Sub Code: BAST-105 **ROLL NO……………..……………..**

EVEN SEMESTER EXAMINATION, 2022 – 23

1st Year, B.Tech

MATHEMATICS-II

Duration: 3:00 hrs **Max Marks: 100**

Note: - Attempt all questions. All Questions carry equal marks. In case of any ambiguity or missing data, the same may be assumed and state the assumption made in the answer.

Q 1.	Answer any four parts of the following. a) Solve $x\frac{dy}{dx} + coty = 0 \quad given\ that\ y = \frac{\pi}{4}, where\ x = \sqrt{2}$. b) Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = Cos2x$. c) Solve $p^2 + 2pycotx - y^2 = 0$. d) Solve $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10(x + \frac{1}{x})$. e) Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x\,e^xCosx$. f) Solve $(1 + y^2)dx = (tan^{-1}y - x)dy$.	5x4=20
Q 2.	Answer any four parts of the following. a) Solve $\frac{d^2y}{dx^2} - \frac{2}{x}\frac{dy}{dx} + (n^2 + \frac{2}{x^2})y = 0$. b) Solve $\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} + 4x^2y = x^4$. c) Express $f(x) = x^3 - 5x^2 + x + 2$ in terms of Legendre's polynomials. d) Prove that $\int_{-1}^{1}[P_n(x)]^2dx = \frac{2}{2n+1}$. e) For Bessel's functions, show that $\frac{d}{dx}[x\,J_n(x)J_{n+1}(x)] = x[\{J_n(x)\}^2 - \{J_{n+1}(x)\}^2]$. f) Apply the method of variation of parameter to solve $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$.	5x4=20
Q 3.	Answer any two parts of the following. a) Solve $(D^3 - 7DD'^2 - 6\,D'^3)z = \sin(x + 2y) + e^{2x+y}$. b) (i) Solve $\frac{\partial^2z}{\partial x^2} + \frac{\partial^2z}{\partial x\partial y} - 6\frac{\partial^2z}{\partial y^2} = y\,Cosx$. (ii) $Form\ the\ partial\ differential\ equation\ from\ z = f_1(y + 2x) + f_2(y - 3x)$. c) (i) Solve the partial differential equation $p^2 + q^2 = n\,p\,q$. (ii) Solve the partial differential equation $p^2 + q^2 = x + y$.	10x2= 20
Q 4.	Answer any two parts of the following. a) (i) Test the convergence of the series $\sum_{n=1}^{\infty}\frac{1}{n}\,Sin\frac{1}{n}$. (ii) Test the convergence of the series $\sum_{n=1}^{\infty}\frac{3^n}{n^3}$. b) Discuss the convergence of the series $x + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + \frac{4^4x^4}{4!} + \cdots \ldots \ldots \ldots \ldots \ldots \ldots$	10x2= 20

	c) Find the half range cosine series to represent the function $f(x)$ *given by* $$f(x) = x\,, 0 \le x \le \pi.$$	
Q 5.	Answer any two parts of the following. a) Using contour integration method, evaluate $\int_0^{2\pi} \frac{cos2\theta}{5+4\,cos\theta}\,d\theta$. b) *Evaluate the integrals using Cauchy integral formula* (i) $\int_C \frac{4-3z}{z(z-1)(z-2)}\,dz$ *where* $\|z\| = \frac{3}{2}$. (ii) $\int_C \frac{z-1}{(z+1)^2\,(z-2)}\,dz$ *where* $\|z-i\| = 2$. c) *Find the values of* a *and* b *such that the function* $f(z) = x^2 + ay^2 - 2xy + i(bx^2 - y^2 + 2xy)$ *is analytic.*	10x2= 20

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