Department of Applied Mechanics
Indian Institute of Technology Delhi
Minor, First Semester 2020-21

Course Title: Advanced Finite Element Method **Course No.:** AML 805
Date: 11 November 2020 **Duration:** 1.5 Hrs (1.00-2.30 PM)
Maximum Marks: 90

Instructions:

(i) You can refer your notes and books. It is an open notes, open book examination.

(ii) Answer all the questions. Marks are indicated against each question. There are total 7 Questions.

(iii) Write answers in your own handwriting.

(iv) Start answer of each question on a new page.

(v) Stop writing at 2.30 PM and start scanning and uploading pdf in gradescope. You should finish uploading answers before 2.50 PM.

(vi) After uploading pdf, select pages for each question.

(vii) If you have any doubt, call me on my mobile: 9213227603.

Q. 1 The coupled differential equations governing transverse deflection (v) and slope (θ) of a beam are as follows:

$$-\frac{d}{dx}\left(GA\left(\frac{dv}{dx}-\theta\right)\right)-q=0 \qquad \text{(Equilibrium of Forces in the transverse direction)}$$

$$-\frac{d}{dx}\left(EI\frac{d\theta}{dx}\right)-GA\left(\frac{dv}{dx}-\theta\right)=0 \quad \text{(Equilibrium of Moments)}$$

where G, E, A, I, q are known constants. Length of the beam is L.

(i) Obtain the weak form of the problem, [7]

(ii) Identify Primary and Secondary variables, [2]

(iii) Obtain the associated Integral Functional. [6]

Hint: Weighted residual statement of the problem can be written by multiplying the first equation with δv, second with $\delta\theta$ and integrating the sum of the resulting expressions from $x = 0$ to $x = L$.

Q. 2: The weak form of the differential equation governing the buckling of a beam under axial compressive force is given by:

$$w\frac{d}{dx}\left(EI\frac{d^2v}{dx^2}\right)\Bigg|_0^L-\frac{dw}{dx}EI\frac{d^2v}{dx^2}\Bigg|_0^L+\int_0^L\frac{d^2w}{dx^2}EI\frac{d^2v}{dx^2}dx+wP\frac{dv}{dx}\Bigg|_0^L-\int_0^L\frac{dw}{dx}P\frac{dv}{dx}dx=0$$

where EI, P are constants, v is an unknown function of x and w is weight function. Identify primary and secondary variables. [5]

Q. 3: Consider a square of side 2 units with initial location of its centre at $\{X\ Y\ Z\}^T = \{0\ \ 0\ 0\}^T$. The coordinates $\{x\ y\ z\}^T$ of the deformed configuration of the square are given by:

$$x = 4 + X + 1.5\,Y; \quad y = 3 + 1.5\,Y; \quad z = Z.$$

(i) Compute the components of the deformation gradient tensor and write it in matrix form. [3]

(ii) Compute the components of the Green-Lagrange and Eulerian Almansi strain tensors. [7]

Q. 4: Suggest the approximate solutions (for u_0, v_0, w_0, θ_x, θ_y) for a 4-noded conforming rectangular plate element based on a higher order plate theory with through the thickness approximation of displacements as:

$$u(x,y,z) = u_0(x,y) + z\theta_x(x,y) - \frac{4}{3h^2}z^3\left(\theta_x + \frac{\partial w_0}{\partial x}\right),\quad v(x,y,z) = v_0(x,y) + z\theta_y(x,y) - \frac{4}{3h^2}z^3\left(\theta_y + \frac{\partial w_0}{\partial y}\right)$$

$$w(x,y,z) = w_0(x,y)$$

You may note that θ_x, θ_y are independent of w_0 and $\frac{\partial w_0}{\partial x}, \frac{\partial w_0}{\partial y}$ are appearing in the displacement approximation above. You need not derive the shape functions. Specify degrees of freedom at each node. [15]

Q. 5: The strain displacement relations for a curved beam element are given as:

$\varepsilon_{xx} = \frac{du_0}{dx} + w/R + z\frac{d\theta}{dx}$; $\gamma_{xz} = \frac{dw}{dx} + \theta$ where R (known constant) is radius of curvature and u_0, w, θ are functions of x. Thin and shallow beam elements depict membrane and shear locking. Membrane locking is due to membrane strain: $\frac{du_0}{dx} + w/R$ approaching to zero when beam becomes thin and shallow. Suggest different methods to eliminate membrane locking. You need to suggest suitable interpolation for u_0 and w to interpolate the membrane strain. [10]

Q. 6: Stress in a viscoelastic rod under uniaxial state of stress is given by: $\sigma_{xx} = \int_0^t E(t-t')\frac{d\varepsilon_{xx}}{dt'}dt'$

where $E(t) = E_\infty + \sum_{i=1}^{N} E_i\, e^{-\frac{t}{\tau_i}}$. Derive the relation between incremental stress $\Delta\sigma_{xx}$ (= $\sigma_{xx}(t+\Delta t) - \sigma_{xx}(t)$) and incremental strain $\Delta\varepsilon_{xx}$ (= $\varepsilon_{xx}(t+\Delta t) - \varepsilon_{xx}(t)$). [15]

Q. 7: Stresses at a point in a body under plane stress condition is given by: σ_{xx} = 250 MPa, σ_{yy} = 150 MPa, τ_{xy} = 100 MPa, $\sigma_{zz} = \tau_{xz} = \tau_{yz} = 0$. Obtain tangent constitutive matrix considering isotropic strain hardening, von Mises yield function and associative flow rule. The material properties are: E = 200 GPa, ν = 0, plastic modulus (H) = 200 MPa, initial yield stress (σ_{Y0}) = 200 MPa. [20]