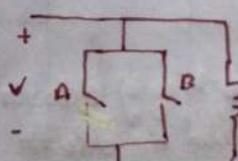
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

NAND Gate :-



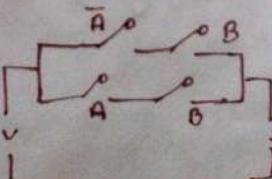
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

NO NOR Gate :-



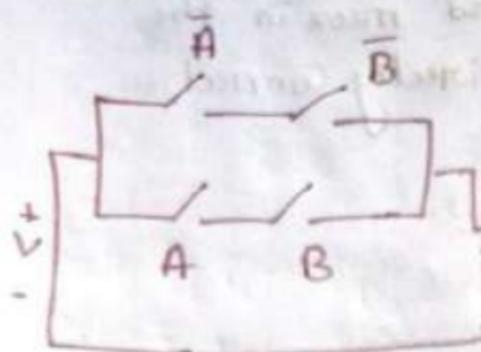
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	0

E xor Gate :-



A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

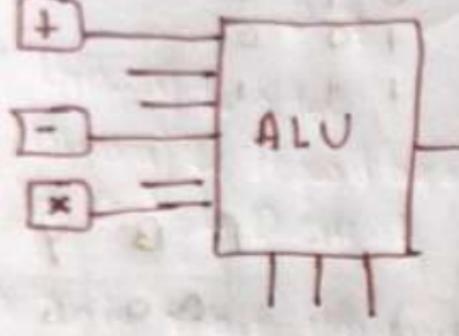
Exnor Gate



A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Application of MUX

- ① data Selector
- ② Serial the parallel data
- ③ ALU (based on select line it perform the operation)



Opcode

Opcode	
000	+
001	-
010	*
110	%

Y R U

1	0	0
0	1	0
0	0	1
0	1	1

Y O N

0 0 0

1 1 0

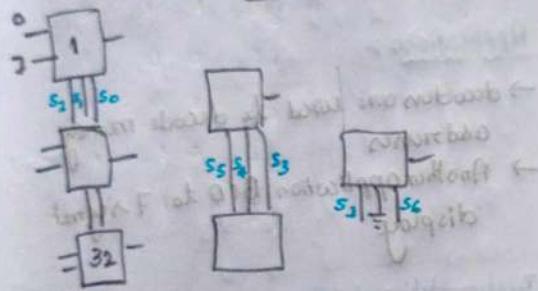
1 0 1

0 1 1

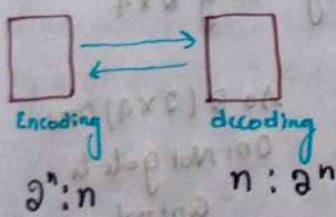
40 1 1

- In Mux if there is a Minuteman use the VEM method.
- If there is a boolean function use the Shannon Expansion.
- The number of 2×1 mux required to implement 256×1 mux is 955.
- 256×1 mux using 8×1 mux.

$$\begin{array}{c} \frac{256}{8} \quad \frac{32}{8} \quad \frac{4}{8} \\ 32 \quad 4 \quad 1 \\ S_7 S_6 S_5 S_4 S_3 S_2 S_1 S_0 \end{array}$$



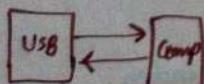
Decoder:



→ decoder has high output lines compared to encoder.

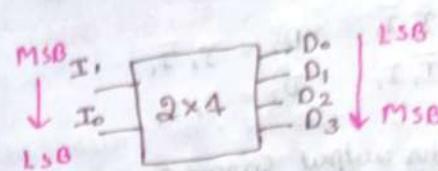
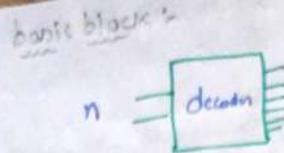
→ decoder has less input lines.

Application:-



Master is controller but slave is receiver

→ Decoder output will be always minterm

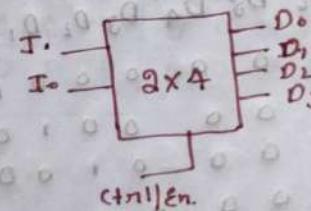


I ₁ (A)	I ₀ (B)	D ₃	D ₂	D ₁	D ₀
0	0	0	0	0	1
0	1	0	0	1	0
1	0	0	1	0	0
1	1	1	0	0	0

$$\begin{aligned} D_0 &= \bar{A}\bar{B} \\ D_1 &= \bar{A}B \\ D_2 &= A\bar{B} \\ D_3 &= AB \end{aligned}$$

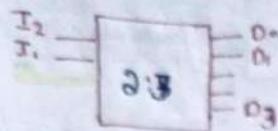
Note:- Implementation of decoder of require of and gate gated or not.

→ We can control the circuit by using the enable circuit.



En	I ₁	I ₀	D ₃	D ₂	D ₁	D ₀
0	0	0	0	0	0	1
	0	1	0	0	1	0
	1	0	0	0	0	0
	1	1	0	0	0	0
1	0	0	0	0	0	1
	0	1	0	0	1	0
	1	0	0	1	0	0
	1	1	1	0	0	0

2×4 decoder.



$$D_0 = I_2 I_1 \quad D_1 = I_2 I_1 \\ D_2 = I_1 I_1 \quad D_3 = I_1 I_1$$

→ Decoder output corresponding to minterm.

→ We can have decoder with 2^n output

3:6 ✓

3:3 ✓

⇒ 3:5 ✗

3:4 ✓

→ Binary to octal
Binary to Hexadecimal
BCD to Decimal

→ In an n-bit ununited don't care combination decoder output will have 2^n output.

I ₂ I ₁ I ₀	D ₇	D ₆	D ₅	D ₄	D ₃	D ₂	D ₁	D ₀
0 0 0	0	0	0	0	0	0	0	1
0 0 1	0	0	0	0	0	0	1	0
0 1 0	0	0	0	0	0	1	0	0
0 1 1	0	0	0	0	0	1	0	0
1 0 0	0	0	0	0	1	0	0	0
1 0 1	0	0	0	1	0	0	0	0
1 1 0	0	0	1	0	0	0	0	0
1 1 1	1	0	0	0	0	0	0	0

What type of questions may ask?

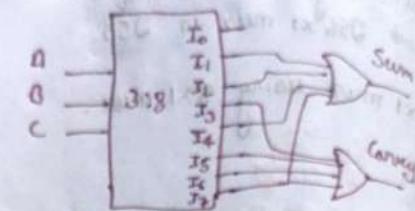
- ① Explain the working of decoder
- ② Boolean function by decoder

→ Bubble will be denoting the active low logic.

Design and implement full adder using decoder.

$$\text{Sum} = \Sigma (1, 2, 4, 7)$$

$$\text{Carry} = \Sigma (3, 5, 6, 7)$$



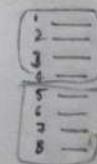
→ Decoder with enable will then have de-multiplexer.

Applications :-

- decoders are used to decode memory addresses
- Another application BCD to 7 segment display

Implementation of higher order decoder by using lower order decoders.

→ 3x8 by using 2x4



No 2 (2x4) and
One Not gate to
control.

⇒ 3x8 by 2x4

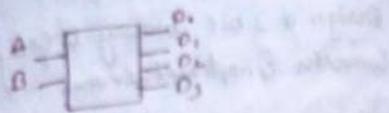
8/4 and 1 for control.

2

MSB → Ctrl Input
LSB → Input of lower decoder.

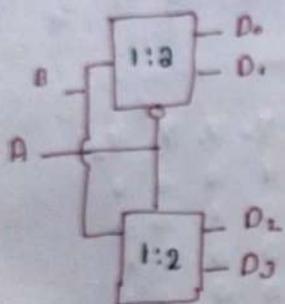
Question

2:4 decoder using 1:2 decoder

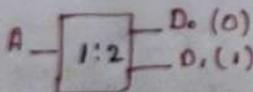


A	B	D ₀	D ₁	D ₂	D ₃
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

We can observe here when A=0
only D₀, D₁ are active and A=1
D₂ & D₃ are active.

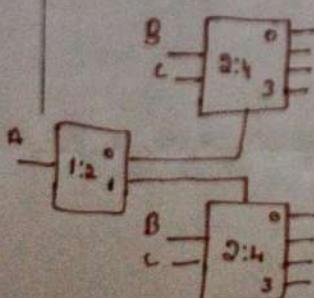


And Not can also be implemented
by a other logic 1:2 decoder



3:8 decoder using 2:4 decoder

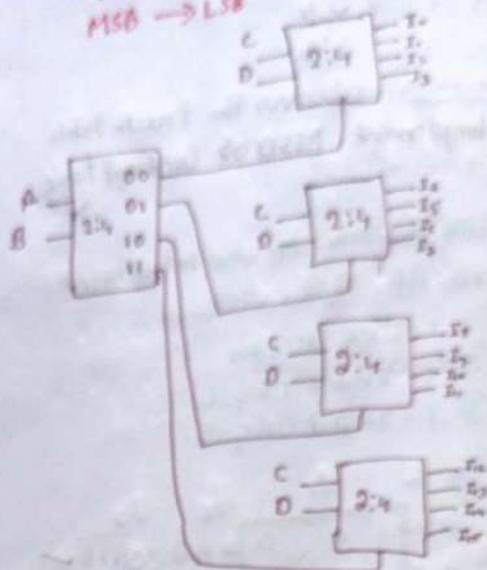
ABC | D₀ D₁ D₂ D₃ D₄ D₅ D₆ D₇



4x16 decoder using ① 2x4 decoder
② 3x8 decoder, 1x2

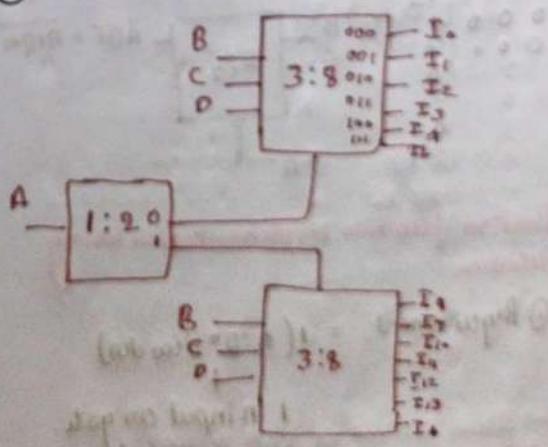
a) $\frac{16}{4} \times 4$

MSB \rightarrow LSB



A	B	C	0	T ₀ T ₁ T ₂ T ₃ T ₄ T ₅ T ₆ T ₇ T ₈ T ₉ T ₁₀ T ₁₁ T ₁₂ T ₁₃ T ₁₄ T ₁₅
0	0	0	0	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0	0	1	0	0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0	1	0	0	0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0	1	1	0	0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
1	0	0	0	0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0

b)



NOTE: Solve & Review this.

H.W

- ① Design a 3-bit binary to Gray converter & implement with minterms
 ② BCD to Excess 3 code converter
 Convert to Boolean Exp & implement by in minterms

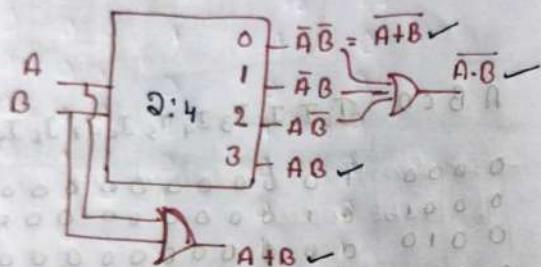
- ① 2:4 \rightarrow 1:2
 ② 3:8 \rightarrow 1:2
 $2:4$
 ③ 4:16 \rightarrow 2:4
 $3:8$

Supertrick: Observe the truth table to implement MSB of control logic.

Q Use three decoder and implement

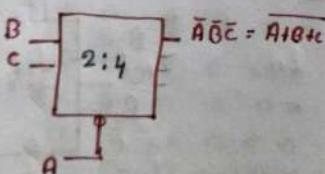
$$A+B, AB, \bar{A}B, \bar{A}+B$$

- ① 1-2x4 decoder
- ② 1-2 input OR Gate
- ③ 1-3 input OR gate.



Design and implement 3 input NOR gate using 2:4 decoder

A	B	C	y
0	0	0	1
0	0	1	0
:		0	0
0			0

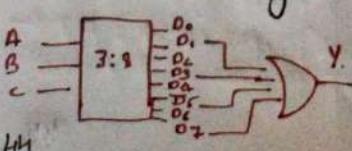


Boolean function implementation using decoder

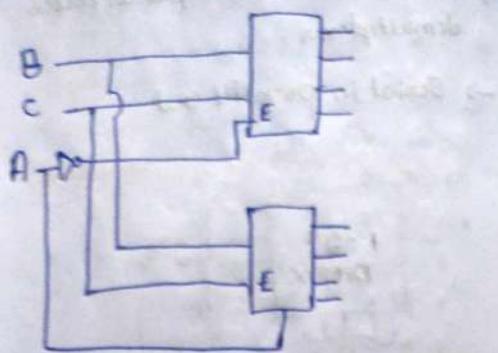
① Requirement = 1 (n:2^n decoder) + 1 n input OR gate.

$$Y(A, B, C) = \Sigma_m(1, 3, 5, 7)$$

② 3:8 decoder + OR gate.



3:8 by using 2:4



number of 1's given as number of 1's

$$\frac{a}{2} + \frac{b}{2}$$

$$c = a + b$$

$$d = (1 - A) = (1 - a)$$

number of 1's given as number of 1's

$$e = X \cdot X + Y \cdot Z \cdot W$$

$$0 \ 0 \ 0 \ 1 \ 0 \ 0$$

$$0 \ 0 \ 1 \ 0 \ 1 \ 0$$

$$1 \ 1 \ 0 \ 0 \ 0 \ 1$$

$$1 \ 0 \ 0 \ 0 \ 1 \ 1$$

a	b	c	d	e
0	1	1	0	0
0	0	0	1	0
0	1	1	0	0
1	0	0	0	1
0	0	1	0	0
1	1	0	1	1
1	0	1	0	1
1	1	0	1	1

$$(F, d, c, b) = e \oplus (d, a, b, c) \oplus d \\ (F, d, b, c) \oplus d$$

x	8x8
x	area
x	area

number of 1's in minterm will be

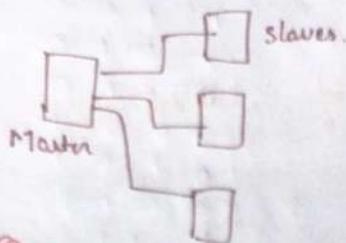
number of 1's in minterm will be 11111111

number of 1's in minterm will be 00000000

[C-11111111] input COMT & ITT
will be 11111111 if we add 00000000

Application of decoders:

→ In Master Slave circuit, where the master want to communicate with slave.

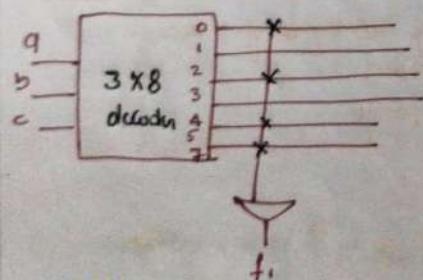


- ④ Design a combinational cir. which accepts 3 inputs and provides 3 output.
- ⑤ High when a combination is even
- ⑥ ABC are palindrome
- ⑦ Only two 8 bit binary one

a	b	c	f_1	f_2	f_3
0	0	0	1	1	0
0	0	1	0	0	0
0	1	0	1	1	0
0	1	1	0	0	1
1	0	0	1	0	0
1	0	1	0	1	1
1	1	0	0	1	1
1	1	1	1	1	1

$$f_1 = \Sigma(0, 2, 4, 6) ; f_3 = \Sigma(3, 5, 6, 7)$$

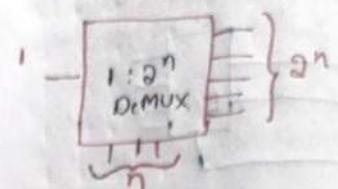
$$f_2 = \Sigma(0, 2, 5, 7)$$

EXTRA:

- Above circuit is ROM circuit
- For PLD & PAL fusing & Antifusing technique
- Mentor graphics buy by SIEMENS
- AMD buy Xilinx.
- TTL & CMOS logic [7400 {54LS}]
- Which IC is used for 7446, 7447

De-Multiplexers

- Decoder with Control input is called demultiplexer
- Serial in parallel out.



→ 1:4 demux using 1:2 demux

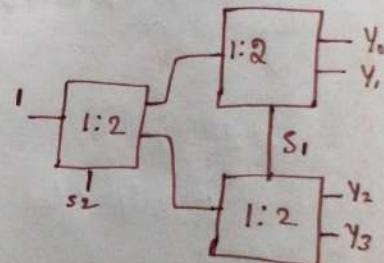
$$\frac{4}{2} = 2$$

$$2 + 1 = 3,$$

$$(n-1) = (4-1) = 3,$$

2x4 Demux using 1x2 demux

$S_2\ S_1$	Y_0	Y_1	Y_2	Y_3
00	1	0	0	0
01	0	1	0	0
10	0	0	1	0
11	0	0	0	1

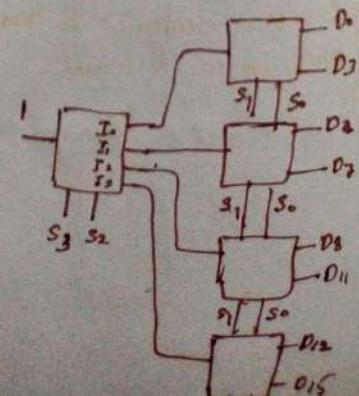


Note: In DMUX MSB → LSB

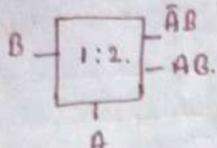
Design 1x16 Demux using 1x4 demux

$$\frac{16}{4} = 4$$

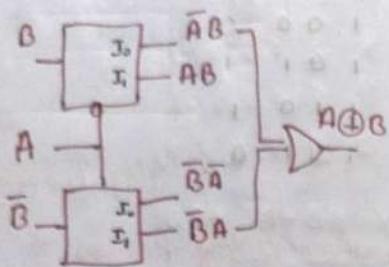
$$4 + 1 = 5$$



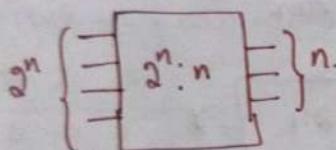
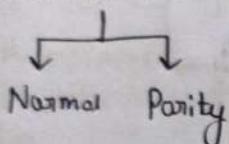
And Gate by Demux.



2 input XOR Gate by Demux (1:2)



Encoder: Secure the date & Compress the date thus Encoder can be used.



I_3	I_2	I_1	I_0	Y_1	Y_0
0	0	0	1	0	0
0	0	1	0	0	1
0	1	0	0	1	0
1	0	0	0	1	1

Limitation: What if the more number of ones are there on single line

Priority Encoder is designed to handle multiple ones in the input sequence.

Priority is given for MSB and then LSB.

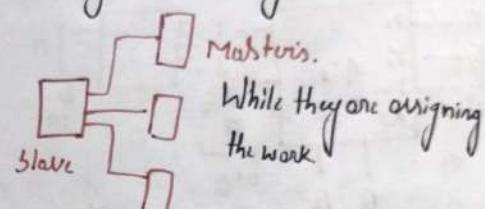
I_3	I_2	I_1	I_0	Y_1	Y_0
0	0	0	1	0	0
0	0	1	0	0	1
0	1	0	X	1	0
1	X	X	X	1	1

Priority orders: higher the subscript higher the priority

Application:

Orbitor: designed using priority encoder

→ For controlling the shared resources and providing the priority.



→ This may create the star vision problem this is the limitation.

→ This can be corrected by using Round Robin Algorithm.

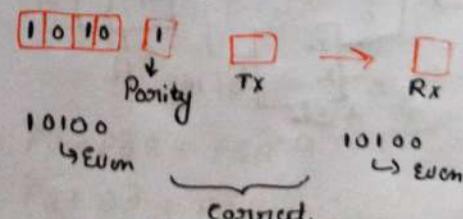
→ Implement using Verilog ↗

Concept of parity:

→ Refers to no of ones in a row is called parity. And they are of 2 types

→ odd parity.

→ even parity.



→ If it is odd it will be wrong.

→ Parity will fail when multiple bit flip.

→ Single bit error detection & correction

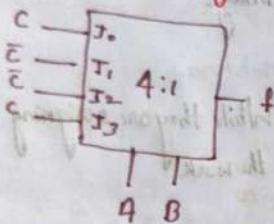
→ CRC, LFSR technique are used

Design 3-bit even parity generator ckt

A	B	C	f_C
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

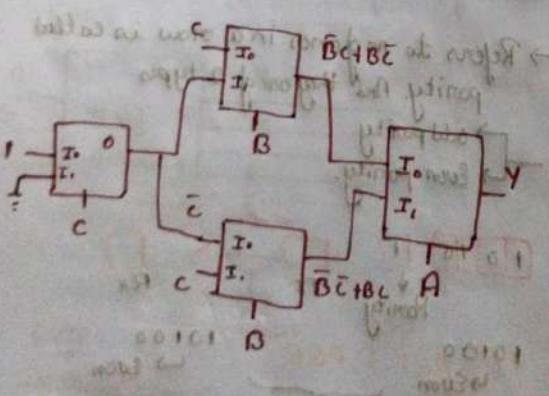
↑
Think how to make even.

implement this using 4:1 mux:



by 2:1 Mux:
 $\bar{B}C + B\bar{C}$ at output
 $\bar{B}\bar{C} + BC$ at output
A → Y
Implementation using 2:1 mux

Implement only by 2:1 Mux



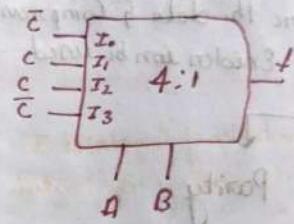
Implementation of this is bit difficult
but still can be done using 2:1 muxes

Implementation of this is bit difficult
but still can be done using 2:1 muxes

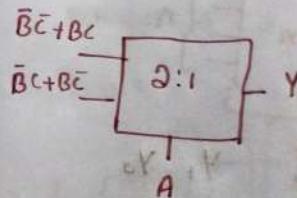
Design 3-bit odd parity generator ckt

A	B	C	f_O
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

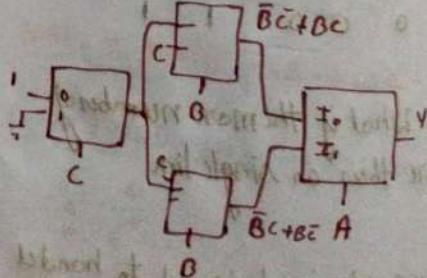
implement using 4:1 mux



By 2:1 mux



implement only by 2:1 mux

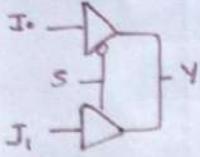


Implementation of this is bit difficult
but still can be done using 2:1 muxes

19/03/2025

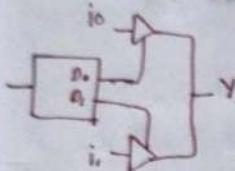
① Implement mux using tristate buffer.

Q.1



S	B ₁	B ₂	Y
0	A _c	J _n	J ₀
1	J _n	A	J ₁

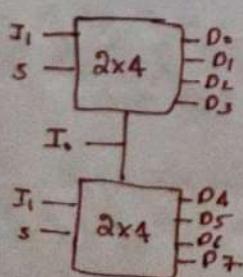
② Decoder using ctrl logic



Hint: In the implementation of any tristate buffer replace the Mux by its tristate buffer.

③ 2:1 Mux using 2x4 decoder

I ₀	I ₁	S	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1



$$\Sigma(3, 4, 6, 7)$$

① Which of the following is not an exclusive OR gate?

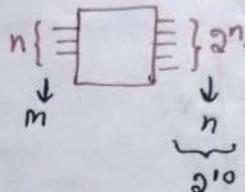
$$\bar{x} \oplus \bar{y} = D \rightarrow z$$

$$\bar{x} \oplus y = D \rightarrow z$$

$$\bar{x} \oplus \bar{y} = D \rightarrow z$$

② If there are m input lines and n output lines for a decoder that is used to uniquely address only 1Kb RAM, where data width 1 byte long, what is the min value of m+n.

$$\text{Size} = M \times n \\ 1\text{Kb} = 1 \times 2^{10} \times 2^3$$

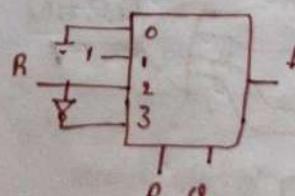


$$= 10 + 3 \\ = 10 + 10 \\ = \underline{\underline{1034}}$$

③ How many 3:8 lines need with 4 input are need to design 6:64 line decoders with only other logic gate.

$$\frac{64}{8} = 8 \\ 8 + 1 = 9$$

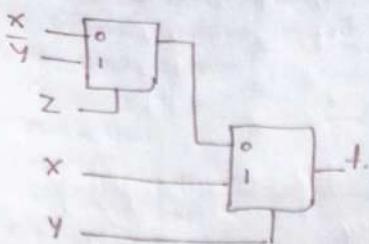
④ Output of Mux



$$= \bar{P}\bar{Q} + P\bar{Q}R + P\bar{Q}\bar{R}$$

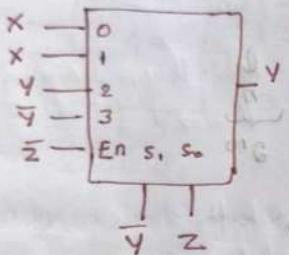
$$= \bar{P}Q + Q\bar{R} + P\bar{Q}R$$

⑤ Output of the Mux



$$\begin{aligned}
 &= (\bar{z}x + z\bar{y})\bar{y} + xy \\
 &= \bar{z}xy + z\bar{y} + xy \\
 &= \bar{y}(x + \bar{z}) + xy \\
 &= \underline{\underline{x + \bar{y}z}}
 \end{aligned}$$

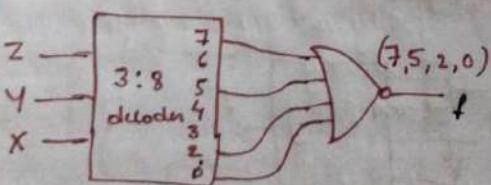
⑥



$$\begin{aligned}
 &= \bar{y}\bar{z}x + \bar{y}zx + \bar{y}\bar{z}y + \bar{y}zy \\
 &= \bar{y}\bar{z}x + yzx + \bar{y}z \\
 &= \bar{y}z + xy \\
 \Rightarrow & \bar{y}z\bar{z} + xy\bar{z}
 \end{aligned}$$

$$f = xyz$$

⑦



$$f = x \oplus z$$

- ⑧ Design a Combinational Circuit for generating 2³ Complement of 3-bit binary number
implement this with ① 2 level AND OR
② using 4x1 Mux.

A	B	C	X	Y	Z
0	0	0	0	0	0
0	0	1	1	1	1
0	1	0	1	1	0
0	1	1	1	0	1
1	0	0	1	0	0
1	0	1	0	1	1
1	1	0	0	1	0
1	1	1	0	0	1

$$X = \Sigma(1, 2, 3, 4)$$

$$Y = \Sigma(1, 2, 5, 6)$$

$$Z = \Sigma(1, 3, 5, 7)$$

$$X = \bar{A}B + \bar{A}C + A\bar{B}C$$

$$Y = BC + \bar{B}C$$

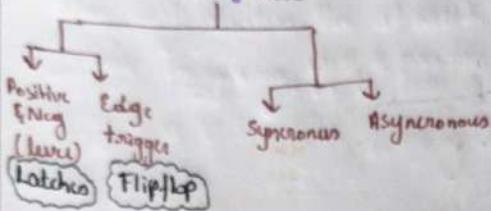
$$Z = C$$

Sequential Circuits:

- Are those which one depends on clock cycles and depends on present or well on past inputs to generate any output.
- Speed is slow.
- because of triggering.
- Designing is tough.
- Time dependent.
- Elementary building blocks are flip flops and latches.
- Mainly used in storing data. (1-bit)
- These circuit has memory.
- It is not easy to use and handle.

Example:- latch, flip flop, counter, shift register.

Classification



→ Latches are level trigger & flip flop is edge trigger device.

→ In single clock the latch can produce the multiple transition. Leads to

Race Condition

→ Flipflops are race free.

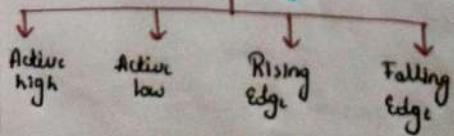
NOTE:- If the sequence are inactive refers to it will be in the previous state.

• But in Combinational the output leads to high impedance state.

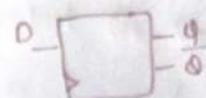
→ Latches are easy to design with acts on combinational.

→ In latch the CLK is acts on a enable signal.

Triggering



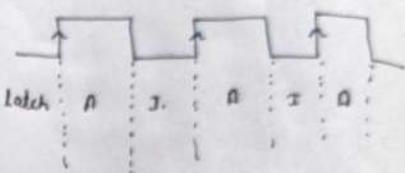
Active low level sensitive



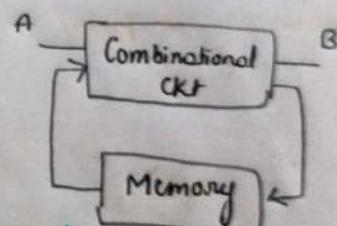
Edge triggering rising



Falling Edge triggering



- At this position it will acts on Combinational circuit.
- We can alter the inputs which may also leads to race around condition.
- Solution is using a flip flop we can manage this.



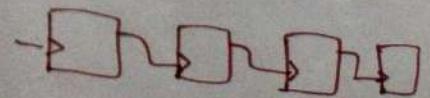
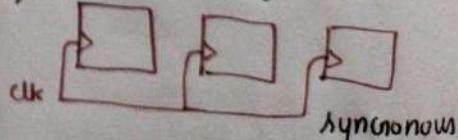
Sequential circuit.

Synchronous vs Asynchronous :-

→ In synchronous single clock to activate

→ Different clock in Asynchronous

→



Asynchronous

- Synchronous output will be predictable
- Most of application we use Asynchronous Ckt.
- In nowadays we are using pipelining technique

→ Total propagation delay in asynchronous is high but individually very low

→ Synchronous → Shift register

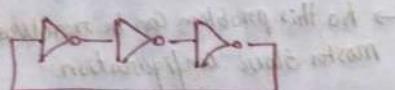
Asynchronous → Counters & frequency dividers

Asynchronous logic is used in low power and high speed applications.

→ Flip flop is also called bistable multivibrator



Ring oscillation:



→ Latches are more prone to noise while flip-flops are less.

↳ because the latches are less prone to the noise by of triggering.

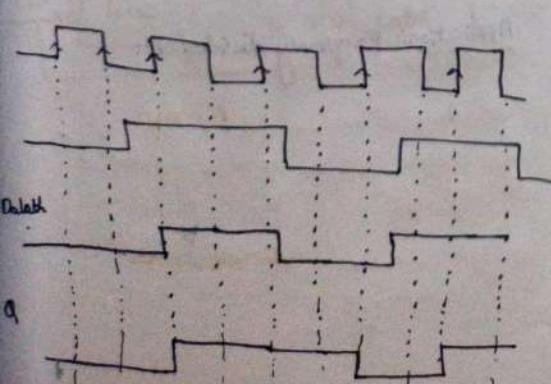
D latch and D flip-flop difference:

Clk	D	Q
0	x	N.C.
1	0	0
1	1	1

D-latch

Clk	D	Q
↑	0	0
↑	1	1
↑	x	0

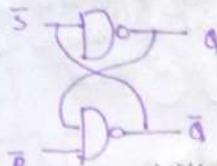
D-flip-flop



S R Latch:

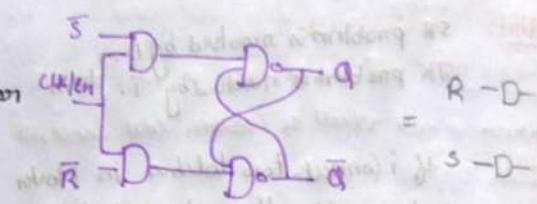
→ It is also called set or Rat-latch

→ Construct using Nand and Nor



without enable

Enable = Clk.



S	R	Q
0	0	N.C.
0	1	0
1	0	1
1	1	X

Reset
Set
Prohibited.



S	R	Q
0	0	N.C.
0	1	Reset
1	0	Set

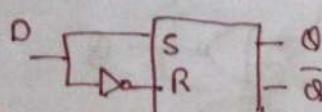
(prohibited) Reset
Reset = R=1
(prohibited) Set
Set = S=1

S	R	Q
0	0	N.C.
0	1	Reset
1	0	Set

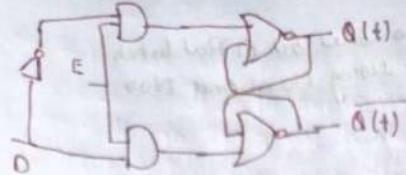
Prohibited / Forbidden

Limitation: Here the both the inputs are high it is creating forbidden condition.

So Now we can complement the two inputs and give



hence it is called D-flip-flop. Name it is D-latch.

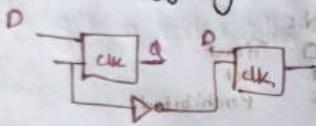


En	D	Q
0	x	Q
1	0	0
1	1	1

(Invertive)

Hint: SR problem is resolved by D. } latch
JK problem is resolved by T. } latch

NOTE: If i connect two latches in Master Slave Configuration then it will be kind of edge triggering. will be produce



→ Flip flop is race free.

Nand (Forbidden)

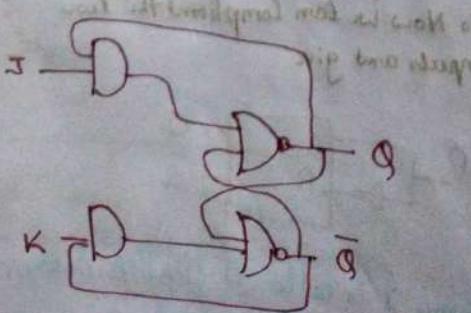
$$S = R = \text{Forbidden} \quad \{ \bar{S}, \bar{R} = 1 \}$$

Non (Forbidden)

$$S = R = \text{Forbidden} \quad \{ S, R = 0 \}$$

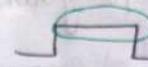
JK - latch

The only difference between SR Latch JK latch is that there is no output feedback towards the input in the SR latch but it is present in JK latch



J	K	Q	Q̄	State
0	0	0t	0̄t	No change
0	1	0	1	Reset
1	0	1	0	Set
1	1	0t	0̄t	Toggle

→ This will create Race Condition at $J = K = 1$ only on $t_{on} > t_{pd}$



large

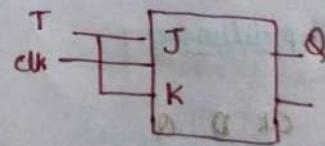
→ Large ON time is because due to the input will change at different level leads to different outputs

Hint: To resolve this to reduce will come down to t_{pd} . And it is no difficult but it can be reduced by master slave config of JK flip flop. (Latch so why)

→ So this problem can be resolved master slave configuration.

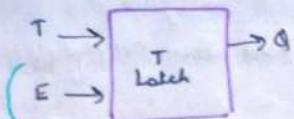
T Latch

at $S = R = 1$ and t_{pd} added $\rightarrow J = K = T$



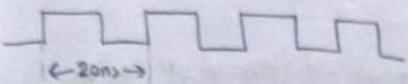
T	0	0	0	
	Q	1	1	
O	Q	0	X.C	
I	Q̄	1	T	

Application: Frequency divider. ckt



output cannot determine if latch or delay

Duty cycle :- Percentage of time for which clock is on.

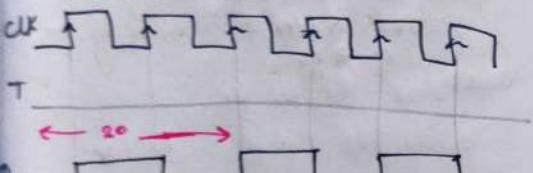


$$T_{on} = 50 \times T_p = 0.5 \times 20 = 10\text{ms}$$

$$T_{off} = 50 \times T_p = 0.5 \times 20 = 10\text{ms}$$



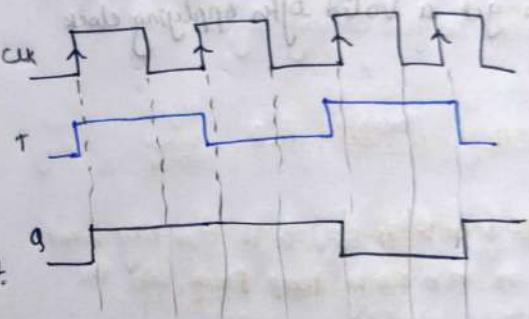
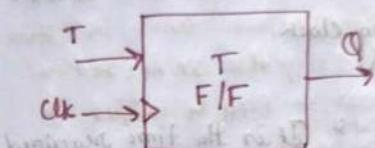
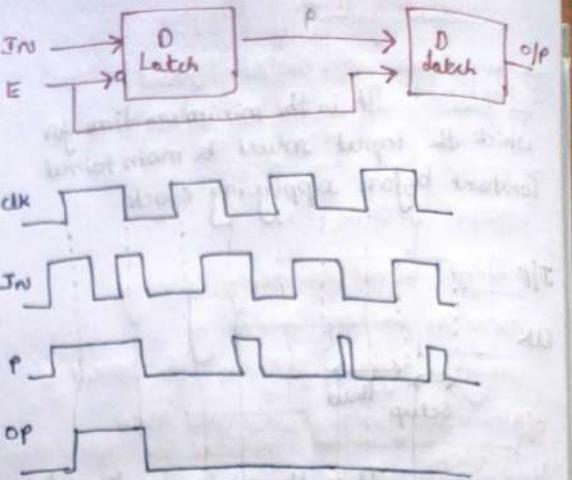
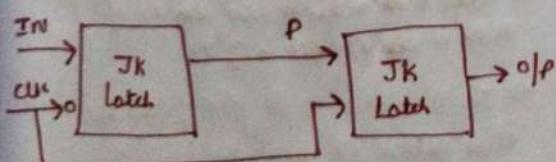
toggle due because it is logic high



Drawbacks of J-K latch:

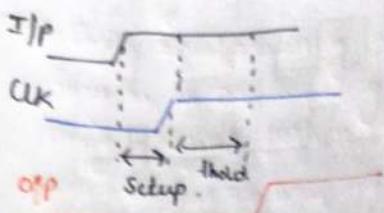
Why to avoid race around condition

1. $T_p(\text{latch}) > \text{Clock}$
2. Master Slave F.F
3. J-K always complemented to each other



Switching times of flip flop

Setup time :- It is the minimum time for which the input must be maintained constant before applying clock



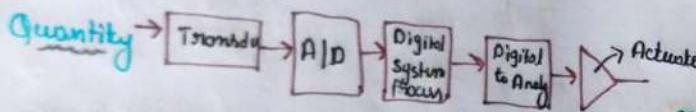
Hold time :- It is the minimum time for which input must be maintained constant after applying clock.

Propagation delay :- It is the time required to get a valid output after applying clock

not to let it propagate
not to let it propagate
not to let it propagate
not to let it propagate

Data Converters:-

- The purpose is to convert analog to digital and digital to analog.
- Digital Signals are defined for all interval of time. - No
- Digital Signal processing is very easy as we are converting Analog-to digital, perform the operation which ever you required and send it back.
- Processing refers to Converting signal into required form.
- Transmitting signal is analog and for processing we need to convert into digital and again for the receive we need to give analog signal only.

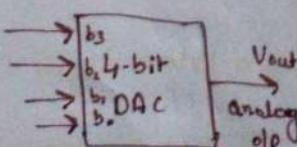


Transducer :- It converts the quantity what we are measuring temp, press, into the Electrical quantity.

Actuator :- To control physical variable

Digital to Analog Conversion :-

- We are giving input in the form of binary. And we are getting analog output for each inputs.
- Output in the form of Voltage (dc), signal it is in the form of Voltage



→ In every DAC the resolution is fix step size

Resolution / Step Size :-

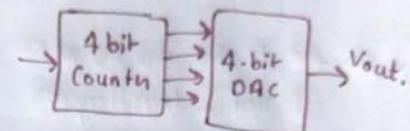
Increment in Vout when binary input increases

$$0000 \rightarrow V$$

$$0001 \rightarrow V'$$

$$\text{Step size} = V' - V$$

It is called pseudo analog because we will not get the between values here.



max^m Vout at full scale

$$O/P = 15V$$

Full Scale refers to the maximum value of the output of the system

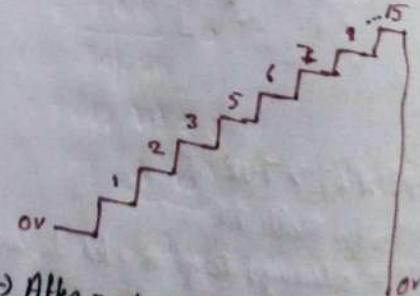
4 bit Counter Counts from

$$0000 \rightarrow 1111$$

$$0 \rightarrow 15$$

$$\text{Resolution} = \frac{\text{Max out}}{\text{Out of Counter (bit)}} = \frac{15}{15} = 1V$$

Since the resolution is one



After reaching this maximum 15 if it is set it will reach to zero.

The output is in the form of Staircase, it is called 'Pseudo analog'.

If a input increases by 1 DAC output will change according to resolution of step size.

gap b/w two steps are called Step size.

→ The output of the DAC will be always multiple of Resolution. Not all values are possible. Only certain values are possible at the output, which are not perfectly continuous.

→ If I reduce the size we can make it analog.

→ By increasing the number of bit we can reduce the stepsize of Resolution.

→ To make the output more analog we keep the full scale output fix but increase number of bits, so that resolution reduces.

Eg: If this is 5 bit, maximum input

11111

$$\frac{15}{32} = 0.5$$

Parameters of DAC :-

① Resolution :- It is the smallest change that occurs in output due to change in digital input.

$$② 0010 \rightarrow V$$

$$0011 \rightarrow V_0 =$$

$$\text{Resolution} = (V - V_0)$$

$$③ 0000 \rightarrow 0 \quad 0 = (V - V_0)$$

$$0001 \rightarrow h \quad V = V_0$$

$$\text{Resolution} = h - 0 = h$$

→ The output when all bits are zero and LSB is one, represents the resolution of the DAC.

$$\text{Resolution} = \frac{\text{Step size}(h)}{\text{Full Scale Value}} \times 100\%$$

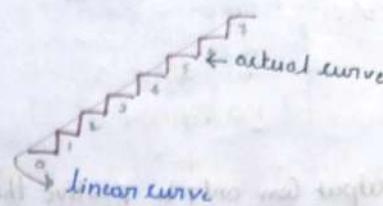
in previously

$$= \frac{1}{15 \text{ (no. of steps)}} \times 100\%$$

$$= \frac{1}{\frac{\text{no. of steps}}{2^r - 1}} \times 100\%$$

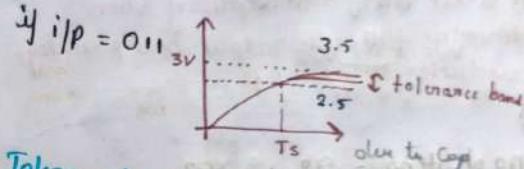
$$= \frac{1}{2^r - 1} \times 100\%$$

Accuracy :- The accuracy of DAC is specified in terms of full scale error, linearity error, expressed as percentage of full scale output.



→ Full scale error is the deviation of the actual output from the linear curve. If we written with % is called accuracy.

Settling time :- The time taken by the output to settle within the tolerance band around the steady state value is called settling time.



Tolerance band :- The nearest value of the required output is called tolerance, so that our work can be done. And time req to reach of that point is called settling time.

$$\text{Tolerance band} = \pm \frac{1}{2} \text{ LSB}$$

$$= \pm \frac{h}{2}$$

$$h = \frac{V}{2^r}$$

Offset Voltage :- The output of DAC at 0 input is called as offset voltage. If we apply a zero voltage but we are getting non-zero because of nonlinearity is called offset voltage.

→ Ideally offset voltage should be zero.

Monotonicity :- If the output increases with increase in binary input then DAC is said to monotonic.

Unipolar and bipolar DAC

- In Unipolar the sign remains same. Either +ve or -ve.
- But 1 bit DAC will produce both the types is called bipolar.

Eg in sign number

$$0001 = +$$

$$1001 = -$$

If the output can only be positive then DAC is said to be Unipolar. In such DAC Unsigned binary numbers should be input.

If output can be both +ve & -ve then it is said to be bipolar DAC. And such DAC we use signed binary numbers.

A 6-bit DAC on a step size of 50mV determine full scale output and percentage resolution.

Sol

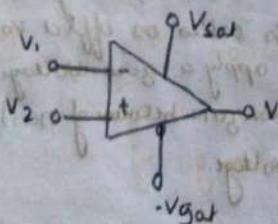
$$\text{No of steps} = 2^6 - 1 = 63.$$

$$\text{full scale o/p} = 0.05 \times 63 = 3.15 \text{ V.}$$

$$\% \text{ resolution} = \frac{0.05}{3.15} \times 100$$

$$= \frac{100}{63} = 1.587\%$$

Virtual ground:



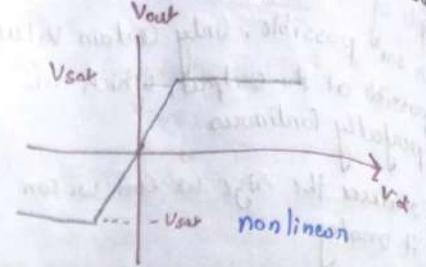
Ad-gain
 V_d : differential input
(+ve - ve terminals)

$$V_{out} = Ad \cdot V_d$$

$$= Ad(V_2 - V_1)$$

$$\text{Open loop gain} = Ad = \text{very high}$$

Rule: V_{out} can't go beyond V_{sat}



→ For small input output changes into saturation

→ In Opamp output should be proportional to input.

→ To make that into equal or proportional, increase the linear region

→ linear region extended by -ve feedback o/p connected to negative terminal of input.

$$V_{out} = Ad(V_2 - V_1)$$

$$V_2 - V_1 = V_A = \frac{V_{out}}{Ad}$$

$$V_{out} = \text{finite } Ad \rightarrow \infty$$

$$= \frac{\text{Finite}}{\text{Infinite}} = 0$$

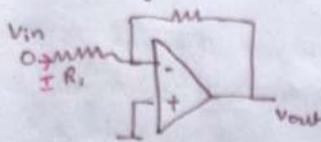
$$= (V_2 - V_1) = 0$$

$$V_1 = V_2$$

By using the -ve feedback one makes the voltage same is called Virtual ground.

The Amplifier made by Opamp of two type
 ① Inverting
 ② Non Inverting.

Inverting: The output will be inverted or negative.



By Virtual ground $V_- = V_+ = 0$

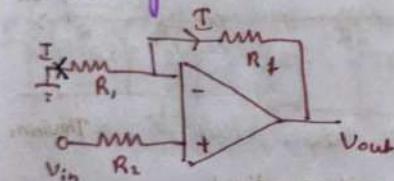
$$I = \frac{V_{in} - 0}{R_1}$$

Opamp has zero input current.

$$I = \frac{0 - V_o}{R_f} = \frac{V_{in}}{R_1}$$

$$V_o = -\frac{R_f}{R_1} V_{in}$$

Non inverting:



Virtual ground $V_- = V_+$

R_2 has no Voltage drop

$$V_+ = V_{in} = V_-$$

$$I = \frac{V_{in} - 0}{R_1} = \frac{V_{in}}{R_1}$$

$$I = \frac{V_{out} - V_{in}}{R_f} = \frac{V_{in}}{R_1}$$

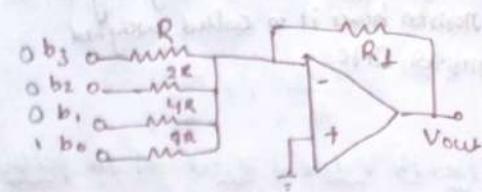
$$V_{out} = V_{in} + V_{in} R_f / A_i$$

$$V_{out} = V_{in} \left(1 + \frac{R_f}{R_1} \right)$$

gain

- ① Weighted Resistor DAC.
- ② R-2R ladder Network.

① Weighted Resistor DAC.



$$\text{If } b_n = 1 \rightarrow b_n = V_{ref} \\ b_n = 0 \quad b_n = 0$$

$$b_n = b_n V_{ref}$$

→ Inverting amplification.

→ The output can be obtained by superposition.

$$\rightarrow V_{out} = -\frac{R_f}{R} (b_3 V_{ref})$$

$$- \frac{R_f}{2R} (b_2 V_{ref})$$

$$- \frac{R_f}{4R} (b_1 V_{ref})$$

$$- \frac{R_f}{8R} (b_0 V_{ref})$$

$$= -\frac{R_f}{8R} V_{ref} (2^3 b_3 + 2^2 b_2 + 2^1 b_1 + b_0)$$

$$= -\frac{R_f}{2^{n-1} R} V_{ref} \times (\text{decimal value of i/p})$$

Resolution.

→ To get resolution $LSB = 1$; others = 0

→ Amplifier output: input \times gain

→ Output we will get by only b_0 .

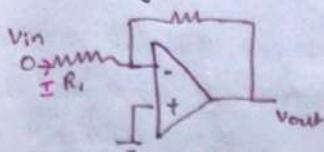
$$V_{out} = -\frac{R_f}{8R} V_{ref} = \text{resolution}$$

$$= V_{out} = \text{resolution} \times \text{decimal value of i/p.}$$

→ They are in a fixed ratio

The Amplifier made by Opamp of two type
 ① Inverting
 ② Non Inverting.

Inverting: The output will be inverted & negative.



By virtual ground, $V_- = V_+ = 0$

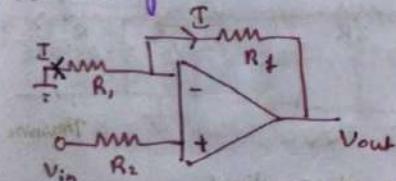
$$I = \frac{V_{in} - 0}{R_1}$$

Opamp has zero input current.

$$I = \frac{0 - V_o}{R_f} = \frac{V_{in}}{R_1}$$

$$V_o = -\frac{R_f}{R_1} V_{in}$$

Non inverting:



Virtual ground $V_- = V_+$

R_2 has no Voltage drop

$$V_+ = V_{in} = V_-$$

$$I = \frac{V_{in} - 0}{R_1} = \frac{V_{in}}{R_1}$$

$$I = \frac{V_{out} - V_{in}}{R_f} = \frac{V_{in}}{R_1}$$

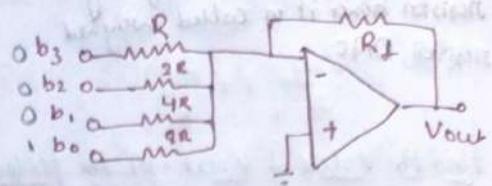
$$V_{out} = V_{in} + V_{in} \frac{R_f}{R_1}$$

$$V_{out} = V_{in} \left(1 + \frac{R_f}{R_1} \right)$$

gain

- ① Weighted Resistor DAC.
- ② R-2R ladder Network.

① Weighted Resistor DAC.



$$\text{If } b_n = 1 \rightarrow b_n = V_{ref}$$

$$b_n = 0 \quad b_n = 0$$

$$b_n = b_n V_{ref}$$

→ Inverting amplifier.

→ The output can be obtained by superposition.

$$\rightarrow V_{out} = -\frac{R_f}{R} (b_3 V_{ref})$$

$$-\frac{R_f}{2R} (b_2 V_{ref})$$

$$-\frac{R_f}{4R} (b_1 V_{ref})$$

$$-\frac{R_f}{8R} (b_0 V_{ref})$$

$$= -\frac{R_f}{8R} V_{ref} (2^3 b_3 + 2^2 b_2 + 2^1 b_1 + b_0)$$

$$= -\frac{R_f}{2^{n-1} R} V_{ref} \times (\text{decimal value of i/p})$$

Resolution.

→ To get resolution $LSB = 1$; Others = 0

→ Amplifier output: input \times gain

∴ Output we will get by only b_0

$$V_{out} = -\frac{R_f}{8R} V_{ref} = \text{resolution}$$

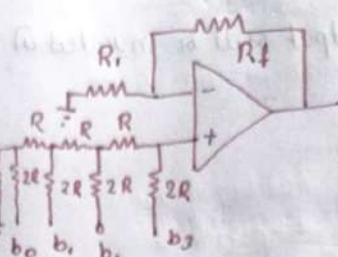
$$= V_{out} = \text{resolution} \times \text{decimal value of i/p.}$$

→ They are in a fixed ratio

Note: highest weighted bit has minimum resistance, minimum bit has maximum resistance, so the output depends on the weights of the resistor hence it is called weighted register DAC.

R-2R ladder networks

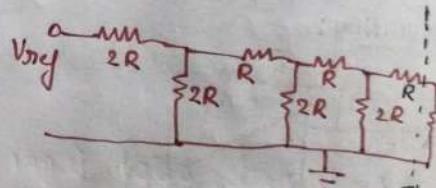
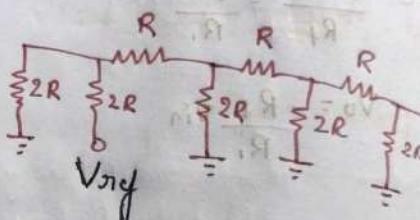
i) Non inverting configuration.



→ MSB = the bit near to opamp.
LSB = Far from opamp

$$\begin{array}{ll} \text{if } b_i = 1 & b_i = V_{ref} \\ b_i = 0 & b_i = 0 \end{array}$$

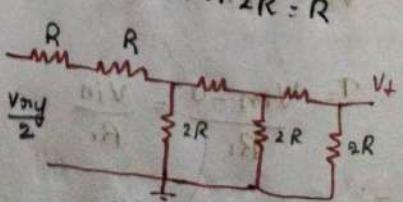
Code i = input 0001



- ① make it open ended
- ② Req by series or parallel

$$V_{th} = V_{ref} \times \frac{3R}{4R} = \frac{V_{ref}}{2}$$

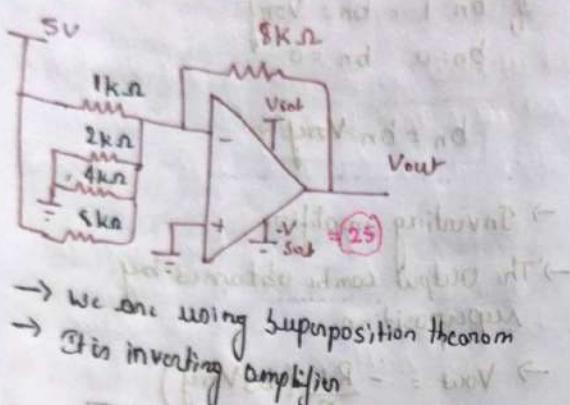
$$R_{th} = 2R \parallel 2R = R$$



$$V_{th} = V_{ref} \times \frac{2R}{4R} = \frac{V_{ref}}{2}$$

$$R_{th} = 2R \parallel 2R = R$$

Find the Value of Vout for the following circuit.



→ We are using superposition theorem
→ It is inverting amplifier

$$V_o = -\frac{R_f}{R_i} V_{in}$$

$$\begin{aligned} V_{out} &= -\frac{8}{1} \times 5 - \frac{8}{8} \times 5 \\ &= -45V \end{aligned}$$

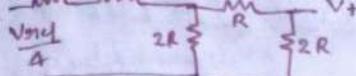
but $V_{sat} = 25$

Opamp output can never exceed V_{sat} limit -25 to 25

So output = -25V

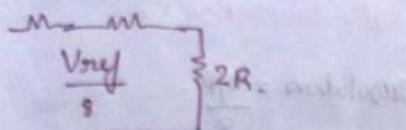
Resolution: Making only LSB bit has logic high, others to zero.

For any input output will be taken by Resolution \times decimal value of input.



$$V_{Th} = \frac{V_{ref}}{4} \times \frac{2R}{4R} = \frac{V_{ref}}{8}$$

$$R_{Th} = 2R \parallel 2R = R$$



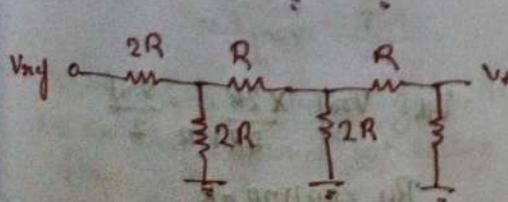
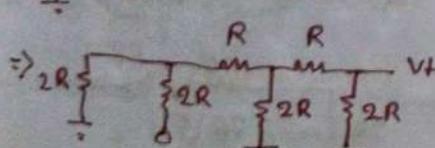
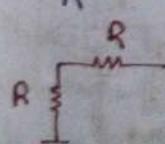
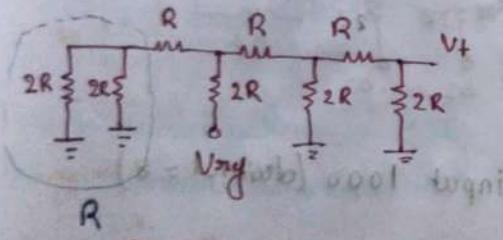
$$V_t = \frac{V_{ref}}{8} \times \frac{2R}{2R+R+R} = \frac{V_{ref}}{16}$$

if D001

$$V_{out} = \frac{V_{ref}}{16} \left(1 + \frac{R_f}{R_i} \right)$$

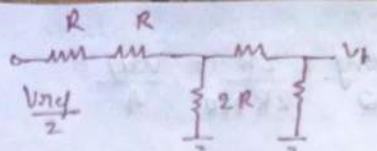
(non-inverting)

Case-2 input 0010



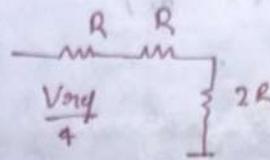
$$V_{Th} = V_{ref} \times \frac{2R}{2R+2R} = \frac{V_{ref}}{2}$$

$$R_{Th} = 2R \parallel 2R = R$$



$$V_{Th} = \frac{V_{ref}}{2} \times \frac{2R}{2R+2R} = \frac{V_{ref}}{4}$$

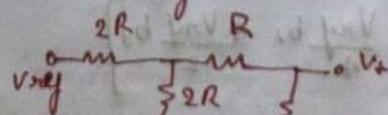
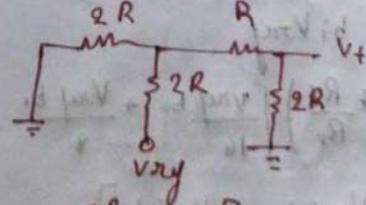
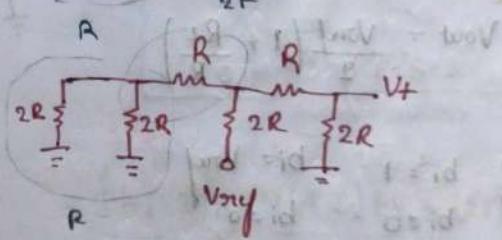
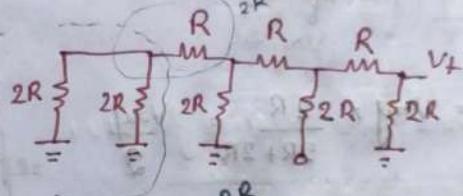
$$R_{Th} = 2R \parallel 2R = R$$



$$V_{Th} = \frac{V_{ref}}{4} \times \frac{2R}{2R+2R} = \frac{V_{ref}}{8}$$

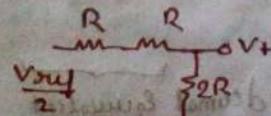
$$V_{out} = \frac{V_{ref}}{8} \left(1 + \frac{R_f}{R_i} \right)$$

Case 3 :- input D100.



$$V_{Th} = V_{ref} \times \frac{2R}{2R+2R} = \frac{V_{ref}}{2}$$

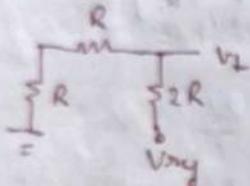
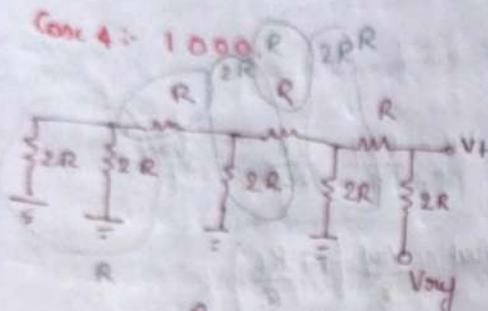
$$R_{Th} = 2R \parallel 2R = R$$



$$V_t = \frac{V_{ref}}{2} \times \frac{2R}{2R+R_f} = \frac{V_{ref}}{4}$$

I/P 0100

$$V_{out} = \frac{V_{ref}}{4} \left(1 + \frac{R_f}{R_i} \right)$$



$$V_t = V_{ref} \times \frac{2R}{2R+2R} = \frac{V_{ref}}{2}$$

$$V_{out} = \frac{V_{ref}}{2} \left(1 + \frac{R_f}{R_i} \right)$$

$$\text{If } b_3 = 1 \quad b_2 = V_{ref} \\ b_3 = 0 \quad b_2 = 0$$

$$b_3 = b_2 V_{ref}$$

$$V_{out} = \left(1 + \frac{R_f}{R_i} \right) \left[\frac{V_{ref} b_0}{16} + \frac{V_{ref} b_1}{8} + \frac{V_{ref} b_2}{4} + \frac{V_{ref} b_3}{2} \right]$$

$$V_{out} = \left(1 + \frac{R_f}{R_i} \right) \frac{V_{ref}}{16} \left[b_0 + 2b_1 + 4b_2 + 8b_3 \right]$$

Resolution

$$\begin{matrix} 2^3 & 2^2 & 2^1 & 2^0 \\ b_3 & b_2 & b_1 & b_0 \end{matrix}$$

$$8b_3 + 4b_2 + 2b_1 + b_0$$

decimal equivalent

Hint: output of DAC = resolution decimal value of I/P
→ finding output at MSB in Vt

If I/P = 1000

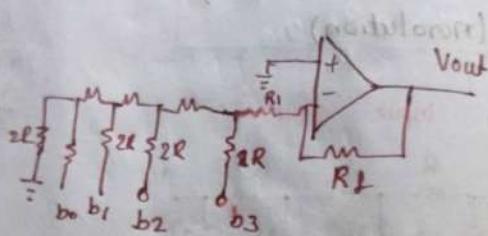
O/P: resolution × 8.

$$\text{resolution} = \frac{\text{O/P}}{8}$$

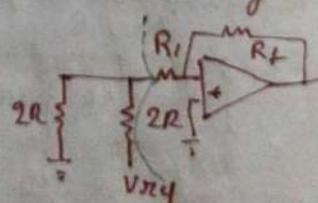
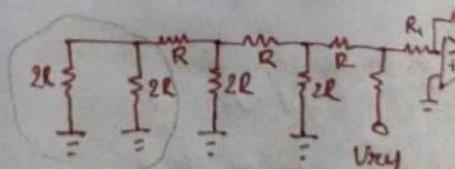
for I/P 1000

$$V_{out} = \frac{V_{ref}}{2} \left(1 + \frac{R_f}{R_i} \right) \\ = \frac{V_{ref}}{16} \left(1 + \frac{R_f}{R_i} \right)$$

Inverting Configuration :-

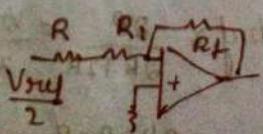


input 1000 (decimal = 8)



$$V_{th} = V_{ref} \times \frac{2R}{2R+2R} = \frac{V_{ref}}{2}$$

$$R_{th} = 2R // 2R = R$$



$$V_{out} = \frac{V_{ref}}{2} \left(-\frac{R_f}{R+R_i} \right)$$

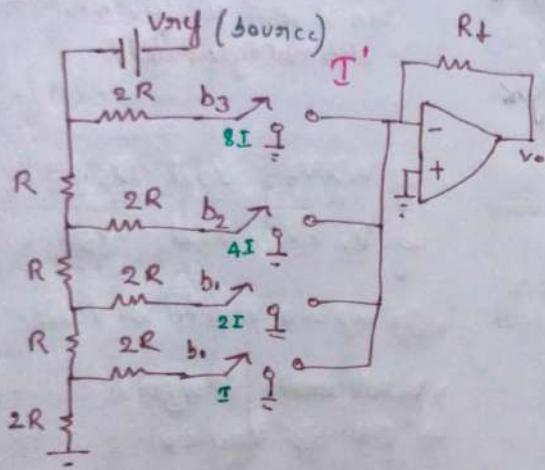
$$V_{out} = 910 \text{ mV} \times 8$$

$$\text{Resolution} = \frac{V_{ref}}{16} \left(-\frac{R_f}{R+R_i} \right)$$

General formula.

$$V_{out} = \frac{V_{ref}}{16} \left(-\frac{R_f}{R+R_i} \right) \left[2^3 b_3 + 2^2 b_2 + 2^1 b_1 + b_0 \right]$$

③ Current Switched R2R ladder Network.



The bit nearest to V_{ref} is MSB.
And the bit nearest to ground is LSB.

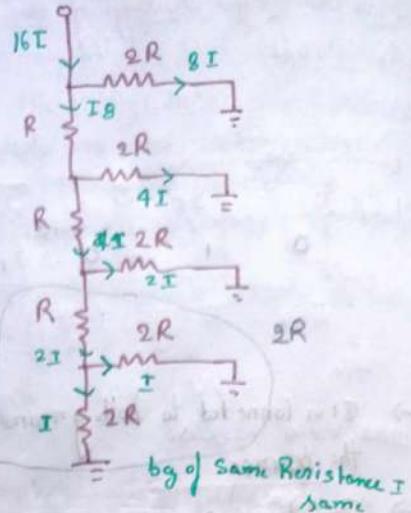
Before to make input one we were connecting with V_{ref} , but here we need to connect for opamp

-ve feedback

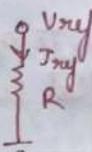
-Virtual ground

$$V_- - V_+ = 0$$

→ Switch voltage is always zero, hence we can change current but voltage is fixed



As the current is Vanishing way where hence it's called, Current switch



$$I_{ref} = \frac{V_{ref}}{R} = 16I$$

$$I = \frac{V_{ref}}{16R}$$

I' is.

$$I' = (8I)b_3 + (4I)b_2 + (2I)b_1 + I b_0$$

$$= I(2^3 b_3 + 2^2 b_2 + 2^1 b_1 + 2^0 b_0)$$

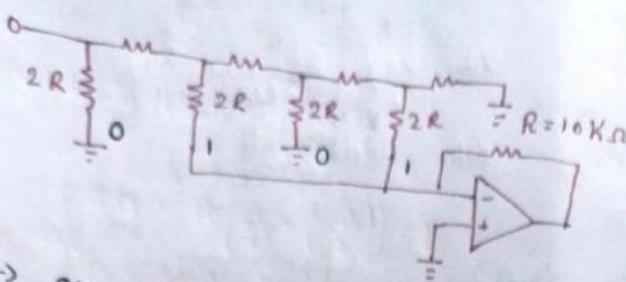
$$= I(\text{decimal equivalent})$$

$$I' = \frac{V_o - V_0}{R_f}$$

$$V_o = -I' R_f$$

$$V_o = -\frac{R_f}{16R} V_{ref} (\text{decimal equivalent})$$

Find the output voltage of the following DAC circuit.



decimal value of 0101 = 5

$$V_o = -\frac{5}{8} \times 5 = -\frac{25}{8}$$

$$= -3.125V$$

Rule:- MSB = 1, Baki bit = 0
find V_o / from V_o we can get resolution. from that find resolution.

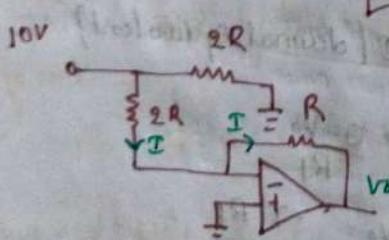
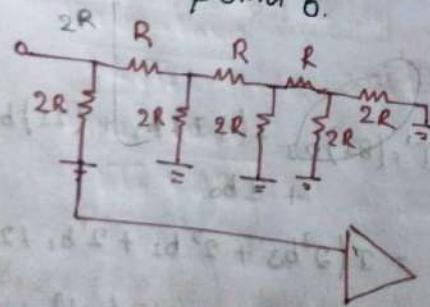
After finding resolution $v_o = m \times$ with the given bit decimal equivalent.

- It is connected to both ground & also with the source.
- Resistance connected with Vref or ground is called Voltage switched.
- Resistor connected with the opamp or ground is called Current switched.
- So it is connected to opamp & ground it is called Current switched.

So we need to find output for 0101.

bit =

make LSB = 1 & other 0.



$$I = \frac{V_{ref}}{2R} = \frac{0-V_o}{R}, V_o = -\frac{V_{ref}}{2} = -\frac{10}{2} = -5V$$

$V_o = 2^{n-1} \times \text{decimal value}$

$$-5 = 2^4 \times 8$$

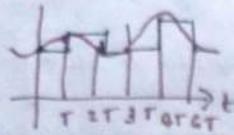
$$\text{over} = -5$$

Analog to Digital converter

Step 1 :- Sampling.

Step 2 :- Quantization.

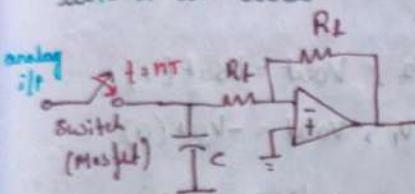
Sampling :- $x(t)$



it is a process in which we collect certain samples of a signal at a particular time instant.

If the samples are periodically placed in time then it is called uniform sampling.

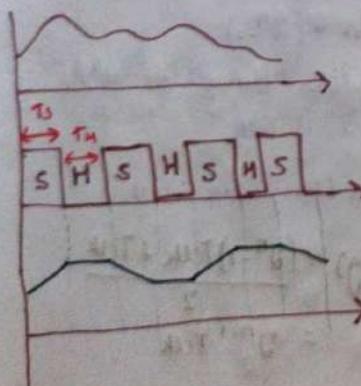
Sample & Hold Circuit



at $t=0$	S closed	momentarily
$t=\tau$	S closed	
$t=2\tau$	S closed	

Not closed at all time

When switch is open $i=0$ $\frac{dv}{dt} = 0$ $V_C = \text{constant}$ (hold)



Acquisition time :- is the time during which capacitor acquires the charge supplied by the source. T_S

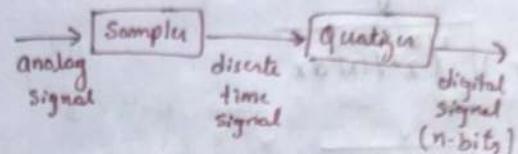
Aperture time :- When the switch is turned off the current flow does not stop instantly and during this interval capacitor acquires a charge from the source.

and this time is called as aperture time.

③ Voltage droop :- due to leakage in the capacitor the voltage across the capacitor during hold time does not remain constant and decreases gradually is called voltage droop.

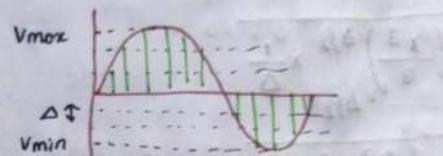
The leakage in the capacitor should be less as possible.

Quantization



$$\text{no of combination} = 2^n.$$

$$\text{no of levels} = 2^n.$$



$$\Delta = \text{step size (resolution)}$$

$$\Delta = \frac{V_{\text{max}} - V_{\text{min}}}{L}$$

→ closing to the nearest Voltage

levels

- 2
- 1.5
- 1.0
- 0.5
- 0
- 0.5
- 1.5

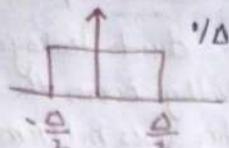
if we get $1.642 \rightarrow 1.5$
 $1.976 \rightarrow 2$

• Quantization is a process of rounding of the sampled values to the nearest level

• By this rounding it may create a error called Quantization error.

$1.642 \rightarrow 1.5$	Error = 0.142
$1.976 \rightarrow 2$	Error = -0.024
$0.75 \rightarrow 1$ $\rightarrow 0.5$	Error = 0.25 Error = -0.25

maximum quantization error = $\frac{\Delta}{2}$
most common always lies in b/w.



$$\text{Probability (total)} = \int_{-\infty}^{\infty} f(x) dx$$

$$\int_{-\Delta/2}^{\Delta/2} A dx = A \left(\frac{\Delta}{2} - \left(-\frac{\Delta}{2} \right) \right) = 1$$

7ms Value of noise

$$= \sqrt{\int_{-\infty}^{7ms} x^2 f(x) dx}$$

$$= \sqrt{\int_{-\Delta/2}^{\Delta/2} x^2 \frac{1}{\Delta} dx}$$

$$\text{noise} = \sqrt{\left(\frac{x^3}{3}\right)_{\Delta/2}^{\Delta/2} \times \frac{1}{\Delta}}$$

$$\sqrt{\frac{1}{3\Delta} \left(\frac{\Delta^3}{8} - \left(-\frac{\Delta^3}{8} \right) \right)}$$

$$= \sqrt{\frac{\Delta^2}{12}} = \frac{\Delta}{\sqrt{12}}$$

$$= \frac{V_{max} - V_{min}}{L\sqrt{12}}$$

$$= \frac{V_{max} - V_{min}}{\sqrt{12} \times 2^n}$$

Signal to noise ratio :-

$$\text{SNR} = 20 \log_{10} \frac{\text{7ms Signal}}{\text{7ms noise.}}$$

$$= 20 \log_{10} \frac{(V_{max} - V_{min})/2 \sqrt{2}}{(V_{max} - V_{min}) \sqrt{2} \times 2^n}$$

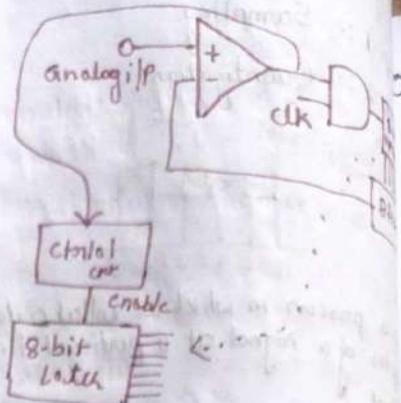
$$= 20 \log_{10} \sqrt{6} \times 2^{n-1}$$

$$\text{SNR} = 20 \log \sqrt{6} + 20 \log \log 2^{n-1}$$

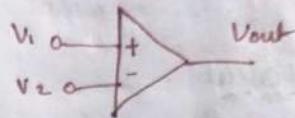
$$= 17.76 + 6.02 n - 6.02$$

$$\text{SNR} = 1.76 + 6.02 n \text{ dB}$$

Counting type ADC / 7ms per ADC



→ Opamp is used as Comparitor



- if $V_1 > V_2$, $V_{out} = V_{sat}(+)$
- if $V_2 > V_1$, $V_{out} = -V_{sat}(-)$

• Opamp is a non linear device

→ Initial Counter O/P is zero.

→ DAC o/p $V_{ref} = 0$

$$V_o > V_{ref} \Rightarrow V_o = V_{sat}(+)$$

$$X = 1, \text{Clk} = \text{Clk.}$$

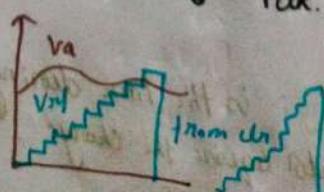
$$\text{Max } ^m \text{ Count} = (2^n - 1)$$

$$\cdot \text{Maximum time taken for conversion} = (2^n - 1) T_{clk}$$

$$\cdot \text{Minimum time for Conv} = T_{clk}$$

$$\cdot T_{conv}(\text{avg}) = \frac{(2^n - 1) T_{clk} + T_{clk}}{2}$$

$$= 2^{n-1} T_{clk}$$



It will take long time from initial
it will reach the maximum

Counts Kippman Counting until the Output of DAC Exceeds the analog input

The drawback is that we have to start the conversion from zero by the Counter will start after the conversion is complete.

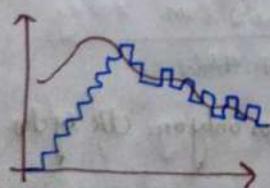
The conversion time of Counter type ADC is directly proportional to the Analog input.

$$T_{\text{conv}} \propto V_a$$

$$\frac{V_{a2}}{V_a} = \frac{T_2}{T_1}$$

Advancement of this is Tracking type ADC.

- Since conversion in Counter type ADC always starts from zero time taken is high, but this type is accurately track, and follow analog input.

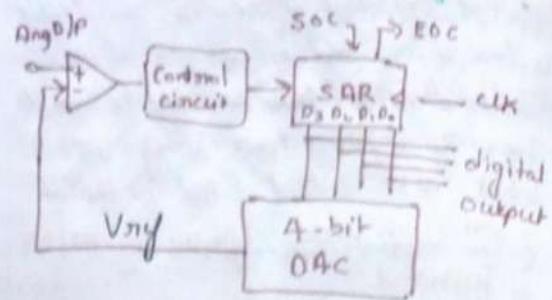


$$f_{\text{clk}} = T$$

$$\frac{f_{\text{clk}}}{f_a} = 8$$

$$f_{\text{clk}} = 8$$

Successive Approximation Register (SAR)



SOC Start of Conversion.
EOC End of Conversion.

SAR

$V_a > V_{\text{ref}}$ SAR will set current bit at clock pulse

$V_a < V_{\text{ref}}$ SAR will reset previous bit at 1st current bit at clock pulse

SAR type ADC will continue to work until all the bits have been occur one by one, and once it happen, it will make EOC signal high to indicate the conversion has ended.

Resolution of DAC = 1V

$$V_a = 10.2V$$

$$t = t_{\text{clk}} \cdot \text{SAR set MSB.}$$

$$\text{SAR} = 1000$$

$$V_{\text{ref}} = 1 \times 8 = 8V$$

$$V_a > V_{\text{ref}}$$

$$\text{at } t = 2t_{\text{clk}} ; \text{ SAR } 1100$$

$$V_{\text{ref}} = 1 \times 12 = 12V$$

$$V_a < V_{\text{ref}}$$

$$\text{at } t = 3t_{\text{clk}} \text{ SAR } 1010$$

$$V_{\text{ref}} = 1 \times 10 = 10V$$

$$V_a = 10.2V > V_{\text{ref}}$$

$$t = 4t_{\text{clk}} \text{ SAR } 1011 \rightarrow \text{final.}$$

The conversion time of SAR type ADC is fixed $(n \times T_{clk})$.

→ Convert to binary
 $(10110011)_2$

A 8bit SAR type ADC is used to convert 3.5V to digital output $V_{ref} = 5V$. What will be output of ADC at the end of 3rd clock pulse after the conversion is initiated.

→ In output of SAR LSB is always high.
→ decimal value is always odd.

$$V_a = 3.5V \text{ [analog input]}$$

$$V_{ref} = 5V$$

8-bit ADC

$$\text{Resolution of DAC} = \frac{5}{2^8 - 1}$$

$$= \frac{5}{255} = \frac{1}{51}.$$

$$T_{clk} \cdot O/P 1000 \ 0000$$

$$V_{ref} = \frac{1}{51} \times 128$$

$$= 2.45V < V_a$$

so present bit one dont touch previous bit

$$O/P = 1100 \ 0000]_2 T_{clk}$$

$$\frac{128+64}{51} = \frac{192}{51} = 3.71 V_a$$

3_{T_{clk}} make previous bit one and next bit one

$$O/P \ 1010 \ 0000$$

For final output (T_{clk})

→ decimal equal of input

$$= \frac{3.5}{1/51}$$

$$= 3.5 \times 51$$

$$= 178.5$$

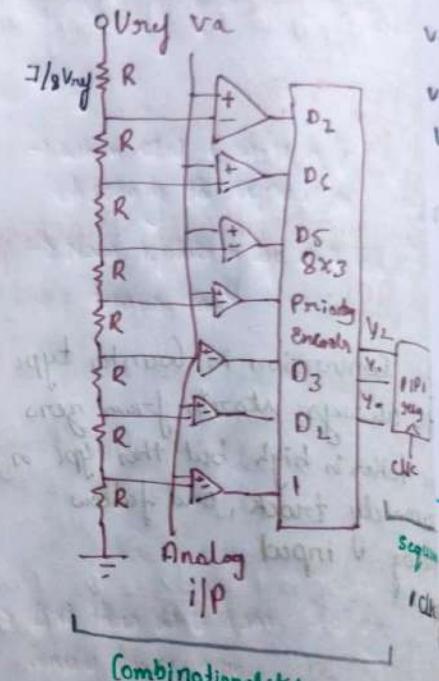
→ round off to nearest odd integer

$$= 179.$$

Flash type ADC :-

→ Faster ADC

→ doesn't require DAC



Combinational logic

→ It requires only one clk cycle.

$$T_{conv} = T_{clk}$$

$$T = V_{ref}/8$$

$$V_7 = -\frac{V_{ref}}{8R} R$$

$$= +V_{ref}$$

$$7/8 V_{ref}$$

$$V_6 = 6/8 V_{ref}$$

$$V_5 = 5/8 V_{ref}$$

$$V_4 = 4/8 V_{ref}$$

$$V_2 = \frac{2}{3} V_{out}$$

$$C \frac{d}{dt} (0 - V_0) = I$$

V_a | $D_7 D_6 D_5 D_4 D_3 D_2 D_1 D_0$

$\frac{1}{8} < V_{in} < \frac{2}{3}$	0 0 0 0 0 0 0 1 → 000
$\frac{2}{3} < V_{in} < \frac{3}{8}$	0 0 0 0 0 0 1 0 → 001
$\frac{3}{8} < V_{in} < \frac{4}{8}$	0 0 0 0 0 1 1 0 → 010
$\frac{4}{8} < V_{in} < \frac{5}{8}$	0 0 0 0 1 1 1 1 → 011
$\frac{5}{8} < V_{in} < \frac{6}{8}$	0 0 0 1 1 1 1 1 → 100
$\frac{6}{8} < V_{in} < \frac{7}{8}$	0 0 1 1 1 1 1 1 → 101
$\frac{7}{8} < V_{in} < 1$	0 1 1 1 1 1 1 1 → 110
$V_a > 1$	1 1 1 1 1 1 1 1 → 111

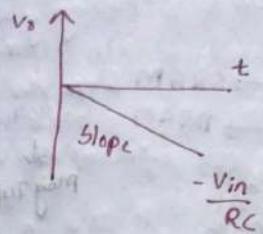
$$C \frac{dV_o}{dt} = \frac{V_{in}}{R}$$

$$\frac{dV_o}{dt} = -\frac{V_{in}}{RC}$$

$$V_o = -\frac{1}{RC} \int V_{in} dt$$

if V_{in} (constant)

$$V_o = -\frac{V_{in}}{RC} t$$



$$V_- > V_+ \quad V_o = 0$$

$$V_- < V_+ \quad V_o = 1$$

$$\text{No. of resistance} = 2^n$$

$$\text{No. of ComPARATOR} = 2^n - 1$$

$$\text{No. of Encoder} = 1$$

$$\hookrightarrow \text{order} (2^n : n)$$

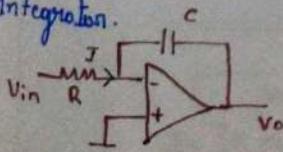
→ If there is no priority encoder the conversion time = 0.

Points :-

- If PIPD register is not present then no clock pulse is required and time required for conversion of propagation delay is almost zero.

Dual Slope integration type ADC :

Integration.



By virtual ground.

$$V_+ = V_o = 0$$

$$I = \frac{V_{in} - 0}{R} = \frac{V_{in}}{R}$$

Initially Counter & FF are clear.

$$S=0, V_{in}=V_a$$

$$V_{out} = -\frac{V_a}{RC} t$$

$$\text{Slope} = -\frac{V_a}{RC}$$

Since $V_{out} < 0$

$$V_+ > V_-, V_1 = \text{logic 1}$$

Counter increases count until reach 0.

Count stops when $V_{out} \rightarrow 0^+$

$$\text{at } t=0 \quad V_{out} = -\frac{V_a}{RC} t = 0$$

$$\Delta t = 15 T_{clk}$$

$$V_{out} = -\frac{V_a + \Delta t}{RC} < 0$$

$V_+, \text{ logic 1 Count} \uparrow$

$$\text{at } t=16 T_{clk} = T_1$$

$$V_{out} = -\frac{V_a + T_1}{RC}$$

out start to increase due to - Very
and when it try to cross 0. the output
of ComPARATOR become low and clock
obligi Enter the Counter, so the Count
stop.

EEPROM

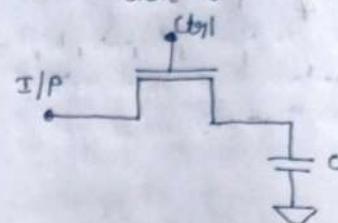
↓
Electrically
Erasable

UV PROM

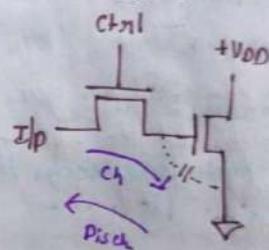
↓
ultra violet
Erasable

D-RAM :-

- Main work to charge & discharge the capacitor
- By changing state of discharge state 0.



- When $\text{Ctrl} = 1$ will help to discharge the capacitor.
- Now replacing Cap with another transistor.

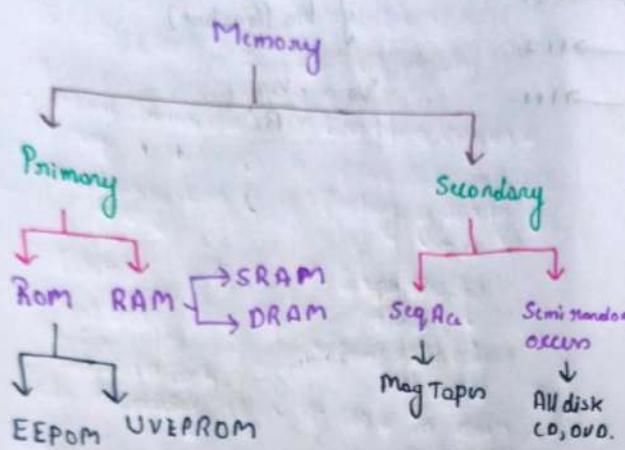


→ But this will have leakage conductance here.

→ So periodic charge (refresh) is:

→ Compared to SRAM it is slow.

Semiconductor Memory

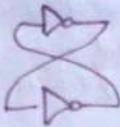


→ Both ROM and RAM are randomly accessible.



cheapest

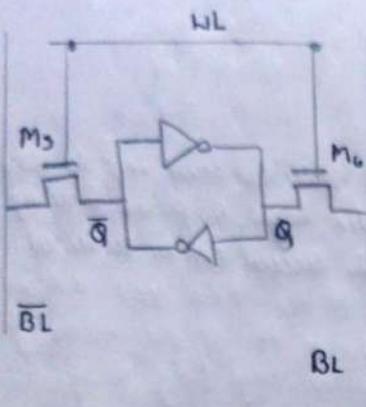
ROM	RAM
• Read only memory	• Read & write memory
• Random Access	• Random Access
• Non volatile	• Volatile memory
• used to store	• used to store temp data
• Permanent data like System prog. OS	



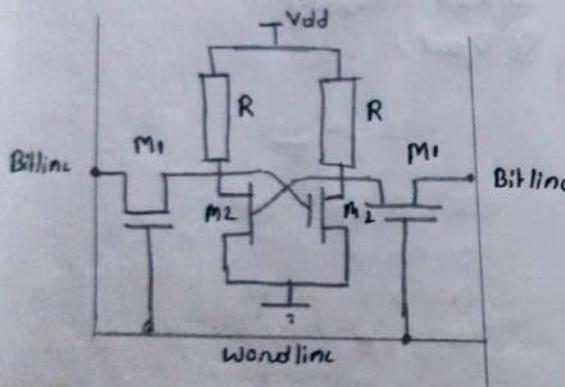
→ This has 2 operation one is the read & another is to write.

Read Operation:

- Now we need to read the data and the data is already there.
- Assume $Q = 0$ & $\bar{Q} = 1$
- $WL = 1$ {Activate}
- And we have 2 capacitors it is called precharged capacitance.



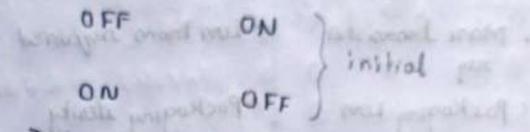
$$\rightarrow \text{Inverter} = \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array}$$



→ It is called AT SRAM.

→ Above is a NMOS version.

→ If we replace by CMOS, it becomes GT SRAM.



→ Capacitor already charged called Precharged.

→ $N_3 = \text{ON}$, $N_1 = \text{OFF}$ } Capacitor is completely discharged

→ But Right side will (Cap) will remain same.

$BL \bar{BL} = 1$ } initially

→ Sensing amplifier will detect

→ N_1 stronger than the N_3 .

→ N_3 Strong resistance to ON than N_1 .

→ For proper operation N_1 on resistance $< N_3$ on resistance

$$R_{N_1} < R_{N_3}$$

$$\left(\frac{W}{L} \right), \left(\frac{W}{L} \right)_3$$

Write Operation:

→ We need to write the data.

→ Initially $Q = 0$ $\bar{Q} = 1$.

OFF ON
ON OFF.

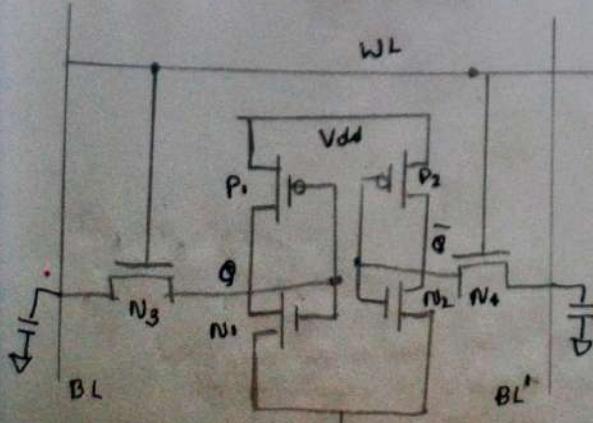
→ Write $Q = 1$; $\bar{Q} = 0$

$$BL = 1 ; \bar{BL} = 0.$$

→ $WL = 1$ (Fix)

clr

data



GT SRAM CELL

$Q = 1$

$\bar{Q} = 0$

N₁ OFF

N₃ OFF

$$RP_2 > RV_A$$

$$R_1 > RN_2$$

Difference b/w SRAM and DRAM

SRAM

- More transistor req
- Packaging less
- Faster
- Costlier
- used in cache memory
- No periodic refresh
- More power consumption
- Data is stored in FIF
- Volatile memory
- Random Access

DRAM

- Less trans required
- Packaging density more
- Comparitively slow
- Comparitively cheap
- Used in main memory.
- periodic refresh
- less power consumption
- Data stored in charging & discharging of capacitor.
- Volatile
- Random access

20/03/2025

- Race Condition : when the output changes multiple time one clock.
- It is impossible to predict the correct output.

SR Latch FF

En/clk	SR	Q	\bar{Q}	
A	00	0	0	(Latch State)
A	01	0	1	(Reset)
A	10	1	0	(Set)
A	11	0	0	(Invalid, prohibited)
X	XX	Q	\bar{Q}^+	[Previous state]

D-Latch / FF

CLK/En	D	Q	Q^+
A	0	0	1
Ac	1	1	0
In	X	Q	\bar{Q}

To avoid this
 $S = D$
 $R = \bar{D}$

J-K Latch FF

Clk/En	J	K	Q	Q^+
Inactive	X	X	0	\bar{Q}
Active	0	0	0	Q^+
A	0	1	0	1
A	1	0	1	0
A	1	1	\bar{Q}	Q

(Toggle)

$$J = K = T$$

T-Latch FF

Clk	T	Q	\bar{Q}
A	0	0	Q^+
A	1	\bar{Q}	0
T	X	Q	Q^+

D-Latch

- Shift registers.
- Memory.
- Counters.

T flip flop for frequency division

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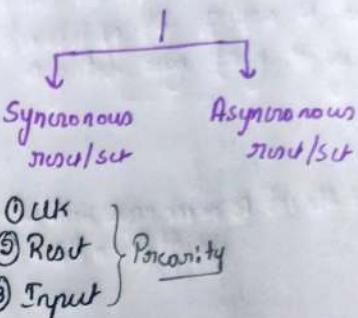
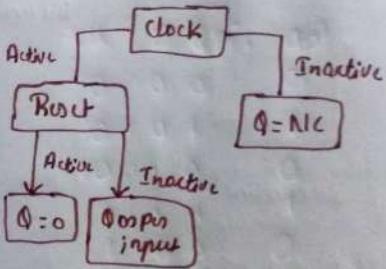
Ckt.

Till now we only assume the initial condition in the graph with low or high, it is not a correct way in the hardware design, what to do and how to solve this problem

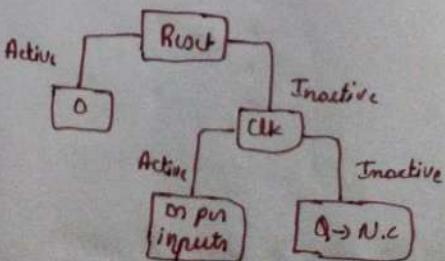
→ By using control inputs

- ① Reset = 0
- ② clear = 0
- ③ Preout = 1
- ④ Set = 1

- And this control inputs uses OR negative logic, so power consumption of the circuit can be reduced.

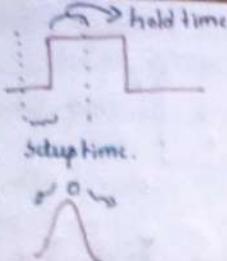
Synchronous (reset)Asynchronous (reset)

→ Not depend on clock



Most of the devices are asynchronous

ctrl inputs.



Q is the condition in which we don't know when it will go.

graph

Q is high in hold time. It will remain high again till returned to low again.

a) most likely to encounter problem with transition

	Q0	Q1	Q2	Q3	Q4	Q5	Q6	Q7
0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
1	1	0	1	0	0	0	0	0
1	1	1	1	0	0	0	0	0
0	0	1	1	0	0	0	0	0

a) how to analyze soft errors of
pulse and require all spaces

which is return address so need to read previous pc value with its last 4 bits

and if code calls or jumps next ip
will get all of same result

current ip address
will be used at next clock

Q is high during hold time because
of propagation of the hold time

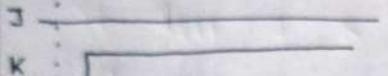
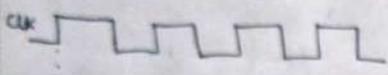
Q is held when initial or ideal with
the data and clock register after 10ns

JK

J	K	Q
0	0	N.C
0	1	Reset
1	0	Set
1	1	Toggle

→ Race around condition.

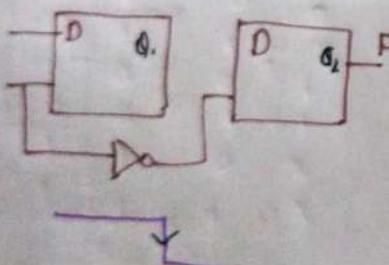
- ① To avoid this problem we need to increase the propagation delay.



$t_{pd} = 3\text{ns}$

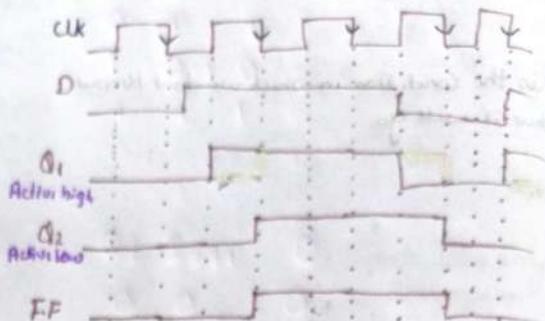
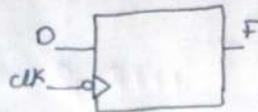
② Master Slave Configuration:-

- Only one of them is either master or slave.
- Equivalent to real implementation by using Flip flop
- Edge triggering is also done by this master slave in the flip flop.



Negative Edge triggering.

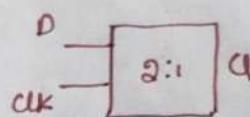
Two latch combine to form a flip flop.



Hence the $Q_2 = \text{FF}$ output at negative Edge, hence it is the proper circuit for negative Edge triggering.

D. Latch using 2x1 Mux

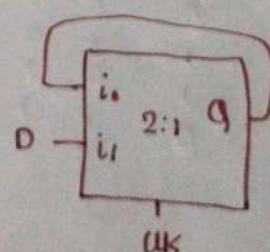
- We need expression or truth table to implement this



clk	D	Q _p	Q _n
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

N.C Q

Active D



- Observe the feedback connectivity by sequential it is compulsory.

- This table is called characteristic table, tells next state output based on clock and the previous state.

Characteristic equation in the next state
Equation in terms of inputs of latch and
flip flop and previous state.

→ Ignore the clock in the characteristic
table.

Characteristic table for D.F.F.

D	Q_t	Q_{t+1}
0	0	0
0	1	1
1	0	1
1	1	1

When clock is active.

D	\bar{Q}	Q
0	1	1

$$Q_{t+1} = D$$

Characteristic Equation.

Characteristic table for T.F.F.

T	Q_t	Q_{t+1}
0	0	0
0	1	1
1	0	1
1	1	0

T	\bar{Q}_t	Q _t
0	0	1
1	1	0

$$Q_{t+1} = T \oplus Q_t$$

Characteristic table for J.K.F.F.

J	K	Q_t	Q_{t+1}
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

\bar{J}	K	Q_t	Q_{t+1}
1	0	1	0
1	1	1	1

$$Q_{t+1} = \bar{K}Q + J\bar{Q}$$

Characteristic table for SR F.F.

S	R	Q	Q_{t+1}
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	X
1	1	1	X

\bar{R}	\bar{S}	Q_t	Q_{t+1}
1	1	1	0
1	1	X	X

$$Q_{t+1} = S + \bar{R}Q$$

Latch F.F	C.E
D	$Q_{t+1} = D$
T	$Q_{t+1} = T \oplus Q_t$
J.K	$Q_{t+1} = \bar{K}Q + J\bar{Q}$
S.R	$Q_{t+1} = S + \bar{R}Q$

What is the use of Excitation table?

Based on the present and the next state we can find the output of the combination.

Why named?

- ① Low power
- ② High fanout
- ③ Low time

In Flash CMOS NAND is used.

D-Flip flop :-

Dt	Qt	Qt+1
0	0	0
0	0	1
1	0	0
1	1	1

Qt	Qt+1	D
0	0	0
0	1	1
1	0	0
1	1	1

$\uparrow = \downarrow$

J.K Excitation table :-

Qt	Qt+1	J	K
0	0	0	x
0	1	x	x
1	0	x	x
1	1	x	x

S.R Excitation table :-

S	R	Qt	Qt+1
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	x
1	1	1	x

Qt	Qt+1	S	R
0	0	0	x
0	1	1	0
1	0	0	1
1	1	x	0

T- Flip flop.

T	Qt	Qt+1
0	0	0
0	1	1
1	0	1
1	1	0

Qt	Qt+1	T
0	0	0
0	1	1
1	0	1
1	1	0

Conversion of one Flip flop

Another

Step 1: Identify what is available & what is required.

Available $\rightarrow X$.

Required $\rightarrow Y$.

Step 2: Characteristic table for the required flip flop interming $\bar{S}P$ at Q_{t+1} .

Step 3: By using above characteristic table, find out the inputs for available flip flop

Step 4: Solve the K-map get the expression of available inputs in terms of required flip flop.

Step 5: Design the circuit as per the expression obtained.

Convert D flip flop into T flip flop

Step 1: Available = D
Required = T

Step 2: Characteristic table of T flip flop

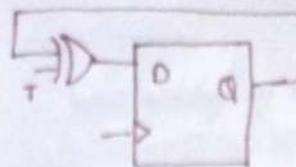
T	Q_t	Q_{t+1}
0	0	0
0	1	1
1	0	1
1	1	0

Step 3: \rightarrow neglect this

T	Q_t	Q_{t+1}	D
0	0	0	0
0	1	1	-
1	0	1	-
1	1	0	0

\bar{T}	\bar{Q}_t	Q_t
0	0	1
1	1	0

$$D = T \oplus Q_t$$

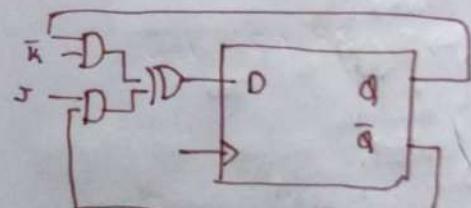


Convert D flip flop into J-K flip flop

J	K	Q_t	Q_{t+1}	D
0	0	0	0	0
0	0	1	1	1
0	1	0	0	0
0	1	1	0	0
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	0	0

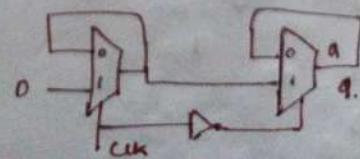
$\bar{K}Q$	$\bar{K}Q$	KQ	KQ
\bar{J}	1	1	1
J	1	1	1

$$Q_{t+1} = J\bar{Q} + \bar{K}Q$$



Note: For any implementation of 2:1 Mux
J will be given for the Q and J_2 will
be its characteristic equation.

\rightarrow Two cascaded latches with opposite
clock will act on the flip flop



\rightarrow Two Master-Slave configuration will
detain the both the edges mean 4
Master slave

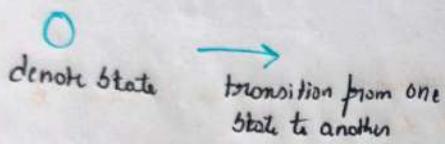
→ Two stable state in latch or flip flop.

Q	Q _{t+1}	T
0	0	0
0	1	1
1	0	1
1	1	0

Change

State table: D denotes the transition from one state to another. And have a format and denotes what is present state, next and exist state.

→ One flip flop has only 2 states.

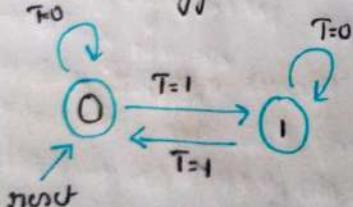


State diagram / table for flip flop:

T.F.F:

0 → N.C

1 → Toggled

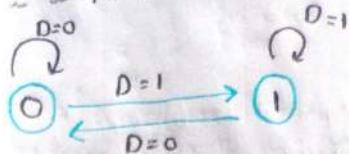


- Active low next = 0
- Active high next = 1.

⑤ No change Ω

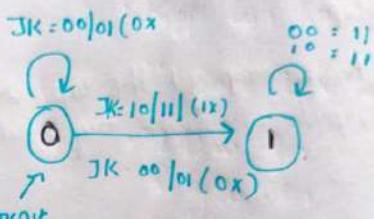
→ Above diagram is for excitation table

D - Flip flop:



D	Q _t	Q _{t+1}
0	0	0
0	1	1
1	0	0
1	1	1

J K Flip flop:



J	K	Q _t
0	0	0
0	1	0
1	0	1
1	1	1

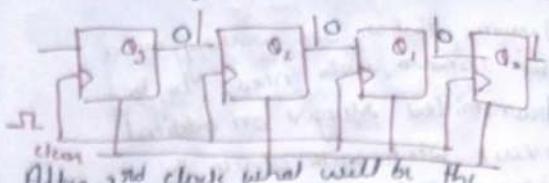
→ Set & preset are same. It makes all 4 values to 111.

P.S	T	N.S
0	0	0
0	1	1
1	0	1
1	1	0

Applications of Sequential Circuits

- Flip flop for 1 bit storage.
- Used for data storage and buffering
- Counters
- Clock division
- Used as registers
- Used in PLDs, CPLD and FPGAs

→ From LSB feeding 1011. Serial in parallel out
 → If MSB feeding 1101.



After 3rd clock what will be the

Output

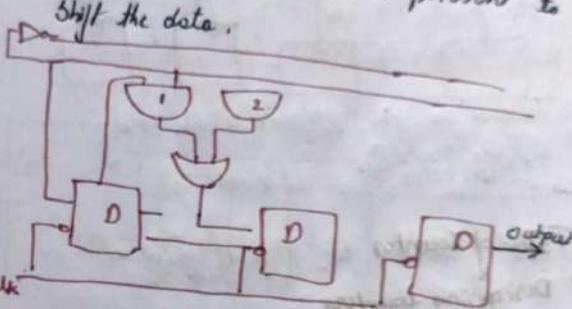
1	0	0	0	1st
1	1	0	0	2nd
0	1	1	0	3rd

→ Here we can take out the data from one clock pulse.

→ Here for feeding we 4 clock but to receive one clock is enough

Parallel in Serial out:

→ One additional clk will be present to shift the data.

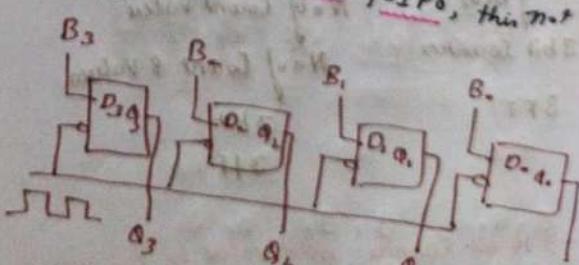


→ It will control the operation if it load condition not allowed to shift if it shift not allowed to load.

S + s.q

Parallel in Parallel out:

→ Max time SISO, PISO, SIPO, this not used



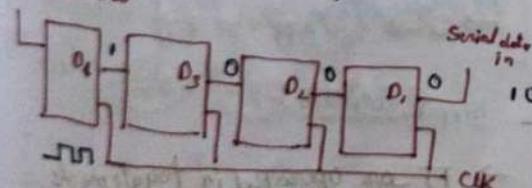
→ Only 2 clk pulses enough

Serial in Serial out:

→ Serial input Serial output.

Serial data in

Serial data out



10110

$Q_3 Q_2 Q_1 Q_0$	$Q_3 Q_2 Q_1 Q_0$
1 0 0 0	1 0 0 0
1 0 0 0	1 0 0 1
1 0 0 0	0 0 1 0
1 0 0 0	0 1 0 1

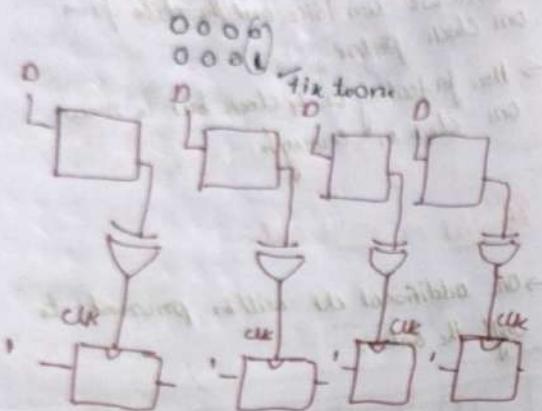
here we need 8 clock pulses to get the data out.

obj: 1s < 10110

Universal Shift registers:-

Amo problem

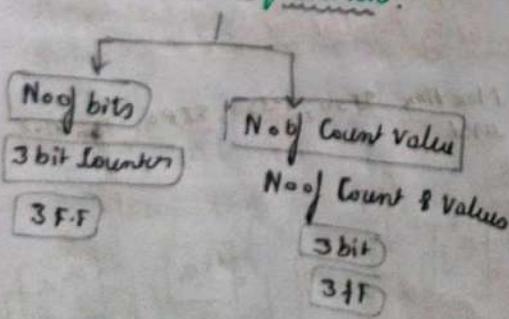
Assume that we are working on a 8 bit random data needs to be transmitted through an additional circuit which will obtain the obtained outputs and procedure of indicate high signal output, when minimum signal time all bits are toggle.



Counters :-

- Type of Counter
- Designing Counters
- Timing Analysis
- Close division (T.F.F)
- Used to count no of clock pulses

Resource Requirement



→ 16 bit Counter mod value is ?

2^{16}

We require 16 flip flop.

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→ Mod 6

$$\begin{aligned} \rightarrow F.F. = 6 \\ \rightarrow B.M. = 6 \end{aligned} \quad \left. \begin{array}{l} \text{Min Count} = 0 \\ \text{Max} = 6^3 \end{array} \right.$$

→ We can design termination counter by using most the value.

→ It is also cascaded chain of F.F.

→ No of States is known as mod.

Types of Counter :-

Asynchronous : Different clock for each

Ripple Counter

Synchronous : Same clock to each flip

- MOD Counter
- Ring Counter
- Johnson Counter.

Asynchronous

- Circuit is simple
- $T_{clk} \geq n t_{pd}$
- Decoding error due to propagation delay
- State skipping is not allowed / Ring

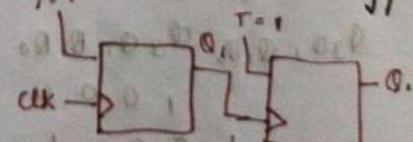
Synchronous

- Circuit is complex
- $T_{clk} \geq t_{pd}$
- No decoding by of some ch
- It is allowed

Ripple Counter :-

→ FF are operated in toggle mode

$T=1$



→ Clk is applied to LSB

$\rightarrow f = f/2^n \text{ or } f/n$

→ For n bit ripple counter, period of clock

$$T_{clk} \geq n t_{pdff}$$

$$\rightarrow T_{clk} = \frac{1}{n T_{pd}}$$

$T_{pd} = 10\text{ns}$; $T_{clk} = ?$ $n = 3$

$$= \frac{1}{(3)(10 \times 10^{-9})}$$

$$= \frac{100 \times 10^6}{3000}$$

$$= 33.3 \text{ MHz}$$

$$\rightarrow T_{clk} \geq 3 \times 10$$

$$\geq 30\text{ns}$$

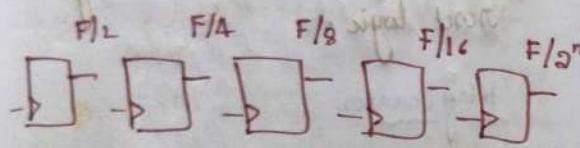
4 bit Ripple Counter $T_{pd} = 5\text{ns}$

$$\cdot T_{clk} \geq n T_{pd}$$

$$= 20\text{ns}$$

$$\cdot F_{max} = \frac{1}{n T_{pd}}$$

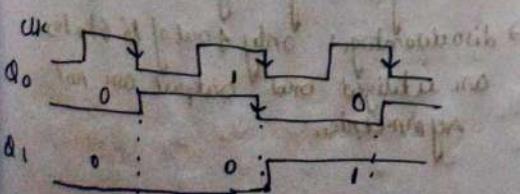
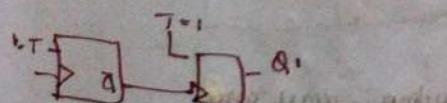
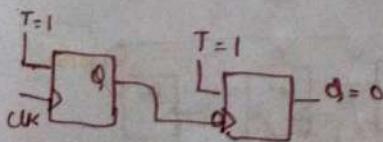
$$= \frac{1}{200 \times 10^{-9}} = 5 \text{ MHz}$$



Possible in Ripple Counter for up and down counting:

① Negative edge triggers F.F & Q's on clock

② Positive edge triggers F.F & Q's on clock



$\{Q_0, Q_1\}$ upcount

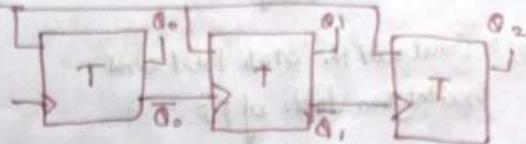
down:

① Negative edge triggers FF & Q's on clock

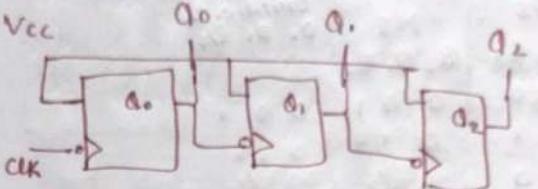
② Positive edge triggers FF & Q's on clock

③ design a 3 bit up counter using Asynchronous.

Vcc

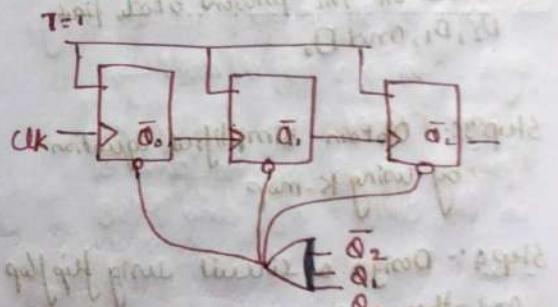


Vcc



Truncation concept.

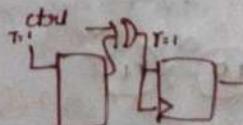
Mod 5 counter should be truncated from 000 to 100.



→ Both up and down counter can be implemented by using EXOR gate

up/down 1 → up

up/down 0 → down



initial	ctrl	Q
0 0 0	0 0 0	0 0 0
1 0 0	1 0 0	0 0 0
0 1 0	0 1 0	1 0 0
1 1 0	1 1 0	0 1 0
0 0 0	0 0 0	1 0 0

Synchronous Counter design:

Step 1:- Identify the required number of FF
Number of inputs and outputs.

Modulo = 4 bit

4 bit $\xrightarrow{\text{Binary}}$ Gray

Step 2:- Construct the State table and
Excitation table of FF.

PS	NS	Excitation D
$Q_2 Q_1 Q_0$	$Q_2 Q_1 Q_0$	$D_2 \quad D_1 \quad D_0$
0 0 0	0 0 1	0 0 1
0 0 1	0 1 0	0 1 0
0 1 0	0 1 1	
0 1 1	1 0 0	
1 0 0	1 0 1	
1 0 1	1 1 0	

→ Based on the present state find D_2 , D_1 , and D_0 .

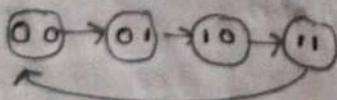
Step 3:- Obtain simplified equation
by using K-map

Step 4:- Design a circuit using flip flop
and other gate corresponding to the
minimized expression

9 3 bit up counter using DFF.

① FF? ② DFF

③



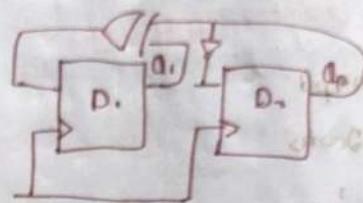
PS	NS	Excitation
$Q_2 Q_1 Q_0$	$Q_2 Q_1 Q_0$	$D_2 \quad D_1 \quad D_0$
0 0 0	0 0 1	0 1
0 0 1	0 1 0	1 0
0 1 0	1 1 0	1 1
0 1 1	0 0 0	0 0

0	0	0	1
0	0	1	0
1	1	0	0

$$D_1 = Q_1 \oplus D_0$$

0	0	0	1
0	1	0	0
1	1	0	0

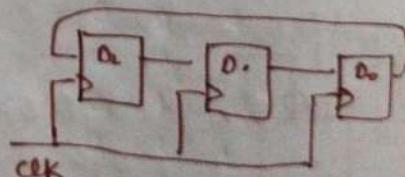
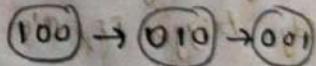
$$D_0 = \overline{Q_0}$$



NOTE:- In Asynchronous we can implement just we need to think reset logic.

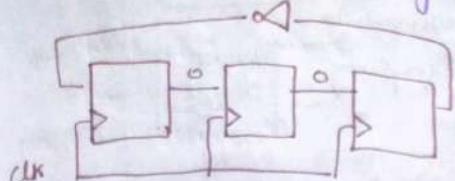
Ring Counter.

- Type of shift register (Dflip)
- For n bit ring counter it has n states. (Lim)
- $2^n - n$ {unused states}



- Decoding is very easy
- disadvantage:- only 4 out of 16 are utilized and output are symmetric.

Johnson Counter / Twisted Ring Counter



clk

0 0 0	0 0 0	0 0 0
1 0 0	1 0 0	1 0 0
1 1 0	1 1 0	1 1 0
1 1 1	1 1 1	1 1 1
0 1 1	0 1 1	0 1 1
0 0 1	0 0 1	0 0 1
0 0 0	0 0 0	0 0 0

6 states.

→ For 3 bit Johnson has $6(2^n)$
n bit 2^n states
 $2^n - 2^n$ {Unused States}

6-bit Ring Counter

Twisted.

No of states = 6

No of unused = $64 - 6 = 58$; $64 - 12 = 52$

5 states	$100000 \rightarrow$	0 0 0 0 0
	$010000 \rightarrow$	1 0 0 0 0
	$001000 \rightarrow$	1 1 0 0 0
	$000100 \rightarrow$	1 1 1 0 0
	$000010 \rightarrow$	1 1 1 1 0
	$000001 \rightarrow$	1 1 1 1 1
	$000000 \rightarrow$	0 0 0 0 0

→ No of flip flop indicates the power consumption in one hot encoding the flip will be less.

Mod n Counter

No of flip flop

Binary

$\rightarrow 4 \log_2 N$

Gray

$\rightarrow 4 \log_2 N$

One hot

$\rightarrow 10 \log_2 N$

Johnson

$\rightarrow 8.5 \log_2 N$

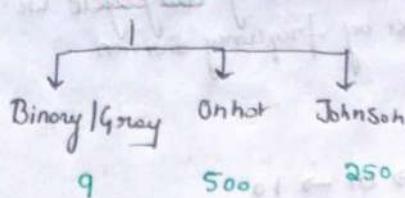
Eg



$$2^n = 2^3 = 8$$

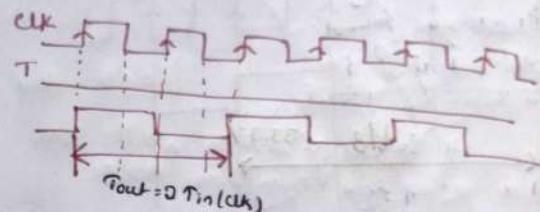
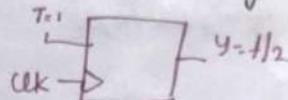
$$\text{bw h m to } 2^3 = 10$$

MoD 500



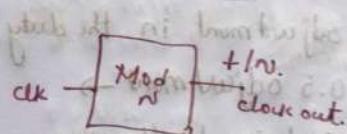
Frequency division

→ T flip flop is majorly used.



$$\bullet T_{out} = Q \times T_{in}$$

$$\bullet f_{out} = f/2$$



Frequency division

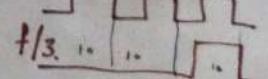
divide by even
 $f/3$

divide by odd
 $f/4$



① 00 → 01 → 10

② 0 0 0 1
0 0 1 0
0 1 0 0
1 0 0 0



0 0
0 1
1 0
MSB

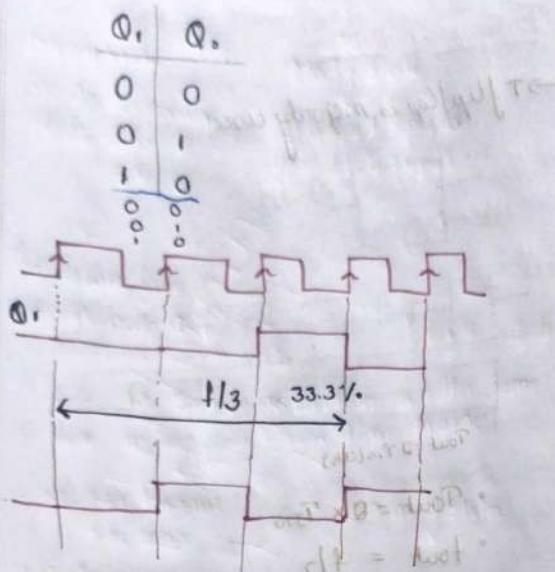
Problem here is duty cycle = $\frac{T_{on}}{T_{off}}$

$$\frac{10}{30}$$

default clock has 50% of duty cycle we need to make by frequency division.

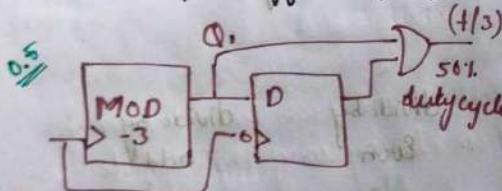
Mod 3:

00 → 01 → 10



→ doing the adjustment in the duty cycle for 0.5 adjustment →

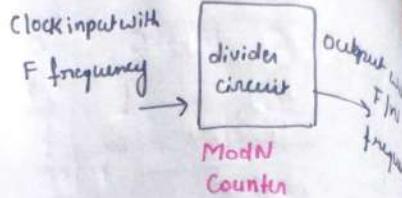
add rugged trigger flip flop whose input is MSB Counter and Q1 in the output dff and MSB



e.g.

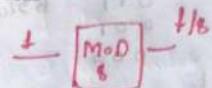
0	0	For this add +ve triggered d flip flop
0	0	
0	1	
1	1	

25 L 50%.



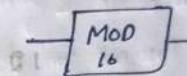
division factor can be 2, 3, ..., N

Mod 8:



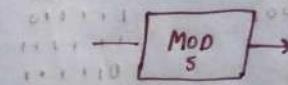
Q_2	Q_1	Q_0
↓	↓	↓

$F/8 \quad F/4 \quad F/2$



Q_3	Q_2	Q_1	Q_0
↓	↓	↓	↓

$F/16 \quad F/8 \quad F/4 \quad F/2$



Q_2	Q_1	Q_0
↓	↓	↓

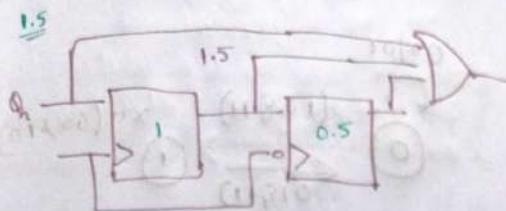
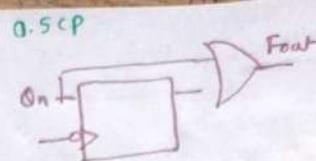
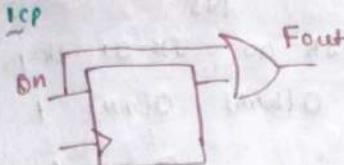
$F/5 \quad F/4 \quad F/2$

0	0	0
0	0	1
0	1	0
0	1	1
1	0	0

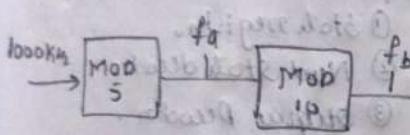
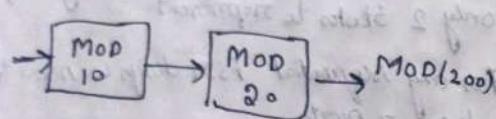
$F/120$

Q_6	Q_5	Q_4	Q_3	Q_2	Q_1	Q_0
↓	↓	↓	↓	↓	↓	↓

$\frac{F}{120} \quad \frac{F}{60} \quad \frac{F}{32} \quad \frac{F}{16} \quad \frac{F}{8} \quad \frac{F}{4} \quad \frac{F}{2}$



Concaded Counters:

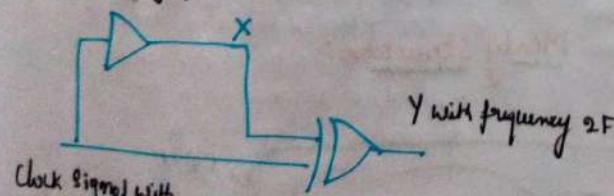


$$f_a = \frac{100\text{kHz}}{5} = 20\text{kHz}$$

$$f_b = \frac{20\text{kHz}}{10} = 2\text{kHz}$$

Frequency Multiplication

Buffer with delay of $t/4$



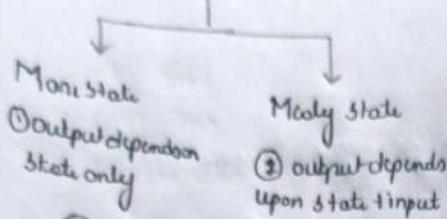
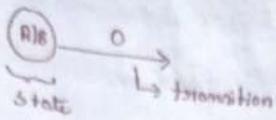
Clock signal with period t , freq f

Notes : Waveform

state	a0	a1	a2
0	0	0	0
0	0	1	0
1	0	0	1
1	1	1	1

state	a0	a1	a2
0	0	0	0
1	0	0	0
1	1	0	0
0	1	0	1

Finite State Machine :-



↳ It will give
stable output (*additional circuitry*) b/c the output
depends only on state.

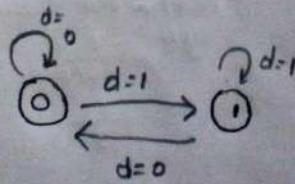
NOTE: We can't say which one is better some
time mealy and some time moore.

- No of states will be more
- Output depends only on present state
- Output is slow
- Output will be stable
- Fun no of states
- Output depends on present state and input
- We will get fast result
- Output affected by the noise.

D-Flipflop :-

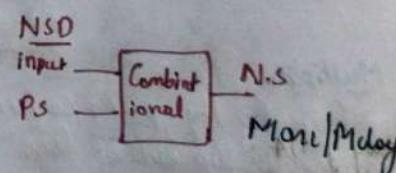
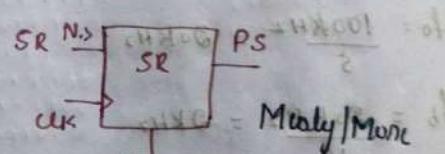
D	Qn	Qn+1
0	0	0
0	1	0
1	0	1
1	1	1

PS	NS
D=0	D=1
0	0
1	0

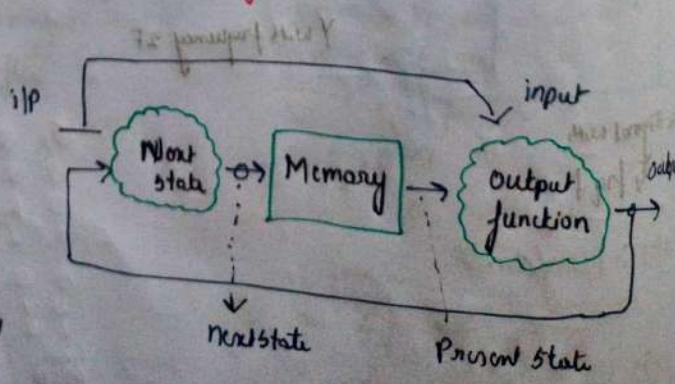


PS	JK = 00	JK = 01	JK = 10	JK = 11
0	0 (latch)	0 (reset)	0	1
1	1	0	0	1
	00/01			
	$(1/0 \& 1/1)$	$x \oplus 0 (S_0, S_1)$	$(01 \& 11)$	
	J K			
	0 0	latch		
	0 1		0	
	1 0	Toggle	1	
	1 1		1	Toggle

- For 4 State State machine we always
only 2 states to represent.
- For any Sequential FSM design we need
design 3 parts
 - State register.
 - Next State decoder.
 - Output Decoder.

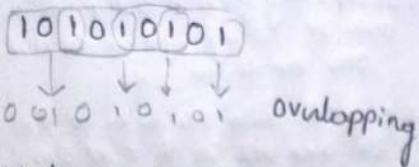


Mealy Structure :-

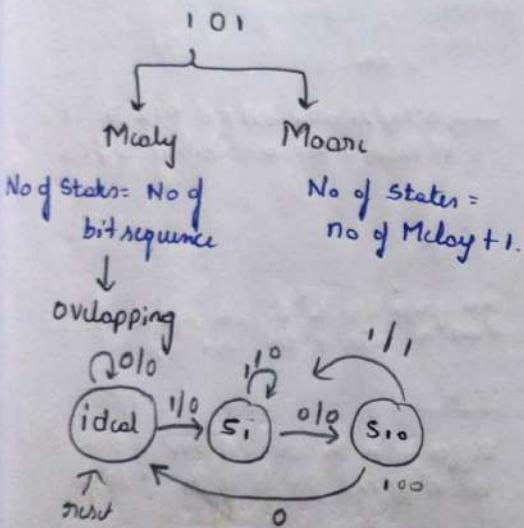


to detect the Sequence

- ↳ overlapping.
- ↳ nonoverlapping.

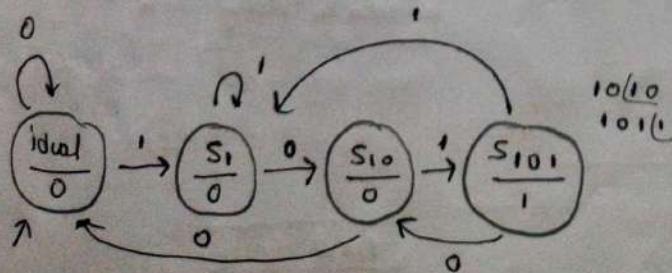


- non overlapping last one will not be considered, if the sequence is deleted it will always go to ideal condition.
 - But in overlapping it will check whether it needs to go for intermediate states or not.



- In non overlapping if the sequence is deleted it will go for initial Condition
 - in overlapping $101 \rightarrow 10$ it will check by removing one by one msB bit

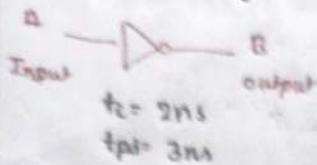
Молс 101



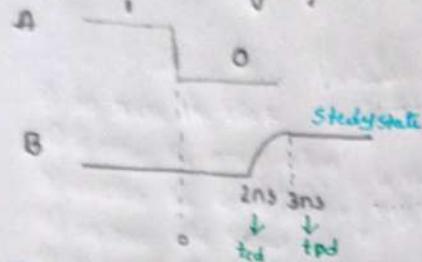
Static timing Analysis:-

① Contamination delay (t_c)

Propagation delay

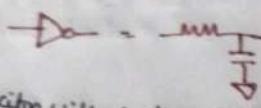


at $t=0$: A is changed from 1 to 0



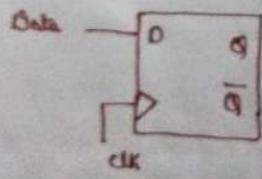
Propagation delay: The time required to reach the steady state.

Contamination delay: It will start going to reach the steady state till that

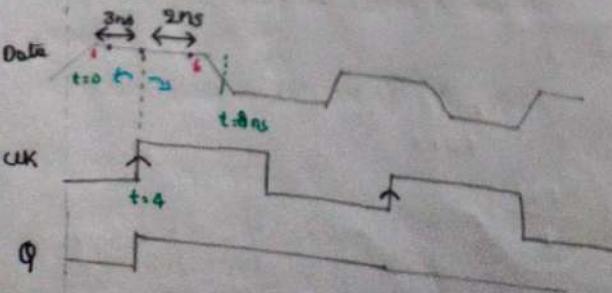


This capacitor will not charge instantly it will take some time to charge.

Setup and Hold time:



Given $t_{setup} = 3\text{ ns}$
 $t_{hold} = 2\text{ ns}$



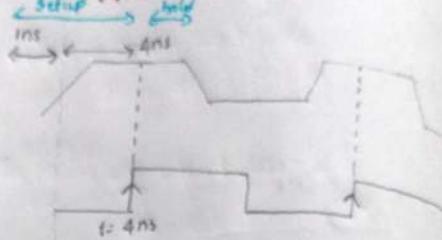
Q in the actual procedure that we will do but for simplicity we need to see back one step.

→ So since they given the setup time, so we make it true if it is constant for 3ns and also for 2 ns.

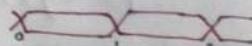
→ Mean the data should be constant for 3ns and also for the next 2 ns, no violation.

② If $t_{setup} = 5\text{ ns}$,

$t_{hold} = 2\text{ ns}$.



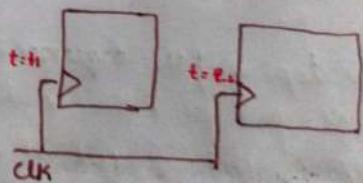
→ Setup Violation but not hold Violation
 It leads to metastability condition



Above graph refers to the data may be one of you it will be constant till $0 \rightarrow 1 \rightarrow 2$.

Clock Skew:

Difference in the arrival time of some at different points in the design



Parasitic refers to the noise that will increase resistance and capacitance which will affect the performance when may be connected to a single clock.

$$T = RC$$

Capacitance of

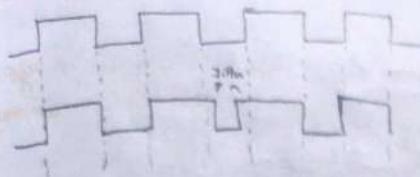
Time

(Assuming $t_2 > t_1$)

$$t_{skew} = t_2 - t_1$$

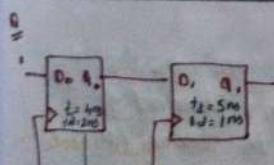
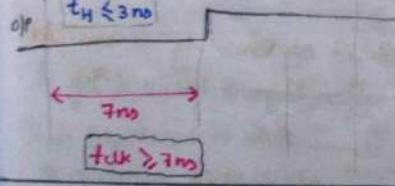
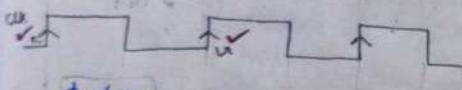
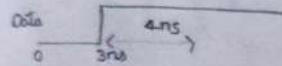
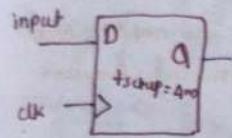
Clock Jitter: Here the starting and reaching time remain same, but the time period will get altered.

ideal clock:

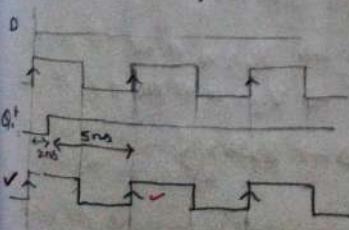


→ clock-to-Q delay / Propagation delay

If input is changed from 0 to 1 at 3ns, and remain constant for proper operation; what should be min clock time period? What should be the maximum hold time of flip flop?

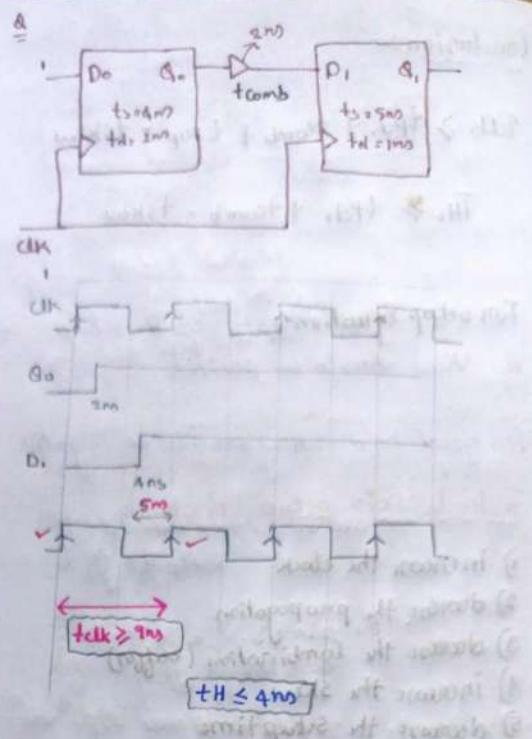


→ No setup & hold Violation.



→ $t_{clk} \geq 7\text{ ns}$

$t_{H2} \leq 2\text{ ns}$

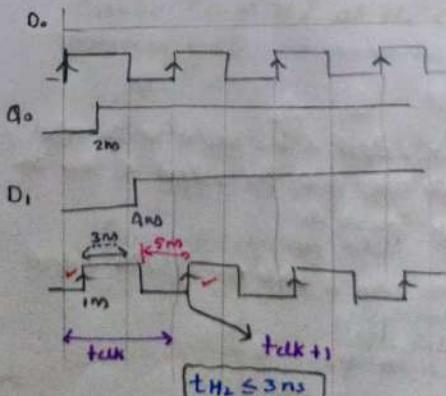
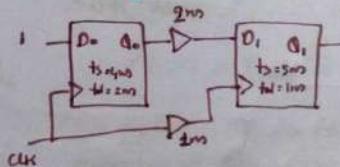


(Conclusion):-

$$t_{clk} \geq t_{pd1} + t_{comb} + t_{setup}$$

$$t_{H2} \leq t_{pd2} + t_{comb}$$

Introducing clock skew:-



$t_{clk} \geq 8\text{ ns}$

$t_{clk+1} \geq 4 + 5\text{ ns}$

$\geq 9 - 1\text{ ns}$

$t_{clk} \geq 8\text{ ns}$

Conclusion :-

$$T_{clock} \geq t_{pd} + t_{comb} + t_{sup} - t_{skew}$$

$$T_{H_L} \geq t_{pd} + t_{comb} - t_{skew}$$

For setup Equation : For good setup
the skew should be positive

For good hold Skew should be negative

Rules to avoid Setup Violation :-

- 1) increase the clock
- 2) decrease the propagation
- 3) decrease the combination (buffer)
- 4) increase the skew
- 5) decrease the setup time

Rules to avoid the hold time :-

- 1) Propagation delay must be less than t_{H_L}
- 2) Combination delay should be high
- 3) Hold will be less
- 4) Tskew should be less

Logic Families

In all the digital electronic concepts we use the gates. Now we need to see the internal parts.

logic family

Evolution

- ① RTL
- ② DCTL {Direct Coupled Transistor logic}
- ③ JTL {Integratied Injunction logic}
- ④ OTL {Diode Transistor logic}
- ⑤ HTL {High Threshold logic}
- ⑥ TTL {Transistor Transistor logic}
- ⑦ ECL {Emissor Coupled logic}

MOS

- PMOS
- NMOS
- CMOS (I_{mp})

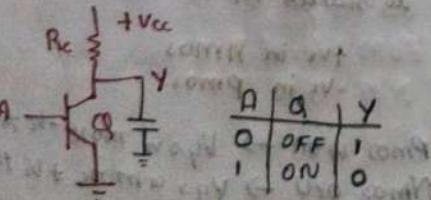
→ Based on the Specification we need to choose the logic family.

Specification:

- 1) Prop delay
- 2) Power dissipation
- 3) Figure of Merit
- 4) Fanout
- 5) Noise Margin

Propagation delay: It is the time required to get a valid output after applying input. It is expressed in milliseconds and it should be as minimum as possible.

It depends on storage time of Transistor & RC time constant.

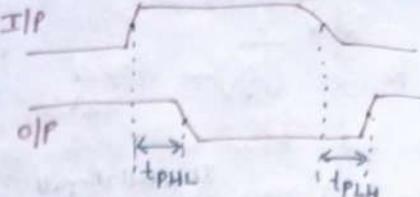


↳ It should operate in cut-off or saturation to operate as a switch.

Since the presence of capacitor it will take more time to charge or discharge.

How to calculate propagation delay

TPD



$$t_{PD} = \frac{t_{PHL} + t_{PLH}}{2}$$

Power dissipation: It is the

average power dissipated by one gate. It is represented by mill watt.

Average power : When gate voltage is logic 1 and logic 0. is called average power.

$$P_d = V_{cc} I_e \quad (\text{Aq})$$

$$I_{cAvg} = \frac{I_{CH} + I_{CL}}{2}$$

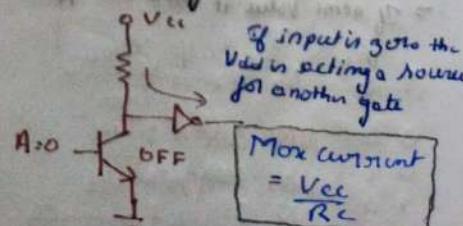
Figure of Merit: It is a product of t_{PD} & P_d . It should be minimum as possible. It is expressed in pico jouls.

$$FOM = t_{PD} P_d \quad (\text{PJ})$$

Fanout: It is the maximum number of logic gate inputs that can be connected at the output of another gate. It must be as high as possible.

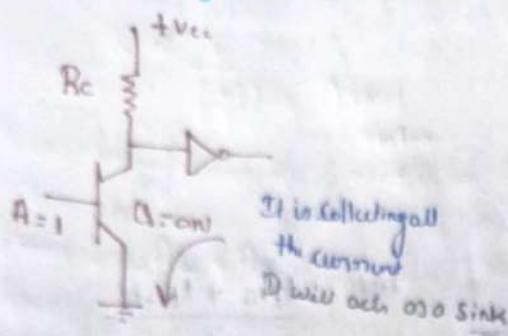
It depends on current sourcing & sinking.

Current Sourcing:



$$\text{Max Current} = \frac{V_{cc}}{R_C}$$

Gurrent Sinking



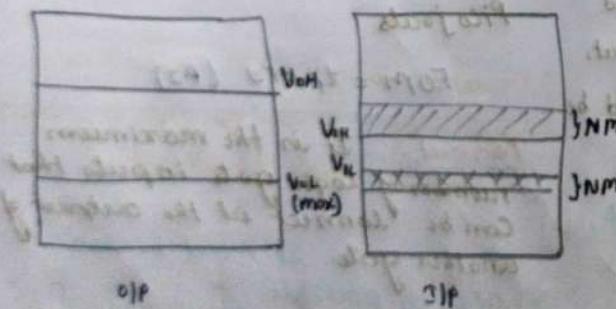
$$\text{Fanout} = \min \{\text{Fan}_1, \text{Fan}_2\}$$

For $\text{Fan}_1 = 8$, $\text{Fan}_2 = 5$ {output low and output high}

$\text{Fan}_1 = 8$ } $\text{Fan}_2 = \min \{8, 5\}$ both should be satisfied

- Final output should be 0 or 1 or both

Noise Margin: how much noise you can add to your system, so still the system must be logic. Which is added at the input and that must not affect the output.



V_{OL}: When output is low and it is connected to next gate. mean the output is low the maximum possibility.

→ If noise value it will not affect the output

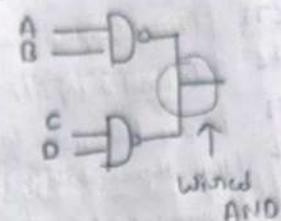
$$V_{OH} > V_{IH} > V_{IL} > V_{OL}$$

$$NMH = V_{OH} - V_{IH}$$

$$NML = V_{IL} - V_{OL}$$

$$NM = \min (NMH, NML)$$

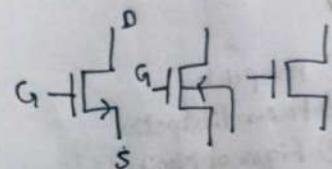
Wired Logic: When two gate outputs are connected with a wire the junction will acts like a wire gate.



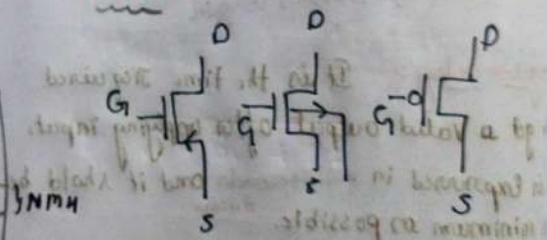
Except in ECL in all other logic families it is wired AND in ECL

In CMOS we avoid wired logic

NMOS:



PMOS:



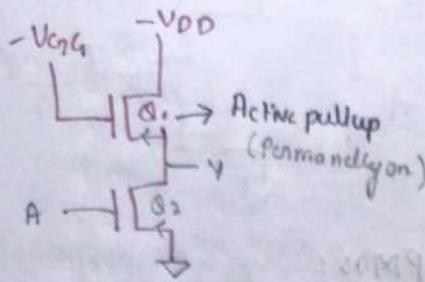
Threshold Voltage: minimum voltage required to form a channel of to switch on the transistor

$+V_{DD}$ in NMOS.
 $-V_{DD}$ in PMOS.

PMOS is on $\rightarrow V_{GS} < 0$ more $-V_{DD}$
NMOS ON $\rightarrow V_{GS} < 0$ more $+V_{DD}$

$$V_{GS} = 0 \rightarrow \text{Both off.}$$

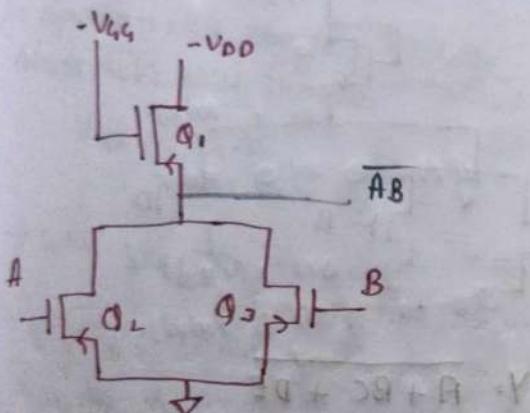
Nmos:
 Logic 0 = -V_{DD}. } +V_C logic.
 Logic 1 = OV.



A	Q ₂	Y
0 (-V _{DD})	ON	1 (OV)
1 (OV)	OFF	0 (V _{DD})

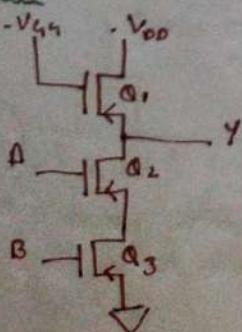
Inverter

NAND:



A B	Q ₂ Q ₃	Y
00	ON ON	(OV) 1
01	ON OFF	(OV) 1
10	OFF ON	(OV) 1
11	OFF OFF	0 (-V _{DD}) 0

NOR:

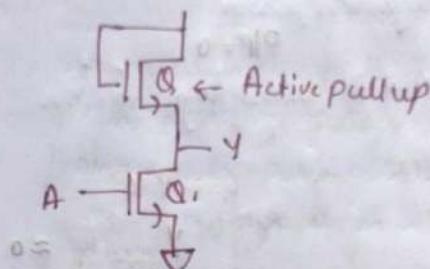


A B	Q ₂ Q ₃	Y
00	ON ON	(OV) 1
01	ON OFF	0 (-V _{DD}) 0
10	OFF ON	0 (-V _{DD}) 0
11	OFF OFF	0 (-V _{DD}) 0

Specification:

t_{pd} = 300ns
 P_{on} = 0.2 MW
 F_{OM} = 60 PJ
 N_M = 1.5 Volts
 Fanout = 15

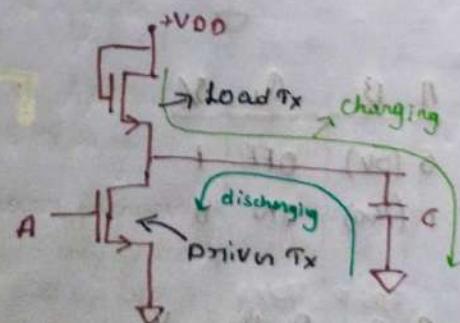
Nmos:
 logic 0 : OV } +V_C logic.
 logic 1 = +V_{DD}



A	Q ₁	Y
0(OV)	OFF	(V _{DD}) 1
1(V _{DD})	ON	(OV) 0

In Nmos Inverter charging time is always greater than discharging time. Which of the following is the reason for it?

- a) Aspect ratio of load Tx more than driver Tx
- b) Aspect ratio of Driver Tx more than load Tx
- c) Charging takes more time than disch
- d) None



$$T_{dL} = R_L \cdot C$$

$$T_D = R_D \cdot C$$

R_L = ON resistance of Q₂

R_D = ON resistance of Q₁

Note ★★

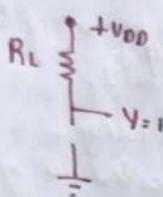
In NMOS Design $R_L > R_o$

$$R_o = \frac{1}{(W/L)} \rightarrow \text{Aspect ratio}$$

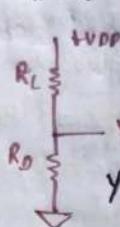
$$A_L < A_o$$

Analysis: $V_{DD} = 5V$, $V_{DD} = 1V$

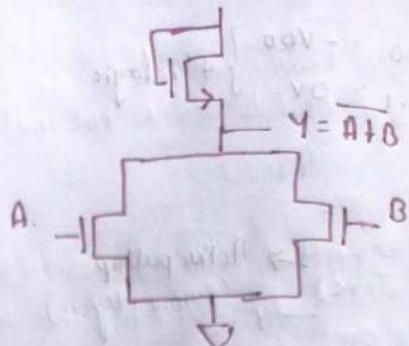
$$O/P = 1$$



$$O/P = 0$$



$$Y = \frac{V_{DD} R_o}{R_L + R_o} \approx 0$$



PMOS:

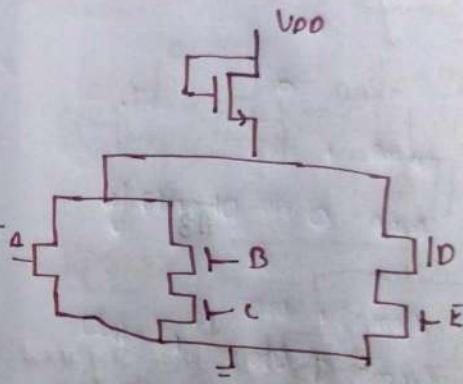
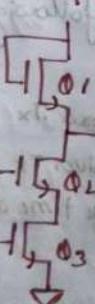
$+V_C \rightarrow \text{Series} \rightarrow \text{NOR}$
 $\text{Parallel} \rightarrow \text{NAND}$

NMOS

$+V_C \rightarrow \text{Series} \rightarrow \text{NAND}$
 $\text{Parallel} \rightarrow \text{NOR}$

In O/P is 0 Condition if 1V is applied in output only 0.5 will be divided.
To avoid that $R_D \ll R_L$

NAND:



$$Y = \overline{A + BC + DE}$$

Specifications:

$t_{PD} : 250\text{ns}$

$P_d : 1\text{mW}$

$FOM : 250\text{PFJ}$

$N.M : 0.15$

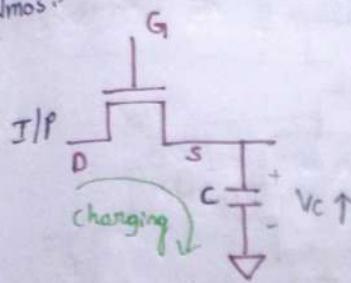
Fanout = 10.

A	B	Q_1	Q_2
0 (OV)	0 (OV)	OFF	1
1 (V_{DD})	1 (V_{DD})	ON	0

A	B	Q_1	Q_2	Y
0 0	0 0	OFF	OFF	1
0 1	0 1	OFF	ON	1
1 0	1 0	ON	OFF	1
1 1	1 1	ON	ON	0

Positive transmission logic :-

Nmos:



→ Input will come when gate = 1.

$$\rightarrow \text{if } I/P = 1 \quad G_1 = 1(V_{DD})$$

When $V_{GS} > V_T = \text{ON}$.

$$\begin{aligned} V_g &= V_{DD} \\ V_s &= 0 \end{aligned} \quad \underbrace{V_{GS} = V_{DD}}_{\text{ON.}}$$

No capacitance will charge, voltage across the V_C will increase.

$$\begin{aligned} V_{GS} &= V_G - V_S \\ &= V_{DD} - V_C > V_T \end{aligned}$$

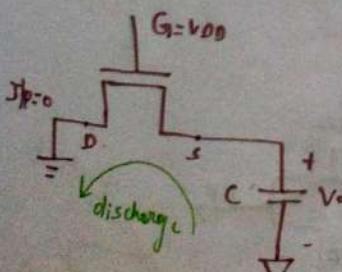
$$\rightarrow \text{When } V_{DD} = V_C \rightarrow \underline{0} \quad \underline{\text{OFF}}$$

$$= V_{DD} - V_T > V_C$$

$$V_o = V_C < V_{DD} - V_T$$

↑
What - 1.

So if i pass V_{DD} input it will not able to give the output V_{DD} in.



$$V_{GS} > V_T$$

$$V_g - V_s > V_T$$

$$V_{DD} - V_s > V_T \rightarrow$$

$$V_{DD} - 0 > V_T \rightarrow \text{on.}$$

So the transistor will stay till it is completely discharged strongly.

Conclusion:-

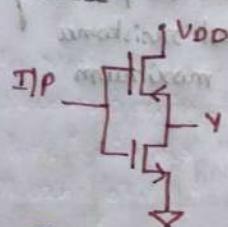
Nmos is strong in '0'
Weak in '1'

Pmos is strong in '1'
Weak in '0'.

CMOS:-

→ In CMOS we will always avoid floating input because its I/P Capacitance is very high. It can easily affected by external magnetic field.

Inverter:-



logic - 0, 0
logic - 1, 3-18V.

$$\begin{aligned} I/P = 0 \quad V_{GS} &= 0 - V_{DD} \quad (\text{PMOS}) \\ V_{GS} &= 0 - 0 \quad (\text{NMOS}) \end{aligned}$$

$$I/P = 1$$

$$\begin{aligned} I/P = 1 \quad V_{GS} &= 1 - V_{DD} = 0 \quad (\text{PMOS}) \\ V_{GS} &= 1 - 0 = 1 \quad (\text{NMOS}) \end{aligned}$$

Advantage:- Compare with the inverting the node by only by nmos or pmos the active pull up will gain none of the current constraint due to R_C .

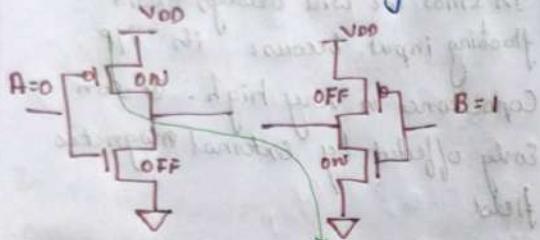
Power efficient, delay will be reduced. Static power dissipation is very low in CMOS.

at any given time only one of them is on static power dissipation is low.

Dynamic power consumption:

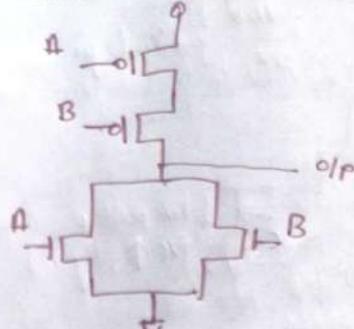
It is a type of power consumption in which when the transition of pmos & nmos from low to high and from high to low. At some point both of them are on will leads to maximum power consumption is called dynamic power.

NOTE:- Avoid the wired logic in CMOS

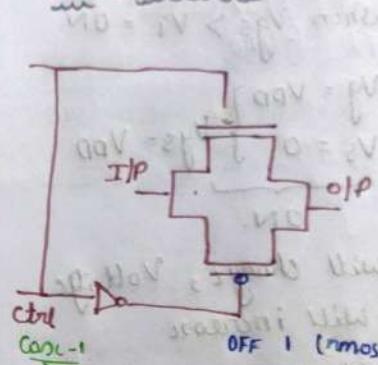


If they connected means we are providing the wired logic with least resistance then it will consume maximum current.

CMOS NOR:

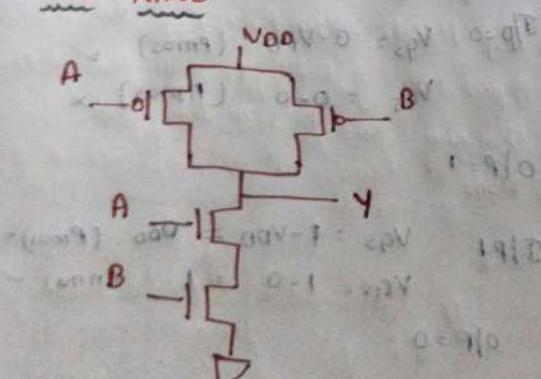


Pass transistors:



if $ctrl = 0$ OFF \ominus (PMOS)

CMOS NAND:



Specifications:

$$t_{pd} = 70 \text{ ns}$$

$$P_d = 0.01 \text{ mW}$$

$$F_{OM} = 0.7 \text{ PJ}$$

$$N.M = 0.35 \text{ VDD}$$

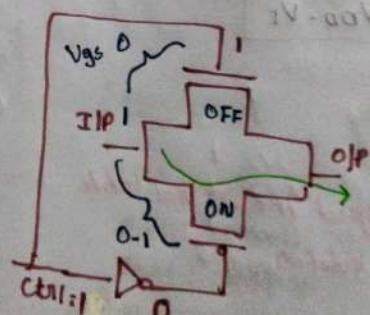
$$F_{out} = 50$$

Cycle 2

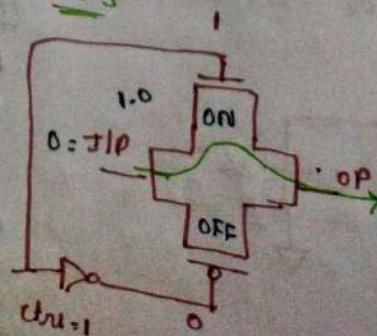
$$ctrl = 1$$

$$ON \oplus (\text{NMOS})$$

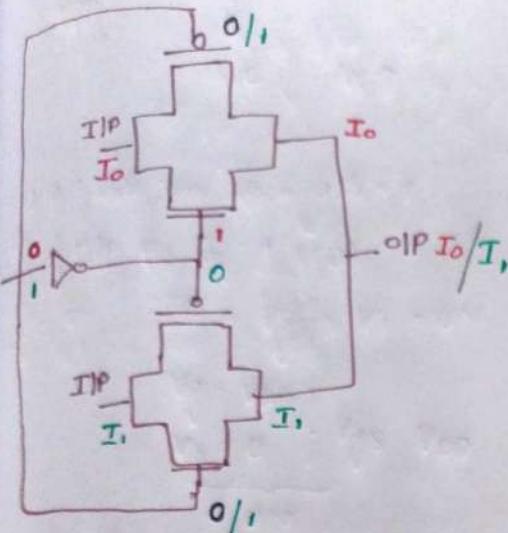
$$ON \ominus (\text{PMOS})$$



Cycle 3



Whenever we want to implement any OR with less no of gates we can use this transmission gates.



$$2:1 \text{ MUX} = 4 + 2 = 6 \text{ Transistors}$$

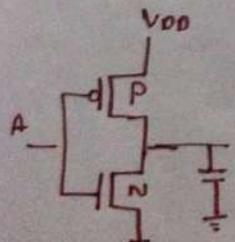
For normal implementation.

$$Y = \bar{S}I_2 + SI_1$$

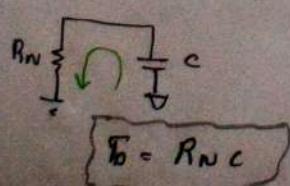
$$\begin{aligned} 2 \text{ AND} &= 2 \times 6 = 12 \\ 1 \text{ OR} &= 1 \times 6 = 6 \\ 1 \text{ IN} &= 1 \times 2 = \underline{\underline{2}} \end{aligned}$$

$\underline{\underline{2}}$ Trans

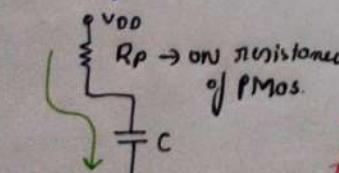
Inverter:-



discharge



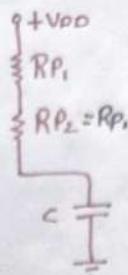
charging



$$T_c = R_p C$$

For NOR Gate:-

charging:-



$$T_c = 2R_p \cdot C$$

in worst case
 $\leq R_p C$.

$$2R_p \leq R_p$$

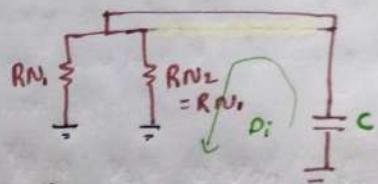
$$R_p \leq \frac{R_p}{2}$$

$P_{mos} \rightarrow R$ half \rightarrow width must be double.

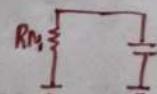
discharging:-

Both N, N2 \rightarrow ON.
(B)

Any one of N1, N2 \rightarrow ON.



Any one ON.



$$T_d = R_{N1} \cdot C$$

$$\text{Worst } R_{N1}C \leq R_{N2}C$$

\rightarrow No change
hence no change in NMOS but change the width in PMOS.

Problem:- Series \rightarrow Multiply
Parallel \rightarrow add } hmos

Series \rightarrow add } PMOS
Parallel \rightarrow mul } PMOS

\rightarrow Add bar at the last.