Real numbers

Proper understanding of real numbers is crucially important but is rather complicated. One could try to produce axiomatic definition:

Axiomatic definition

Assume that you have a set of objects and you want to demand something from them to turn them into the set of real numbers. The simplest idea here is that you should be able to do two basic operation with elements of the set: **addition** and **multiplication**, thus producing some other element of the set. What one should expect from those operations?

1. $x+y=y+x$, $x \cdot y=y \cdot x$ (it is easy: the **commutative** rule)
2. $(x+y)+z=x+(y+z)$, $(x \cdot y) \cdot z=x \cdot (y \cdot z)$ (**associativity**)
3. $(x+y) \cdot z=x \cdot z+y \cdot z$ (**distributivity**)
4. Existence of two **neutral** elements 1 and 0: $a \cdot 1=a$, $a+0=a$.
5. Existence of the **inverse** elements (except 0): $a+(-a)=0$, $a \cdot a^{-1}=1$
6. Non-triviality $0 \neq 1$

In the same manner we can add more, because not only we expect from real numbers this to arithmetic operations, but some order: the ability to compare two numbers.

1. For any two real numbers one is able to say $a>b$, $a<b$ or $a=b$.
2. This oder is **transitive**: if $a<b$, $b<c$, then $a<c$.