










## Partial Derivatives and Differentiability

### Chain Rule for Multivariate Case

### Second Partial Derivatives

-  **Video:** Second Partial Derivatives  
9 min
-  **Video:** Differentials of Multivariate Functions  
13 min
-  **Video:** Convexity  
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-  **Video:** Second Partial Derivatives: Convexity  
11 min
-  **Reading:** Convexity: clearing the air  
10 min
-  **Practice Quiz:** Practice Quiz #2  
2 questions
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6 questions



# Convexity: clearing the air

Let us for a moment summarise all we have learned about convexity.

## Definition and corrections

Let us address the definition for the single-variate case first (the multi-variate is a simple generalization as we discussed): *the function is convex on the segment  $[a; b]$  if the function's graph on any subsegment  $[c; d]$  is below the segment connecting points  $(c, f(c))$  and  $(d, f(d))$ .*

We came up with a more algebraic way to write it:

- if we have a segment  $[c; d]$ , then any point there could be written as  $\alpha c + (1 - \alpha)d$  where  $0 \leq \alpha \leq 1$ ; why is it so? One way to think about it is the following: imaging a seesaw which starts at  $c$  and end at  $d$ : if we put a weight  $\alpha$  on one side and  $1 - \alpha$  on the other, then where should we put the foundation (the fulcrum) so it is in balance? Exactly in  $\alpha c + (1 - \alpha)d$  (it is known in physics as the center of mass). Now try to change  $\alpha$  and you will see how we can get the fulcrum to any point of the segment
- the similar thing applies to the two dimensional segment (e.g. between  $(c, f(c))$  and  $(d, f(d))$ ) -- we still need to sum them with  $\alpha$  and  $1 - \alpha$  weights
- now all we need is to write an inequality for the word "below" in the definition:  $f(\alpha c + (1 - \alpha)d) \leq \alpha f(c) + (1 - \alpha)f(d)$

**Attention:** there is a correction in comparison with our