Lecture 1

Time Value of Money: A Little Overview

Introduction to Corporate Finance

Professor Michael R. Roberts

If you have not already done so, please read “Big Picture Motivation” to better understand the motivation for the course. In this note, I want to briefly speak to our first topic – the time value of money.

The lectures will teach you the tools and mechanics of moving money through time via discounting and compounding, as well as how this gets applied in practice. Taking a step back, one might ask: Why? Why does money have a time unit and why do we need to account for it?

The answer is that there is an opportunity cost associated with not having money today. Specifically, with money in hand, we can “put it to work” in a financial sense. That is, we can invest it in a savings account, a certificate of deposit (CD), government bonds, corporate bonds, stocks, real estate, etc. By doing so, we earn money, or a “return,” on our investment. So, in the future, the money we had today will be worth something different and, on average, more than what we had today. Thus, when we consider money, we have to recognize that money received at different points in time really does have different value.

Let me give you an example to emphasize this point. If I have $100.00 today, I can put the money in a savings account at my bank and at the end of one year I will have about $100.01 – sad but true in today’s low interest rate environment. Alternatively, if I put the $100.00 in a stock-based mutual fund – an investment in a large portfolio of stocks – then I will have, on average, approximately $112.00 at the end of the year. (Yes, you don’t really know exactly what you will have in the future and you could lose money but, on average, you will make about 12% per year.)

The implication of these investment opportunities is that $100 today is not the same thing as $100 one year from today because the money received in the future misses out on the investment opportunity. As such, $100 today is worth more than $100 in the future. And, this distinction has nothing to do with inflation, which we will discuss in the lectures as well.

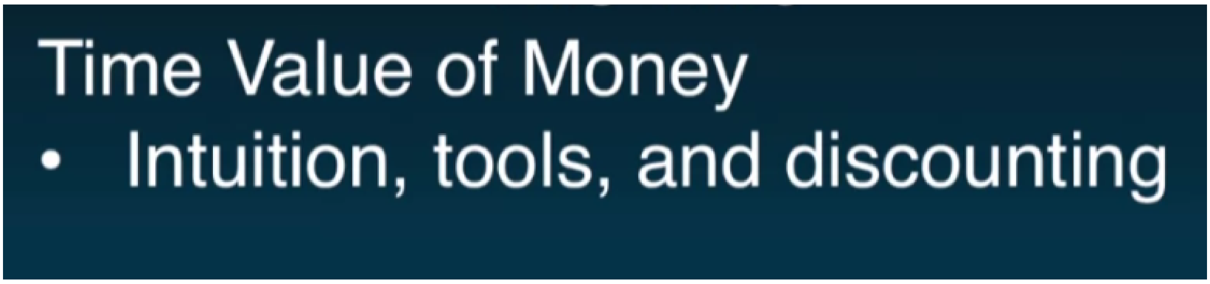
Precisely what the $100 today will be worth, $100.01 or $112.00 in our example, will depend on in what you decide to invest. If you invest in something relatively safe, like a bank savings account, the opportunity cost is relatively low – you will only lose out on $0.01 if you have to wait a year to get the $100. Alternatively, if you invest in something relatively risky, like a portfolio of stocks, the opportunity cost can be quite high – you will lose out on $12.00 if you have to wait a year to get the $100. Thus, the return on your investment represents the opportunity cost you face, and the risk of the investment determines the precise cost – low risk, low cost; high risk, high cost.

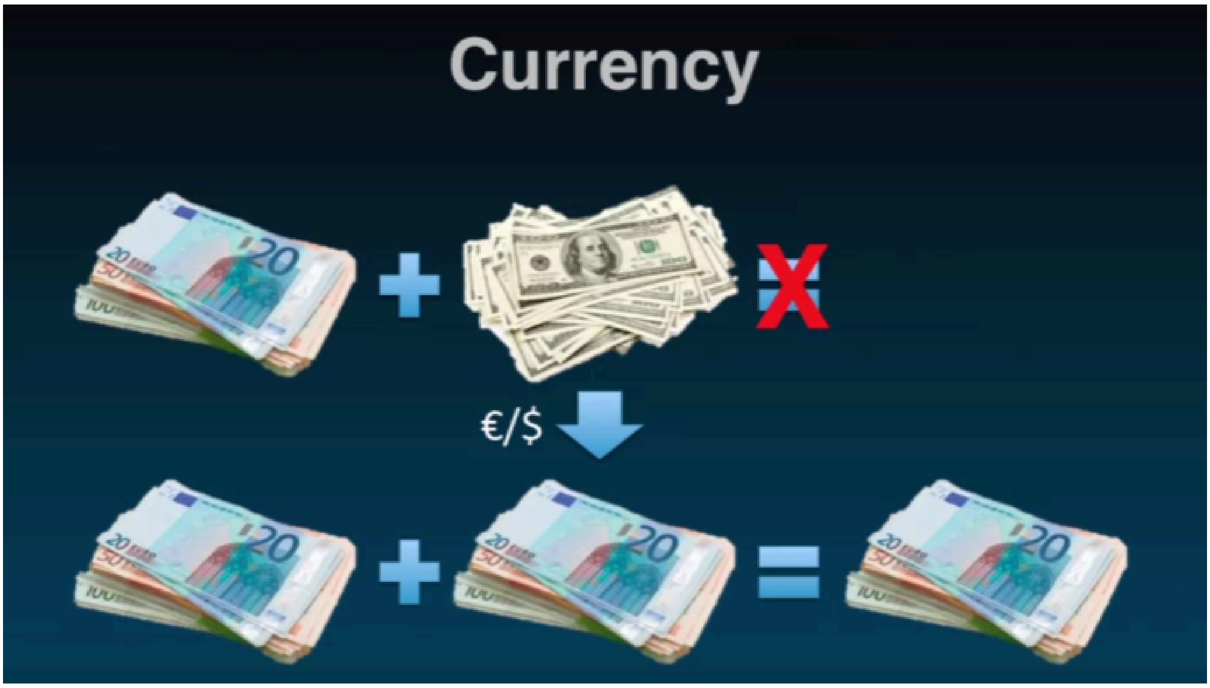
At the risk of redundancy, let me summarize. We have to recognize that money has a time unit identifying when it is received/paid. The reason is that there is an opportunity cost associated with money in the future; we miss out on the opportunity to invest it today. This cost is determined by the nature of the investment. If we invest the money in something relatively safe, then the opportunity cost is low. That is, the $100 received today will be similar to the amount of money we would receive in the future if we make the safe investment (savings account example). If we invest the money in something relatively risky, then the opportunity cost is high. That is, the $100 received today will be quite different from (less than) the amount of money we would receive, on average, in the future if we make the risky investment (stock portfolio example).

Finance Books (Light Reading)

Below is a brief list of interesting and often entertaining books related to Finance.

- *Against the Gods: The Remarkable Story of Risk*, Peter L. Bernstein
- *Barbarians at the Gate: The Fall of RJR Nabisco*, Bryan Burrough and John Helyar
- *Too Big to Fail*, Andrew Ross Sorkin
- *When Genius Failed: The Rise and Fall of Long-Term Capital Management*, Roger Lowenstein
- *Liar's Poker*, Michael Lewis
- *Capital Ideas: The Improbably Origins of Modern Wall Street*, Peter L. Bernstein





Messages (Look up)

1. Can't add/subtract different currencies

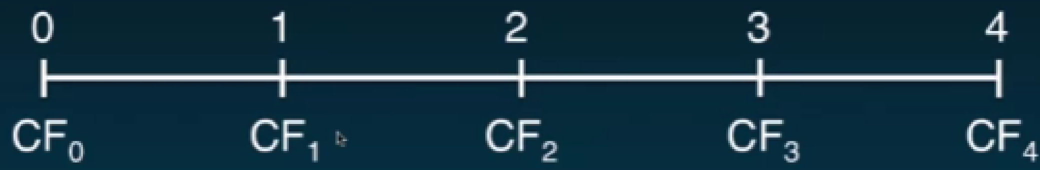
2. Must convert currencies to common (base) currency using exchange rate

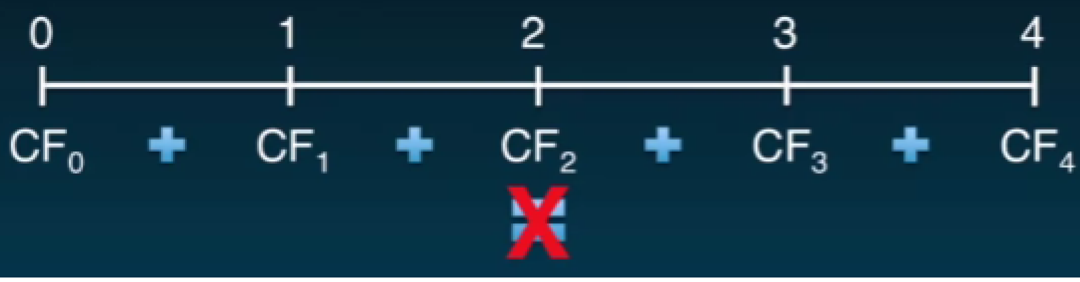
Time Value of Money

- Money received/paid at different times is like different currencies
  - Money has a time unit

- Must convert to common/base unit to aggregate
  - Need exchange rate for time

Time Line





Discount Factor

The discount factor is our exchange rate for time

$$(1+R)^{\boxed{t}}$$

t = time periods into future ($t > 0$) or past ($t < 0$) to move CFs

Definition: R is the rate of return offered by investment alternatives in the capital markets of equivalent risk.

A.k.a., discount rate, hurdle rate, opportunity cost of capital

To determine R, consider the risk of the cash flows that you are discounting.

Investment	Average Annual Return, R
Treasury-Bills (30-Day)	3.49%
Treasury-Notes (10-Year)	5.81%
Corporate Bonds (Investment Grade)	6.60%
Large-Cap Stocks	11.23%
Mid-Cap Stocks	15.15%
Small-Cap Stocks	25.32%

Riskier investment, higher return

Discounting CFs moves them back in time

0	1	2	3	4
CF_0	CF_1	CF_2	CF_3	CF_4

$CF_1 \times (1+R)^{-1}$

$CF_2 \times (1+R)^{-2}$

$CF_3 \times (1+R)^{-3}$

$CF_4 \times (1+R)^{-4}$

We can add/subtract these CFs because they are in the same time units (date 0)

Present Value

Present value, $PV_t(\bullet)$ of CFs is discounted value of CFs as of t

0	1	2	3	4
CF_0	CF_1	CF_2	CF_3	CF_4

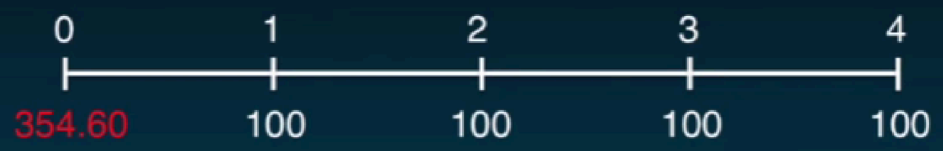
$CF_1 \times (1+R)^{-1} = PV_0(CF_1)$

$CF_2 \times (1+R)^{-2} = PV_0(CF_2)$

$CF_3 \times (1+R)^{-3} = PV_0(CF_3)$

$CF_4 \times (1+R)^{-4} = PV_0(CF_4)$

These are present values of future CFs as of today (period 0)



Interpretation 1: We need $354.60 today in an account earning 5% each year so that we can withdraw $100 at the end of each of the next four years

Interpretation 2: The present value of $100 received at the end of the next four years is $354.60 when the discount rate is 5%.

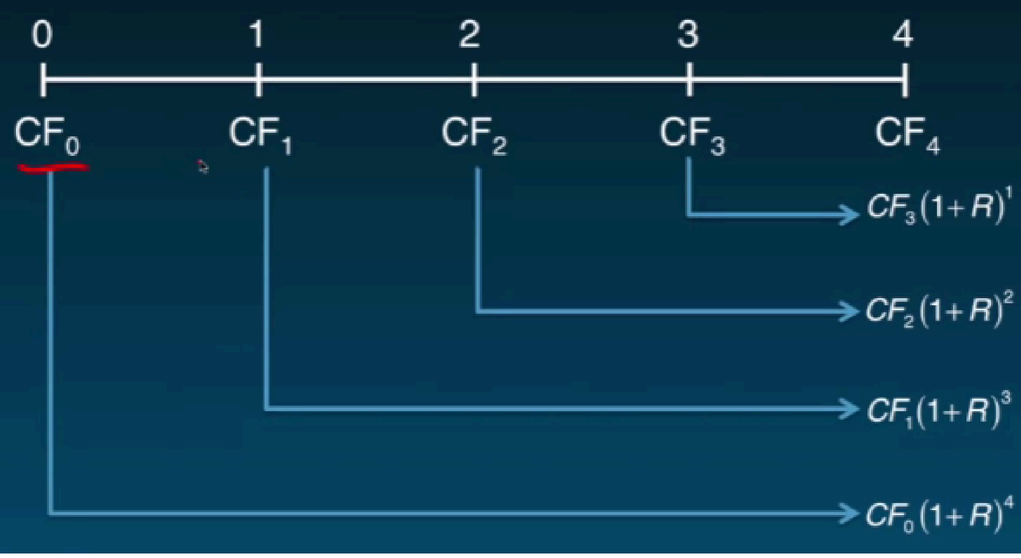
Interpretation 3: Today's price for a contract that pays $100 at the end of the next four years is $354.60 when the discount rate is 5%.

Example 2 – Savings (Account)

Year	Interest	Pre-Withdrawl Balance	Withdrawal	Post-Withdrawl Balance
0				$354.60
1	$17.73	$372.32	$100.00	$272.32
2	$13.62	$285.94	$100.00	$185.94
3	$9.30	$195.24	$100.00	$95.24
4	$4.76	$100.00	$100.00	$0.00

Compounding

Compounding CFs moves them forward in time



Future Value

Future value, $FV_t(\bullet)$ of CFs is compounded value of CFs as of t

0	1	2	3	4
CF_0	CF_1	CF_2	CF_3	CF_4

These are future values of CFs as of year 4

$$CF_3(1+R)^1 = FV_4(CF_3)$$

$$CF_2(1+R)^2 = FV_4(CF_2)$$

$$CF_1(1+R)^3 = FV_4(CF_1)$$

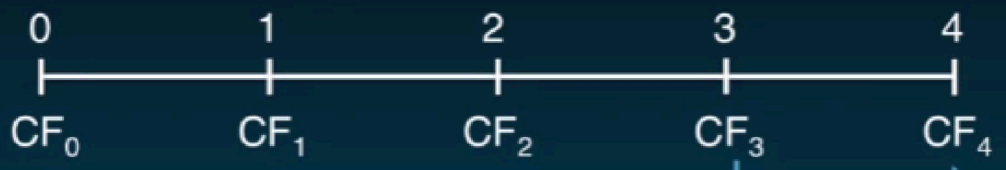
$$CF_0(1+R)^4 = FV_4(CF_0)$$

0	1	2	3	4
100	100	100	100	452.564

Interpretation 1: We will have $452.56 at the end of four years if we save $100 starting today for the next three years and our money earns 5% per annum.

More Generally

Can add CFs at any point in time if same units



$$CF_3(1+R)^{-1} = PV_2(CF_3)$$

$$CF_4(1+R)^{-2} = PV_2(CF_4)$$

$$CF_1(1+R)^{1} = FV_2(CF_1)$$

$$CF_0(1+R)^{2} = FV_2(CF_0)$$

Annuity

An annuity is a finite stream of cash flows of identical magnitude and equal spacing in time

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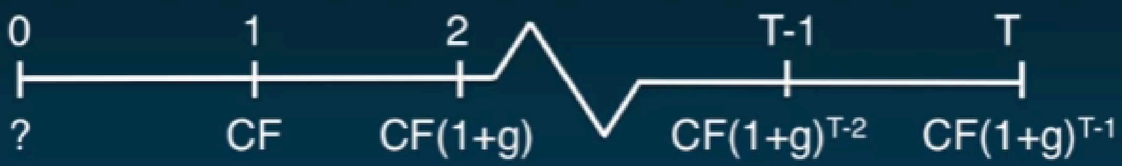
$$\text{PV of Annuity} = \frac{CF}{R}\left(1-(1+R)^{-T}\right)$$

*The first cash flow arrives one period from today

Growing Annuity

A growing annuity is a finite stream of cash flows that grow at a constant rate and that are evenly spaced through time



E.g., Income streams, savings strategies, project revenue/expense streams

Growing Annuity

A growing annuity is a finite stream of cash flows that grow at a constant rate and that are evenly spaced through time

0	1	2		T-1	T
?	CF	CF(1+g)		$CF(1+g)^{T-2}$	$CF(1+g)^{T-1}$

$$\text{PV of Growing Annuity} = \frac{CF}{R-g}\left(1-\left(\frac{1+R}{1+g}\right)^{-T}\right)$$

*The first cash flow arrives one period from today

How much do you have to save today to withdraw $100 at the end of this year, 102.5 next year, 105.06 the year after, and so on for the next 19 years if you can earn 5% per annum?

0	1	2		19	20
?	100	102.5		155.966	159.865

$$\text{PV of Growing Annuity} = \frac{CF}{R-g}\left(1-\left(\frac{1+R}{1+g}\right)^{-T}\right)$$

$$= \frac{100}{0.05-0.025}\left(1-\left(\frac{1+0.05}{1+0.025}\right)^{-20}\right) = 1{,}529.69$$

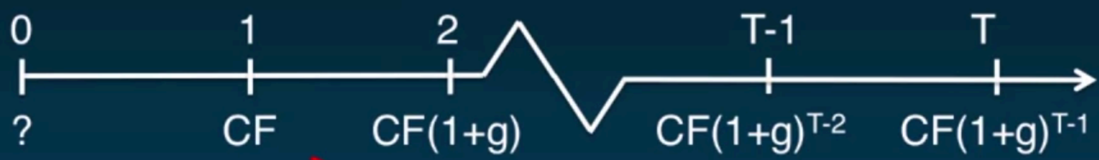
Perpetuity

An perpetuity is an infinite stream of cash flows of identical magnitude and equal spacing in time

$$\text{PV of Perpetuity} = \frac{CF}{R}$$

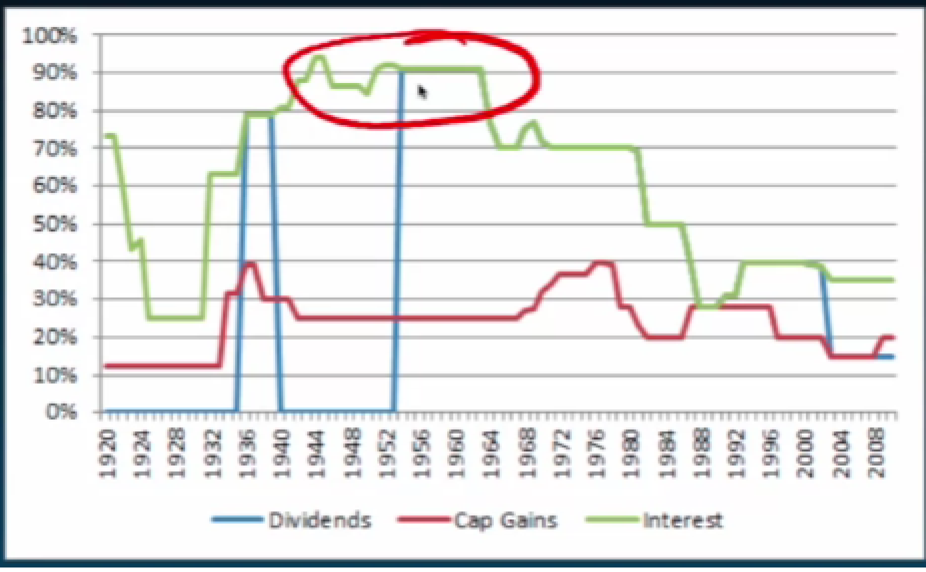
Growing Perpetuity

A growing perpetuity is an infinite stream of cash flows that grow at a constant rate and that are evenly spaced through time



$$\text{PV of Growing Perpetuity} = \frac{CF}{R-g}$$

Tax Rates



Savings with Taxes (Account)

Year	Interest	Taxes (35%)	Pre-Withdrawal Balance	Withdrawal	Post-Withdrawal Balance
0					\$354.60
1	\$17.73	-\$6.21	\$366.12	\$100.00	\$266.12
2	\$13.31	-\$4.66	\$274.77	\$100.00	\$174.77
3	\$8.74	-\$3.06	\$180.45	\$100.00	\$80.45
4	\$4.02	-\$1.41	\$83.06	\$83.06	\$0.00

We are \$100 – \$83.06 = \$16.94 short. Taxes reduce funds available for withdrawal. We run out of money early

Lesson: Taxes reduce the return on our investment, *R*

After-tax Discount Rate

$$Rt = R \times (1 - t)$$

- For our example:

$$5\% \times (1 - 35\%) = 3.25\%$$

Savings with Taxes

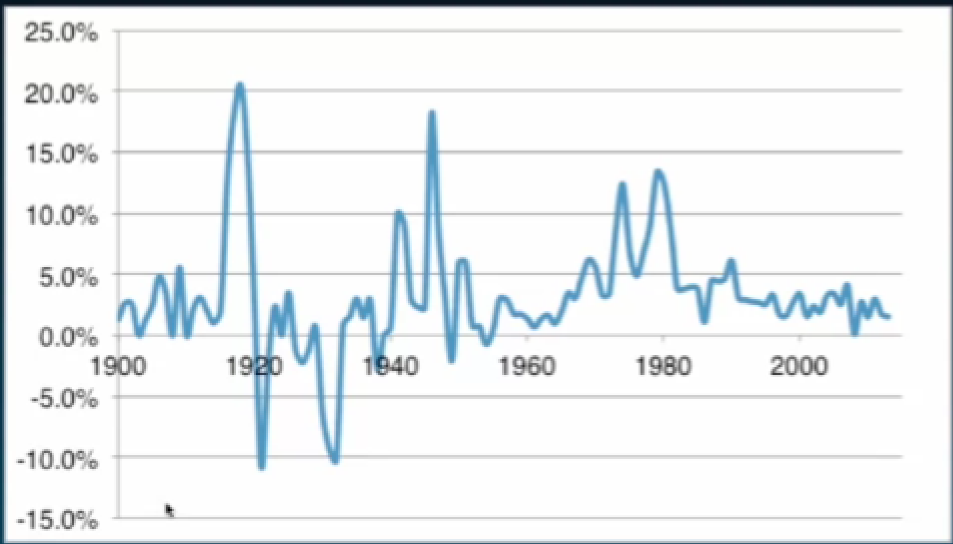
Year	Interest	Taxes	Pre-Withdrawal Balance	Withdrawal	Post-Withdrawal Balance
0					$369.50
1	$18.47	-$6.47	$381.51	$100.00	$281.51
2	$14.08	-$4.93	$290.66	$100.00	$190.66
3	$9.53	-$3.34	$196.85	$100.00	$96.85
4	$4.84	-$1.69	$100.00	$100.00	$0.00

Implication: We need to save more to ($369.50 > $354.60) to withdraw $100 each year *after taxes*

Note: $369.50 – $354.60 = $14.90 which also equals the present value of the taxes at 5%. (Check this!)

where R is the nominal return and t is the tax rate

Inflation



Lesson: Inflation will affect what we can buy with the money

Lesson: Inflation won't affect the money we earn

Real Discount Rate

$$1+\boxed{RR} = (1+\boxed{R}) / (1+\boxed{\pi})$$

RR is the real discount rate
π is expected inflation

- Commonly used approximation:

$$RR = R - \pi$$

Savings with Inflation

- Difference:
  - –taxes affect $
  - –Inflation affects consumption, not $
    - Earn nominal return but can't buy as much

Savings with Inflation

Year	Interest	Pre-Withdrawal Balance	Withdrawal	Post-Withdrawal Balance
0				$376.75
1	$18.84	$395.59	$100.00	$295.59
2	$14.78	$310.37	$100.00	$210.37
3	$10.52	$220.89	$100.00	$120.89
4	$6.04	$126.93	$100.00	$26.93

Implication: We have extra money(?).
We need to change withdrawal amount.
(Increase to buy costlier goods.)

Savings with Inflation

354.60

0 1 2 3 4

$$\boxed{\$376.75} = \frac{CF}{(1+0.05)} + \frac{CF}{(1+0.05)^2} + \frac{CF}{(1+0.05)^3} + \frac{CF}{(1+0.05)^4}$$

$$CF = \$376.75\left(\frac{1}{(1+0.05)} + \frac{1}{(1+0.05)^2} + \frac{1}{(1+0.05)^3} + \frac{1}{(1+0.05)^4}\right)^{-1}$$

$$= \underline{\$106.25} > 100$$

Savings with Inflation

Year	Interest	Pre-Withdrawal Balance	Withdrawal	Post-Withdrawal Balance
0				\$376.75
1	\$18.84	\$395.59	\$106.25	\$289.34
2	\$14.47	\$303.81	\$106.25	\$197.56
3	\$9.88	\$207.44	\$106.25	\$101.19
4	\$5.06	\$106.25	\$106.25	\$0.00

Ideally withdrawals grow each year to accommodate inflation

Year	Withdrawal
0	
1	$100 \times (1 + 0.025)^1 = \$102.50$
2	$100 \times (1 + 0.025)^2 = \$105.06$
3	$100 \times (1 + 0.025)^3 = \$107.69$
4	$100 \times (1 + 0.025)^4 = \$110.38$

This sequence of withdrawals maintains purchasing power of $100 in todays terms

These are “nominal” values corresponding to the real $100 purchasing power in year 0.

Year	Withdrawal
0	
1	$102.50
2	$105.06
3	$107.69
4	$110.38

PV at 5% discount rate = $376.75

We discount nominal cash flows by the nominal rate to get the price.

Note: PV of nominal CFs at nominal discount rate = PV of real cash flows at real rate

Savings with Inflation

Year	Interest	Pre-Withdrawal Balance	Withdrawal	Post-Withdrawal Balance
0	5%			$376.75
1	$18.84	$395.59	$102.50	$293.09
2	$14.65	$307.74	$105.06 2.5%	$202.68
3	$10.13	$212.81	$107.69	$105.13
4	$5.26	$110.38	$110.38	$0.00

- Real return, *RR*

$$RR = \frac{1+R}{1+\pi} - 1 \simeq R - \pi$$

where R is the nominal return and π is the rate of inflation

- Inflation does not affect $ return
- Inflation does purchasing power of $

Lesson: The relation between APR and EAR is:

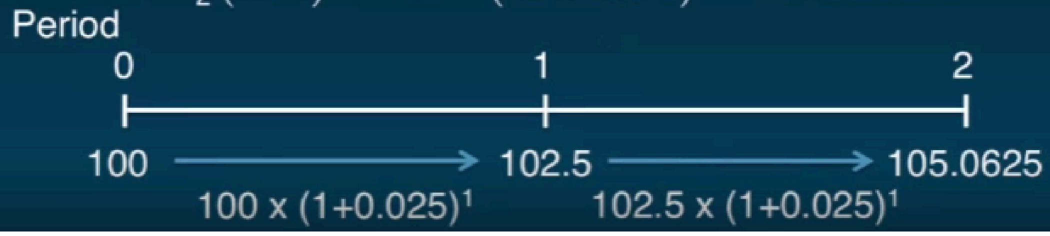
$$EAR = \left(1 + \frac{APR}{k}\right)^k - 1$$

$$= (1 + i)^k - 1$$

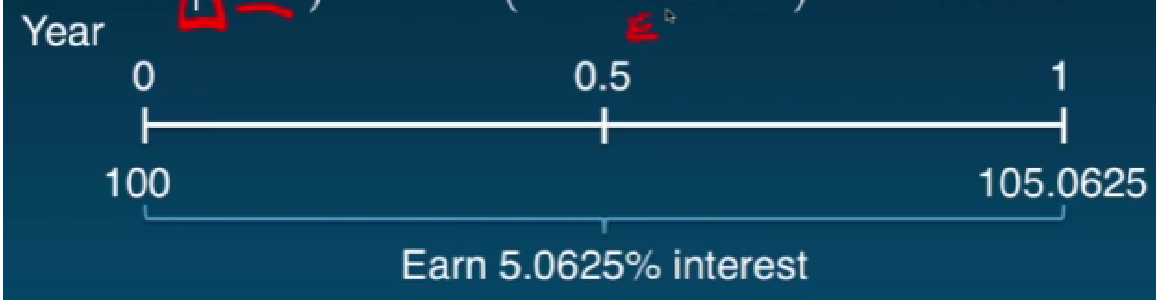
i is the periodic interest rate, or periodic discount rate

- Invest $100 in CD offering 5% APR with semi-annual compounding. How much money will you have in one year?

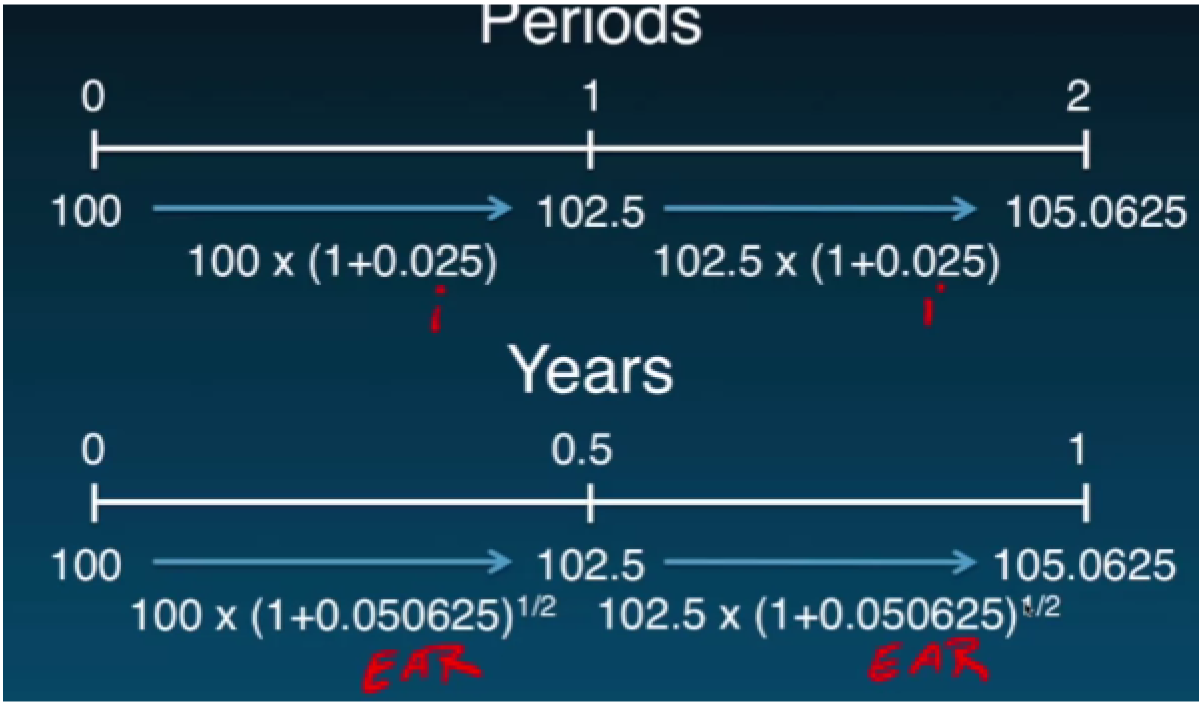
$$FV_2(100) = 100 \times (1 + 0.025)^2 = 105.0625$$



$$FV_{\boxed{1}}(100) = 100 \times (1 + 0.050625)^1 = 105.0625$$



Lesson: If you discount cash flows using EAR, then measure time in years. If you discount cash flows using periodic interest rate, then measure time in periods.



$$
\begin{aligned}
(1+EAR)^T &= \left(1+(1+i)^k-1\right)^T \\
&= \left((1+i)^k\right)^T \\
&= (1+i)^{kT} \\
&= (1+i)^N
\end{aligned}
$$

where $N = kT = \#$ of periods

effective annual rate

Current 5-Year Jumbo CD Rates

Insitition	APY	Rate
CIT Bank ★★★★	2.40% Tue Dec 16	2.37% Compounded daily
Maximize growth and savings. Member FDIC. Apply Now!		
synchrony BANK ★★★★★	2.25% Tue Dec 16	2.23% Compounded daily
Great Rates + Safety = Peace of Mind		
EverBank ★★★★	2.17% Tue Dec 16	2.15% Compounded daily
Nationwide Bank ★★★★	2.30% Tue Dec 16	2.27% Compounded daily

*Bankrate.com as of 12/16/2014

APR = 2.37%

k = 365 (or 360, 252)

→ i = 2.37% / 365
= 0.006714%

→ EAR =
$(1+0.006714\%)^{365}-1$
= 2.398%

- EAR is a discount rate
  – Measures cash flows in years

- Period interest rate, i, is a discount rate
  – Measures cash flows in periods

- APR is not a discount rate