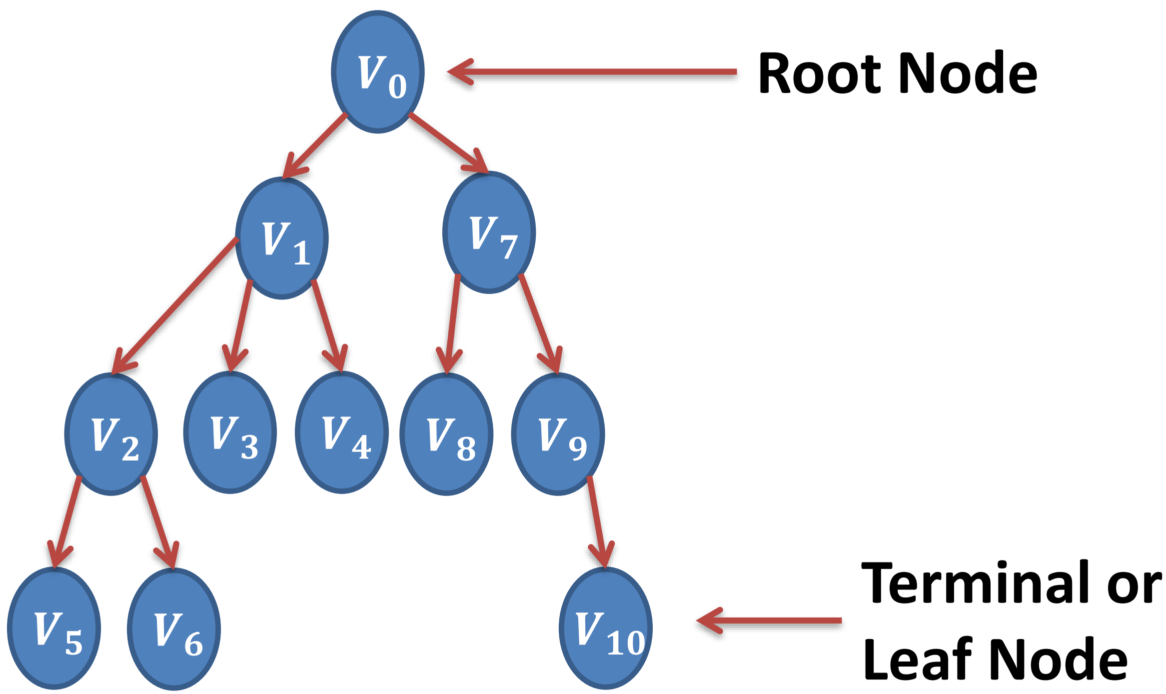
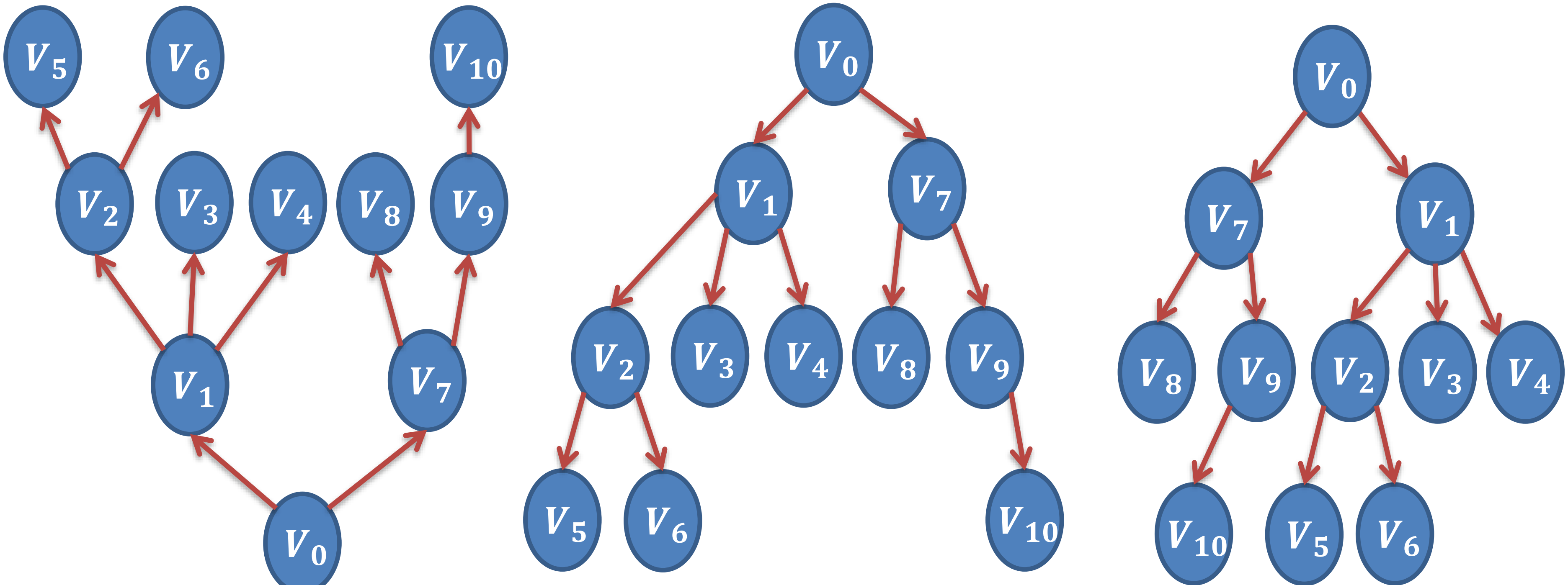
Tree– Concepts & Definitions

- **Directed Tree**
  - A directed tree is an acyclic digraph which has one node called its root with in degree 0, while all other nodes have in degree 1.
  - Every directed tree must have at least one node.
  - An isolated node is also a directed tree.



Tree– Concepts & Definitions



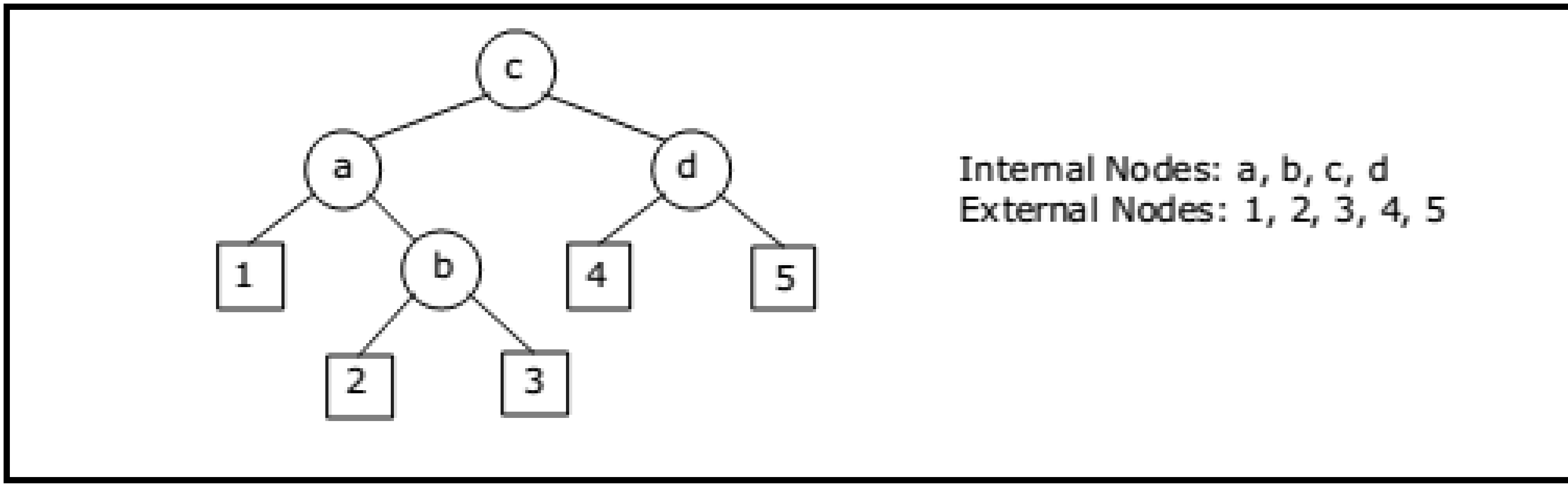
(a) (b) (c)

Tree– Concepts & Definitions

- **Terminal Node (Leaf Node)**
  - In a directed tree, any **node** which **has out degree 0** is called terminal node or leaf node.
- **Level of Node**
  - The level of any node is the length of its path from the root.
- **Ordered Tree**
  - In a directed tree an ordering of the nodes at each level is prescribed then such a tree is called ordered tree.
  - The diagrams (b) and (c) represents same directed tree but different ordered tree.
- **Forest**
  - If we delete the root and its edges connecting the nodes at level 1, we obtain a set of disjoint tree. A set of disjoint tree is a forest.

External and Internal node

- An external node is one without child branches, while an internal node has at least one child branch.

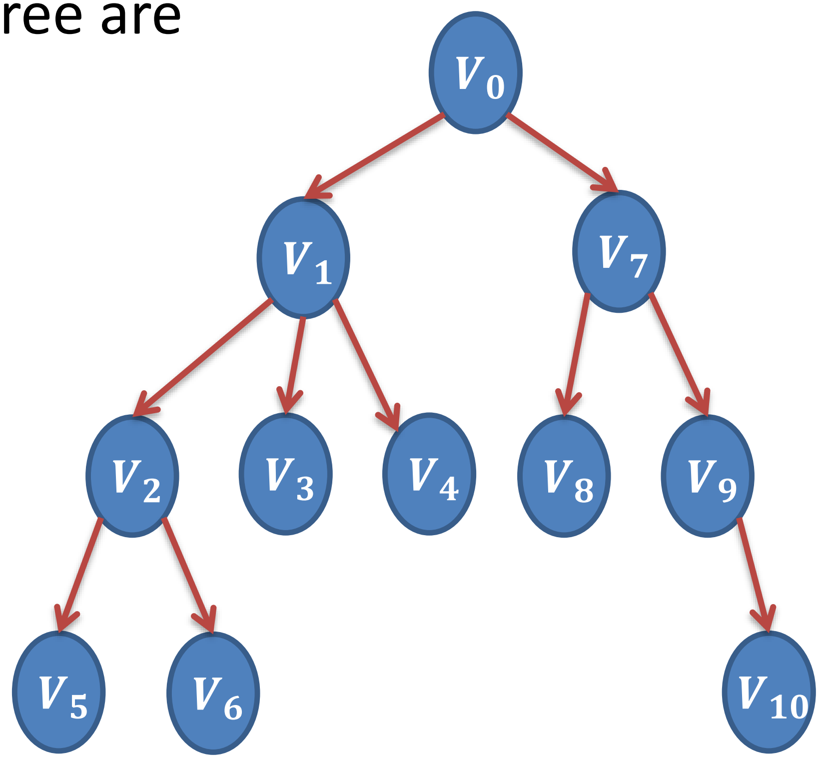


Tree– Concepts & Definitions

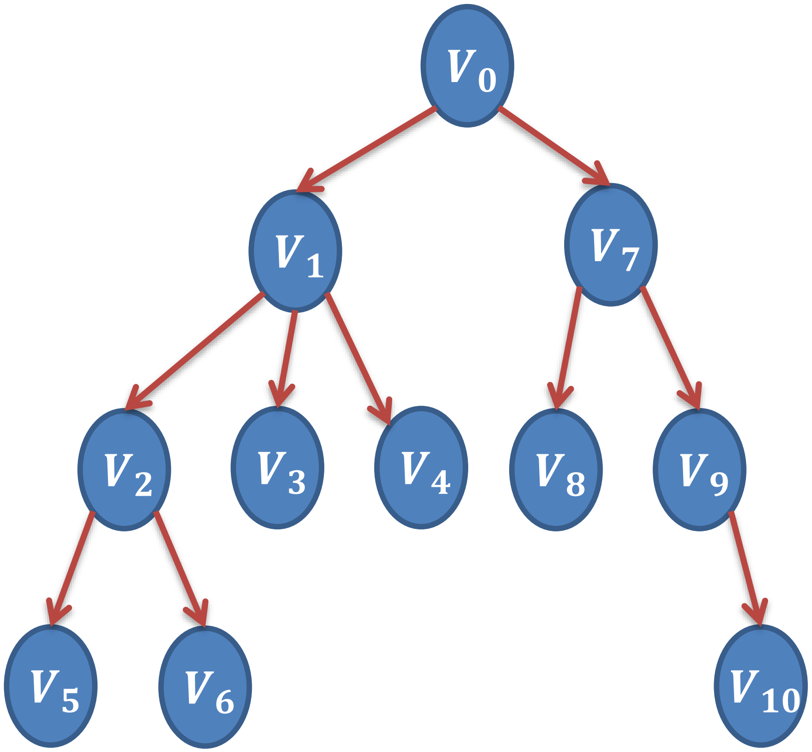
- ***Descendant node***
  - ***A descendant node is any successor node on any path from*** the node to a leaf node. Leaf nodes do not have any descendants. I
- ***Level number***
  - ***Every node in the tree is assigned a level number in such a way*** that the root node is at level 0, children of the root node are at level number 1. Thus, every node is at one level higher than its parent. So, all child nodes have a level number given by parent’s level number + 1.
- ***Degree***
  - ***Degree of a node is equal to the number of children that a node has.*** The degree of a leaf node is zero.
- ***In-degree***
  - ***In-degree of a node is the number of edges arriving at that node.***
- ***Out-degree***
  - ***Out-degree of a node is the number of edges leaving that node.***

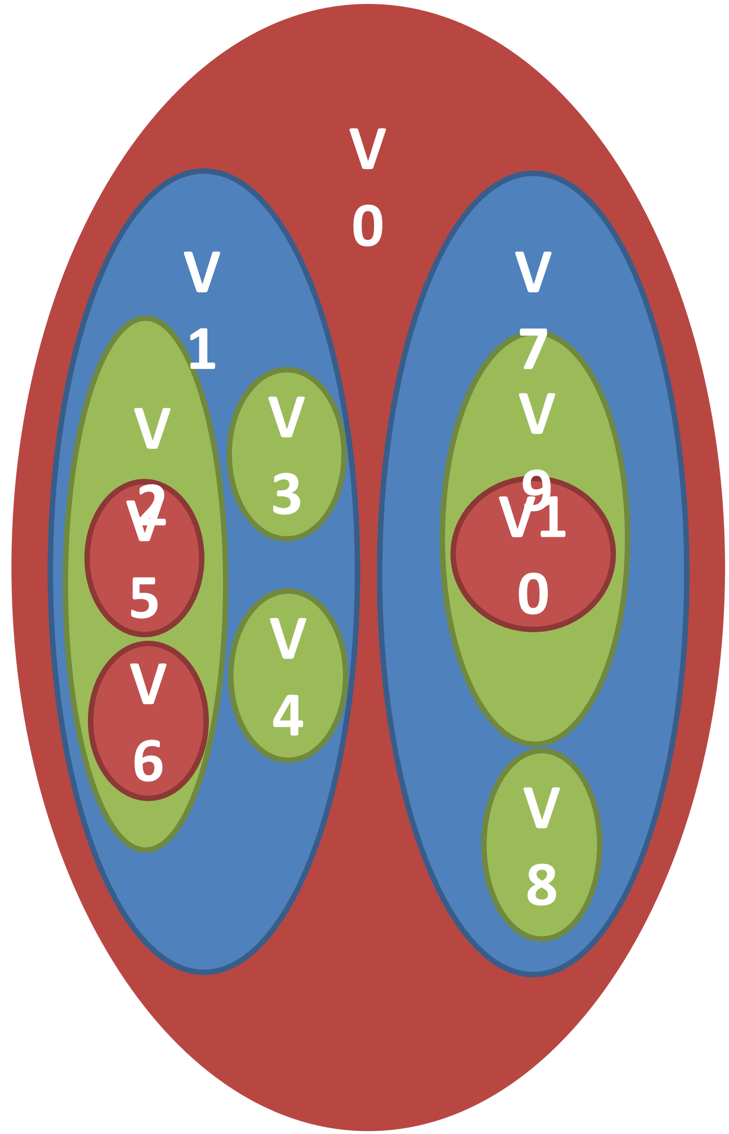
Representation of Directed Tree

- Other way to represent directed tree are
  - Venn Diagram
  - Nesting of Parenthesis
  - Like table content of Book
  - Level Format

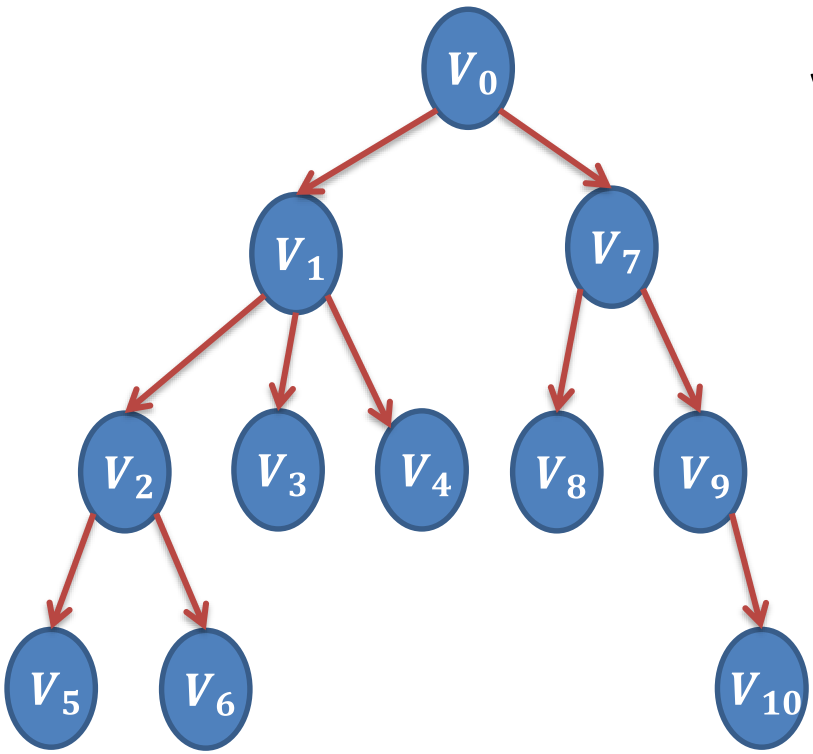


Venn Diagram

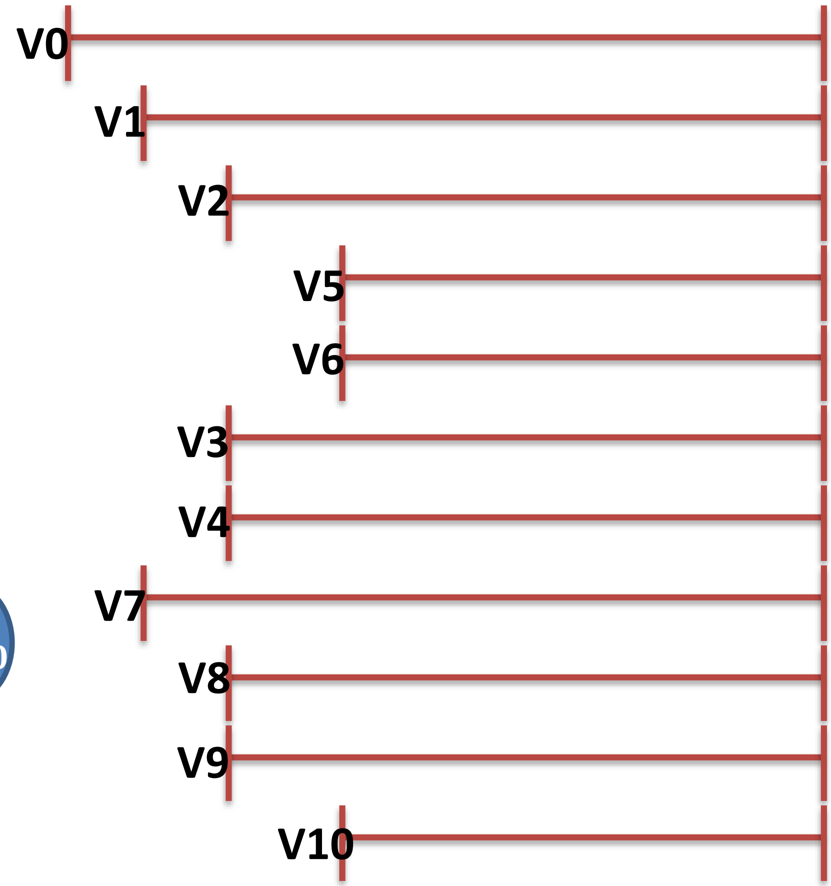




Nesting of Parenthesis



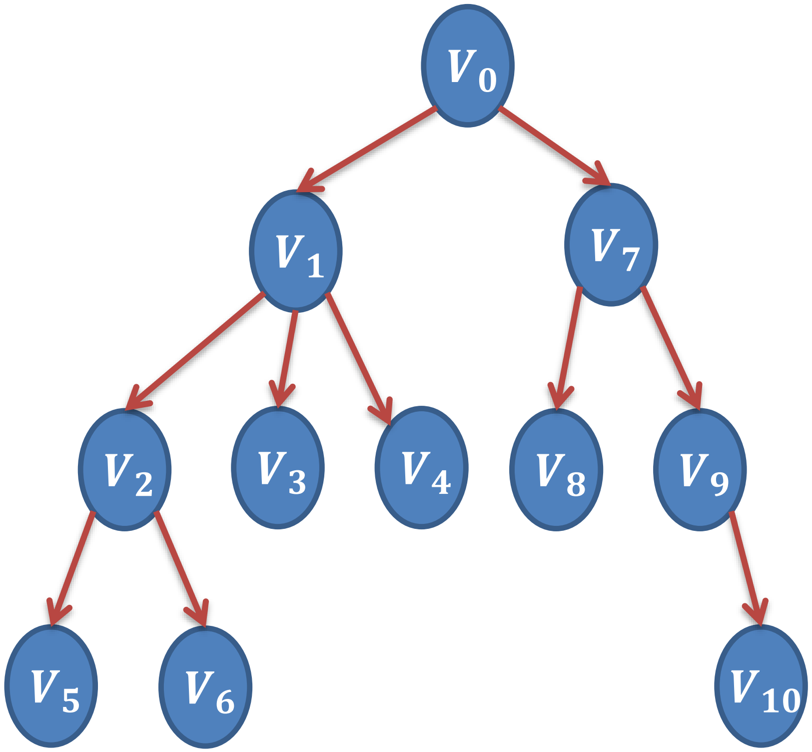
Like a table Content of Book



$(V_0(V_1(V_2(V_5)(V_6))\ (V_3)(V_4))\ (V_7(V_8)(V_9(V_{10}))\ )\ )$

Nesting of Parenthesis

Level Format



$1V_0$

$2V_1$

$3V_2$

$4V_5$

$4V_6$

$3V_3$

$3V_4$

$2V_7$

$3V_8$

$3V_9$

$4V_{10}$

TYPES OF TREES

- Trees are of following 6 types:

1. General trees
2. Forests
3. Binary trees
4. Binary search trees
5. Expression trees
6. Tournament trees

TYPES OF TREES

1. General trees

- General trees are data structures that store elements hierarchically. The top node of a tree is the root node and each node, except the root, has a parent.
- A node in a general tree (except the leaf nodes) may have zero or more sub-trees. General trees which have 3 sub-trees per node are called ternary trees.
- However, the number of sub-trees for any node may be variable. For example, a node can have 1 sub-tree, whereas some other node can have 3 sub-trees.

2. Forests

- A forest is a disjoint union of trees. A set of disjoint trees (or forests) is obtained by deleting the root and the edges connecting the root node to nodes at level 1.

TYPES OF TREES

3. Binary trees

- A binary tree is a data structure that is defined as a collection of elements called nodes.
- In a binary tree, the topmost element is called the root node, and each node has 0, 1, or at the most 2 children.
- A node that has zero children is called a leaf node or a terminal node. Every node contains a data element, a left pointer which points to the left child, and a right pointer which points to the right child.
- The root element is pointed by a 'root' pointer. If root = NULL, then it means the tree is empty.

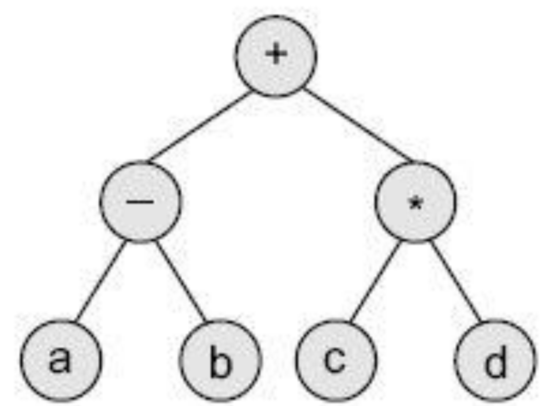
TYPES OF TREES

4. Binary search trees

- A binary search tree, also known as an ordered binary tree, is a variant of binary tree in which the nodes are arranged in an order.

5. Expression trees

- Binary trees are widely used to store algebraic expressions. For example, consider the algebraic expression given as:
- Exp = (a – b) + (c * d)



TYPES OF TREES

6. Tournament trees

- In a tournament tree (also called a *selection tree), each external node represents a player and* each internal node represents the winner of the match played between the players represented by its children nodes.
- There are 8 players in total whose names are represented using a, b, c, d, e, f, g, and h.
- In round 1, a and b; c and d; e and f; and finally g and h play against each other.
- In round 2, the winners of round 1, that is, a, d, e, and g play against each other.
- In round 3, the winners of round 2, a and e play against each other. Whosoever wins is declared the winner.
- In the tree, the root node a specifies the winner.

TYPES OF TREES

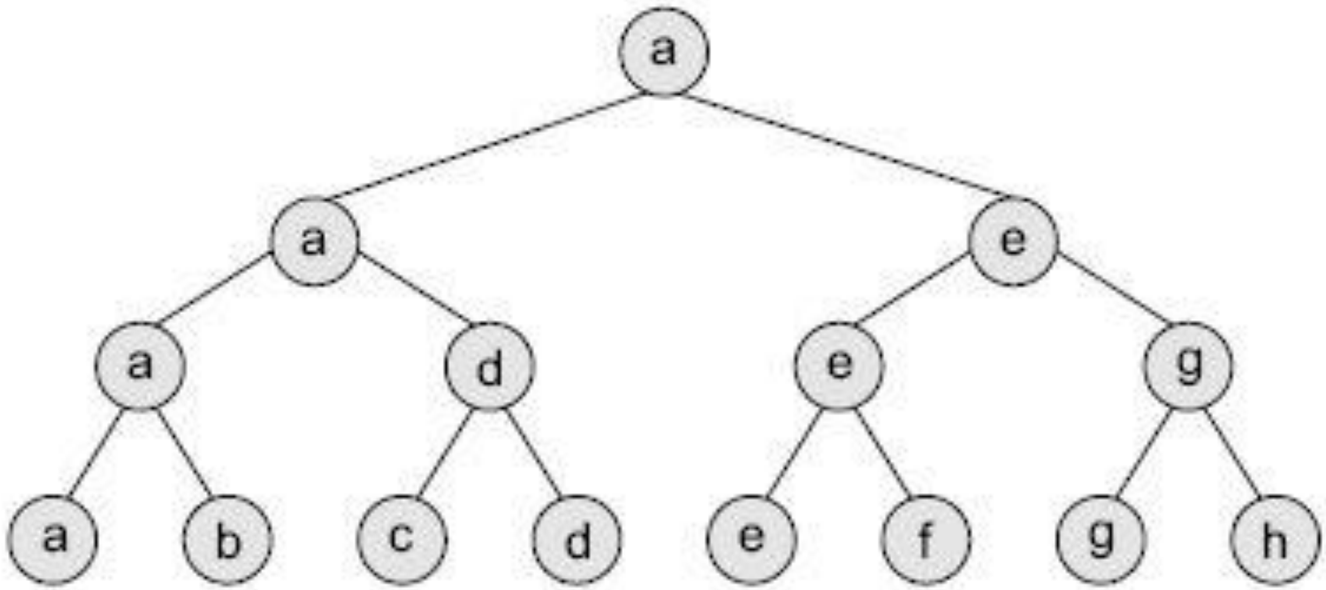


Figure 9.14 Tournament tree

Tree– Concepts & Definitions

- The node that is reachable from a node is called **descendant** of a node.
- The nodes which **are reachable from a node through a single edge** are called the **children of node**.
- **M-ary Tree**
  - If in a directed tree the **out degree of every node** is **less than or equal to m** then tree is called an m-ary tree.
- **Full or Complete M-ary Tree**
  - If **the out degree of each and every node** is **exactly equal to m or 0** and their **number of nodes at level i is m(i-1)** then the tree is called a full or complete m-ary tree.
- **Positional M-ary Tree**
  - If we consider m-ary trees in which the m children of any node are assumed to have m distinct positions, if such positions are taken into account, then tree is called positional m-ary tree.

Tree– Concepts & Definitions

Height of the tree

- The height of a tree is the length of the path from the root to the deepest node in the tree.

Binary Tree

- If in a directed tree the **out degree of every node** is **less than or equal to 2** then tree is called binary tree.

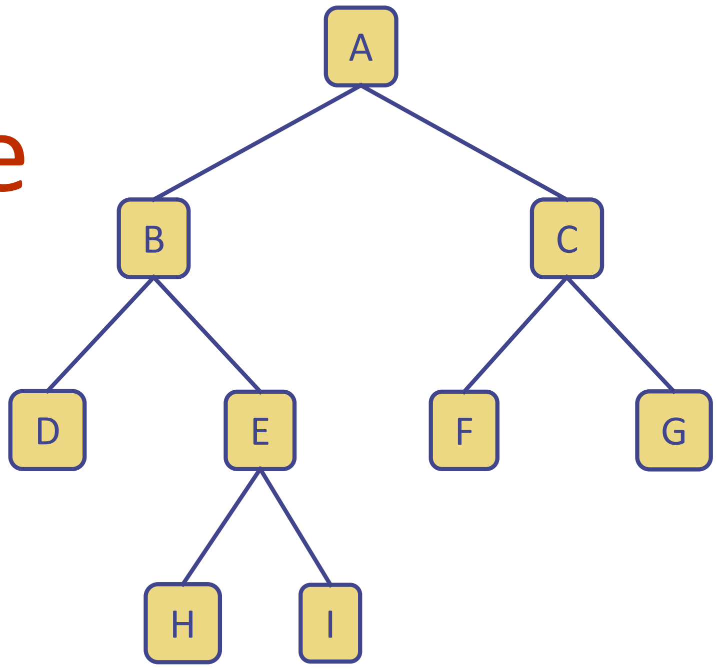
Strictly Binary Tree

- A strictly binary tree (sometimes proper binary tree or 2-tree or full binary tree) is a tree in **which every node other than the leaves has two children.**

Complete Binary Tree

- If the **out degree of each and every node is exactly equal to 2 or 0** and **their number of nodes at level i is 2(i-1)** then the tree is called a full or complete binary tree.

Extended Binary Tree



- A binary tree can be converted to an extended binary tree by adding new nodes that have only one child
- New nodes are added in such a way that all the nodes in resultant tree have either zero or 2 children.
- Also know as 2-Tree
- Original tree nodes are internal, added are external

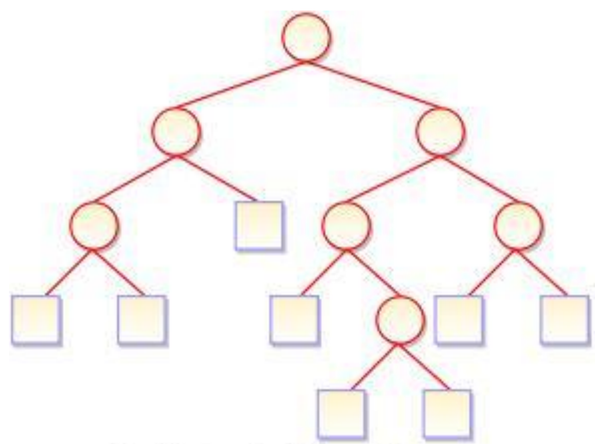
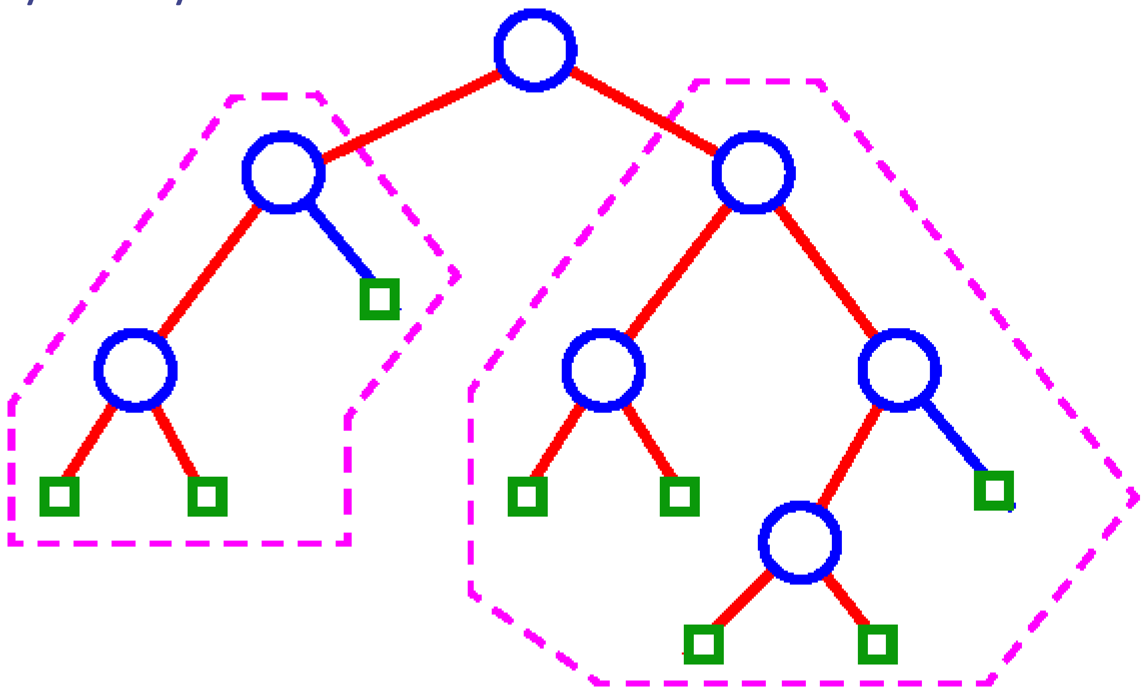


Fig. Extended Binary Tree

Binary Tree

- If a tree has n nodes, number of branches is n-1
- Only one parent to each node except root node
- A single path connects any two nodes
- For a binary tree of height h, the maximum number of nodes can be $2^{h+1} – 1$
- Any binary tree with n internal nodes has n+1 external nodes



Applications of Binary Tree

- Searching algorithm
- arithmetic expressions
- decision processes
- Populating voluminous, relational, hierarchical data into memory
- Expression evaluation
- AI

Binary Tree Creation: Recursive Algorithm

Create_Tree (Info, Node)

Step 1: [Check whether the tree is empty]

If Node = NULL

Node = Create a node

Left_child[Node] = NULL

Right_child[Node] = NULL

Step 2 : [Test for the left child]

If Info[Node] >= Info

Left_child[Node] = Call Create_Tree (Info, Left_Child[Node])

Else

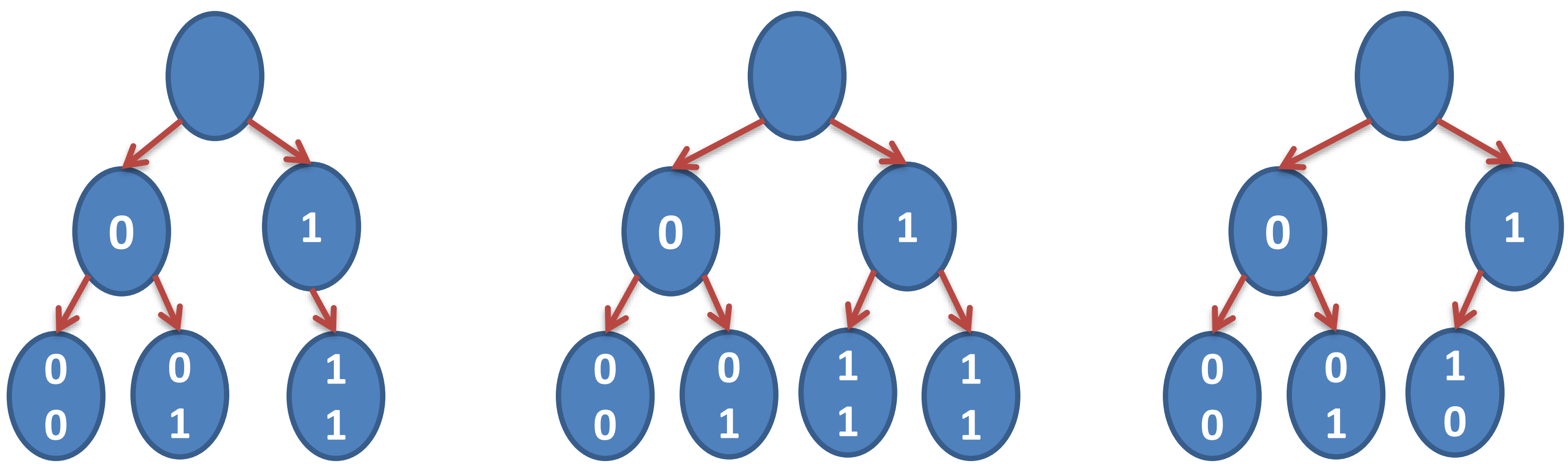
Right_child[Node] = Call Create_Tree (Info, Right_Child[Node])

Step 3:

Return (Node)

Tree– Concepts & Definitions

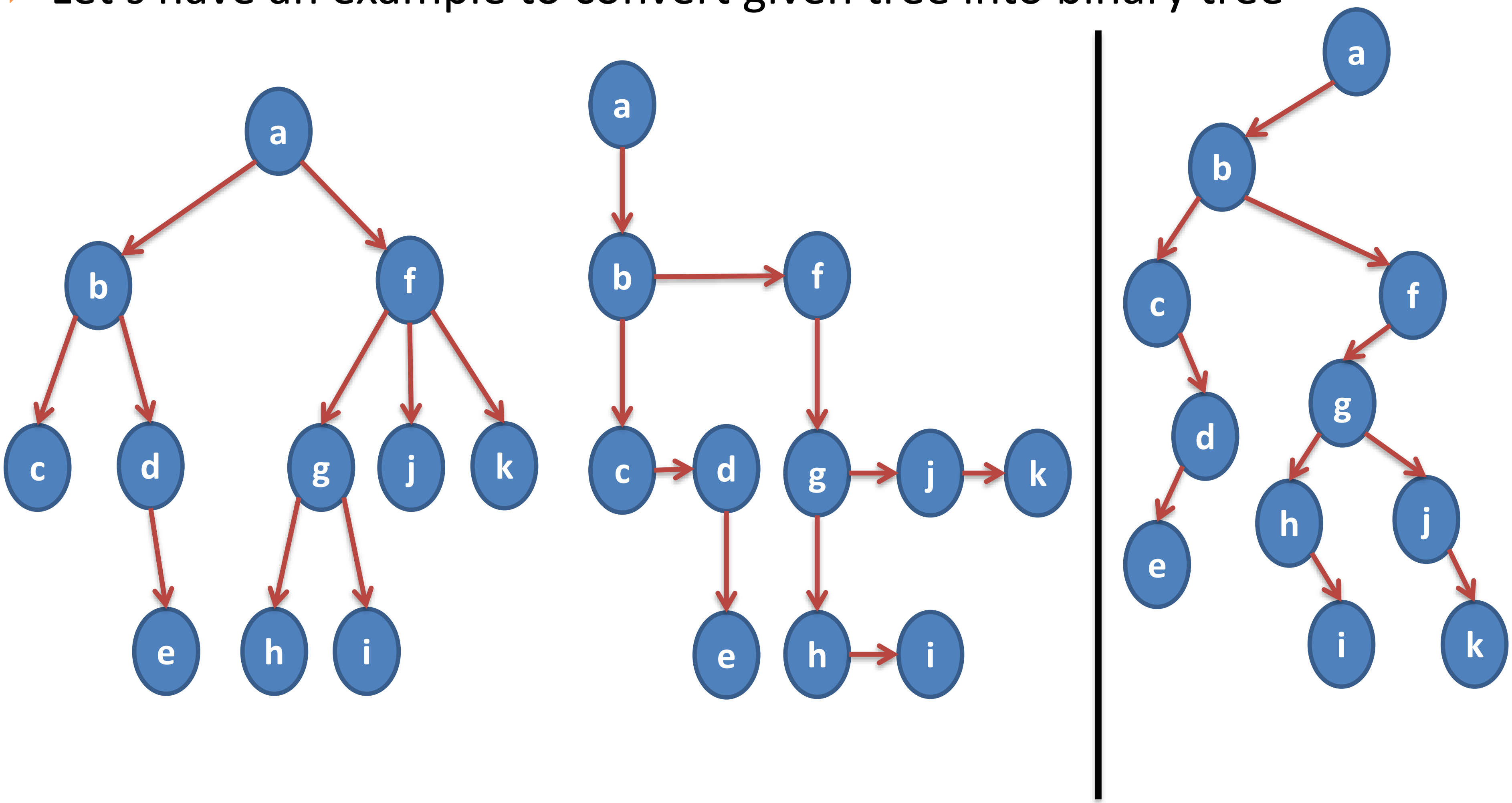
- **Sibling**
  - Siblings are nodes that share the same parent node
- **Positional m-ary Tree**
  - If we consider m-ary trees in which the **m children of any node** are assumed **to have m distinct positions**, if such positions are taken into account, then tree is called positional m-ary tree



(a) Binary tree **(b) Complete binary tree** **(c)**

Convert any tree to Binary Tree

- Every Tree can be Uniquely represented by binary tree
- Let’s have an example to convert given tree into binary tree



Convert Forest to Binary Tree