AML 713 9:30-10:30 45

1. (10) **Use index notation** to prove that:

$$\nabla\times(\underline{u}\times\underline{w}) = \underline{u}(\nabla\cdot\underline{w}) - (\underline{u}\cdot\nabla)\underline{w} + (\underline{w}\cdot\nabla)\underline{u} - \underline{w}(\nabla\cdot\underline{u})$$

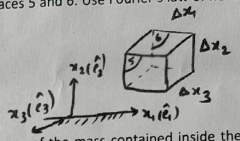
Useful relations: $\varepsilon_{ijk}\varepsilon_{imn} = \delta_{jm}\delta_{kn} - \delta_{jn}\delta_{km}$

$\hat{e}_i \times \hat{e}_j = \varepsilon_{ijk}\hat{e}_k$

2. (10) Prove that (i) Viscous heating is independent of rotation-rate tensor.
(ii) Viscous heating process can never be negative.

Assume the fluid to be a liquid. $\underline{\sigma} = -p\underline{I} + 2\mu\underline{S}$. *Viscous heating is* $\underline{\tau} : (\underline{S} + \underline{R})^T$. ***If you wish, you may use index notation.***

3. (6)(i) Consider a small cuboidal control volume, and derive the expression for the net rate of heat addition to the mass contained inside the CV due to conduction through Surfaces 5 and 6. Use Fourier's law of heat conduction to arrive at the final form of your answer.



(2) (ii) Can this heat-addition rate ***directly*** increase kinetic energy of the mass contained inside the CV. Briefly justify your answer? No need to derive any equations.

4. (4) A liquid flow is being observed by two observers – one fixed to Frame I and another fixed to Frame S. Frame I is an inertial reference frame, but S is not.
(i) What are the independent and dependent variables for the S-fixed observer?
(ii) What are the independent and dependent variables for the I-fixed observer?
Clearly describe all the symbols you use in your answer.

5. *(1 point for each correct and -1 point for each incorrect answer. Max: 4, Min 0.)* Fill in the blanks.

(i) *The pressure-dilatation work increases internal energy of a fluid element, if the fluid element is* _____ *(contracting/expanding/volume preserving).*
(ii) *A Newtonian fluid has a linear relationship between the* _____(viscous/pressure/total) stress tensor and the strain-rate tensor.
(iii) The viscous stress tensor in a Newtonian fluid has_____ ("normal components also"/"shear components only").
(iv) For a liquid at rest, $p_A - p_B = \rho g h$ is statement of _____(continuity equation/momentum equation/total energy equation). ***Symbols have their usual meanings.***