Additional material on matrix differentiation.

- A, B - constant matrices, α - constant scalar;
- X, Y - differentiable matrix function;
- ϕ - differentiable scalar function.

Transformation rules
$dA = 0$
$d(\alpha X) = \alpha(dX)$
$d(AXB) = A(dX)B$
$d(X+Y) = dX + dY$
$d(X^T) = (dX)^T$
$d(XY) = (dX)Y + X(dY)$
$d\langle X, Y\rangle = \langle dX, Y\rangle + \langle X, dY\rangle$
$d\left(\frac{X}{\phi}\right) = \frac{\phi dX - (d\phi)X}{\phi^2}$

Table of the derivatives
$d\langle A, X\rangle = \langle A, dX\rangle$
$d\langle Ax, x\rangle = \left\langle \left(A + A^T\right) x, dx\right\rangle$
$d\langle Ax, x\rangle = 2\langle Ax, dx\rangle \quad (\text{if } A = A^T)$
$d(Det(X)) = Det(X)\langle X^{-T}, dX\rangle$
$d(X^{-1}) = -X^{-1}(dX)X^{-1}$

Composition rule

If $\phi(X) = g(f(X))$, where ϕ, g, f are functions, then $d(\phi(X)) = d(g)[f(X)] \cdot d(f(X))$.

Example:

$$\phi(x) := \ln\langle Ax, x\rangle, A \in \mathbb{S}^n_{++}$$

$$g(y) := \ln(y) \qquad f(x) := \langle Ax, x\rangle$$

$$dg(y) = \frac{dy}{y} \qquad df(x) = 2\langle Ax, dx\rangle$$

$$d(\phi(X)) = \frac{dy}{y}[y = f(X)] = \frac{2\langle Ax, dx\rangle}{\langle Ax, x\rangle}$$