Sub Code: AHT-003 **ROLL NO……………..……………..**

I SEMESTER EXAMINATION, 2022 – 23
Ist Year , B.Tech. –(Common to All Branch)
Introduction to Engineering Mathematics

Duration: 3:00 hrs **Max Marks: 100**

Note: - Attempt all questions. All questions carry equal marks. In case of any ambiguity or missing data, the same may be assumed and state the assumption made in the answer.

Q 1.	**Answer any four parts of the following.** a) Verify Roll's Theorem for the function $f(x)$ =log$\left(\frac{x^2+ab}{x(b+a)}\right)$ $in\ [a,b]$. b) Discuss the applicability of Lagrange's mean value theorem for the function $f(x) = \begin{cases} 2+x^2 & if\ x \leq 1 \\ 3x & if\ x > 1 \end{cases}$ on $[-1,2]$. c) If $x^x y^y z^z$ =c. Show that at $x = y = z$, $\frac{\partial^2 z}{\partial x \partial y}$ = - $(xlogex)^{-1}$. d) Find the minimum value of $x^2+y^2+z^2$, given that $ax+by+cz=p$. e) Find the first six terms of the expansion of the function e^x $log(1+\ y\ )$ in a Taylor series in the neighbourhood of the point (0,0). f) Expand $sinx$ in power of $\left(x-\frac{\pi}{2}\right)$. Hence find the value of $sin91^0$. Given $1^0 = 0.0174\,rad.$	**5x4=20**
Q 2.	**Answer any four parts of the following.** a) Prove that $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx$ = β(m,n). b) Change the order of integration in $\int_0^\infty \int_0^x xe^{-\frac{x^2}{y}}dydx$ and hence evaluate it. c) Change to polar coordinates and then evaluate $\iint \frac{x^2y^2dxdy}{x^2+y^2}$ over the annular region between circles $x^2+y^2=a^2$ and $x^2+y^2=b^2$; $a>b>0$. d) Trace the curve $y^2(a-x)=x^2(a+x)$ by giving all its features in detail. e) Find the possible percentage error in computing the parallel resistance r of three resistances $r_1,\ r_2, r_3$ from the formula $\frac{1}{r}=\ \frac{1}{r_1}+\ \frac{1}{r_2}+\ \frac{1}{r_3}$ if $r_1,\ r_2, r_3$ are each in error by + 1.2 %. f)Find the directional derivative of $\emptyset(x,y,z)=x^2yz+4xz^2$ at $(1,-2,1)$ in the direction of $2\hat{\imath}-\hat{\jmath}-2\hat{k}$.	**5x4=20**

Q 3.	**Answer any two parts of the following.** a) If $u^3+v^3+w^3=x+y+z$, $u^2+v^2+w^2=x^3+y^3+z^3$ and $u+v+w=x^2+y^2+z^2$, then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. b) Find the volume and surface generated by the revolution of the Astroid $x=a\cos^3\theta, y=a\sin^3\theta$, about the x – axis. c) A triangular thin plate with vertices $(0,0),(2,0)$, and $(2,4)$ has density $\rho=1+x+y$. Then find: (i) The mass of the plate (ii) The position of its centre of gravity G.	**10x2= 20**
Q 4.	**Answer any two parts of the following.** a)Show that the vector field $\vec{F}=\frac{\vec{r}}{\lvert\vec{r}\rvert^3}$ where $\vec{r}=x\hat{\imath}+y\hat{\jmath}+z\hat{k}$ is irrotational as well as solenoidal. Find the scalar potential $\emptyset$ such that $\vec{F}=\nabla\emptyset$. b) Apply Divergence Theorem to evaluate $\iint_S \vec{F}.\hat{n}ds$,where $\vec{F}=4x^3\hat{\imath}-x^2y\hat{\jmath}+x^2z\hat{k}$ and S is the surface of the cylinder $x^2+y^2=a^2$ bounded by the planes $z=0$ and $z=b$. c)Verify Stoke's theorem for $\vec{F}=(x^2+y^2)\vec{\imath}-2xy\vec{\jmath}$ taken around the rectangle bounded by the lines $x=\pm a, y=0$ and $y=b$.	**10x2= 20**
Q 5.	**Answer any two parts of the following.** a) Find the eigen values and the corresponding eigen vectors of the matrix $\begin{bmatrix}-2 & 5 & 4\\ 5 & 7 & 5\\ 4 & 5 & -2\end{bmatrix}$. b) Reduce the matrix $\begin{bmatrix}2 & 3 & -2 & 4\\ 3 & -2 & 1 & 2\\ 3 & 2 & 3 & 4\\ -2 & 4 & 0 & 5\end{bmatrix}$ to normal form and hence find the rank. c)Test the consistency of the system of equations and solve, if consistent: $x_1+2x_2-x_3-5x_4=4$ $x_1+3x_2-2x_3-7x_4=5$ $2x_1-x_2+3x_3=3$	**10x2= 20**

****