

III SEMESTER EXAMINATION, 2023 – 24
IInd yr B.Tech. – (Civil Eng/CS&E/AI&ML/IT/EE/E&EE/ME/Robotics &Automation)
ADVANCE APPLIED MATHEMATICS

Duration: 3:00 hrs**Max Marks: 100**

Note: - Attempt all questions. All Questions carry equal marks. In case of any ambiguity or missing data, the same may be assumed and state the assumption made in the answer.

Q 1.	<p>Answer any four parts of the following.</p> <p>a) Find the Laplace transform of the function $F(t) = (\sin t - \cos t)^2$.</p> <p>b) Find the Fourier sine transform of $\frac{1}{x}$.</p> <p>c) Find the third divided with arguments 2,4,9,10 of the function $f(x) = x^3 - 2x$.</p> <p>d) The first three moments about the origin are given by</p> $\mu_1' = \frac{n+1}{2}, \mu_2' = \frac{(n+1)(2n+1)}{6}, \mu_3' = \frac{n(n+1)^2}{4}$ <p>Examine the skewness of the data.</p> <p>e) For a group of 20 items, $\sum x_i = 200$, $\sum x_i^2 = 500$ and median 1.5, find Karl person's coefficient of skewness.</p> <p>f) Find $\frac{dy}{dx}$ at $x = 0.1$ from the following data</p> <table><tr><td>x</td><td>0.1</td><td>0.2</td><td>0.3</td><td>0.4</td></tr><tr><td>y</td><td>0.9975</td><td>0.9900</td><td>0.9776</td><td>0.9604</td></tr></table>	x	0.1	0.2	0.3	0.4	y	0.9975	0.9900	0.9776	0.9604	5x4=20
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Q 2.	<p>Answer any four parts of the following.</p> <p>a) Find the inverse Laplace transform of $\frac{1}{9s^2 + 6s + 1}$.</p> <p>b) Find the Fourier cosine transform of $f(x) = e^{-ax}$</p> <p>c) Find the first four moments for the following individual data:</p> <table><tr><td>x</td><td>1</td><td>3</td><td>9</td><td>12</td><td>20</td></tr></table> <p>d) Find a root of the equation $x^3 - 4x - 9 = 0$. Using Bisection method is four stages.</p> <p>e) The first three moments of a distribution, about the value 2 of the variable are 1, 16 and -40. Show that the mean is 3, Variance is 15 and $\mu_3 = -86$.</p>	x	1	3	9	12	20	5x4=20				
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	f) Use Trapezoidal rule to evaluate $\int_0^1 x^3 dx$ considering five intervals.																															
Q 3.	<p>Answer any two parts of the following.</p> <p>a) State convolution theorem and hence find $L^{-1}\left[\frac{1}{s^3(s^2+1)}\right]$.</p> <p>b) Using Laplace transform, solve the following equation</p> $\frac{d^2x}{dt^2} + 9x = \cos 2t, \text{ if } x(0) = 1 \text{ and } x\left(\frac{\pi}{2}\right) = -1.$ <p>c) Use Fourier Sine transform to solve the equation $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$, under the condition</p> <p>(i) $u(0,t) = 0$ (ii) $u(x,0) = e^{-x}$ and (iii) $u(x,t)$ is bounded.</p>	10x2= 20																														
Q 4.	<p>Answer any two parts of the following.</p> <p>a) Solve $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$ for $x \geq 0, t \geq 0$ under the given condition $u = u_0$ at $x = 0, t > 0$ with initial condition $u(x,0) = 0, x \geq 0$.</p> <p>b) Find the root of the equation $\tan x + \tanh x = 0$, which lies in the interval (1.6, 3.0) correct to four significant digits using method of False position.</p> <p>c) The function $y = f(x)$ is given at the point (7,3), (8,1), (9,1) and (10,9). Find the value of y for $x = 9.5$, using Lagrange's interpolation formula.</p>	10x2= 20																														
Q 5.	<p>Answer any two parts of the following.</p> <p>a) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using (i) Simpson's one-third rule (ii) Simpson's three-eight rule (iii) Trapezoidal rule</p> <p>b) Calculate the coefficient of the Skewness from the following data:</p> <table border="1"><tr><td>Wage in rupees</td><td>0-10</td><td>10-20</td><td>20-30</td><td>30-40</td><td>40-50</td><td>50-60</td><td>60-70</td></tr><tr><td>No. of labours</td><td>185</td><td>77</td><td>34</td><td>180</td><td>136</td><td>23</td><td>50</td></tr></table> <p>c) Use least square method to fit a curve of the form $y = ae^{bx}$ to the following data:</p> <table border="1"><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>y</td><td>7.209</td><td>5.265</td><td>3.846</td><td>2.809</td><td>2.052</td><td>1.499</td></tr></table>	Wage in rupees	0-10	10-20	20-30	30-40	40-50	50-60	60-70	No. of labours	185	77	34	180	136	23	50	x	1	2	3	4	5	6	y	7.209	5.265	3.846	2.809	2.052	1.499	10x2= 20
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