



The Derivative and Differentiability

- ▶ **Video:** Introduction to the Week
2 min
- ▶ **Video:** Derivative: Definition
3 min
- ▶ **Video:** Differentiability
3 min
- ⌂ **Interactive Plot:** Definition of the Derivative
15 min
- ▶ **Video:** Derivatives: Examples
8 min
- ▶ **Video:** Arithmetic of Derivatives
7 min
- ▶ **Video:** Derivatives: Chain Rule
8 min
- 📖 **Reading:** Derivatives: Logarithmic Rule
10 min
- 📖 **Reading:** Derivatives: Inverse Functions
10 min
- 📋 **Practice Quiz:** Practice Quiz #1
6 questions

Linear Objects associated with Differentiability

Derivatives of Higher Order

Survey



Derivatives: Logarithmic Rule

Sometimes given function does not qualify either as exponential example consider $f(x) = x^x$. How to find the derivative in this approaches:

By the Definition of the Logarithm

Remember: logarithm of some b with a base a is such a number

$$a^{\log_a b} = b$$

Let us apply the same to the input function:

$$x^x = e^{\ln x^x} = e^{x \ln x}$$

This is manageable since it is simple composition of exponential. Thus we get:

$$(x^x)' = (e^{x \ln x})' = e^{x \ln x} \cdot (x \ln x)' = x^x \cdot ((x)' \cdot \ln x + x \cdot$$

By logarithmic equation

Consider we have written input as:

$$y(x) = x^x$$

Then we can in advance take a logarithm of both sides:

$$\ln y(x) = x \ln x$$

Since both sides coincide, their derivatives coincide too:

$$(\ln y(x))' = \ln x + 1$$

In the left part we clearly see the composition and thus we apply

$$\frac{1}{y(x)} \cdot y'(x) = \ln x + 1$$