Simple Guide for Using VUMAT_LocalModel_HTC.for Subroutine in Enhanced Local Model

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1. Hsieh-Ting-Chen Equivalent Strain

The Hsieh–Ting–Chen failure criteria is four parameter criteria. It provides a smooth and convex representation of concrete strength under multiaxial stress states. The equivalent strain for Hsieh-Ting-Chen criterion is expressed as

$$\varepsilon_{eq} = \frac{1}{2k}\left(B_{HTC}\sqrt{\frac{J_2}{1+\nu}} + \frac{C_{HTC}\sigma_1}{E} + \frac{D_{HTC}I_1}{1-2\nu} + \sqrt{\left(B_{HTC}\sqrt{\frac{J_2}{1+\nu}} + \frac{C_{HTC}\sigma_1}{E} + \frac{D_{HTC}I_1}{1-2\nu}\right)^2 + 4A_{HTC}\frac{J_2}{(1+\nu)^2}}\right) \tag{1}$$

where I_1 and J_2 are the strain invariants, k is the ratio of compressive strength to tensile strength, and σ_1 is the maximum principal stress with a positive value and it is given as

$$\sigma_1 = \frac{1}{3}I_1 + \sqrt{\frac{4J_2}{3}}\cos 3\theta \tag{2}$$

To determine the four material constants $(A_{HTC}, B_{HTC}, C_{HTC}, D_{HTC})$ closed-form expressions are derived by enforcing the criterion at four representative stress states: (i) uniaxial compressive strength, f_c, (ii) uniaxial tensile strength, $f_t = 0.1 f_c$, (iii) triaxial compressive strength $f_{cc} = 4.2 f_c$, and (iv) shear to tension ratio $\tau_o = 1.35\, f_t$. The values of the four constants $A_{HTC}, B_{HTC}, C_{HTC}, D_{HTC}$ are 7.6924, 6.3689, 0.8252, and 5.2412 respectively.

Implementation in Abaqus

In the Abaqus finite element framework, user-defined material models are implemented through FORTRAN subroutines. The UMAT (User Material) subroutine allows the user to define a constitutive model by explicitly specifying the stress–strain relation, its consistent tangent operator $\partial\sigma/\partial\varepsilon$, and storage of history variables through the STATEV array. These state variables are crucial for tracking damage evolution and are accessible during post-processing for visualization of material degradation. A schematic pseudocode for the UMAT implementation of the enhanced local damage model is provided in Algorithm 1.

For explicit simulations, Abaqus provides the VUMAT interface. Unlike UMAT, the explicit solver employs the central difference time integration scheme, which directly updates the solution at each increment without requiring the consistent tangent operator. Consequently, in VUMAT, the computation of $\partial\sigma/\partial\varepsilon$ and the tangent stiffness matrix is no required. This simplification not only reduces computational cost but also enhances numerical robustness. The pseudocode for the explicit implementation of the enhanced local damage model is summarized in Algorithm 2.

Algorithm 1: UMAT Procedure for the Enhanced Local Damage Model Based on the Hsieh–Ting–Chen Equivalent Strain

Step 1: Compute elastic stiffness $\mathbb{C}$

Step 2: Update strain tensor: $\varepsilon_{n+1} = \varepsilon_n + \Delta\varepsilon$

Step 3: Evaluate the strain invariants $(I_1,\ J_2,\ J_3)$ and maximum principal stress (σ_1).

- $I_1 = tr(\varepsilon)$, $J_2 = \frac{1}{2} s : s$, $J_3 = \det(s)$
- $\sigma_1 = \frac{E}{3(1-2\nu)} I_1 + \frac{E}{(1+\nu)}\sqrt{\frac{4J_2}{3}}\cos 3\theta$; $\cos 3\theta = \frac{3\sqrt{3}}{2}\frac{J_3}{J_2^{3/2}}$
- where $s = \varepsilon - \frac{1}{3} I_1 \mathrm{I}$ and I is the identity matrix.

Step 4: Compute the Hsieh-Ting-Chen equivalent strain (ε_{eq}).

$$\varepsilon_{eq} = \frac{1}{2k}\left(B_{HTC}\sqrt{\frac{J_2}{1+\nu}} + \frac{C_{HTC}\sigma_1}{E} + \frac{D_{HTC}I_1}{1-2\nu} + \sqrt{\left(B_{HTC}\sqrt{\frac{J_2}{1+\nu}} + \frac{C_{HTC}\sigma_1}{E} + \frac{D_{HTC}I_1}{1-2\nu} \right)^2 + 4A_{HTC}\frac{J_2}{(1+\nu)^2}} \right)$$

Step 5: Update the scalar history variable:

$$\kappa_{n+1} = \max\left(\kappa_n, \varepsilon_{eq}\right).$$

Step 6: Evaluate the softening coefficient β.

$$\beta = \frac{E\kappa_o h_e}{G_f - \frac{1}{2}\kappa_o f_t h_e}$$

Step 7: Evaluate the damage variable, $\omega(\kappa)$ and its derivative:

- $$\omega(\kappa) = \begin{cases} 0, & if\, \kappa < \kappa_o \\ 1 - \frac{\kappa_o}{\kappa}\left[1 - \alpha \exp^{-\beta(\kappa - \kappa_o)}\right], & if\, \kappa \geq \kappa_o \end{cases}$$

- $$\frac{\partial \omega}{\partial \kappa} = \frac{\kappa_o}{\kappa}\left(\beta + \frac{1}{\kappa}\right)\exp\left(-\beta\left(\kappa - \kappa_o\right)\right)$$

Step 8: Compute the derivative of Hsieh-Ting-Chen equivalent strain with respect to strain:

$$\frac{\partial \varepsilon_{eq}}{\partial \varepsilon} = \frac{\partial \varepsilon_{eq}}{\partial I_1}\frac{\partial I_1}{\partial \varepsilon} + \frac{\partial \varepsilon_{eq}}{\partial J_2}\frac{\partial J_2}{\partial \varepsilon} + \frac{\partial \varepsilon_{eq}}{\partial \sigma_1}\frac{\partial \sigma_1}{\partial \varepsilon}$$

where,

- $$\frac{\partial \varepsilon_{eq}}{\partial I_1} = \frac{\omega}{1-2\nu}\left[1 + C_1 C_2^{-0.5}\right]$$

- $$\frac{\partial \varepsilon_{eq}}{\partial J_2} = \frac{B_{HTC} J_2^{-0.5}}{2(1+\nu)}\left[1 + C_1 C_2^{-0.5}\right] + \frac{2AC_2^{-0.5}}{(1+\nu)^2}$$

- $$\frac{\partial \varepsilon_{eq}}{\partial \sigma_1} = \frac{C_{HTC}}{E}\left[1 + C_1 C_2^{-0.5}\right]$$

- $\frac{\partial I_1}{\partial \varepsilon_{ij}} = \delta_{ij}$, $\frac{\partial J_2}{\partial \varepsilon_{ij}} = s_{ij}$, $\frac{\partial J_3}{\partial \varepsilon_{ij}} = \left(s^2\right)_{ij} - \frac{1}{3}tr\left(s^2\right)\delta_{ij}$

- $$\frac{\partial \sigma_1}{\partial \varepsilon_{ij}} = \frac{1}{3}\delta_{ij} + \frac{\cos\theta}{\sqrt{6J_2}}\frac{\partial J_2}{\partial \varepsilon_{ij}} + \sqrt{\frac{2J_2}{3}}\frac{\partial \cos 3\theta}{\partial \varepsilon_{ij}}$$

- $$\frac{\partial \cos 3\theta}{\partial \varepsilon_{ij}} = \frac{3\sqrt{3}}{2}\left[J_2^{-3/2}\frac{\partial J_3}{\partial \varepsilon_{ij}} - \frac{3}{2}J_3 J_2^{-5/2}\frac{\partial J_2}{\partial \varepsilon_{ij}}\right]$$

- $$C_1 = B_{HTC}\sqrt{\frac{J_2}{1+\nu}} + \frac{C_{HTC}\sigma_1}{E} + \frac{D_{HTC} I_1}{1-2\nu}$$

- $C_2 = \left(B_{HTC}\sqrt{\frac{J_2}{1+\nu}} + \frac{C_{HTC}\sigma_1}{E} + \frac{D_{HTC}I_1}{1-2\nu} \right)^2 + 4A_{HTC}\frac{J_2}{(1+\nu)^2}$

Step 9: Compute the derivative of the damage variable with respect to strain:

$$\frac{\partial \omega}{\partial \varepsilon} = \frac{\partial \omega}{\partial \kappa}\frac{\partial \kappa}{\partial \varepsilon_{eq}}\frac{\partial \varepsilon_{eq}}{\partial \varepsilon}$$

Step 10: Update the stress tensor considering stiffness degradation:

$$\sigma_{n+1} = (1-\omega_{n+1})\mathbb{C} : \varepsilon_{n+1}$$

Step 11: Compute the consistent tangent stiffness matrix:

$$DDSDDE = \frac{\partial \sigma}{\partial \varepsilon} = \left[(1-\omega_{n+1})\mathbb{C} - (\mathbb{C} : \varepsilon_{n+1}) \otimes \frac{\partial \omega}{\partial \varepsilon} \right]$$

Algorithm 2: VUMAT Procedure for the Enhanced Local Damage Model Based on the Hsieh–Ting–Chen Equivalent Strain

Step 1: Update strain tensor: $\varepsilon_{n+1} = \varepsilon_n + \Delta\varepsilon$

Step 2: Evaluate the strain invariants (I_1, J_2, J_3) and maximum principal stress (σ_1).

- $I_1 = tr(\varepsilon)$, $J_2 = \frac{1}{2}s : s$, $J_3 = \det(s)$

- $\sigma_1 = \frac{E}{3(1-2\nu)}I_1 + \frac{E}{(1+\nu)}\sqrt{\frac{4J_2}{3}}\cos 3\theta$; $\cos 3\theta = \frac{3\sqrt{3}}{2}\frac{J_3}{J_2^{3/2}}$

where $s = \varepsilon - \frac{1}{3}I_1\mathrm{I}$ and I is the identity matrix.

Step 3: Compute the Hsieh-Ting-Chen equivalent strain (ε_{eq}).

$$\varepsilon_{eq} = \frac{1}{2k}\left(B_{HTC}\sqrt{\frac{J_2}{1+\nu}} + \frac{C_{HTC}\sigma_1}{E} + \frac{D_{HTC}I_1}{1-2\nu} + \sqrt{\left(B_{HTC}\sqrt{\frac{J_2}{1+\nu}} + \frac{C_{HTC}\sigma_1}{E} + \frac{D_{HTC}I_1}{1-2\nu} \right)^2 + 4A_{HTC}\frac{J_2}{(1+\nu)^2}} \right)$$

Step 4: Update the scalar history variable:

$$\kappa_{n+1} = \max(\kappa_n, \varepsilon_{eq}).$$

Step 5: Evaluate the softening coefficient β.

$$\beta = \frac{E\kappa_o h_e}{G_f - \frac{1}{2}\kappa_o f_t h_e}$$

Step 6: Evaluate the damage variable, $\omega(\kappa)$.

$$\omega(\kappa)=\begin{cases} 0, & if\,\kappa<\kappa_o \\ 1-\dfrac{\kappa_o}{\kappa}\left[1-\alpha\exp^{-\beta(\kappa-\kappa_o)}\right], & if\,\kappa\geq\kappa_o \end{cases}$$

Step 7: Update the stress tensor considering stiffness degradation:

$$\sigma_{n+1}=(1-\omega_{n+1})\left[\lambda tr(\varepsilon)\mathrm{I}+2\mu\varepsilon\right]$$

Where, I is the identity tensor and λ, μ are the Lame constant and it can be written in terms of young's modulus (E) and poisson's ratio (ν) as:

$$\lambda=\frac{E\nu}{(1+\nu)(1-2\nu)}, \qquad \mu=\frac{E}{2(1+\nu)}$$

Step 7: Compute the new internal energy per unit mass:

$$enerInternNew=\frac{1}{2\rho}(\sigma:\varepsilon)$$

Where ρ is the density of the material.

2. Files

File Name	Description
`UMAT_LocalModel_HTC.for`	An implicit time integration user defined material subroutine implementing the Enhanced Local Model for 2D and 3D analyses using the Hsieh-Ting-Chen equivalent strain formulation.
`VUMAT_LocalModel_HTC_Exponential.for`	An explicit user-defined material subroutine implementing an enhanced local damage model for 2D and 3D analyses based on the Hsieh–Ting–Chen equivalent strain formulation with exponential softening.
`VUMAT_LocalModel_HTC_Bilinear.for`	An explicit user-defined material subroutine implementing an enhanced local damage model for 2D and 3D analyses based on the

	Hsieh–Ting–Chen equivalent strain formulation with bilinear softening.
`VUMAT_LocalModel_HTC_Linear.for`	An explicit user-defined material subroutine implementing an enhanced local damage model for 2D and 3D analyses based on the Hsieh–Ting–Chen equivalent strain formulation with linear softening.
`UniaxialTension_UMAT.inp`	Abaqus Implicit time integration input file for 2D uniaxial tension simulation
`UniaxialTension_VUMAT.inp`	Abaqus Explicit time integration input file for 2D uniaxial tension simulation

3. Material Model Definition

The constitutive behavior of the material is defined through six user-specified parameters in the UMAT/VUMAT subroutine:

Symbol	Description
E	Young's modulus
ν	Poisson's ratio
k	Ratio of compressive to tensile strength
κ_o	Damage initiation threshold
f_t	Tensile strength
G_f	Fracture energy

4. State Variables (DEPVAR)

The number and definition of state-dependent variables (`DEPVAR`) vary depending on the analysis type, as summarized below:

Case1: An implicit user-defined material (UMAT) subroutine

Analysis Type	DEPVAR	State Variables
Plane Stress	3	$\kappa, D, \partial D/\partial \varepsilon$

Plane Strain	3	$\kappa, D, \partial D/\partial \varepsilon$
3D	3	$\kappa, D, \partial D/\partial \varepsilon$

Case2: An explicit user-defined material (VUMAT) subroutine

Analysis Type	**DEPVAR**	**State Variables**
Plane Stress	5	$\kappa, D, \varepsilon_{11}, \varepsilon_{22}, \varepsilon_{12}$
Plane Strain	6	$\kappa, D, \varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, \varepsilon_{12}$
3D	8	$\kappa, D, \varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, \varepsilon_{12}, \varepsilon_{23}, \varepsilon_{13}$

5. Running a simulation

To execute a simulation using the `UMAT_LocalModel_HTC.for` subroutine, proceed as follows:

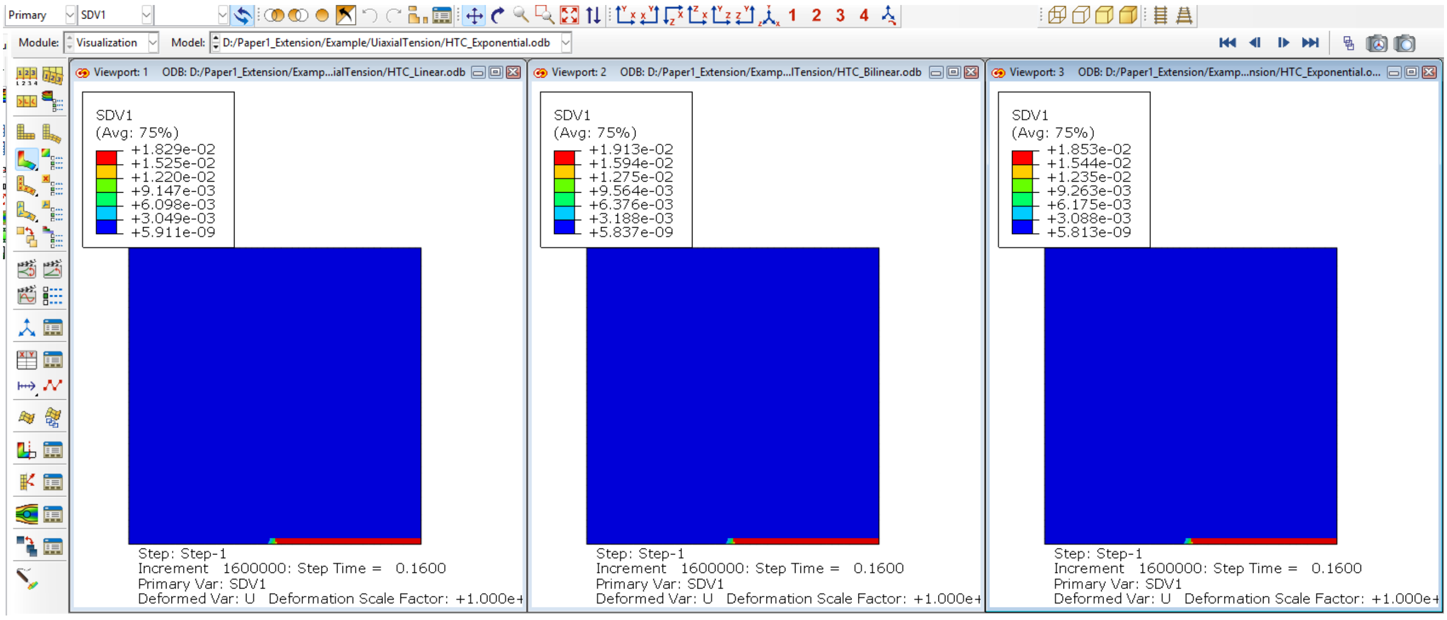
1. Ensure that both the Fortran subroutine (`UMAT_LocalModel_HTC.for`) and the relevant Abaqus input file (e.g., `UniaxialTension_UMAT.inp`) are placed within the same working directory.
2. Open the **Abaqus Command Window**.
3. Execute the following command:

```
abaqus -job UniaxialTension_UMAT -user  UMAT_LocalModel_HTC.for
```

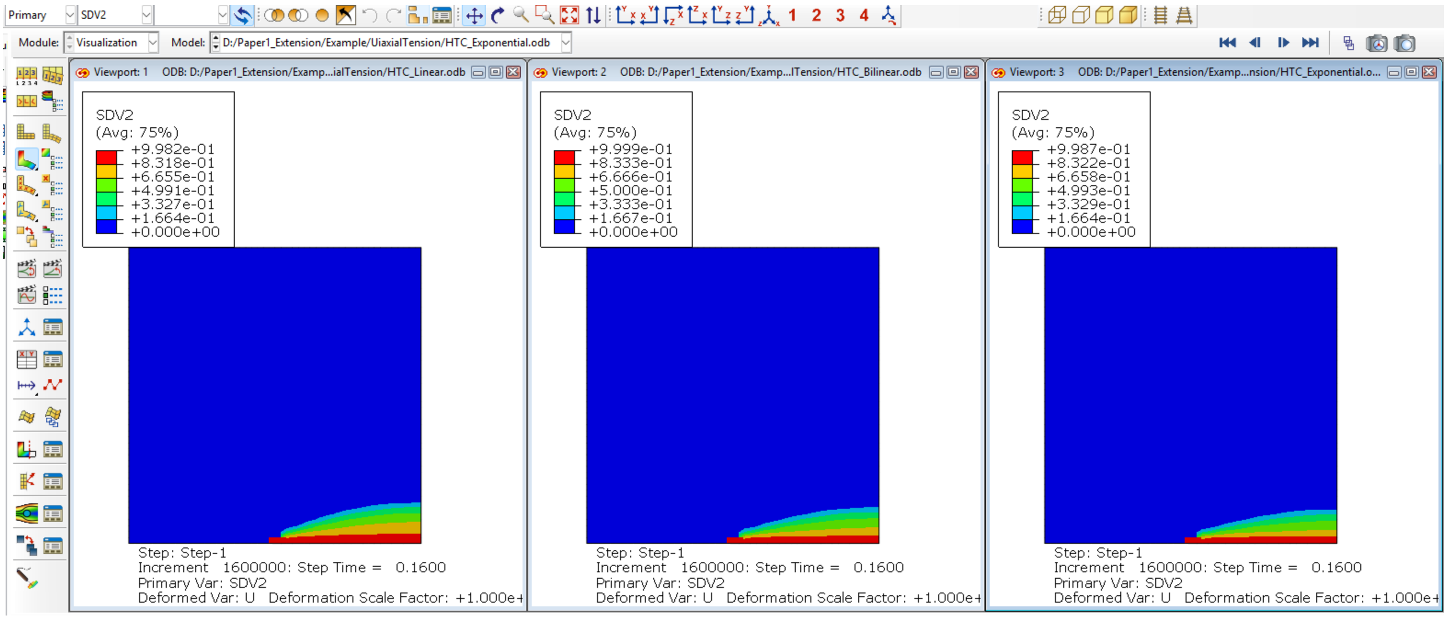
6. Post Processing

Upon completion of the simulation, open the output database (ODB) file in Abaqus/Viewer to examine and interpret the results.

i. Equivalent strain (κ): (Stored as SDV1)



ii. Damage: (Stored as SDV2)



iii. Load-displacement curve