



## Introduction to the Course

- Reading:** About University  
10 min
- Video:** About the University  
1 min
- Video:** Welcome to the Course!  
2 min
- Reading:** Rules on the academic integrity in the course  
10 min

## Numerical Sets and Mappings: Counting Oranges

- Pre-survey "Calculus and Optimization for Machine Learning"  
10 min
- Video:** Introduction to the Week  
1 min
- Video:** Numerical Sets  
13 min
- Video:** Mappings and Quantifiers  
3 min
- Reading:** Real numbers  
10 min

## Functions: Definitions and Graphs

- Video:** Functions : Definitions  
10 min
- Interactive Plot: Domain and Range  
15 min



# Newton's Binomial Theorem

Here we remind you what is Newton's Binomial Theorem is all about.

Firstly, let us remind ourselves what is the **binomial coefficient**  $\binom{n}{k}$  the number of ways to choose  $k$  objects (the order is not important) from the set of  $n$  objects, without replacement, and thus can be found as

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

The simplest idea to prove it is to subsequently consider the number of ways to choose  $k$  objects from  $n$  objects: the first object can be chosen by  $n$  ways; the second object by  $n-1$  ways; the third object by  $n-2$  ways and so on until we choose the  $k$ th object amongst  $n-k+1$  objects. If we care about objects' order, we should divide produced number by  $k!$  (the number of possible permutations of  $k$  element set, which is similarly  $k \cdot (k-1) \cdot \dots \cdot 1 = k!$ ), thus we get:

$$\frac{n \cdot (n-1) \cdot \dots \cdot (n-k+1)}{k!} = \frac{n!}{k!(n-k)!}$$

**Newton's binomial theorem**, basically, states the relation between binomial coefficients and  $(a+b)^n$ :

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{n}b^n$$

It is quite easy to come up with a proper proof for it. Consider the binomial expression:

$$(a+b)^n = (a+b) \cdot (a+b) \cdot \dots \cdot (a+b)$$

Thus, further transformations require us to choose in each multiplication either  $a$  or  $b$ . As a result, terms like  $a^l b^{n-l}$  are formed. To state the coefficient of  $a^l b^{n-l}$  consider that it is simply the number of times this exact term has been formed during the multiplication. In other words, it is the number of ways to choose  $l$   $a$  terms from whilst the rest of the power of  $b$  forms inevitably. This number is the binomial coefficient  $\binom{n}{l}$  by definition and we have proven the theorem.