



The Derivative and Differentiability

- ▶ **Video:** Introduction to the Week
2 min
- ▶ **Video:** Derivative: Definition
3 min
- ▶ **Video:** Differentiability
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- ⌂ **Interactive Plot:** Definition of the Derivative
15 min
- ▶ **Video:** Derivatives: Examples
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- ▶ **Video:** Arithmetic of Derivatives
7 min
- ▶ **Video:** Derivatives: Chain Rule
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- 📖 **Reading:** Derivatives: Logarithmic Rule
10 min
- 📖 **Reading:** Derivatives: Inverse Functions
10 min
- 📋 **Practice Quiz:** Practice Quiz #1
6 questions

Linear Objects associated with Differentiability

Derivatives of Higher Order

Survey



Derivatives: Inverse Functions

To finalise let us consider the derivative of the **inverse** function $\arctan x$). The function is called inverse in case its composition function results into the initial argument:

$$f^{-1} \circ f = x$$

The rule for derivative here is:

$$(f^{-1}(y))' = \frac{1}{f'(x)}$$

Let us use it, e.g., for $\arctan x$:

$$(\arctan x)' = \frac{1}{(\tan(y))'} = \cos^2 y$$

This is nice, but we were hoping for the answer in terms of x , not y . We remember the main trigonometric mantra:

$$\sin^2 y + \cos^2 y = 1 \quad \Rightarrow \quad \tan^2 y + 1 = \frac{1}{\cos^2 y} \quad \Rightarrow \quad \cos^2 y = \frac{1}{1 + \tan^2 y}$$

Since $y = \arctan x$, $\tan y = \tan \arctan x = x$. Thus

$$(\arctan x)' = \frac{1}{1 + x^2}$$

