EE 547 (PMP) Project Assignment

Assigned: 10/31/17 (Tue)
Due: 12/05/17

Professor Linda Bushnell UW EE

1. Project Guidelines

1. In this project, you will work in groups of two.
2. Submit a **written report** as well as the corresponding **MATLAB code** and **Simulink models** (through Canvas) by the due date. Write in complete sentences, give equations, and explain what you are doing. Choose your figures and tables carefully and show the relevant details of any computation/derivation. Use a font size of 11. The write-up is limited to 10 single-sided pages (or 5 double-sided pages). You may wish to, for example, scale your figures and tables such that several can fit onto one page. Follow the steps given below to organize your written report.
3. Prepare a **presentation** to give to the class on the last day of class (and upload PPT to Canvas). This should be a demonstration of your robot and a PPT presentation of your model and results. Limit the presentation and demo to 10 minutes. Make sure your PPT presentation covers the following topics: model, analysis of stability, controllability, observability, your controller and estimator, your minseg robot, what worked and what did not, what were some challenges you faced, any extra analysis you did beyond the given steps.
4. **Grading**: Report 25pts; Presentation 15pts (40% of your course grade).
5. **Bonus**: max 4pts for demonstrating your robot balances.

2. Introduction

The goal of the project is to implement concepts learned in the class with the MinSeg robot, and to control the movement and body attitude by state-space control methods.

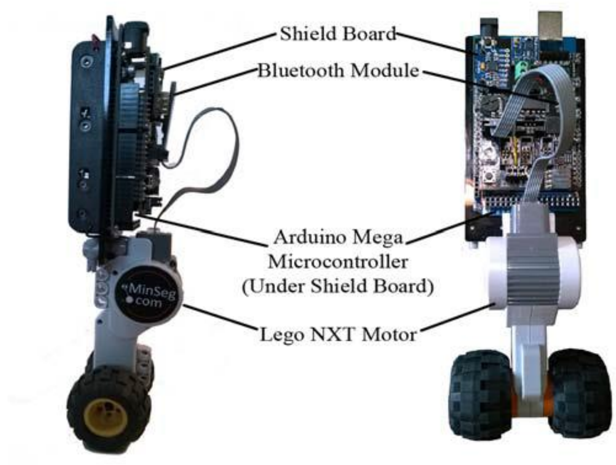

Figure 1 The test platform: MinSeg.

The focus of the project is the MinSeg mathematical model and applications of Matlab/Simulink to balance the MinSeg robot. From the state-space representation of the system, the state variables can be defined and the stability can be evaluated via Matlab. Since the MinSeg is equipped with accelerometers and gyroscope, appropriate controller design via Simulink can improve the stability of MinSeg and achieve balancing.

3. A Dynamical Model of the MinSeg Robot

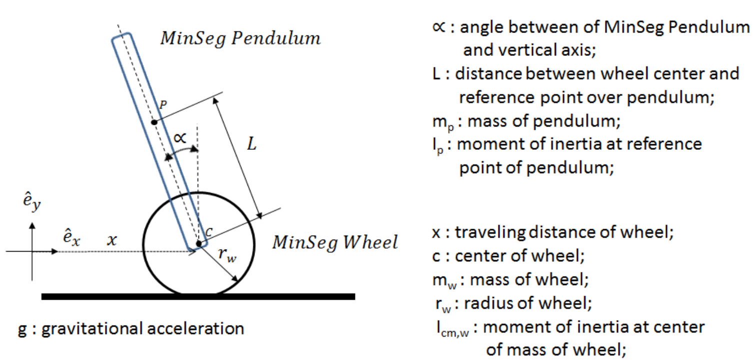

Figure 2 Mathematical model of MinSeg around the zero-equilibrium system.

Figure 2 depicts the motion of a MinSeg over flat ground. In our first homework assignment, the equation of motion was given as (1).

$$\begin{bmatrix} -(I_p + m_p L^2) & m_p L \cos\alpha \\ m_p L\, r_w^2 \cos\alpha & -(I_{cm,w} + m_w r_w^2 + m_p r_w^2) \end{bmatrix} \begin{bmatrix} \ddot{\alpha} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} T_m - m_p L g \sin\alpha \\ T_m r_w + m_p L\, r_w^2 \dot{\alpha}^2 \cos\alpha \end{bmatrix} \tag{1}$$

The torque T_m was provided by the DC motor of the MinSeg.

The final relation between torque, input voltage and state variables was given as (2).

$$T_m = \frac{k_t}{R} V + \frac{k_t k_b}{R r_w} \dot{x} + \frac{k_t k_b}{R} \dot{\alpha} \tag{2}$$

where V is applied voltage, R is the resistance of DC motor, k_t is torque constant and k_b is back-EMF constant.

With (1) and (2), the state-space matrices A and B of this system are derived as (3) and (4).

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{gLm_p(i_{\mathrm{cmw}}+(m_p+m_w)r_w^2)}{i_{\mathrm{cmw}}(i_p+L^2m_p)+(L^2m_pm_w+i_p(m_p+m_w))r_w^2} & -\frac{k_bk_t(i_{\mathrm{cmw}}+r_w(m_wr_w+m_p(L+r_w)))}{R(i_{\mathrm{cmw}}(i_p+L^2m_p)+(L^2m_pm_w+i_p(m_p+m_w))r_w^2)} & 0 & -\frac{k_bk_t(i_{\mathrm{cmw}}+r_w(m_wr_w+m_p(L+r_w)))}{Rr_w(i_{\mathrm{cmw}}(i_p+L^2m_p)+(L^2m_pm_w+i_p(m_p+m_w))r_w^2)} \\ 0 & 0 & 0 & 1 \\ \frac{gL^2m_p^2r_w^2}{i_{\mathrm{cmw}}(i_p+L^2m_p)+(L^2m_pm_w+i_p(m_p+m_w))r_w^2} & -\frac{k_bk_tr_w(i_p+Lm_p(L+r_w))}{R(i_{\mathrm{cmw}}(i_p+L^2m_p)+(L^2m_pm_w+i_p(m_p+m_w))r_w^2)} & 0 & -\frac{k_bk_t(i_p+Lm_p(L+r_w))}{R(i_{\mathrm{cmw}}(i_p+L^2m_p)+(L^2m_pm_w+i_p(m_p+m_w))r_w^2)} \end{bmatrix} \tag{3}$$

$$B=\begin{bmatrix} 0 \\ -\frac{k_t(i_{\mathrm{cmw}}+r_w(m_wr_w+m_p(L+r_w)))}{R(i_{\mathrm{cmw}}(i_p+L^2m_p)+(L^2m_pm_w+i_p(m_p+m_w))r_w^2)} \\ 0 \\ -\frac{k_tr_w(i_p+Lm_p(L+r_w))}{R(i_{\mathrm{cmw}}(i_p+L^2m_p)+(L^2m_pm_w+i_p(m_p+m_w))r_w^2)} \end{bmatrix} \tag{4}$$

4. Assignment

The project content and required steps are listed below. All of them are related to what you learned in the course. Your report should be organized in the same order as these steps.

4.1 Linear Dynamical Model of the MinSeg Robot

Step 1 Since matrices A and B are determined as above, develop a linear continuous time state-space representation of the system.

$$\dot{\mathrm{x}} = \mathrm{A}\,\mathrm{x} + \mathrm{B}\,\mathrm{u}$$

$$\mathrm{y} = \mathrm{C}\,\mathrm{x} + \mathrm{D}\,\mathrm{u}$$

The state variables, x, is defined as x $:= [\,\alpha\ \dot{\alpha}\ x\ \dot{x}]^T$, the input vector as u := V and the output vector as y:= x $=[\,\alpha\ \dot{\alpha}\ x\ \dot{x}]^T$.

Step 2 Measure the physical parameters of your MinSeg as shown in Figure 2 (find creative ways to measure the weight in grams (home kitchen scale, post office, grocery store, etc.) Create a table to list the values of your measurement (SI unit). A set reference values of (k_t, K_b, R) are given as (k_t, K_b, R) = (0.3233 Nm/A, 0.4953 Vs/rad, 5.2628 ohms).

Step 3 Find the transfer function matrix of the linearized system.

Step 4 Find the characteristic polynomial and eigenvalues of matrix A.

Step 5 Is the system asymptotically stable? Is it marginally stable? Explain why.

Step 6 Find the poles of the transfer function. Is the system BIBO stable? Explain why.

4.2 Controllability and Observability of the System

Step 7 Find the controllability matrix of the linearized system. What is the rank of the controllability matrix? Is the linearized system completely controllable?

Step 8 Analyze the observability of the linearized system with the output vector as y:= x $=[\,\alpha\ \dot{\alpha}\ x\ \dot{x}]^T$.

Step 9 Transform the linearized system into a *controllable canonical form* and *observable canonical form*.

4.3 State Estimator

Step 10 Develop a closed-loop state estimator (full-dimensional observer) for the open-loop system (no feedback yet) such that the poles of the observer are stable and that the dynamics of the observer is at least 6-8 times faster than the dynamics of the linearized model. Include in your report the value of the estimator gain, L.

Step 11 Develop a Simulink model of the linearized system (open-loop system with full-dimensional observer designed above). Add the state estimator derived in previous step to your Simulink model and set the initial conditions of the state estimator to $\hat{x}_0 = [0\ 0\ 0\ 0]$. Simulate the behavior of the system when a unit-step u(t) = 1, t ≥ 0 is applied at the input. Plot the estimated state-variables and output variables on the same graph.

4.4 Feedback Control

Step 12 Consider the case when the linearized system is stabilized through the use of feedback control. Using the *pole placement method*, develop in Matlab a proportional controller such that the poles of the closed-loop system are stable and that the dynamics of the closed-loop model is at least 4-6 times faster than the dynamics of the open-loop model. Include in your report the value of the proportional gain, K.

Step 13 Derive the state-space representation of the closed-loop system. Find the characteristic polynomial and the eigenvalues of the closed-loop system. Is this closed-loop system asymptotically stable?

Step 14 Develop a Simulink model of the linearized closed-loop system (no estimator here) when the output of the system equals state variables y:= x =$[\ \alpha\ \dot{\alpha}\ x\ \dot{x}]^T$. Add the proportional controller developed above to the Simulink model and simulate the response of the closed-loop system when a unit step u(t) = 1, t ≥ 0 is applied. Plot all the outputs on the same graph.

4.5 Feedback Control using State Estimator

Step 15 Combine the feedback controller with the state estimator. This should be an easy step since you have designed both parts separately. Combine the two in Simulink and simulate the system to see how it performs. Try using the estimator for states not measured (this will change your y(t)). Plot the error function and discuss your results.

4.6 Bonus

Step 16 **(Bonus – 4 points)** You will get up to 4 bonus points if you demonstrate the MinSeg balancing with one motor. This can be done with no batteries (teathered), or with batteries (unteathered). You can use the given LQR controller or design your own PID controller.