**Department of Applied Mechanics**
**Indian Institute of Technology Delhi**
**Major, First Semester 2021-22**

**Course Title: Advanced Finite Element Method** **Course No.:** AML 805
**Date:** 18 November 2021 **Duration:** 2.5 Hrs (9.00-11.30 AM) **Maximum Marks:** 150

**Instructions:**

(i) You can refer your notes and books. It is an open notes, open book examination.

(ii) Answer all the questions. Marks are indicated against each question. There are total 9 Questions.

(iii) Write answers in your own handwriting.

(iv) Start answer of each question on a new page.

(v) Stop writing at 11.30 AM and start scanning and uploading pdf in gradescope. You should finish uploading answers before 11.50 AM.

(vi) After uploading pdf, select pages for each question.

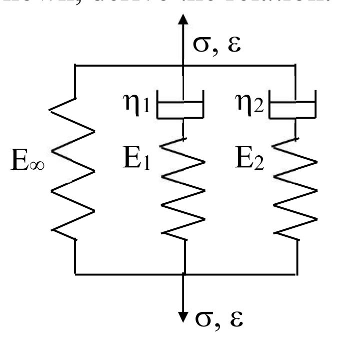
(vii) If you have any doubt, call me on my mobile: 9213227603.

**Q.1:** The weak form of equilibrium equations for a 2D plane stress problem is given by: $\int_{V^0} \delta\{\mathbf{E}\}^T \{\mathbf{S}\} dV^0 - \int_{V^0} \delta\{\mathbf{U}\}^T \{\mathbf{f_b}\} dV^0 = 0$ where

$$\{\mathbf{E}\} = \begin{Bmatrix} E_{XX} \\ E_{YY} \\ E_{XY} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial U}{\partial X} + \dfrac{1}{2}\left(\dfrac{\partial V}{\partial X}\right)^2 \\ \dfrac{\partial V}{\partial Y} + \dfrac{1}{2}\left(\dfrac{\partial V}{\partial Y}\right)^2 \\ \dfrac{\partial U}{\partial Y} + \dfrac{\partial V}{\partial X} + \dfrac{\partial V}{\partial X}\dfrac{\partial V}{\partial Y} \end{Bmatrix}; \ \{\mathbf{S}\} = \begin{Bmatrix} S_{XX} \\ S_{YY} \\ S_{XY} \end{Bmatrix}; \ \{\mathbf{U}\} = \begin{Bmatrix} U \\ V \end{Bmatrix}; \ \{\mathbf{f}_b\} = \begin{Bmatrix} f_{bX} \\ f_{bY} \end{Bmatrix}; \ \{\Delta\mathbf{S}\} = [\mathbf{C_T}]\{\Delta\mathbf{E}\}; \ U$$

and $V$ are displacements along $X$ and $Y$ directions. Derive the incremental element level governing equation considering 4-noded isoparametric element. [20]

**Q. 2:** For the viscoelastic model shown, derive the relation: $E(t) = E_\infty + \sum_{i=1}^{2} E_i\, e^{-\frac{t}{\tau_i}}$. [10]




**Q. 3:** State of stress at a point in a body is given by: $\sigma_{xx}$ = 120 MPa, $\sigma_{yy}$ = -80 MPa, $\tau_{xy}$ = 0 MPa, $\sigma_{zz}$ $=\tau_{xz}=\tau_{yz}$=0. Consider isotropic strain hardening, von Mises yield function and associative flow rule ($E$ = 200 GPa, $\nu$ = 0, plastic modulus ($H$) = 200 MPa, initial yield stress ($\sigma_Y$) = 200 MPa).

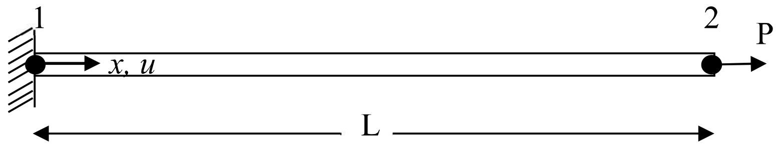
(i) Is the current state of stress elastic or elasto-plastic? [3]

(ii) For the next load step, finite element solution gives incremental strains: $\Delta\varepsilon_{xx} = 0.0009$, $\Delta\varepsilon_{yy} = 0.0009$, $\Delta\gamma_{xy} = \Delta\gamma_{zz} = \Delta\gamma_{xz} = \Delta\gamma_{yz} = 0$. Show that the new state of stress calculated assuming elastic behavior lies outside the yield surface. [5]

(iii) Calculate the factor $\beta$ such that state of stress at the point reaches to the yield surface for strain increments: $\Delta\varepsilon_{xx} = 0.0009\,\beta$, $\Delta\varepsilon_{yy} = 0.0009\,\beta$, $\Delta\gamma_{xy} = \Delta\gamma_{zz} = \Delta\gamma_{xz} = \Delta\gamma_{yz} = 0$. [12]

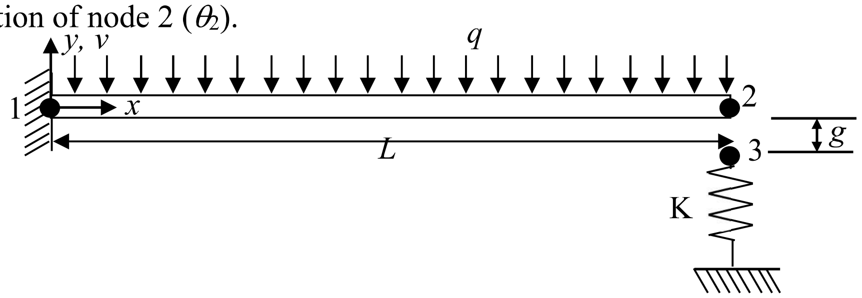
**Q. 4:** Derive the expressions for first ($I_1$) and second ($I_2$) invariants of Right Cauchy Green deformation tensor in terms of Green Lagrange Strain tensor components. For incompressible material ($I_3$=1) with $E_{YY}$=$E_{ZZ}$, express $E_{YY}$ and $E_{ZZ}$ in terms of $E_{XX}$ if $E_{XY} = E_{YZ} = E_{ZX} = 0$. [10]

**Q. 5:** Perform one iteration for the solution of the truss member as shown (EA = 100 N, L = 1 m, P = 10 N, $q$ = 0). Model the truss using one two-noded truss element considering Finite Deformation. The initial guess is given by $\{u_1\ u_2\}^T = \{0.0\ \ 0.1\}^T$. [20]

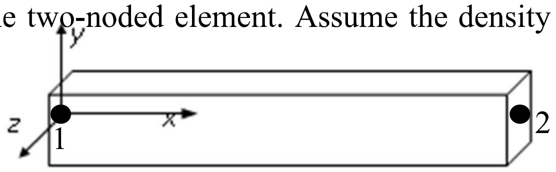



You may note that due to boundary conditions: $\Delta u_1 = 0$.

**Q. 6:** For the cantilever beam as shown ($EI$ = 1 N m$^2$, $L$ = 1 m, $q$ = 0.16 N/m, $K$ = 12 N/m, $g$ = 0.01 m). Model the beam using one Euler-Bernoulli beam element and find displacements of nodes 2 ($v_2$) and 3 ($v_3$) and rotation of node 2 ($\theta_2$). [20]




**Q. 7:** Derive the mass matrix of a two-noded frame element with the displacements: $u(x,y,t) = u_0(x,t) - y\,\theta_z(x,t)$; $v(x,y,t) = v(x,t)$. $\theta_z$ is an independent variable. Interpolate $u_0$, $v$ and $\theta_z$ using shape functions for the two-noded element. Assume the density and cross-section constant within the element. [20]




**Q. 8:** For a 4-noded isoparametric Mindlin plate bending reduced integration element with nodal coordinates as: $(x_1, y_1) = (0, 0)$; $(x_2, y_2) = (1, 0)$; $(x_3, y_3) = (1, 1)$; $(x_4, y_4) = (0, 1)$, find the hourglass stabilization matrix corresponding to transverse displacement $w$. You can leave your answer in terms of stabilization constant $c_1$. [10]

**Q. 9:** Set-up the element level governing equation for the solution of nonlinear differential equation: $-\frac{d^2u}{dx^2}+\left(\frac{du}{dx}\right)^2-4=0$. Use two-noded element for the interpolation of $u$ and take element length as $l$. In the weak form, do the integration by parts only for the first term involving second derivative of $u$.

[20]