Department of Applied Mechanics
Indian Institute of Technology Delhi
Major, First Semester 2020-21

Course Title: Advanced Finite Element Method **Course No.:** AML 805
Date: 11 January 2020 **Duration:** 2.5 Hrs (9.00-11.30 AM)
Maximum Marks: 130

Instructions:

(i) You can refer your notes and books. It is an open notes and open book examination.

(ii) Answer all questions. Marks are indicated against each question. There are total 9 Questions.

(iii) Write answers in your own handwriting.

(iv) Start answer of each question on a new page.

(v) Stop writing at 11.30 AM and start scanning and uploading pdf in gradescope. You should finish uploading answers before 11.50 PM.

(vi) After uploading pdf, select pages for each question.

(vii) If you have any doubt, call me on my mobile: 9213227603.

Q. 1: Consider a square of side 2 units with initial location of its centre at $\{X\ Y\ Z\}^T = \{0\ \ 0\ 0\}^T$. The coordinates $\{x\ y\ z\}^T$ of the deformed configuration of the square are given by:

$x = 4 + X + 1.5\,Y; \quad y = 3 + 1.5\,Y; \quad z = Z.$

If the components of Cauchy Stress tensor are: $\sigma_{xx} = 100$ MPa, $\sigma_{yy} = 200$ MPa, $\sigma_{zz} = -50$ MPa, $\tau_{xy} = 100$ MPa, $\tau_{yz} = \tau_{zx} = 0$; Find the components of First Piola-Kirchhoff Stress and Second Piola-Kirchhoff Stress tensors. [10]

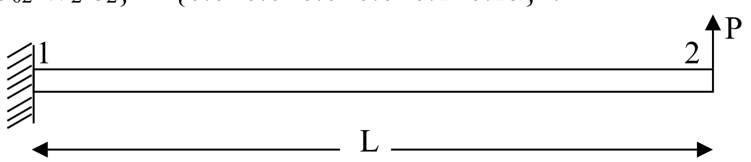
Q. 2: The Green-Lagrange strain in a truss member oriented in 2D space is given by:

$$\varepsilon_{XX} = \frac{dU}{dX} + \frac{1}{2}\left(\left(\frac{dU}{dX}\right)^2 + \left(\frac{dV}{dX}\right)^2\right)$$ where components of the displacement vector $\{U\ \ V\}^T$ with respect to initial un-deformed configuration are function of X coordinate only. The strain energy (π) stored in the member is given by: $\pi = \frac{1}{2}\int_0^L EA\left(\varepsilon_{XX}\right)^2 dX$. For a two-noded element, give the following:

(i)Strain-displacement matrices: $\mathbf{B}$ and $\mathbf{B}_{\mathbf{NL}}$, (ii) Final expression for tangent stiffness matrix: $\mathbf{K}_{\mathbf{T}}^{\mathbf{e}}$, (iii) Right hand side of element level incremental equation: $\mathbf{F}^{\mathbf{e}}$. [20]

Q. 3: Write the steps to perform one iteration for the solution of the cantilever beam as shown (EA = 100 N, EI = 1 N m^2, L = 1 m, P = 0.3 N). Model the beam using one Euler-Bernoulli beam element considering geometric nonlinearity. The starting solution is given by

$\{U_{01}\ W_1\ \theta_1\ U_{02}\ W_2\ \theta_2\}^T = \{0.0\ \ 0.0\ \ 0.0\ \ 0.0\ \ 0.1\ \ 0.15\}^T.$ [20]



Q. 4: The total potential energy for a beam is given by:

$$I = \frac{1}{2}\int_0^L \left[a\left(\frac{du_0}{dx}\right)^2 + b\left(\frac{d^2v}{dx^2}\right)^2 + c\left(\frac{d\theta}{dx}\right)^2 + 2e\frac{d^2v}{dx^2}\frac{d\theta}{dx} + f\left(\frac{dv}{dx}+\theta\right)^2 \right] dx .$$

For a two noded beam element, suggest the approximate solutions for u_0, v, θ, and state the degrees of freedom at each node. How are the continuity and inter-element compatibility conditions satisfied by your assumed solution? You need not derive the shape functions. [10]

Q. 5: Equation of motion for a system is given below:

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}\begin{Bmatrix} \dot{u}_1 \\ \dot{u}_1 \end{Bmatrix} + \begin{bmatrix} 20 & -10 \\ -10 & 10 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} t \\ 0.5t \end{Bmatrix}$$

(i) Find the undamped natural frequencies of the system. [5]

(ii) For the system, estimate Δt for numerically stable central difference scheme. [2]

(iii) Find displacement vector at $t = 0.1$ second using Newmark's time integration scheme taking Δt = 0.1 seconds and zero initial conditions. [8]

Q. 6: The differential equation governing the axial deformation (u) of an elastic rod of length L subjected to distributed load q and temperature change T is given by:

$$\frac{d}{dx}\left[EA\left(\frac{du}{dx} - \alpha\, T\right)\right] + q = 0,$$ subject to the boundary conditions: $u = 0$ at $x = 0$ and L.

E, A, α, T and q are known and u is an unknown. Derive the associated Integral Functional. [10]

Q. 7: For a three-noded truss element, the potential energy (V), kinetic energy (T) and virtual work of non-conservative forces (δW) are given by:

$$V = \frac{1}{2}\{\mathbf{d}^e\}^T[\mathbf{K}^e]\{\mathbf{d}^e\};\quad T = \frac{1}{2}\{\dot{\mathbf{d}}^e\}^T[\mathbf{M}^e]\{\dot{\mathbf{d}}^e\};\quad \delta W = -\delta\{\mathbf{d}^e\}^T[\mathbf{C}^e]\{\dot{\mathbf{d}}^e\} + \delta\{\mathbf{d}^e\}^T\{F^e\}.$$

Derive the equation of motion using Hamilton's principle. [15]

Q.8: In the associated flow rule, incremental plastic strains are given by $\{d\varepsilon^p\} = \lambda\left\{\frac{\partial F}{\partial \sigma}\right\}$. Taking von Mises yield function with isotropic strain hardening, show that $\lambda = d\varepsilon_{eq}^p$ where

$$d\varepsilon_{eq}^p = \sqrt{\tfrac{2}{3}\left(\left(d\varepsilon_{xx}^p\right)^2 + \left(d\varepsilon_{yy}^p\right)^2 + \left(d\varepsilon_{zz}^p\right)^2 + \left(\left(d\gamma_{xy}^p\right)^2 + \left(d\gamma_{yz}^p\right)^2 + \left(d\gamma_{zx}^p\right)^2\right)/3\right)}.$$ [15]

Q. 9: Consider two elements that are in potential contact as shown. The nodal coordinates are:

Node	x	y	Node	x	y
1	0	0	5	0.5	1.5
2	1.5	0	6	1.8	2.3
3	1.5	1	7	1.3	3.1
4	0	1	8	0	2.4

For a point located at the center of contactor edge 5-6, determine its contact point on the target line. Also compute the gap distance. [15]