N-body simulations with Jordan-Brans-Dicke

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Introduction

The N-body equations are quite simple once we use some approximations. Note that there is no problem not performing these approximation (apart from the quasi-static one) if desirable. The full equation set for a very similar model has been derived before, see e.g. Li et al. [1] - the JBD model corresponds to taking $f(\varphi) = \frac{\kappa^2\varphi^2}{4\omega} - 1$ and redefining $\frac{\kappa^2\varphi^2}{4\omega} \to \varphi$. Below I will give the equations in their simplest form with all approximations included. Using same notation as in Lima & Ferreira [2] and Avilez & Skordis [3][1].

Approximations

The main approximations used is that perturbations of ϕ will always be small. Splitting $\phi = \overline{\phi} + \delta\phi$ then in the quasi-static approximation we have

$$\frac{1}{a^2}\nabla\delta\phi \simeq \frac{\kappa^2}{2(3+2\omega)}\delta\rho_m$$

so $\delta\phi \simeq \frac{1}{\phi(3+2\omega)}\Phi_N$ where Φ_N is the standard Newtonian gravitational potential in GR. In a cosmological simulation we have $\Phi_N \lesssim 10^{-5}$ and also since $\omega \gtrsim 100$ this means we can neglect terms $\phi_{,\mu}\phi_{,\nu}$ relative to ρ_m in the Einstein equation.

N-body equations

The 00-component of the Einstein equation

$$G_{\mu\nu} = \frac{\kappa^2}{\phi}T_{\mu\nu} + \frac{\omega}{\phi^2}(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}g_{\mu\nu}(\partial\phi)^2) + \frac{1}{\phi}(\phi_{,\nu;\mu} - g_{\mu\nu}\Box\phi) - \frac{\kappa^2\Lambda}{\phi}$$

gives us the Poisson equation

$$\nabla^2\Phi \simeq \frac{\kappa^2}{2}a^2\delta\rho_m \cdot \frac{1}{\overline{\phi}}\frac{4+2\omega}{3+2\omega}$$

[1]References:
[1]: https://arxiv.org/pdf/1009.1400.pdf
[2]: https://arxiv.org/pdf/1506.07771.pdf
[3]: https://arxiv.org/pdf/1303.4330.pdf

The N-body (geodesic) equation becomes

$$\ddot{x} + 2H\dot{x} = -\frac{1}{a^2}\nabla\Phi$$

Thus the main modification is the presence of an effective (time-dependent) gravitational constant instead of G in the Poisson equation

$$\frac{G_{\rm eff}}{G} = \frac{1}{\overline{\phi}}\frac{4+2\omega}{3+2\omega}$$

The field should be normalized such that $\frac{\kappa^2}{\overline{\phi}_0}\frac{4+2\omega}{3+2\omega} = 8\pi G_N$. If $\kappa^2 = 8\pi G_N$ then we simply take $\overline{\phi}_0 \equiv \frac{4+2\omega}{3+2\omega}$.

Background cosmology and evolution of ϕ

The background cosmology and ϕ is determined by

$$E^2(a)\left(1 + \frac{d\log\overline{\phi}}{d\log a} - \frac{\omega}{6}\left(\frac{d\log\overline{\phi}}{d\log a}\right)^2\right) = \frac{\phi_0}{\overline{\phi}}\left[\frac{\Omega_{m0}}{a^3} + \frac{\Omega_{r0}}{a^4} + \Omega_{\Lambda 0}\right]$$

$$E(a)\frac{d}{d\log a}\left[E(a)a^3\frac{d\log\overline{\phi}}{d\log a}\frac{\overline{\phi}}{\phi_0}\right] = \frac{3a^3}{3+2\omega}\left[\frac{\Omega_{m0}}{a^3} + 4\Omega_{\Lambda 0}\right]$$

where $\Omega_{\Lambda 0} = 1 + \frac{d\log\phi_0}{d\log a} - \frac{\omega}{6}\left(\frac{d\log\phi_0}{d\log a}\right)^2 - \Omega_{m0} - \Omega_{r0}$ and $\phi_0 = \frac{4+2\omega}{3+2\omega}$. We solve these by starting the integration deep inside in the radiation era with the initial conditions ϕ_i and $\frac{d\log\phi_i}{d\log a} = 0$ and tune the value of ϕ_i such that $\phi(a=1) = \phi_0$ (this is what Avilez & Skordis called restricted Brans-Dicke models). The value of $\frac{d\log\phi_0}{d\log a}$ needed to determine $\Omega_{\Lambda 0}$ is set to $\frac{1}{1+\omega}$ and corrected at every step until we have convergence (the true value is roughly $\simeq (1.9 - 1.1\Omega_{m0})/(1+\omega)$ for realistic values of Ω_{m0}). See the plots below for the results when $\omega = 50$. The results I find gives a reasonable match the Fig. 1 in Lima & Ferreira (though $\phi(a=1) = \phi_0$ is not imposed there).

Initial conditions

The IC are generated using second order lagrangian perturbation theory. We need a $P(k)$ or $T(k)$ from CAMB or similar at $z = 0$. This is then scaled backwards to the initial redshift using the (1LPT) growth-factor which is determined by

$$\ddot{D} + 2H\dot{D} = \frac{3}{2}\frac{\Omega_{m0}}{a^3H^2}\frac{\phi_0}{\overline{\phi}}D$$

For 2LPT it should be enough to simply use the EdS approximation $D_2 = -\frac{3}{7}D_1^2$.

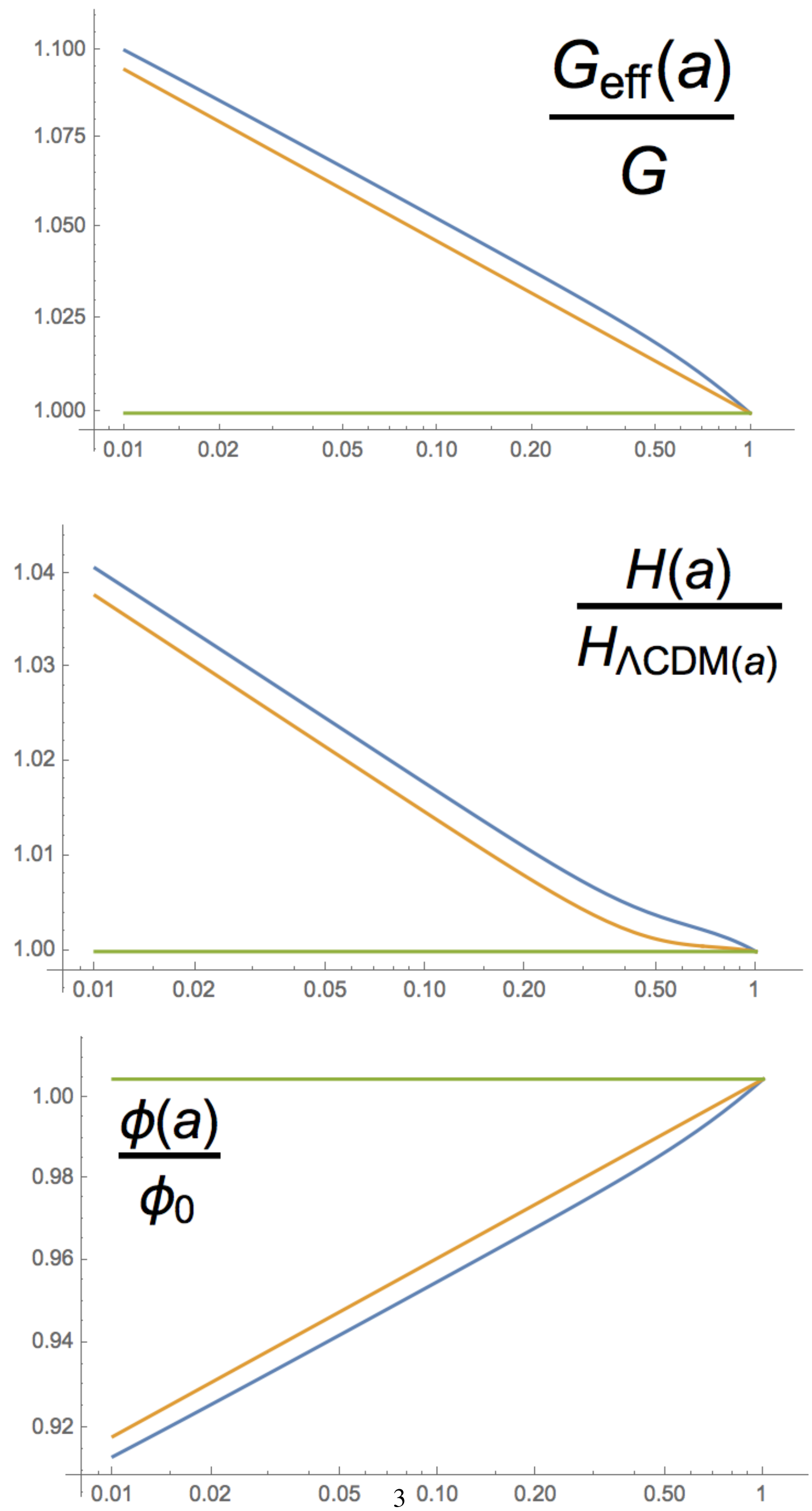

Figure 1: The evolution of $\phi(a)$, $H(a)/H_{\Lambda CDM}(a)$ and $\frac{G_{\rm eff}(a)}{G}$ for $\omega = 50$, $\Omega_{m0} = 0.3$ and $\Omega_{r0} = 8.4 \cdot 10^{-5}$. The orange lines shown the results of using the approximation $\phi(a) = \phi_0 a^{\frac{1}{1+\omega}}$ and the blue lines are the full numerical solution. The error of using this approximation is $\sim 0.5\%$ for $\omega = 50$ and drops to 0.05% for $\omega = 500$.

Summary

The equation we solve in the *N*-body:

$$\ddot{x} + 2H\dot{x} = -\frac{1}{a^2}\frac{\phi_0}{\overline{\phi}}\nabla\Phi_N, \qquad \nabla^2\Phi_N = \frac{3}{2}\Omega_{m0}H_0^2 a^{-1}\delta_m$$

where the background is determined by:

$$E^2(a)\left(1 + \frac{d\log\overline{\phi}}{d\log a} - \frac{\omega}{6}\left(\frac{d\log\overline{\phi}}{d\log a}\right)^2\right) = \frac{\phi_0}{\overline{\phi}}\left[\frac{\Omega_{m0}}{a^3} + \Omega_{\Lambda 0}\right]$$

$$E(a)\frac{d}{d\log a}\left[E(a)a^3\frac{d\log\overline{\phi}}{d\log a}\frac{\overline{\phi}}{\phi_0}\right] = \frac{3a^3}{3+2\omega}\left[\frac{\Omega_{m0}}{a^3} + 4\Omega_{\Lambda 0}\right]$$

and these are solved with initial conditions set in the deep radiation era with $\frac{d\log\phi}{d\log a} = 0$ and ϕ_i tuned as to give $\phi(a=1) = \phi_0$ implying $\frac{G_{\rm eff}}{G} = 1$ at the present time.