

DAA
Chapter - 17
Homework

①. Given a dynamic table that doubles in size when it needs more space. Find the amortized runtime for inserting n elements

② use the aggregate method.

To insert ' n ' elements using the aggregate method with can be done in 2 ways.

For case 1:

If we don't take and need to allocate new memory then

1	2	3	4	5	6	7	8	...
$O(1)$	$O(1)$	$O(1)$	$O(1)$	$O(1)$	$O(1)$	$O(1)$	$O(1)$	

So the sequence of n inserts

$$O(n) + O(2n) = O(n)$$

So, replace $O(1)$ in above example

$$O(1) + O(2n) = O(1)$$

thus the amortized runtime is $O(n)$ for inserting n elements is $O(1)$

For case 2:

If we allocate new memory

$$i = 2^k + 1, \quad k = 1, 2, 3, \dots$$

to include the capacity & double the size of array

then we need to allocate new memory

For inserting the element n in the new array

$$\left\{ \begin{array}{ll} \text{Running time} = z^k + 1 & \text{if } i = z^{k+1}, \text{ case 1} \\ = 1 & \text{otherwise, case 2} \end{array} \right.$$

⑥ use the accounting method.

using the accounting method. charge z units to each insertion.

when the table doubles in size from m to $2m$ credit m units.

The credit exactly pay for the copy cost of $O(m)$

Total credit is $m + 2m + 4m + \dots$

$$n/2 * m = O(n)$$

Pseudo code:

initialize table with capacity = 1

for $i = 1$ to n :

if table is new table with size

$2 * \text{current size}$

insert element i into table

initialize charges = 0

initialize credits = 0

for $i = 1$ to n

charges $+$ = 2

credit $+$ = m

Total charges = $2 * n = O(n)$

Total credit = $m + 2m + \dots n/2 + m = O(n)$

cost per insertion = $\text{total}/n = O(n)/n = O(1)$

\therefore runtime per insertion = $O(1)$

Total time for inserting n elements is $O(n)$