

SECOND SEMESTER EXAMINATION, 2022 – 23
B.Tech, FIRST Year (All Braches)
ANALYTICAL MATHEMATICS

Duration: 3:00 hrs

Max Marks: 100

Note: - Attempt all questions. All Questions carry equal marks. In case of any ambiguity or missing data, the same may be assumed and state the assumption made in the answer.

Q.1 Answer any four parts of the following.

5x4=20

a) Solve the differential equation

$$\frac{dy}{dx} = \frac{x(2 \log x + 1)}{\sin y + y \cos y}.$$

b) Solve $(D^2 - 6D + 9)y = 6e^{3x} + 7e^{-2x} - \log 2$.

c) Solve the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin x.$$

d) Show that the function $e^x(\cos y + i \sin y)$ is an analytic, find its derivative.e) Evaluate by Cauchy's integral formula $\int_C \frac{dz}{z(z + \pi i)}$, where C is circle $|z + 3i| = 1$.f) Examine the convergence of the series $\sum (\sqrt[3]{n^3 + 1} - n)$.

Q.2 Answer any four parts of the following.

5x4=20

a) Solve $y = px + \frac{q}{p}$ and obtain the singular solution.b) Solve the differential equation $\frac{d^2 y}{dx^2} - 4y = \cos^2 x$.c) Discuss singularity of $\frac{\cot \pi z}{(z - a)^2}$ at $z = a$ and $z = \infty$.d) Test for convergence the series whose n^{th} term is $\frac{2^n}{n^3}$.e) Expand for $f(x) = k$ for $0 < x < 2$ in a half range Sine series.f) Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \sin x \cos 2y$.

Q.3 Answer any two parts of the following.

10x2= 20

a) Solve the differential equation

$$\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x}.$$

b) Solve $(2xy^4 e^y + 2xy^3 + y)dx + (x^2 y^4 e^y - x^2 y^2 - 3x)dy = 0$.

c) Using Clairaut's method and solve the differential equation $y = px + \sqrt{a^2 p^2 + b^2}$.

Q.4 Answer any two parts of the following.

10x2= 20

a) Find the complete solution of the differential equation

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 2y = \sinh x + \sin \sqrt{2} x$$

b) Solve the method of variation of parameters

$$\frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} + 2y = e^{-x} \sec^3 x.$$

c) Solve the simultaneous differential equation

$$\begin{aligned} \frac{dx}{dt} + 2x - 3y &= t \\ \frac{dy}{dt} - 3x + 2y &= e^{2t} \end{aligned}$$

Q.5 Answer any two parts of the following.

10x2= 20

a) Solve completely the equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$, representing the vibration of a string of length l , fixed at both ends, given that $y(0, t) = 0$, $y(l, t) = 0$, $y(x, 0) = 0$ and $\frac{\partial}{\partial t} y(x, 0) = 0$, $0 < x < l$.

b) Find the sum of the residues of the function $f(z) = \frac{\sin z}{z \cos z}$ at its poles inside the circle $|z| = 2$.

c) Obtain the Fourier cosine series expansion of the periodic function defined by $f(t) = \sin\left(\frac{\pi t}{l}\right)$, $0 < t < l$.
