BACK PAPER

Sub Code: BAST203 **ROLL NO……..……..**

SECOND SEMESTER EXAMINATION, 2022 – 23

B.Tech, FIRST YEAR (All Braches)

MATHEMATICS-II

Duration: 3:00 hrs **Max Marks: 100**

Note: - Attempt all questions. All Questions carry equal marks. In case of any ambiguity or missing data, the same may be assumed and state the assumption made in the answer.

Q.1 Answer any four parts of the following. 5x4=20

a) Solve the differential equation

$$\sin^{-1}\left(\frac{dy}{dx}\right) = x + y\,.$$

b) Solve the differential equation

$$x\frac{dy}{dx} + y = y^2 \log x$$

c) Prove that $\sum_{n=0}^{\infty} P_n(x) = \frac{1}{\sqrt{2-2x}}$.

d) Test the convergence of the series $\frac{1}{2}+\frac{3}{4}+\frac{4}{9}+\frac{5}{16}+\ldots\ldots\ldots+\frac{n+1}{n^2}+\ldots\ldots\ldots$

e) Evaluate $\int \frac{e^z}{z^2+1}dz$ over the circular path $|z| = 2$.

f) Solve $25r - 40s + 16t = 0$.

Q.2 Answer any four parts of the following. 5x4=20

a) Solve $\left\{y\left(1+\frac{1}{x}\right)+\cos y\right\}dx + (x + \log x - x\sin y)dy = 0$.

b) Solve the differential equation $\left(D^6 - 1\right)y = 0$.

c) Using the recurrence relation, show that $4J_n''(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)$.

d) Find the singularities of the function $\frac{1}{\sin z - \cos z}$ at $z = \frac{\pi}{4}$.

e) Evaluate the integral $\int_c |z|dz$, where c is the straight line from $z = -i$ to $z = i$.

f) Solve $\left(D^4 - D'^4\right)z = 0$.

Q.3 Answer any two parts of the following. 10x2= 20

a) Solve the differential equation

$$\frac{d^2y}{dx^2} + y = x^2 \sin 2x .$$

b) Solve by the method of variation of parameters

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \sin x .$$

c) Prove that $\int_{-1}^{+1} x^2 P_n^2(x)dx = \frac{2(n+1)^2}{(2n+1)^2(2n+3)} + \frac{2n^2}{(2n+1)^2(2n-1)}$.

Q.4 Answer any two parts of the following. 10x2= 20

a) Solve $(D^2 - DD' - 2D)z = \sin(3x+4y) + x^2 y$.

b) Test for convergence the series $\frac{2^p}{1^q} + \frac{3^p}{2^q} + \frac{4^p}{3^q} + \ldots\ldots\ldots\ldots$ p and q being positive numbers.

c) Test for convergence the series

$$1 + \frac{3}{7}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.10.13}x^3 + \frac{3.6.9.12}{7.10.13.16}x^4 + \ldots\ldots\ldots$$

Q.5 Answer any two parts of the following. 10x2= 20

a) Using complex variable techniques evaluate real integral

$$\int_0^{2\pi} \frac{\sin^2\theta}{5 - 4\cos\theta} d\theta .$$

b) Evaluate the following integral using residue theorem

$$\int_C \frac{e^z}{(z^2 + \pi^2)^2} dz$$, where C is $|z| = 4$.

c) Let $f(z) = u(x,y) + iv(x,y)$ be an analytic function. If $u = 3x - 2xy$ then find v and corresponding $f(z)$ in terms of z .

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