TNO innovation for life

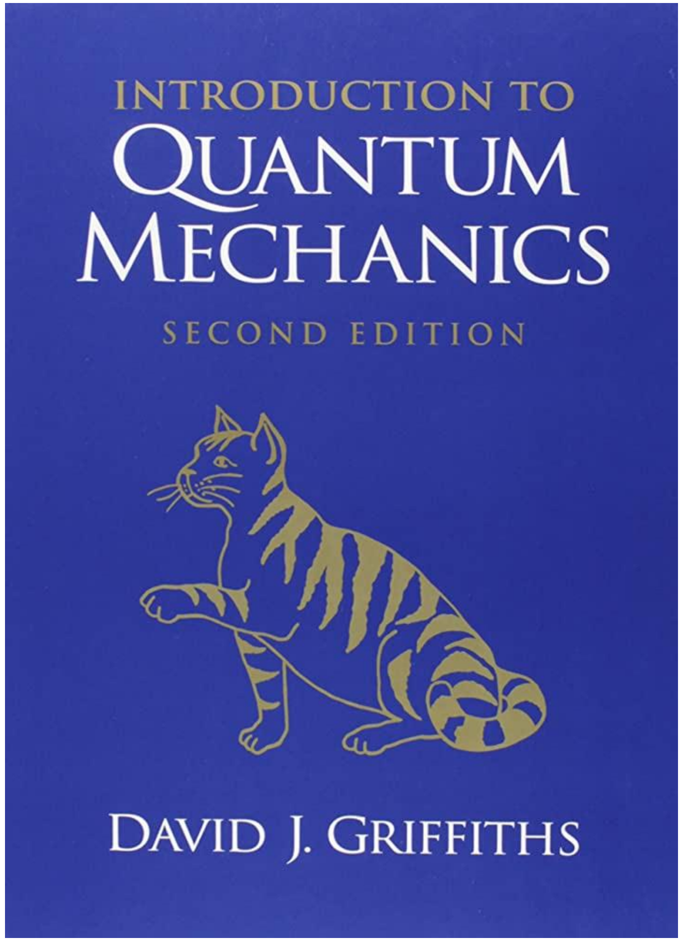


# QUANTUM COMPUTING STARTER GUIDE

TUMI MAKINWA AND YORAM VOS

# COURSE PREREQUISITES

- Strong fundamentals in differential analysis
- Advanced knowledge on thermodynamics
- Some practical ability in electronic engineering
- Deep knowledge of statistics and statistical physics
- (Preferred) knowledge of optics and waves
- 1 copy of the book to the right ->



# QUANTUM COMPUTATION
## SECOND TITLE OF THE SLIDE

While a lot of knowledge is needed to fully understand quantum computers
The quantum computation itself is mostly maths and logic, nicely separatable from the other stuff

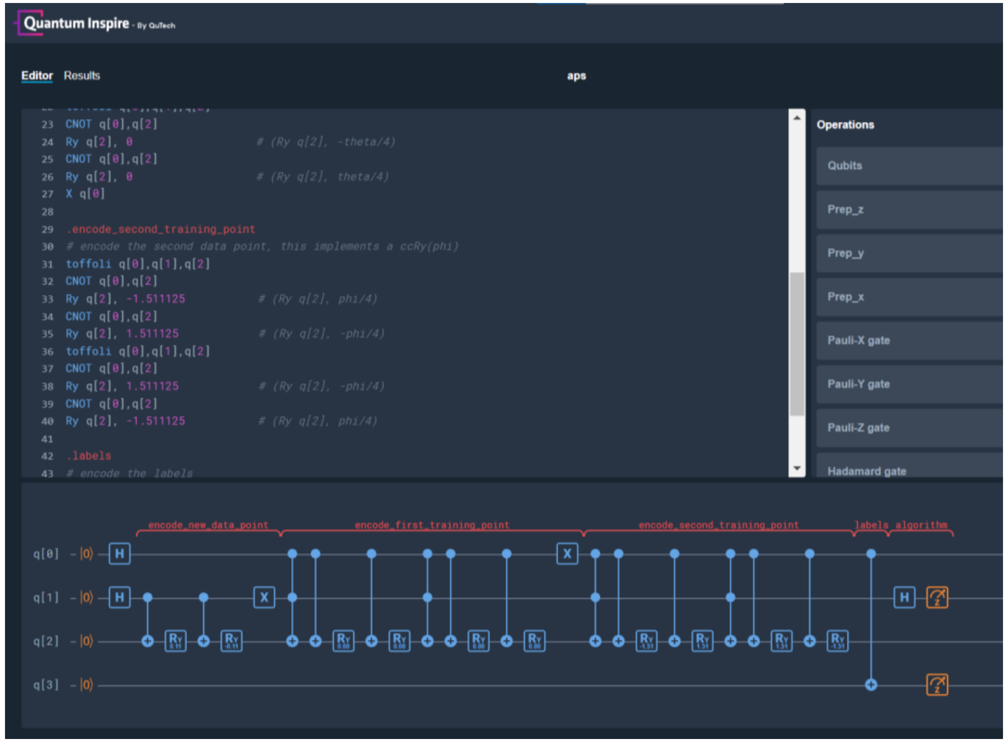
Quantum computation

- Physics
- Quantum mechanics
- Electrical engineering
- Cryo-engineering
- Etc

# QUANTUM INSPIRE

For those that feel particularly inspired, you can run the quantum algorithms and read more about them here:

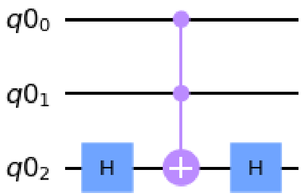
→ Give quantum inspire a spin at **quantum-inspire.com**




```python
n=3
N=2**n
q = QuantumRegister(n)
qc = QuantumCircuit(q)

if n==1:
    def black_box(qc, q):
        qc.z(q)
elif n==2:
    def black_box(qc, q):
        for i in range(n):
            qc.s(q[i])
        qc.h(q[1])
        qc.cx(q[0], q[1])
        qc.h(q[1])
        for i in range(n):
            qc.s(q[i])
else:
    def black_box(qc, q):
        qc.h(q[2])
        qc.ccx(q[0], q[1], q[2])
        qc.h(q[2])
black_box(qc,q)
cplot=qc.draw(output='mpl')
display(cplot)

print('black box circuit:')
run_circuit(qc, q)
```



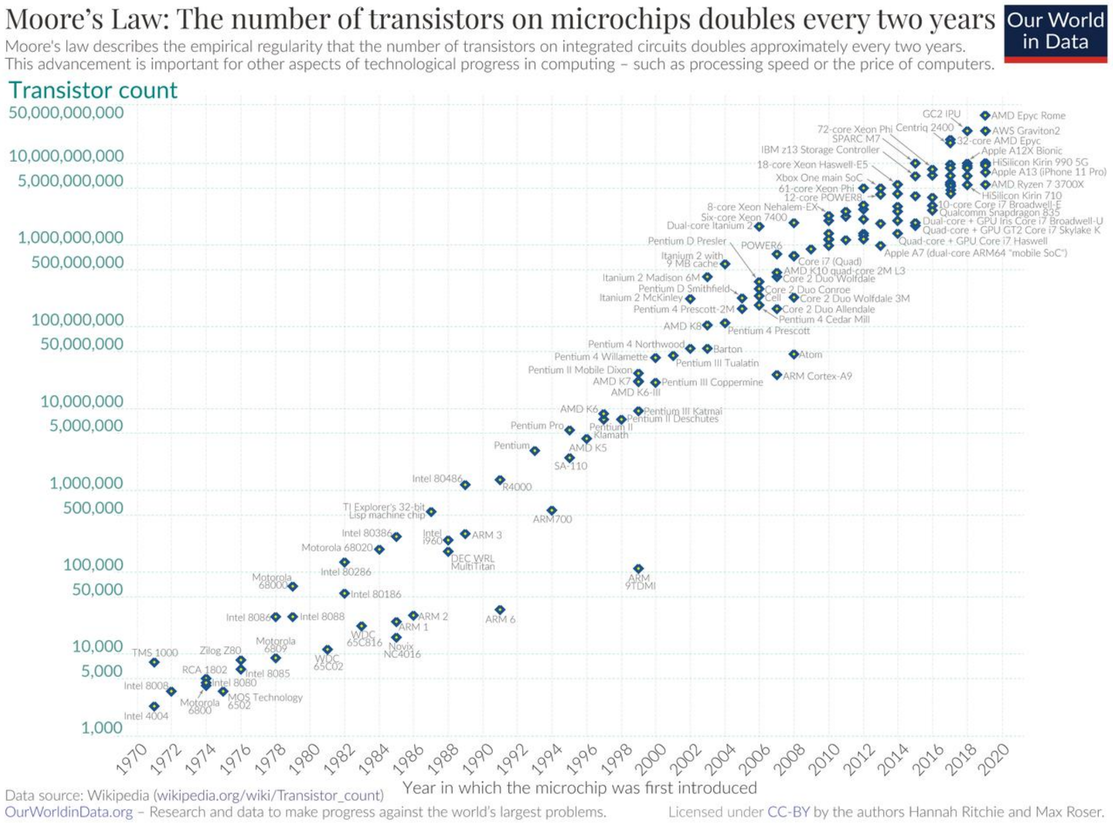
# MOORE'S LAW

“Moore's law is the observation that the number of transistors in an integrated circuit (IC) doubles about every two years.”

Law kept through creation of smaller and smaller transistors

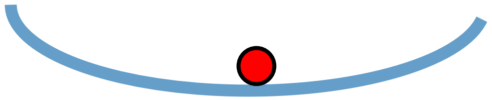
However, when things get too small quantum effects show up:

- Current stops working nicely
- Electrons start randomly clipping out of bounds

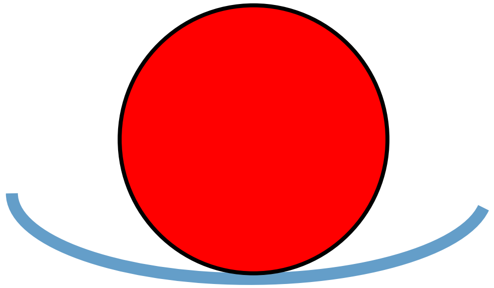



Electron large compared to circuit

Electron small compared to circuit



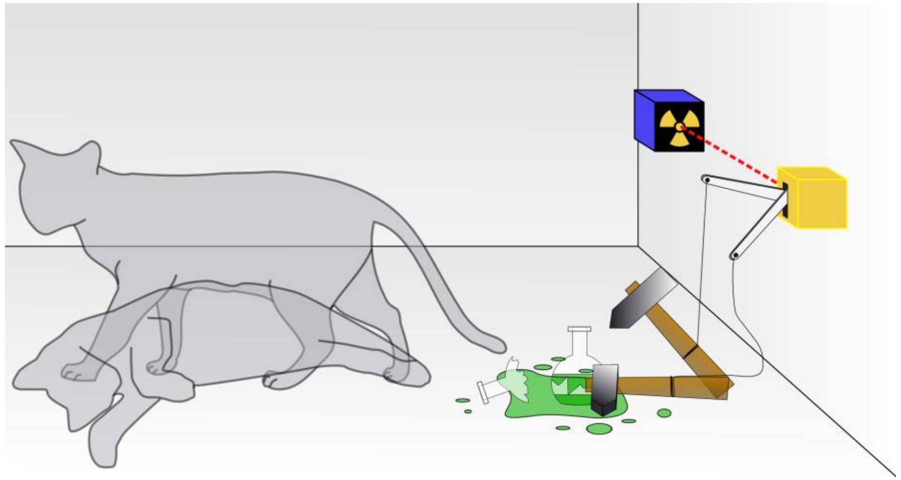
vs



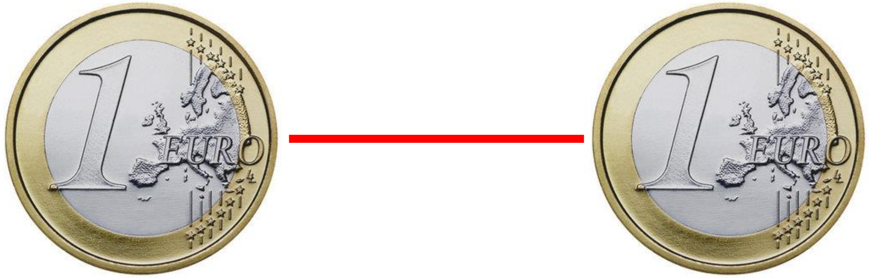
# › WHY NOT USE THESE QUANTUM EFFECTS?

Useful quantum effects that could be use for computation:

1) Quantum superposition



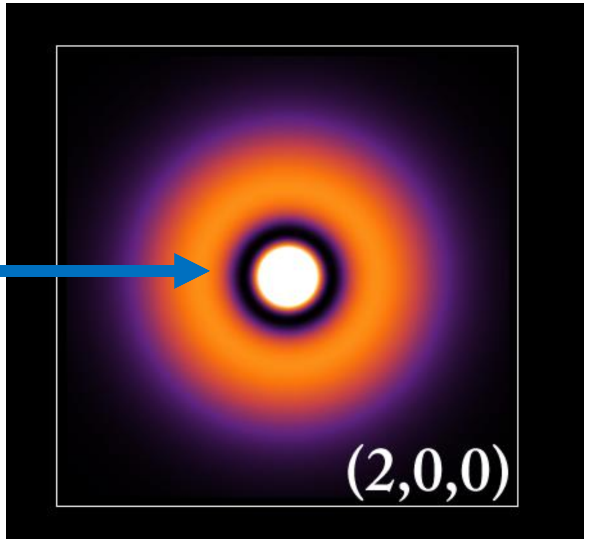
2) Quantum entanglement



# QUANTUM SUPERPOSITION

- In the quantum limit (very very small), things like position/speed/state are no longer fixed definite properties

Electron is
somewhere in
the cloud here




- This is not fully random however, these are still bound by a distribution of probabilities

Chance of electron being
Here, is much larger than it being here

# Quantum Superposition

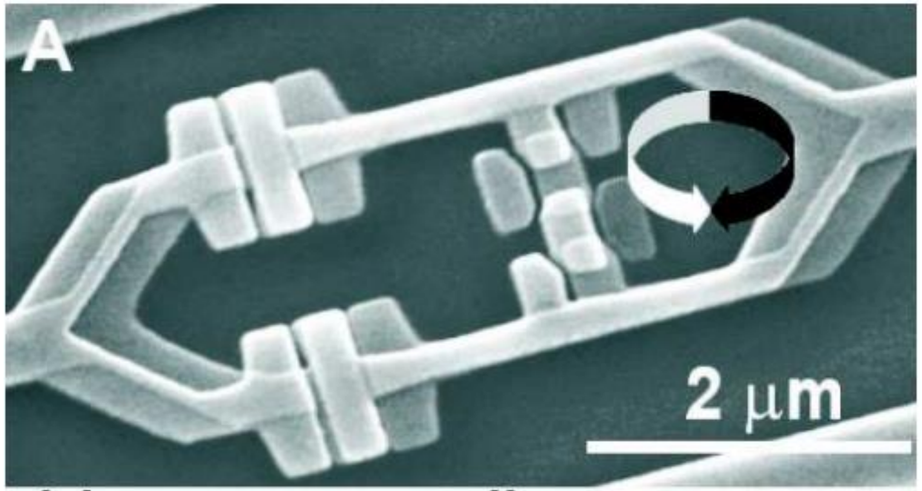
# QUANTUM STATES

Things become even more interesting when we construct systems that limit this probability distribution to just two states, eg:

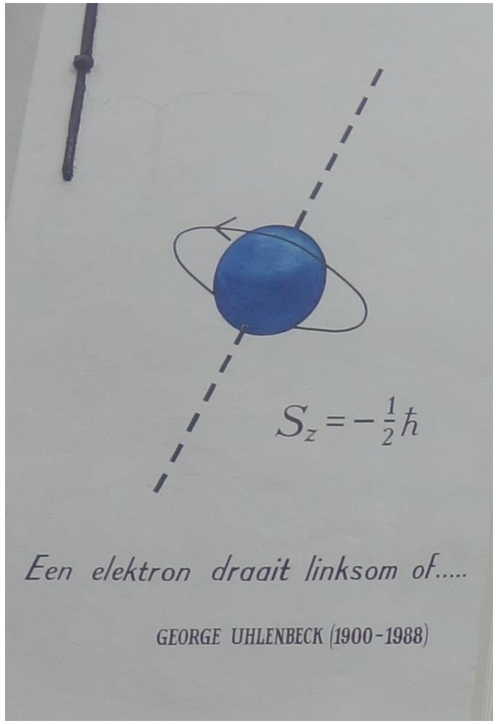
1. the direction of a superconducting current

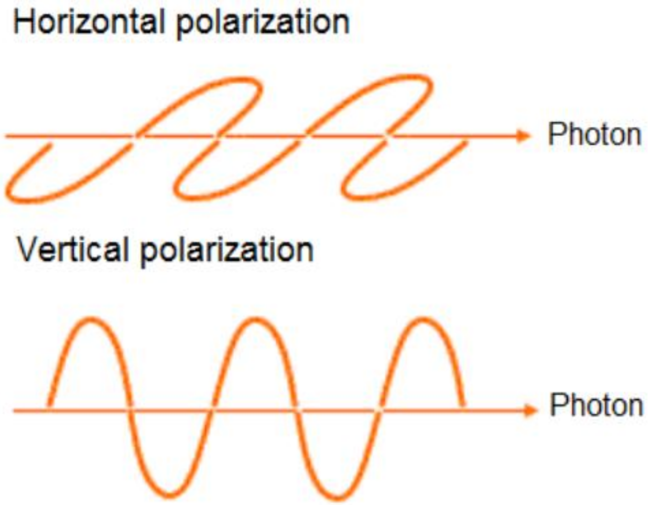
2. The direction of an electron spin

3. The polarization of a photon



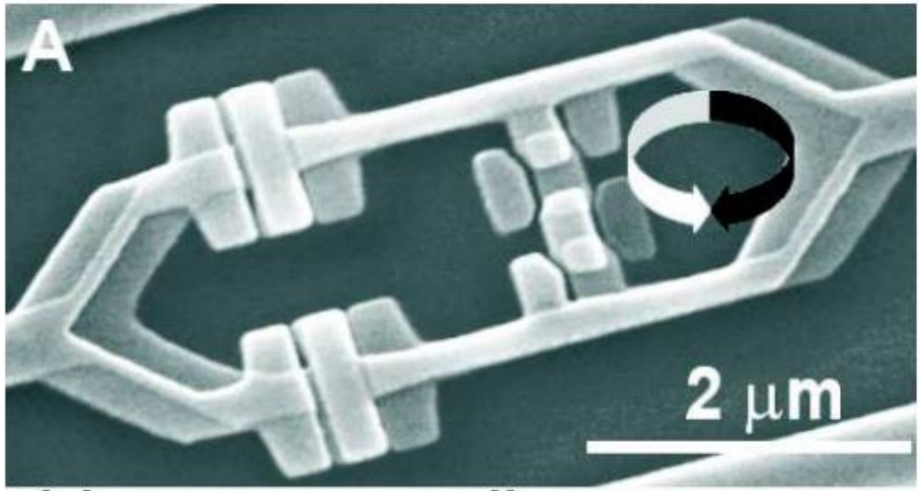

I. Chiorescu (2003)



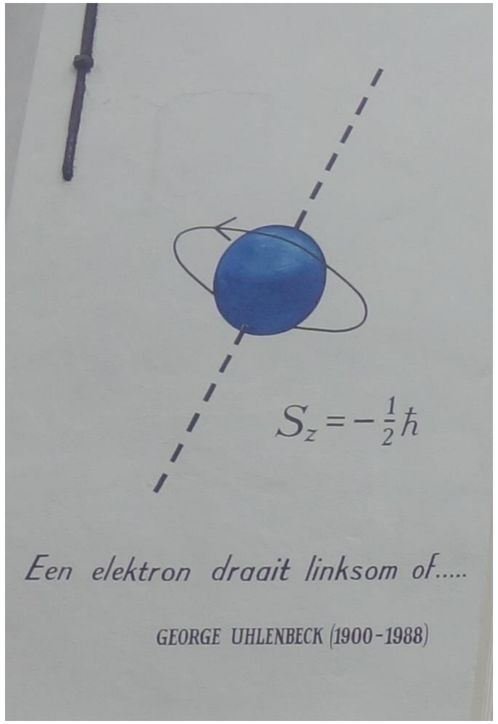



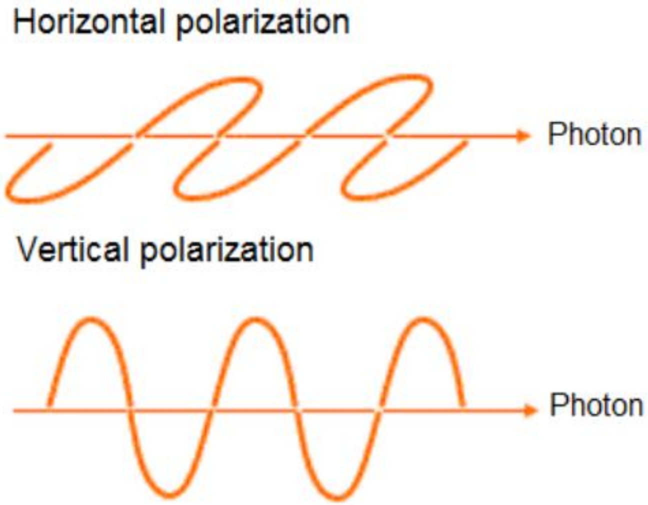
# QUANTUM STATES

We now have a system of which its probability can get stuck somewhere between the 2 states



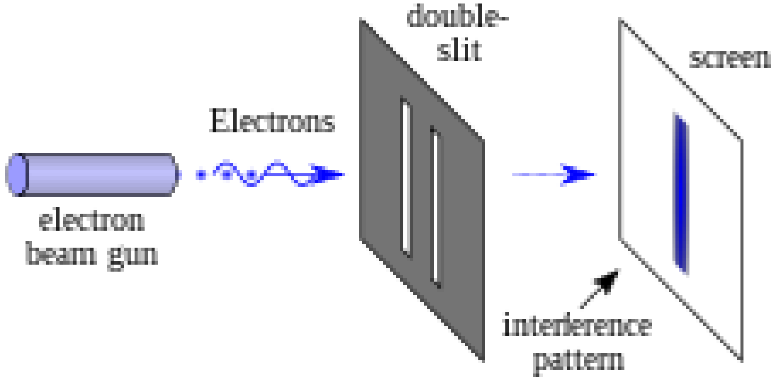

I. Chiorescu (2003)

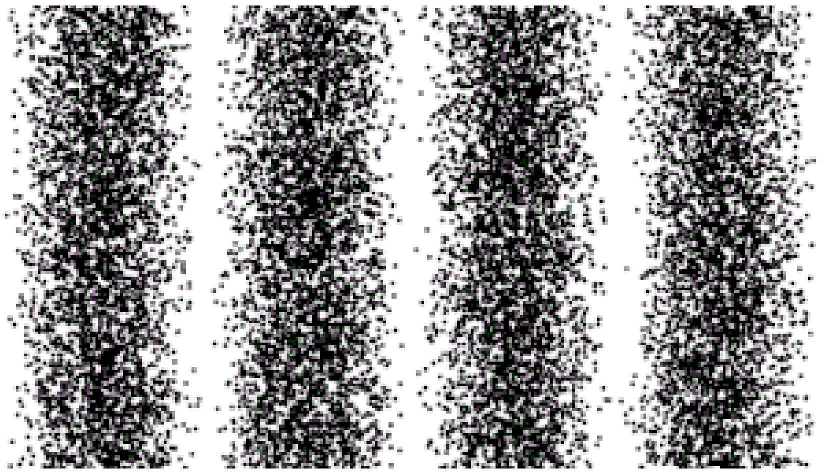


# MEASUREMENT

- Superposition states “collapse” into one of the possible states when measured/observed
- Superposition is not just a complicated explanation random process, before measurement the both exist and can even interfere with each other
- See: the double split experiement



# MATHEMATICAL REPRESENTATION

Due to the various ways of making a 2 level quantum system (qubit), a standarized method of describing these is used: $|0\rangle$ and $|1\rangle$

$|0\rangle$

- Clockwise current flow
- Clockwise spin
- Vertical polarization
- etc

$|1\rangle$

- Anti-clockwise current flow
- Anti-clockwise spin
- Horizontal polarization
- etc

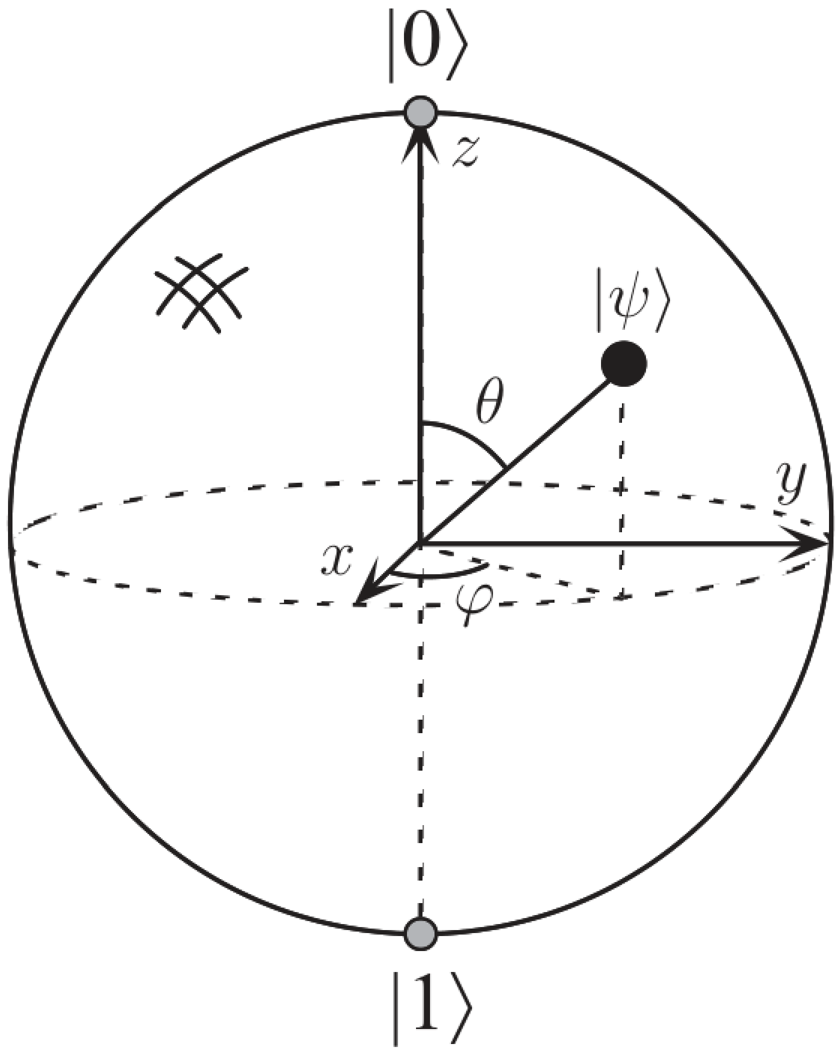
State that's a perfect 50/50 of 0 and 1 is noted as:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

# MATHEMATICAL DESCRIPTION: BLOCH SPHERE

To help understand qubit states and their operations
visualize as states and rotations on a sphere:

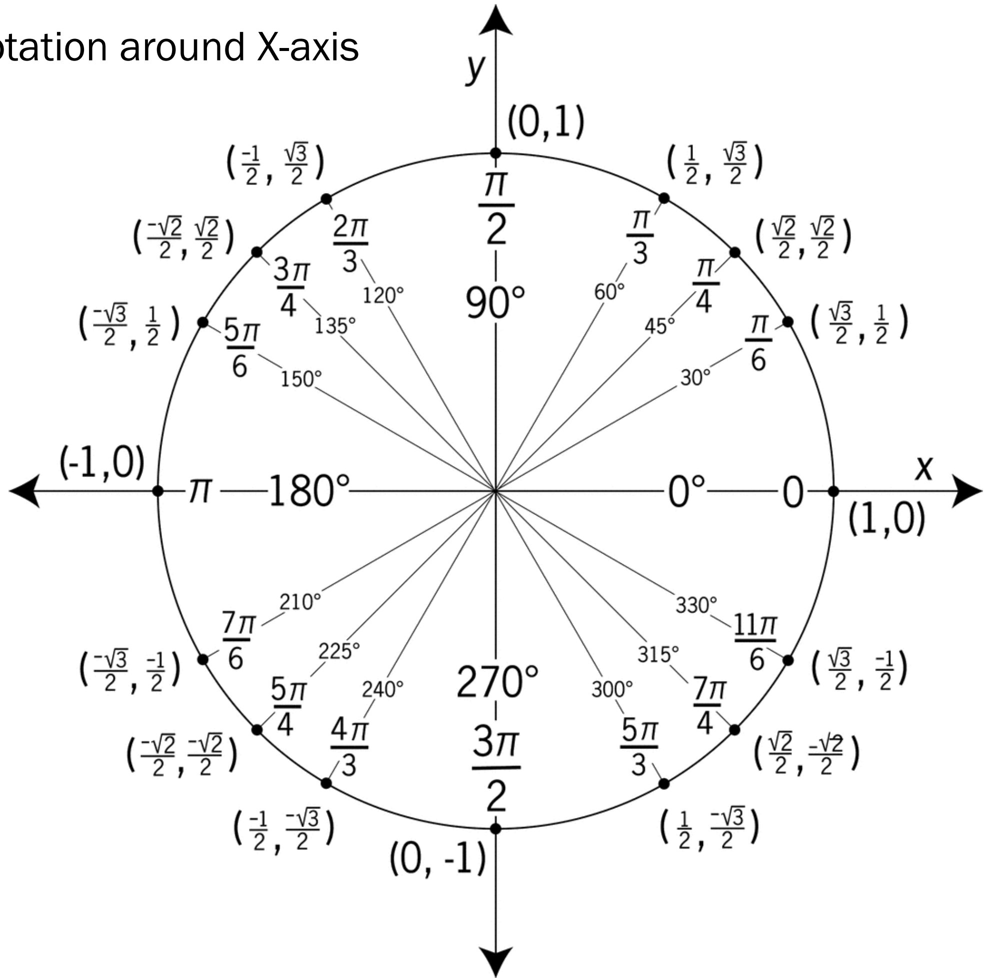
$$\left|\Psi(\theta,\varphi)\right\rangle = \cos\left(\frac{\theta}{2}\right)\left|0\right\rangle + e^{i\varphi}\sin\left(\frac{\theta}{2}\right)\left|1\right\rangle$$



# CIRCLES CIRCLES CIRCLES

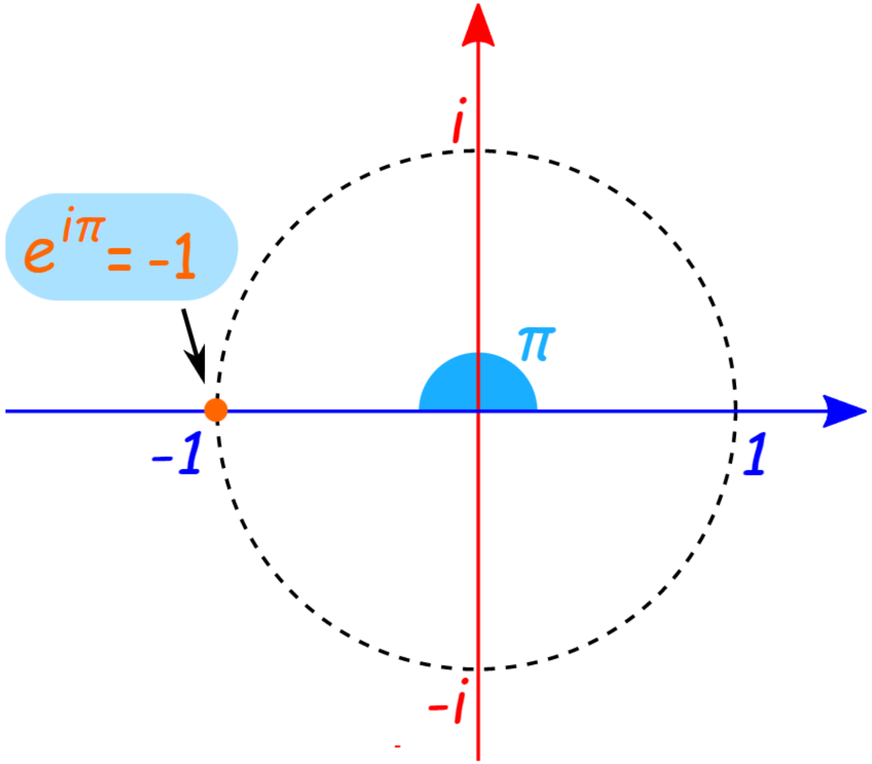
## Unit circle

Rotation around X-axis




## Complex number circle

Rotation around Z-axis



# CIRCLES CIRCLES CIRCLES

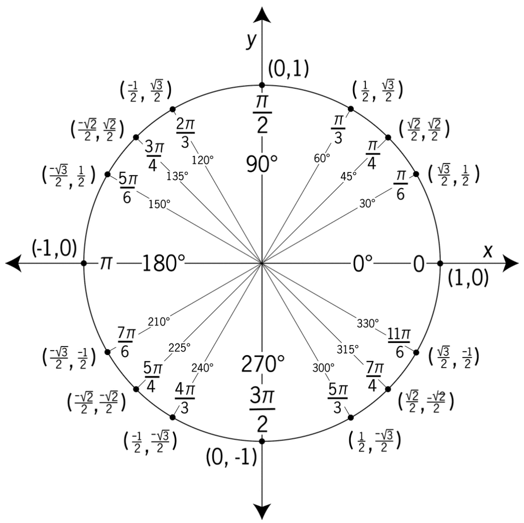
This is convenient because you can just multiply the two numbers without them interfering

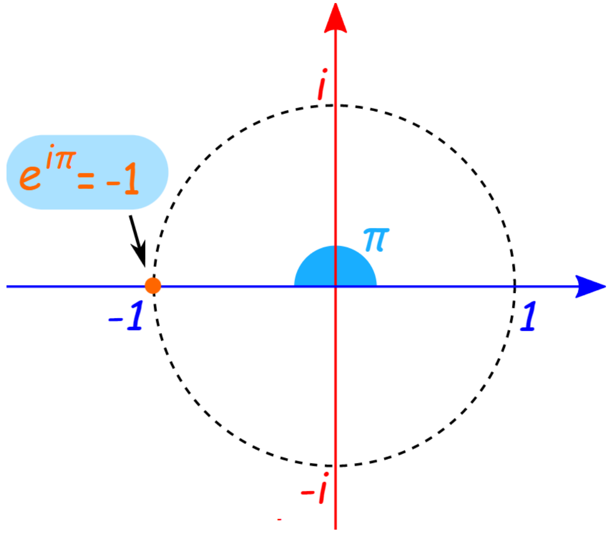
2x unit circle:

$$\frac{1}{\sqrt{2}} * \frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$\frac{3}{\sqrt{2}} * \frac{1}{\sqrt{2}} = \frac{3}{2}$$

Impossible to keep the two separate






Unit circle + imaginary circle

$$i * \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} i$$

$$-1 * \frac{3}{\sqrt{2}} = -\frac{3}{\sqrt{2}}$$

Can still (mostly) tell them apart!

# MATHEMATICAL DESCRIPTION: BLOCH SPHERE

Example 1:
Full rotation (pi) over purely the Y axis: $Y(\pi)$

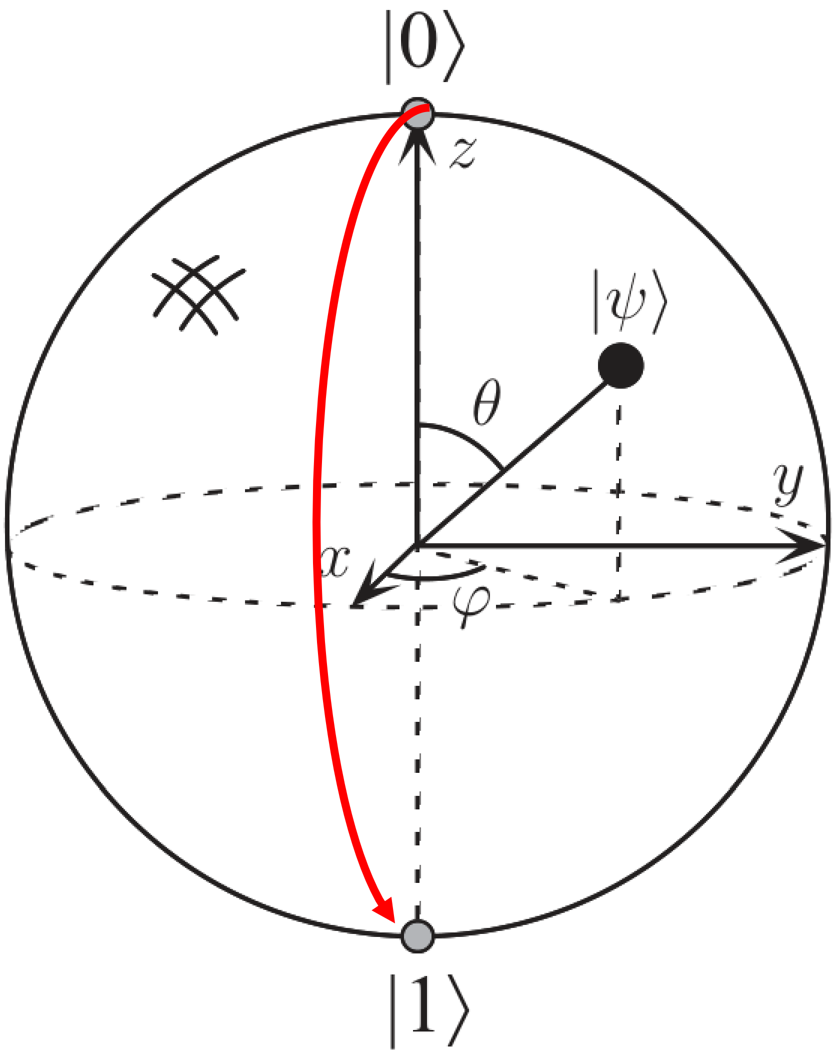
$$\left|\Psi(\theta,\varphi)\right\rangle = \cos\left(\frac{\theta}{2}\right)\left|0\right\rangle + e^{i\varphi}\sin\left(\frac{\theta}{2}\right)\left|1\right\rangle$$

$$\left|\Psi(\pi,0)\right\rangle =$$

$$\cos\left(\frac{\pi}{2}\right)\left|0\right\rangle + e^{i0}\sin\left(\frac{\pi}{2}\right)\left|1\right\rangle =$$

$$0*\left|0\right\rangle + 1*1\left|1\right\rangle =$$

$$\left|1\right\rangle$$



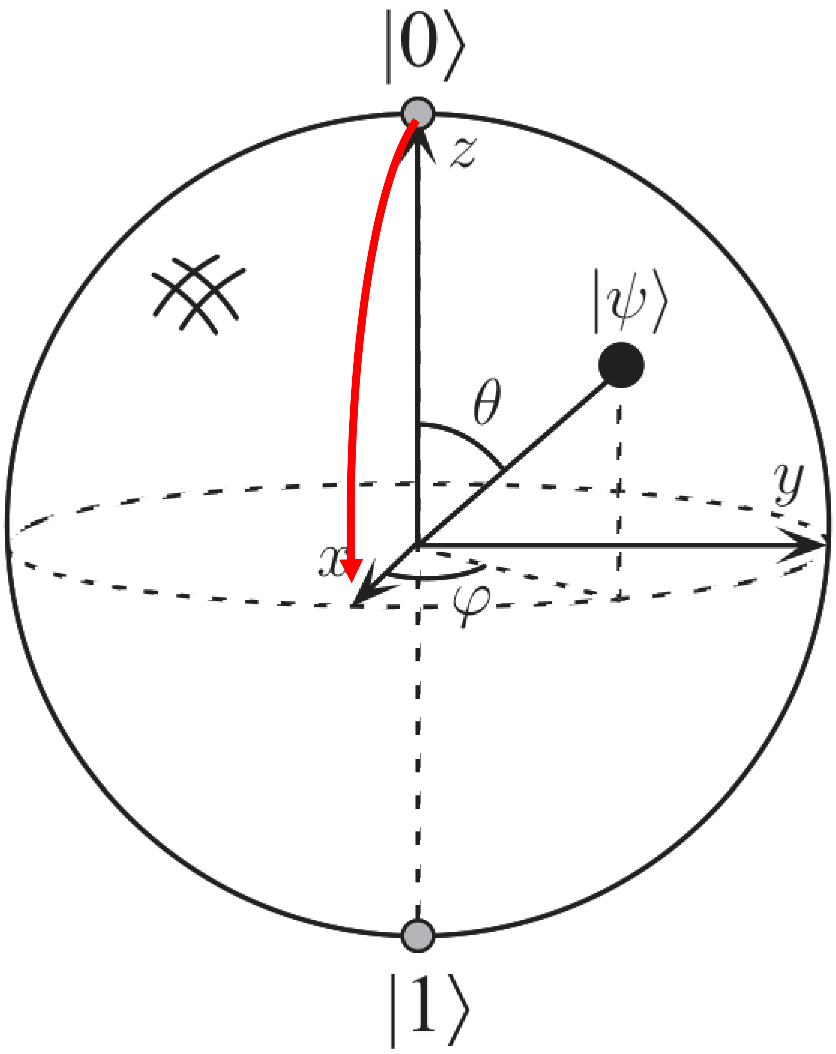
# MATHEMATICAL DESCRIPTION: BLOCH SPHERE

Example 2:
Half rotation (half pi) over purely the Y axis $Y\left(\frac{\pi}{2}\right)$

$$\left|\Psi\left(\frac{\pi}{2},0\right)\right\rangle =$$

$$\cos\left(\frac{\pi}{4}\right)|0\rangle + e^{i0}\sin\left(\frac{\pi}{4}\right)|1\rangle =$$

$$\frac{1}{\sqrt{2}}|0\rangle + 1 * \frac{1}{\sqrt{2}}|1\rangle =$$



# MATHEMATICAL DESCRIPTION: BLOCH SPHERE

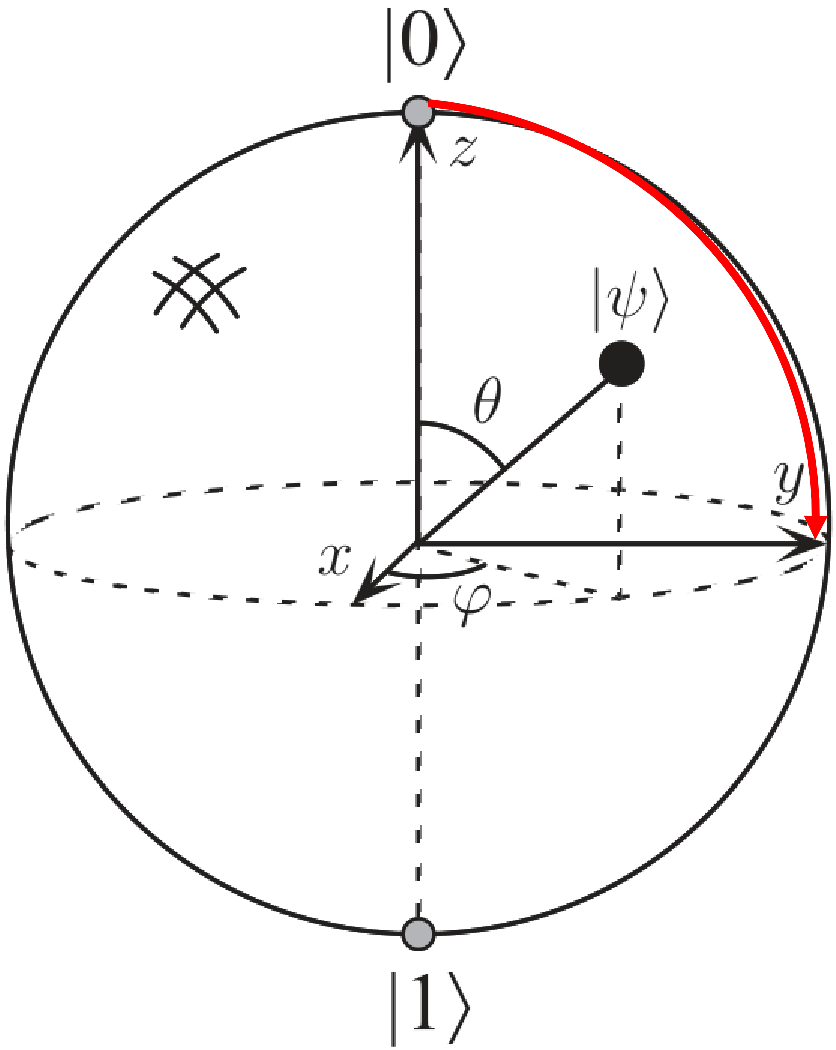
Example 3:
Half rotation (half pi) over purely the X axis $X\left(\frac{\pi}{2}\right)$

$$\left|\Psi\left(\frac{\pi}{2},\frac{\pi}{2}\right)\right\rangle =$$

$$\cos\left(\frac{\pi}{4}\right)|0\rangle + e^{i\frac{\pi}{2}}\sin\left(\frac{\pi}{4}\right)|1\rangle =$$

$$\frac{1}{\sqrt{2}}|0\rangle + i*\frac{1}{\sqrt{2}}|1\rangle =$$

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle =$$

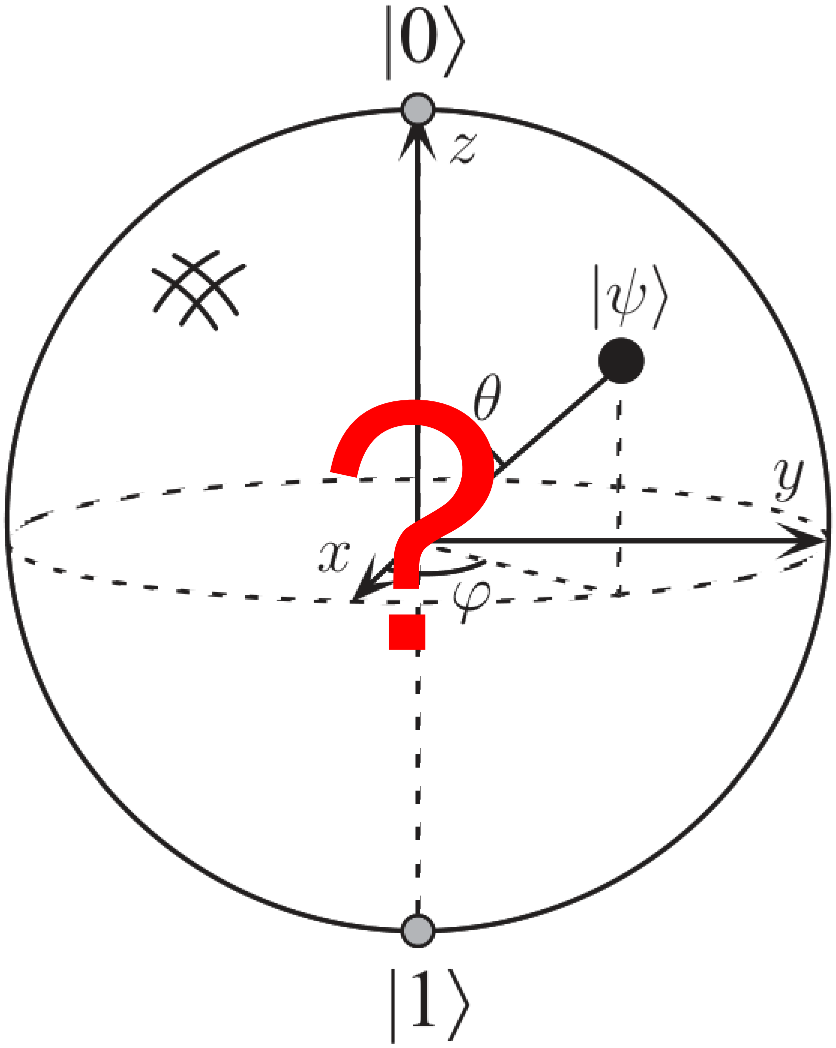


# QUIZ TIME

Question:
Opposite direction half rotation (minus half pi) over purely the Y axis

$$Y\left(-\frac{\pi}{2}\right)$$

$$\left|\Psi(?,?)\right\rangle =$$




Remember:

$$\left|\Psi(\theta,\varphi)\right\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\varphi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

# QUIZ TIME

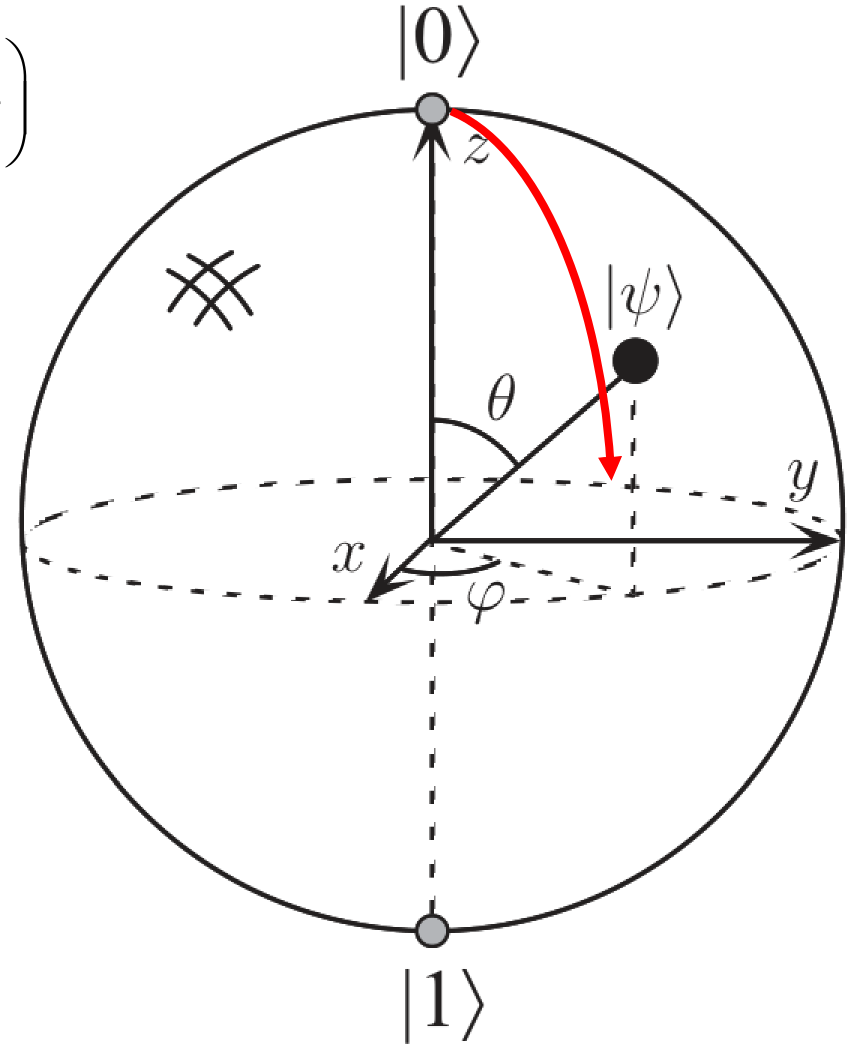
Question:
Rotation of minus half pi over purely the Y axis $Y\left(-\frac{\pi}{2}\right)$

$$\left|\Psi\left(-\frac{\pi}{2},0\right)\right\rangle =$$

$$\cos\left(-\frac{\pi}{4}\right)|0\rangle + e^{i0}\sin\left(-\frac{\pi}{4}\right)|1\rangle =$$

$$\frac{1}{\sqrt{2}}|0\rangle + 1 * \frac{-1}{\sqrt{2}}|1\rangle =$$

$$\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$



# Quantum Entangement

# ENGLEMENT = CORRELATION

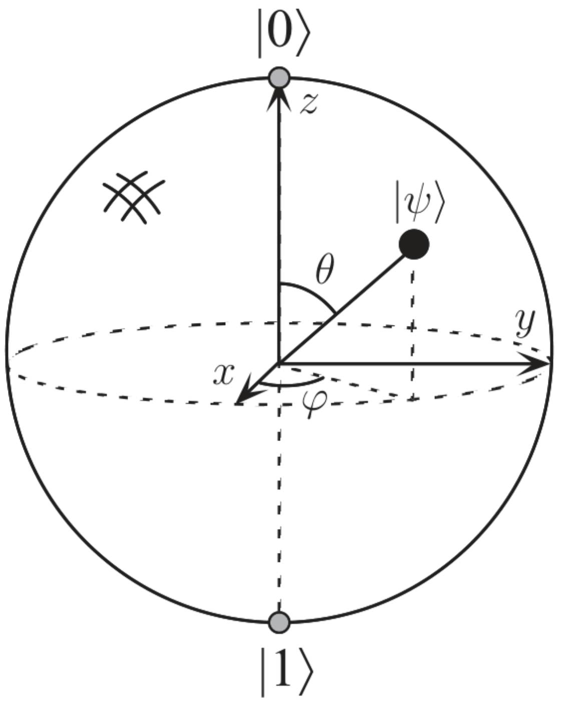
- Due to superposition, qubits have a probabilistic element to their states
- This means that reading out a qubit state can lead to random outputs
- We can work around this randomness by linking 2 qubits (Entanglement), making their states correlated
- This correlation can let us ensure that a qubits pair will always return the same (random) result
- Once entangled, qubits are capable of remaining in this state for any amount of distance, until measured

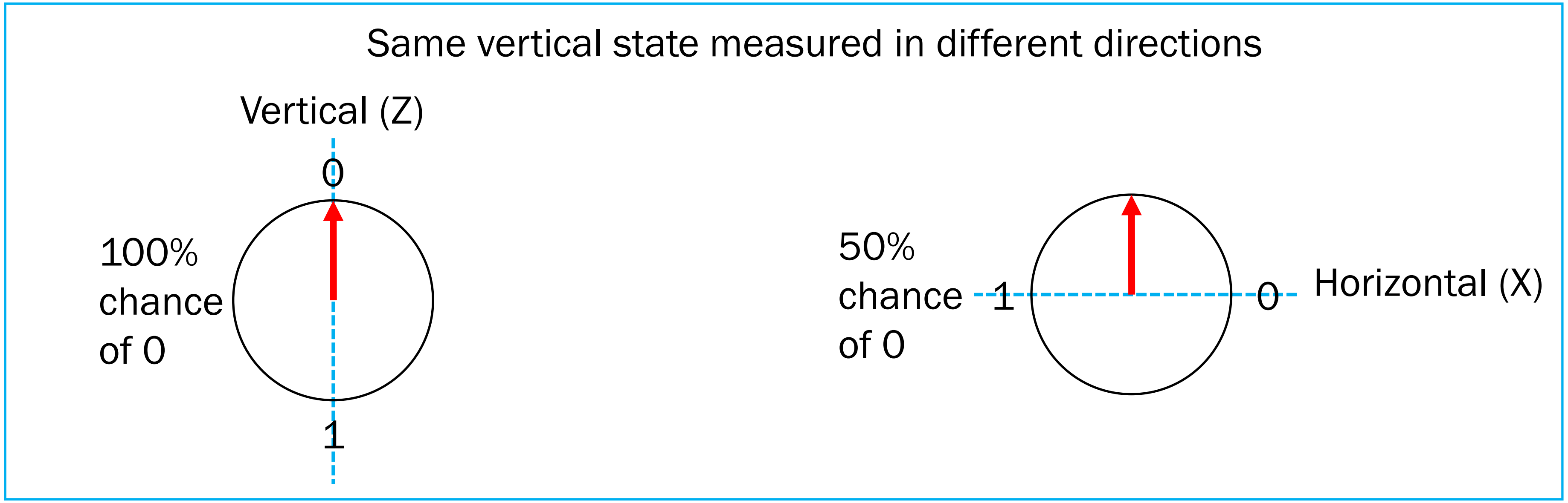
# QUANTUM ENCRYPTION

- Beyond just being a probabilty, a qubit's state also has a direction

- The direction (basis) a qubit is **measured** in also has effect on the result of the measurement

- **Takeaway:**
  Measuring in the wrong direction changes the state of the qubit

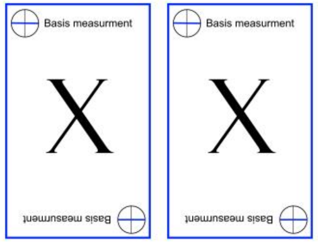


# ENTANGLED MEASUREMENTS

In case of state: $\left|\Psi_1\Psi_2\right\rangle = \frac{1}{2}\left|0_1\right\rangle\left|0_2\right\rangle + \frac{1}{2}\left|1_1\right\rangle\left|1_2\right\rangle$

Qubits measured in same direction

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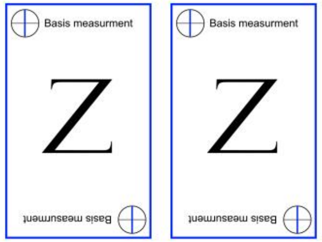



Both players get the same random result

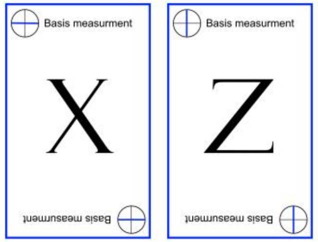
50%

50


or
differe

50%


Qubits measured in different directions

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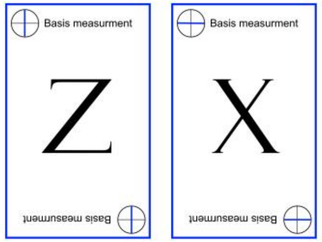


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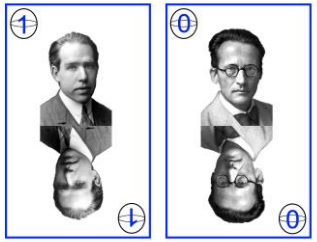


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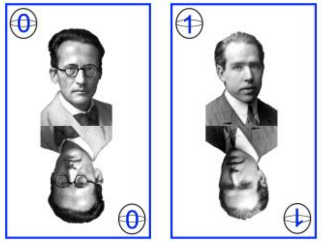


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# NOW INTRODUCING:

TNO presents:

# The Quantum Gameshow

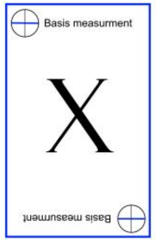
## QKD Edition

Learn quantum key distribution in 15 minutes

# GAME RULES

1. Team up with the person facing you

2. Choose a Z or X card

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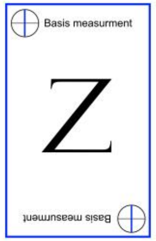
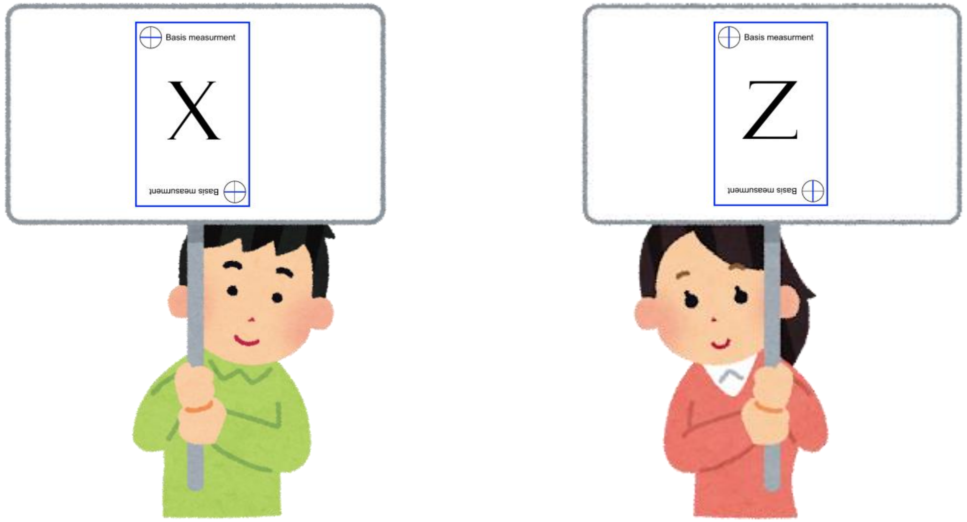



3. Receive a 0 or 1 card, depending on your and your teammates choice


or


4. Players reveal their choice of Z/X card

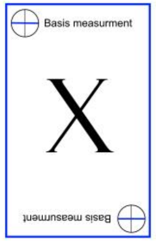



Game continues to round 2

# GAME(?) RULES

1.~~Team up with the person facing you~~
Share an entangled pair of qubits

2. ~~Choose a Z or X card~~
Choose a basis (direction) to meausure your qubit in

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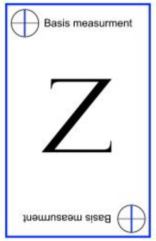
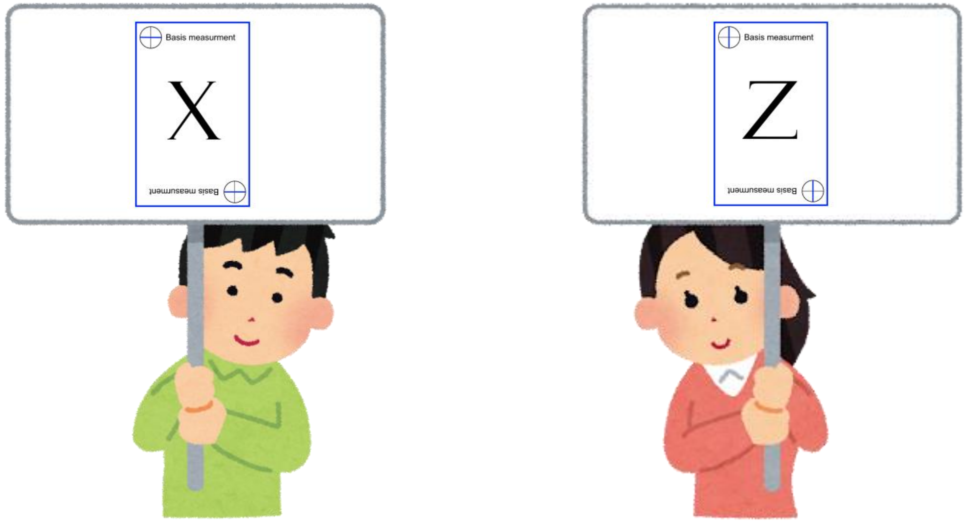



3. ~~Receive a 0 or 1 card, depending on your and your teammates choice~~
Receive the result of your qubit measurement


or


4. ~~Players reveal their choice of Z/X card~~
Comunicate the direction you looked at qubit in




~~Game continues to round 2~~
Measurement results and comunicated directions used to create final bitstring

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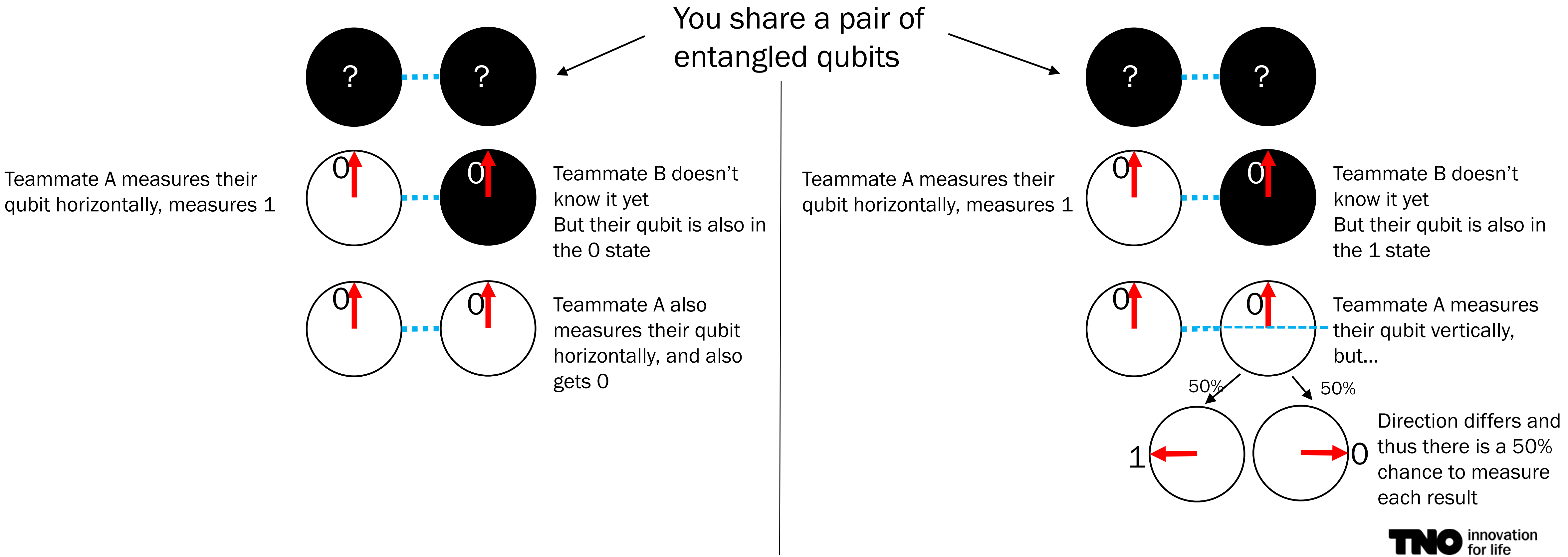
# 3. RECEIVE THE RESULT OF YOUR QUBIT MEASUREMENT

- Combining the effects of superposition and entanglement we know that:
  1. Measuring qubits is probabilitic in nature
  2. You and your teammate will measure the same value
  3. Measuring in a different bases changes the measurement result
- This leads to the following situation



# SAMPLE QKD RUN

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Tumi directions | Z | X | X | Z | Z | Z | X | X |
| Yoram directions | Z | X | Z | X | Z | Z | Z | X |
| Tumi measurement | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 |
| Yoram measurement | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |

Throw away the results in differing directions

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Tumi basis | Z | X | X | Z | Z | Z | X | X |
| Yoram basis | Z | X | Z | X | Z | Z | Z | X |
| Tumi measurement | 0 | 1 | | | 1 | 1 | | 0 |
| Yoram measurement | 0 | 1 | | | 1 | 1 | | 0 |

The remaining measurement results are your key: 01110 (14)

| |
|---|
| Tumi basis |
| Yoram basis |
| Tumi measurement |
| Yoram measurement |

# Quantum Coding time

# MATRIX MULTIPLICATIONS

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \bullet \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} A*X+B*Y \\ C*X+D*Y \end{bmatrix}$$

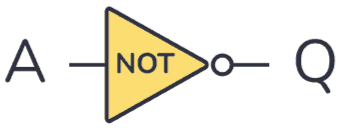
# MATRIX MULTIPLICATIONS

$$\begin{array}{|c|c|}\hline X & Y \\ \hline\end{array}$$

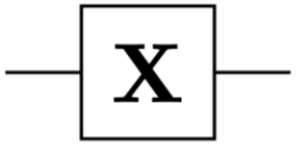
$$\begin{array}{|c|c|}\hline A & B \\ \hline C & D \\ \hline\end{array} = \begin{array}{|c|}\hline A*X+B*Y \\ \hline C*X+D*Y \\ \hline\end{array}$$

# QUANTUM LOGIC GATES

Not logic gate – X gate




| A | Q |
|---|---|
| 0 | 1 |
| 1 | 0 |

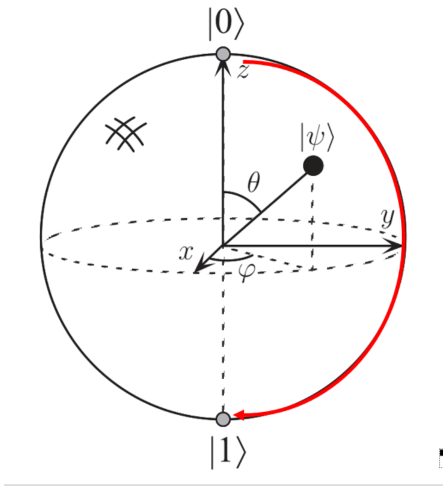



$$|0\rangle \rightarrow |1\rangle$$

$$|1\rangle \rightarrow |0\rangle$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha|0\rangle \\ \beta|1\rangle \end{bmatrix}$$

$$X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$$



## REMEMBER:

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0*1+1*0 \\ 1*1+0*0 \end{bmatrix}$$

# 2 QUBITS

Descriptor of a state with 2 qubits is done as following:

$$|\Psi_1\rangle = |0_1\rangle$$

$$|\Psi_2\rangle = |1_2\rangle$$

$$|\Psi_1\rangle \otimes |\Psi_2\rangle = |\Psi_1\Psi_2\rangle = |0_1\rangle \otimes |1_2\rangle = |0_1 1_2\rangle d$$

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}|0_1\rangle + \frac{1}{\sqrt{2}}|1_1\rangle$$

In case of 2 qubits in 50/50 superposition:

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}}|0_2\rangle + \frac{1}{\sqrt{2}}|1_2\rangle$$

$$|\Psi_1\rangle \otimes |\Psi_2\rangle = \left(\frac{1}{\sqrt{2}}|0_1\rangle + \frac{1}{\sqrt{2}}|1_1\rangle\right) \otimes \left(\frac{1}{\sqrt{2}}|0_2\rangle + \frac{1}{\sqrt{2}}|1_2\rangle\right) = \frac{1}{2}|0_1\rangle|0_2\rangle + \frac{1}{2}|1_1\rangle|1_2\rangle + \frac{1}{2}|0_1\rangle|1_2\rangle + \frac{1}{2}|1_1\rangle|0_2\rangle$$

$${}^{6}\!/_{4}$$

$${}^{3}\!/_{16}$$

Equal chance of all 4 combinations

# SUPERPOSITION

Lets take our 2 qubit superposition state:

$$|\Psi_1\Psi_2\rangle = \frac{1}{2}|0_1\rangle|0_2\rangle + \frac{1}{2}|1_1\rangle|1_2\rangle + \frac{1}{2}|0_1\rangle|1_2\rangle + \frac{1}{2}|1_1\rangle|0_2\rangle$$

We can simplify the notation a bit, remind you of anything?

$$|\Psi_1\Psi_2\rangle = \frac{1}{2}|00\rangle + \frac{1}{2}|11\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle$$

Binary conversion:

$$|\Psi_1\Psi_2\rangle = \frac{1}{2}|0\rangle + \frac{1}{2}|3\rangle + \frac{1}{2}|1\rangle + \frac{1}{2}|2\rangle$$

Our 2 qubits are now simultaneously in 4 different binary number states at once!

# SUPERPOSITION

Lets add one more superpositioned qubit

$$|\Psi_1\Psi_2\Psi_3\rangle = \frac{1}{4}|000\rangle + \frac{1}{4}|001\rangle + \frac{1}{4}|010\rangle + \frac{1}{4}|011\rangle + \frac{1}{4}|100\rangle + \frac{1}{4}|101\rangle + \frac{1}{4}|110\rangle + \frac{1}{4}|111\rangle$$

Binary conversion:

$$|\Psi_1\Psi_2\Psi_3\rangle = \frac{1}{4}|0\rangle + \frac{1}{4}|1\rangle + \frac{1}{4}|2\rangle + \frac{1}{4}|3\rangle + \frac{1}{4}|4\rangle + \frac{1}{4}|5\rangle + \frac{1}{4}|6\rangle + \frac{1}{4}|7\rangle$$

Our 3 qubits are now simultaneously in 8 different binary number states at once!
(2x increase)

# QUANTUM SPEEDUP

- For each extra qubit you add, you can double the amount of states you can have simultaneously

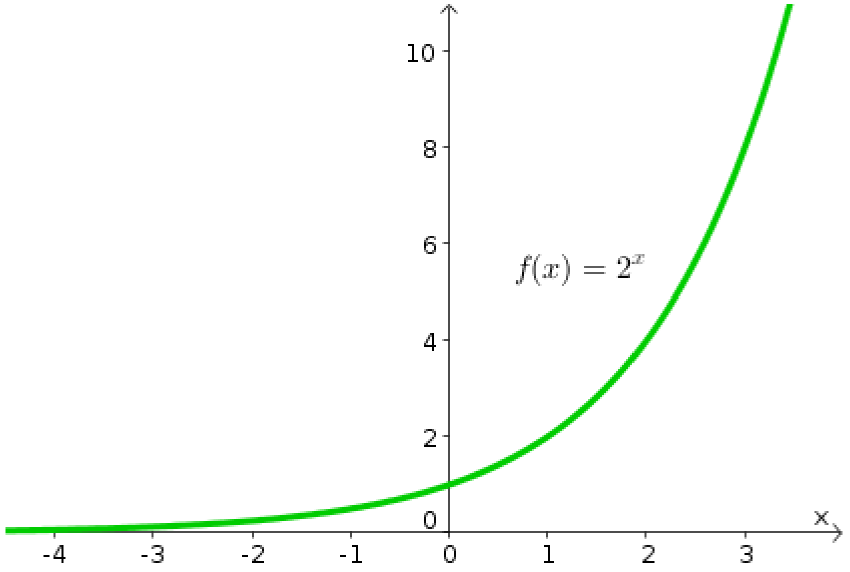
- Aka, states = 2^x

- “just”100 qubits = 1.267.650.600.228.229.401.496.703.205.376 states

- Before measurement each state exists simultaneously -> all operations take place on each state at once

- Very powerful for the class of problems where its easy to check **if** an answer is a correct but **finding** an answer is hard

- See: traveling salesman problem, prime factorization



# QUANTUM CONTRADICTION?

- When measured, your huge simultaneous state will end up randomly falling into a random state

$$|\Psi_1\Psi_2\Psi_3\rangle = \frac{1}{4}|0\rangle + \frac{1}{4}|1\rangle + \frac{1}{4}|2\rangle + \frac{1}{4}|3\rangle + \frac{1}{4}|4\rangle + \frac{1}{4}|5\rangle + \frac{1}{4}|6\rangle + \frac{1}{4}|7\rangle$$

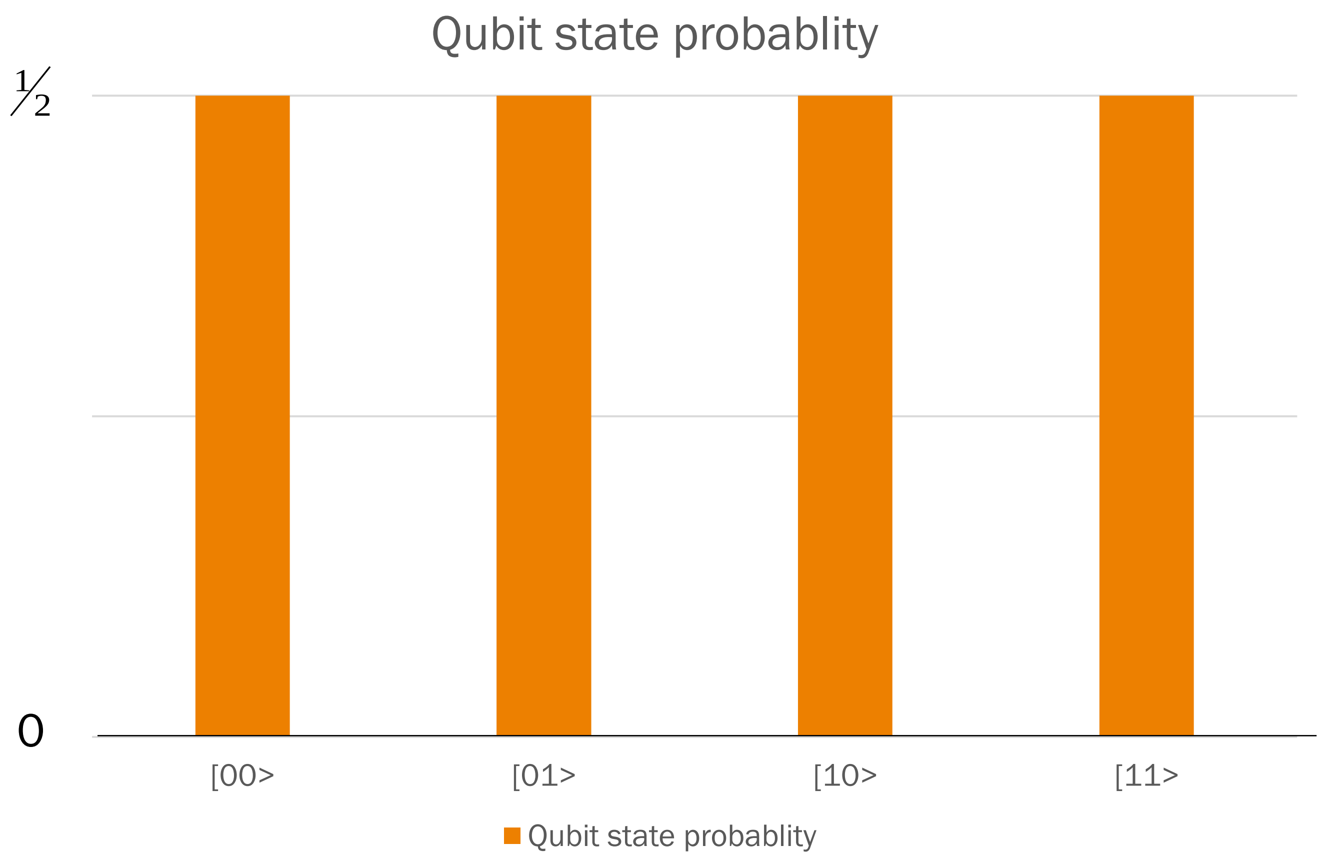
- Even if your quantum algorithm finds the correct answer, how are you going to read it?

- Well, there's a trick for that

# GROVER'S ALGORITHM

- To avoid getting random results we don't want, we simply need to decrease the probability of the wrong answers in the superposition.
- Thankfully this is possible, (phase amplification):
- To help explain we make a graph of the probabilities of the state:

$$|\Psi_1\Psi_2\rangle = \frac{1}{2}|00\rangle + \frac{1}{2}|11\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle$$

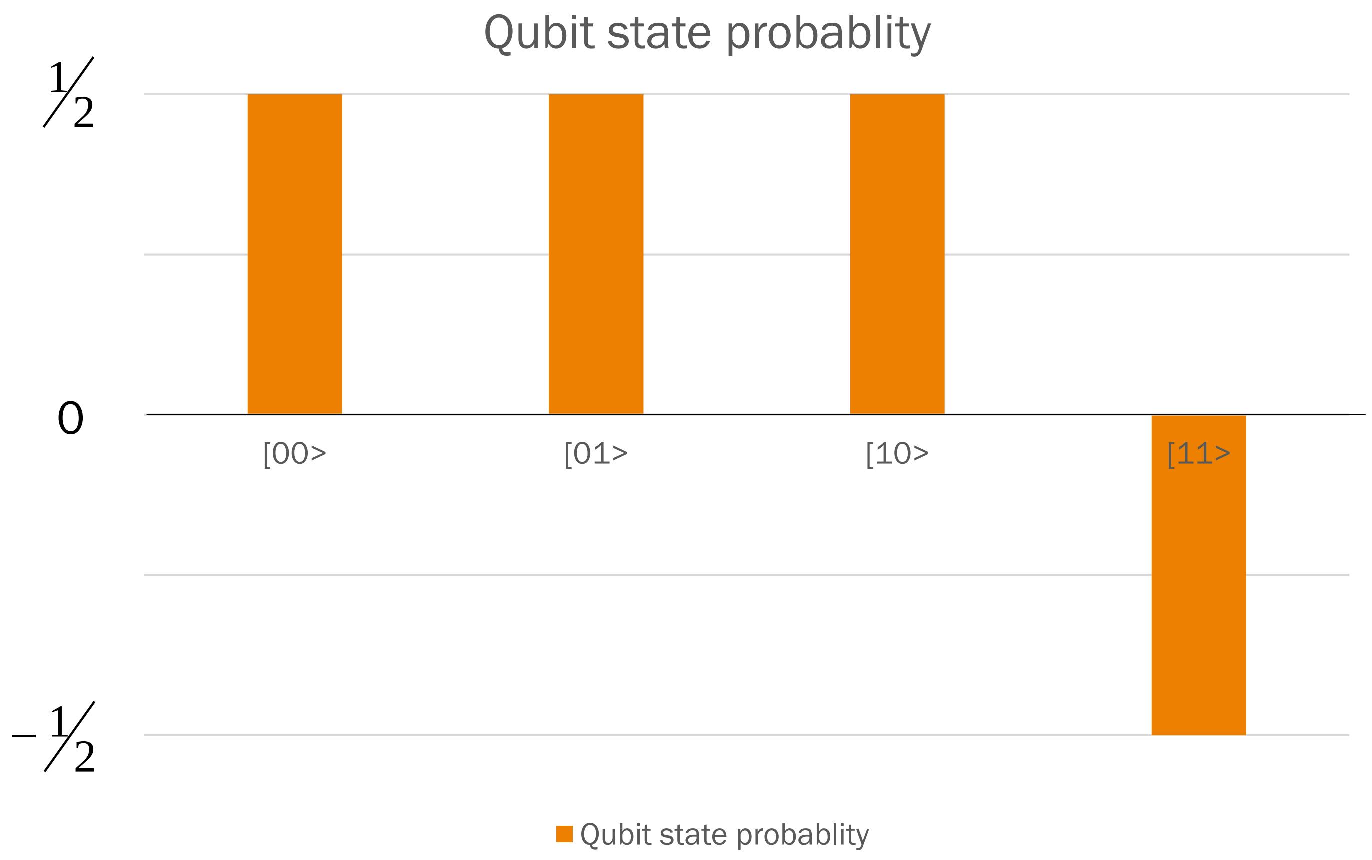


# GROVER'S ALGORITHM: PHASE FLIP

- We run a qubit operation that flips the phase of the correct answer to negative
- (correct answer here is 11 here)

- New state is:

$$\left|\Psi_1\Psi_2\right\rangle = \frac{1}{2}\left|00\right\rangle - \frac{1}{2}\left|11\right\rangle + \frac{1}{2}\left|01\right\rangle + \frac{1}{2}\left|10\right\rangle$$

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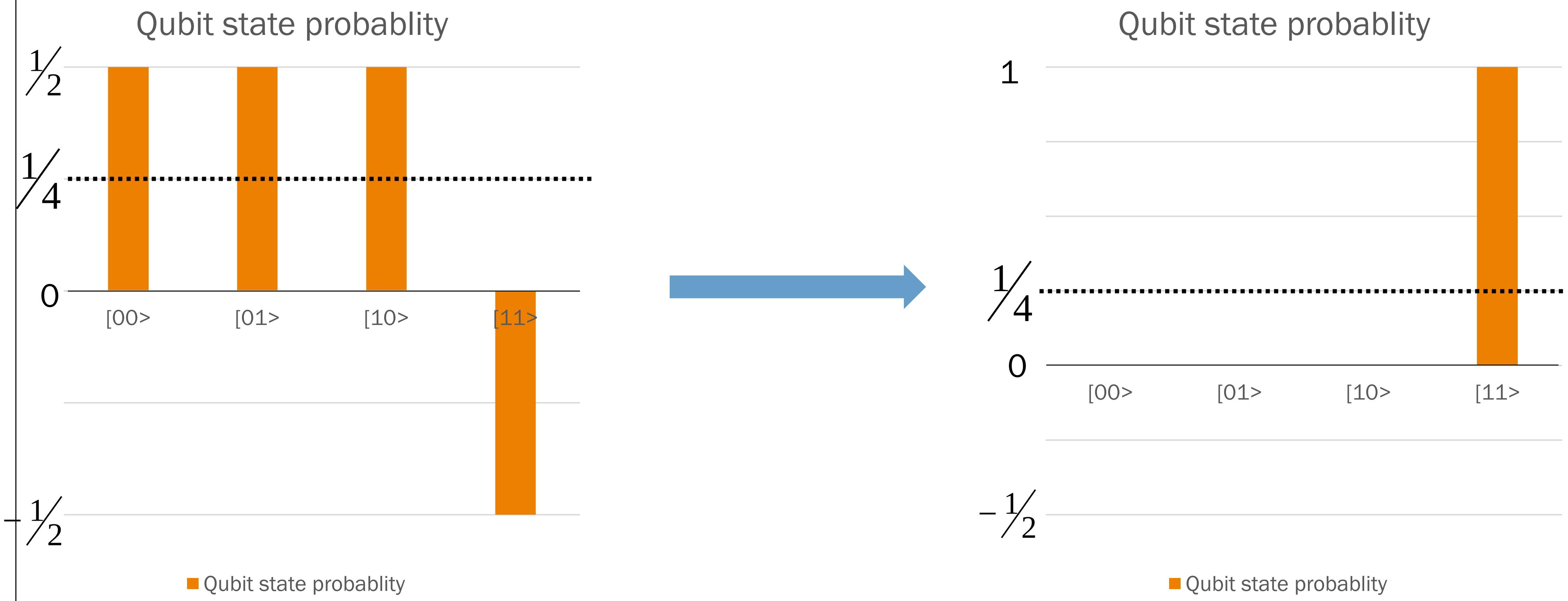
# SHOR’S ALGORITHM: AVERAGE INVERSION

- We next run an operation that inverts the state around the mean of probabilities



# QUANTUM OPERATION: PHASE FLIP

- Perhaps the previous explanation sounded a bit too convenient
- However these specific operations can be demonstrated to be possible to do with quantum circuits:
- First up, the conditional phase flip (Oracle):

Remember the matrix formulation?

$$\alpha|0\rangle + \beta|1\rangle = \begin{bmatrix} \alpha|0\rangle \\ \beta|1\rangle \end{bmatrix}$$

$$X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$$

A similar formulation can be made for 2 or more qubits

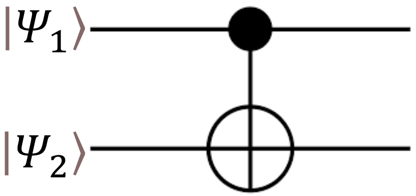
$$|\Psi_1\Psi_2\rangle = \alpha|00\rangle + \beta|11\rangle + \sigma|01\rangle + \omega|10\rangle = \begin{bmatrix} \alpha|0_1 0_2\rangle \\ \beta|0_1 1_2\rangle \\ \sigma|1_1 0_2\rangle \\ \omega|1_1 1_2\rangle \end{bmatrix}$$

$$CNOT|\Psi_1\Psi_2\rangle = CNOT|10\rangle = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \bullet \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = |11\rangle$$

# CNOT GATE

Gate where if the state of qubit 1 is 1 it flips the state of qubit 2:

| | |
|---|---|
| $\|0_1\rangle\|0_2\rangle$ | $\|0_1\rangle\|0_2\rangle$ |
| $\|0_1\rangle\|1_2\rangle$ | $\|0_1\rangle\|1_2\rangle$ |
| $\|1_1\rangle\|0_2\rangle$ | $\|1_1\rangle\|1_2\rangle$ |
| $\|1_1\rangle\|1_2\rangle$ | $\|1_1\rangle\|0_2\rangle$ |




Matrix representation

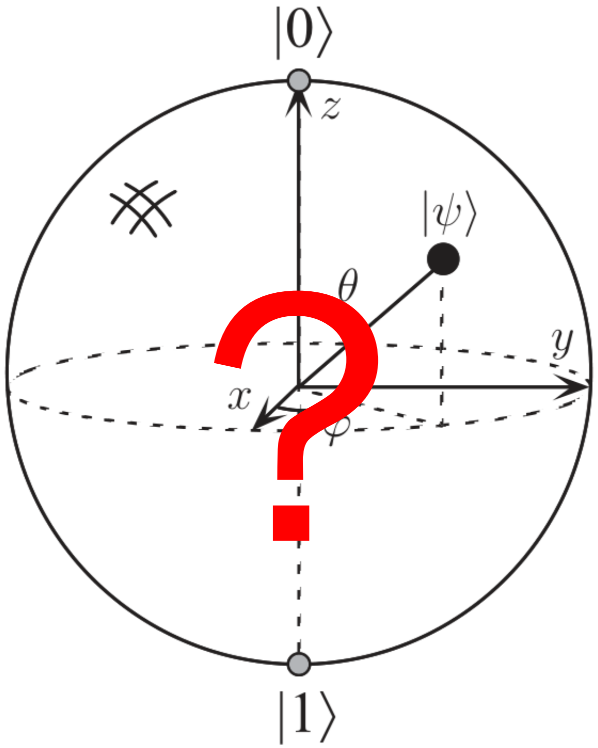
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \bullet \begin{bmatrix} |0_1\rangle|0_2\rangle \\ |0_1\rangle|1_2\rangle \\ |1_1\rangle|0_2\rangle \\ |1_1\rangle|1_2\rangle \end{bmatrix}$$

# HADAMARD

**Hadamard (H)** $\boxed{\mathbf{H}}$ $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$

$$\left|\Psi(?,?)\right\rangle =$$

Starting from the [0> state




Remember:

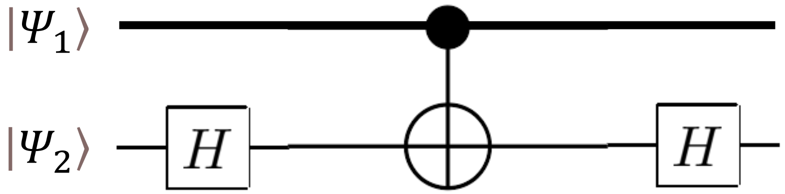
$$\left|\Psi(\theta,\varphi)\right\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\varphi}\sin\left(\frac{\theta}{2}\right)|1\rangle$$

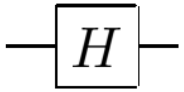
# QUANTUM OPERATION: PHASE FLIP

Back to our example, the operation we'd need to only flip the phase of the 11 state would be:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \bullet \begin{bmatrix} \alpha|0_1 0_2\rangle \\ \beta|0_1 1_2\rangle \\ \sigma|1_1 0_2\rangle \\ \omega|1_1 1_2\rangle \end{bmatrix} \longrightarrow$$

Which is a operation that can be created through 2 single qubit gates and one CNOT gate




Hadarmard gate: 


$$H|\Psi\rangle = H|0\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

# QUIZ 2:

If you wanted to make an oracle that flips the phases of all odd numbers, what would it look like?

$$\begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \bullet \begin{bmatrix} \alpha |0_1 0_2\rangle \\ \beta |0_1 1_2\rangle \\ \sigma |1_1 0_2\rangle \\ \omega |1_1 1_2\rangle \end{bmatrix}$$

# QUIZ 2:

If you wanted to make an oracle that flips the phases of all odd numbers, what would it look like?

Hint: since all you want is to change the phase, you only need the diagonals:

$$\begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \bullet \begin{bmatrix} \alpha|0_1 0_2\rangle \\ \beta|0_1 1_2\rangle \\ \sigma|1_1 0_2\rangle \\ \omega|1_1 1_2\rangle \end{bmatrix}$$

$$\begin{bmatrix} ? & 0 & 0 & 0 \\ 0 & ? & 0 & 0 \\ 0 & 0 & ? & 0 \\ 0 & 0 & 0 & ? \end{bmatrix} \bullet \begin{bmatrix} \alpha|0_1 0_2\rangle \\ \beta|0_1 1_2\rangle \\ \sigma|1_1 0_2\rangle \\ \omega|1_1 1_2\rangle \end{bmatrix}$$

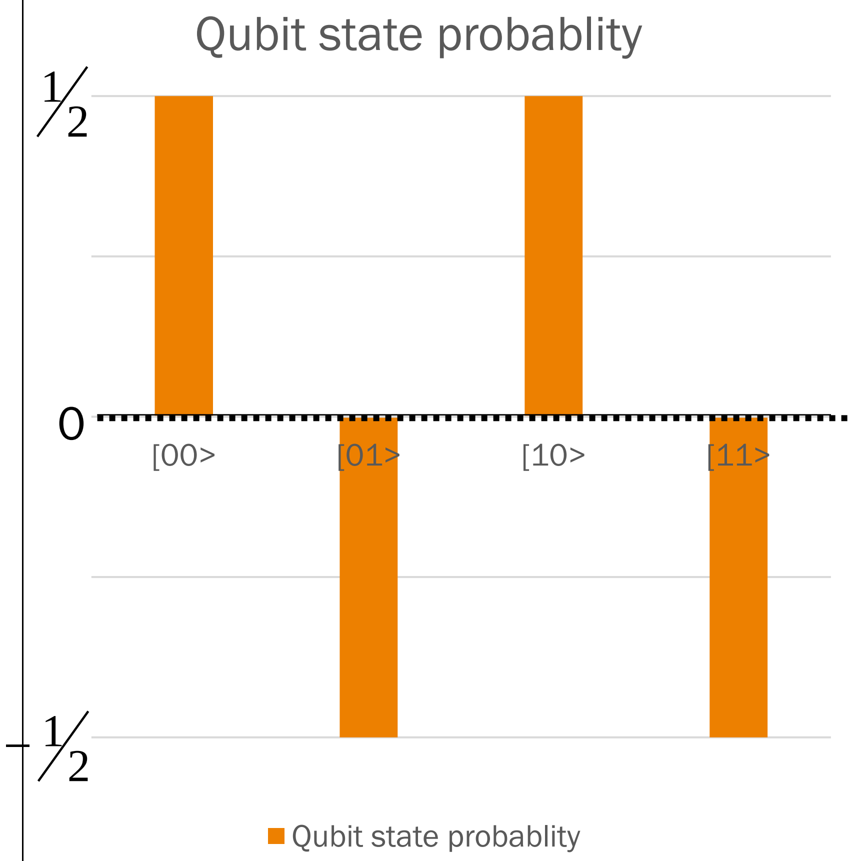
# QUIZ 2:

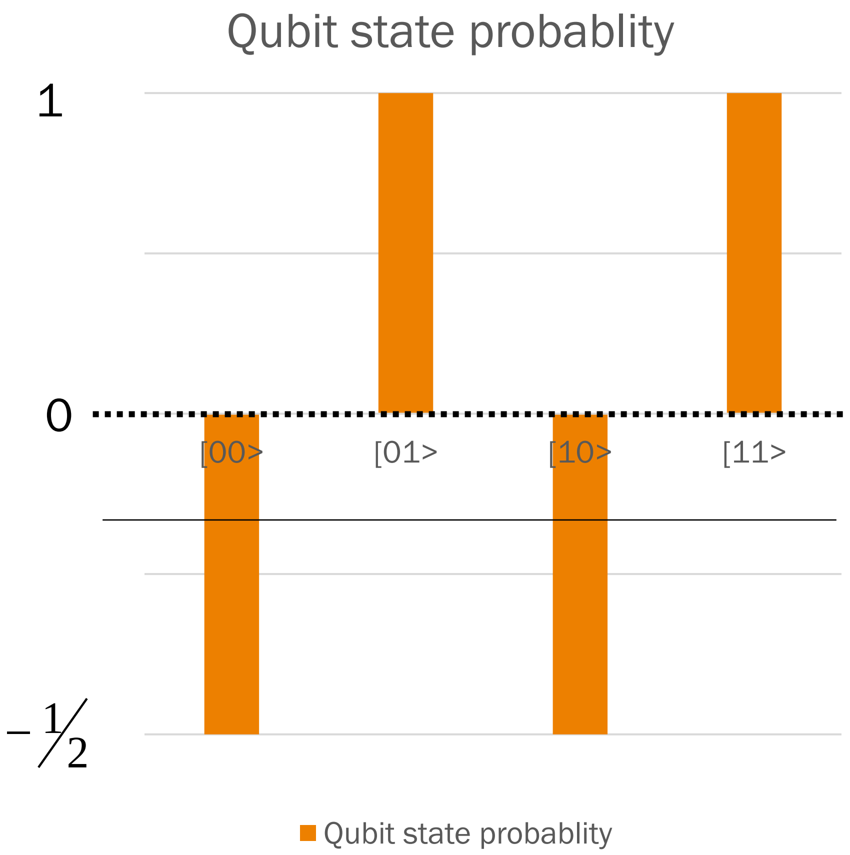
Answer:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \bullet \begin{bmatrix} \alpha\left|0_1 0_2\right\rangle \\ \beta\left|0_1 1_2\right\rangle \\ \sigma\left|1_1 0_2\right\rangle \\ \omega\left|1_1 1_2\right\rangle \end{bmatrix}$$

# SHOR'S ALGORITHM: AVERAGE INVERSION

- We next run an operation that inverts the state around the mean of probabilities



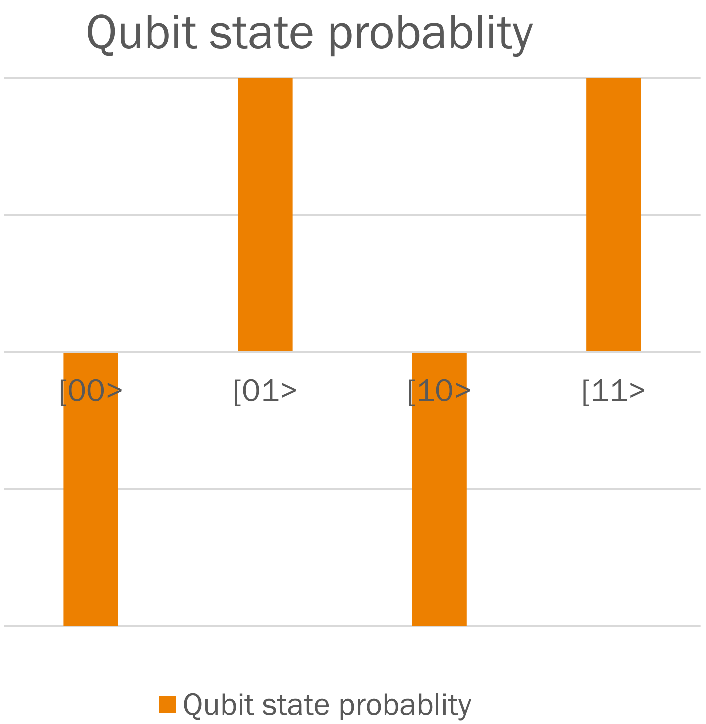




Probabilities fail to amplify!

# LIMITATIONS OF GROVER

Performance of grover's algorithm will depend on the amount of possible solutions and possible correct answers

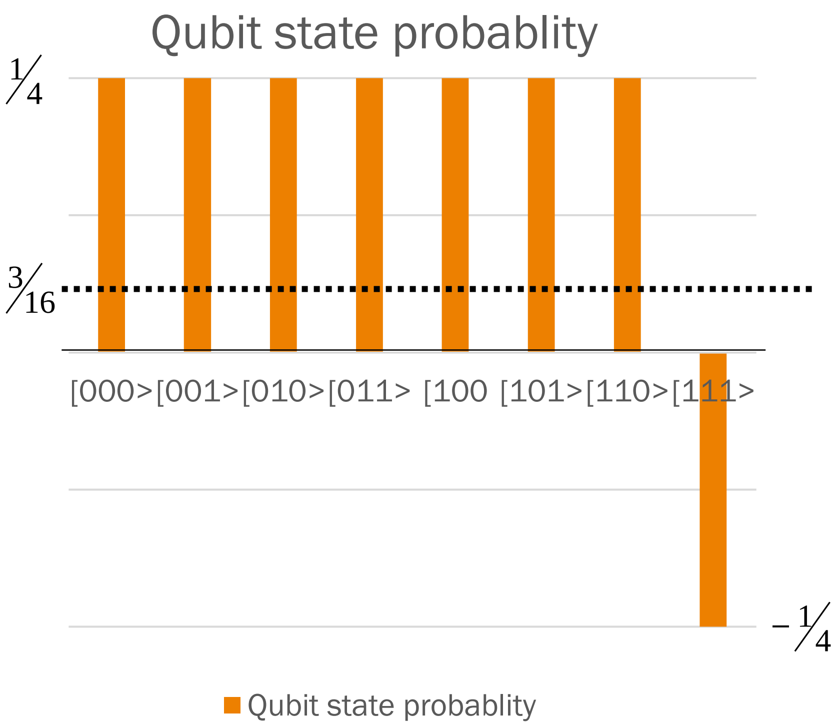
Case: exactly half the answers are correct



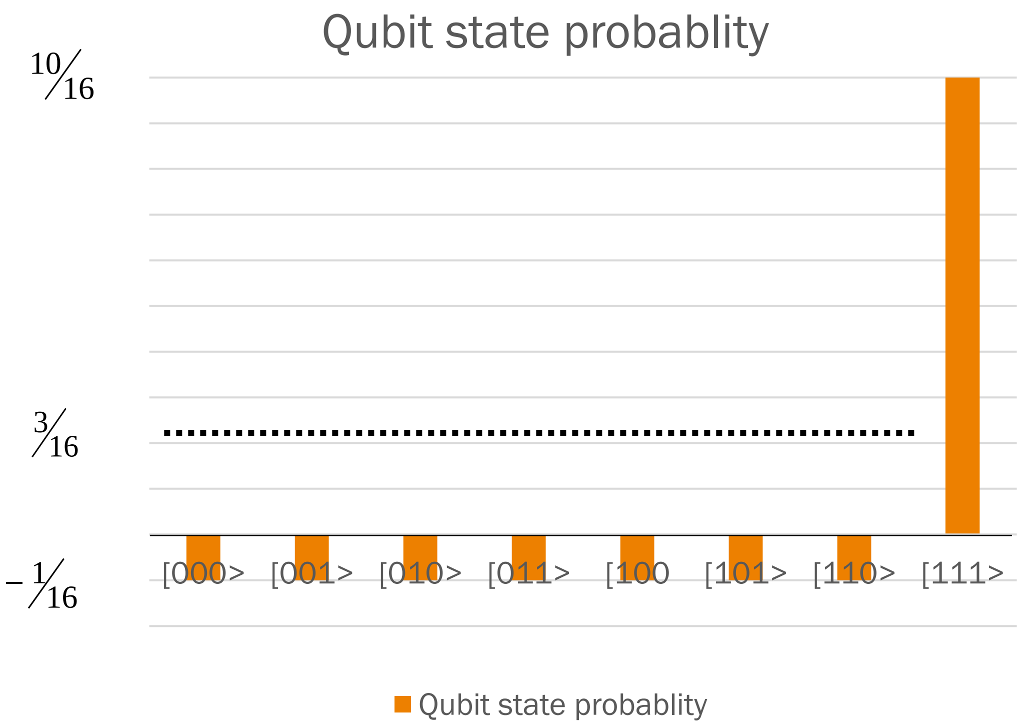

Grover will not converge

# LIMITATIONS OF GROVER

Case, out of 8 possible answers, 1 is correct




One loop of phase amplification isn't quite cutting it

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# LIMITATIONS

- No problem, we can just repeat the process and amplify the correct process further

- With some maths you can determine that the amount of loops needed scales with:
- M = amount of correct answers
- N = total amount of possible answers

amount of iterations needed is

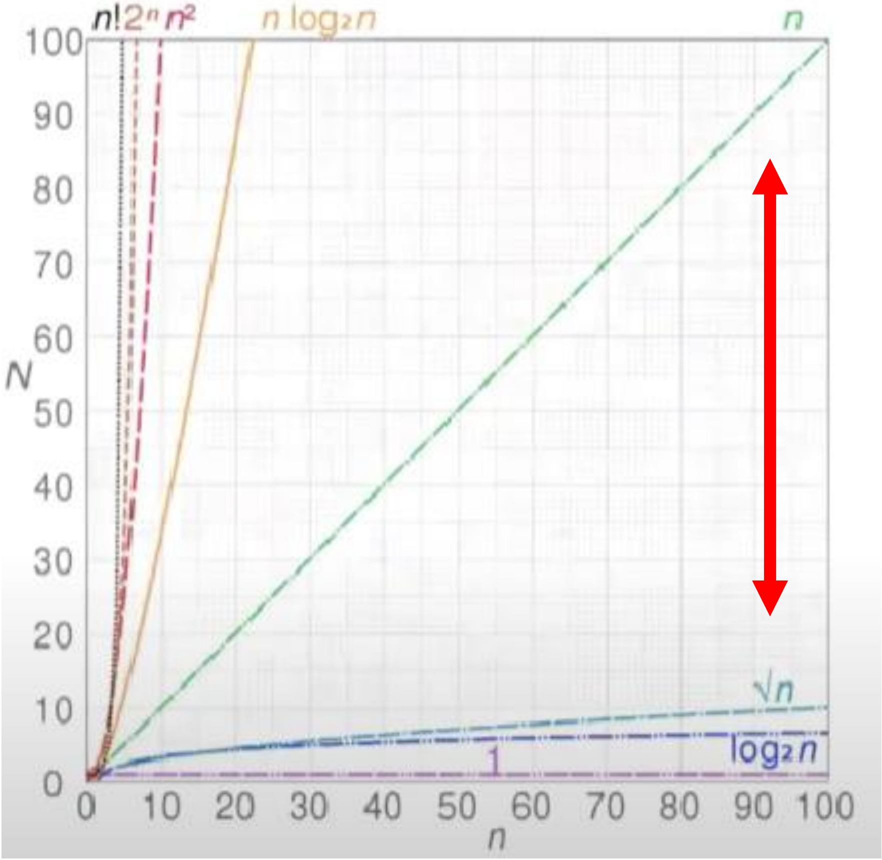
| Quantum: | Classical: |
| --- | --- |
| $\sqrt{\frac{N}{M}}$ | $\frac{N}{M}$ |

In this example:

$$\sqrt{\frac{8}{1}} = 2\sqrt{2} = 2{,}82842 \sim 3$$

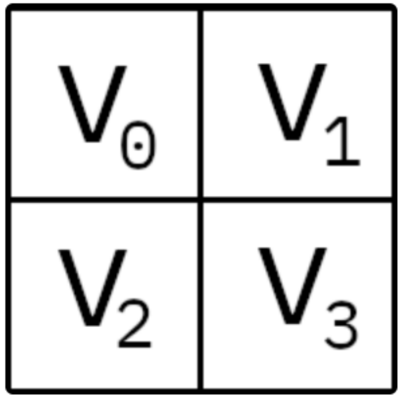
$$\frac{8}{1} = 8$$

Quadratic speedup!



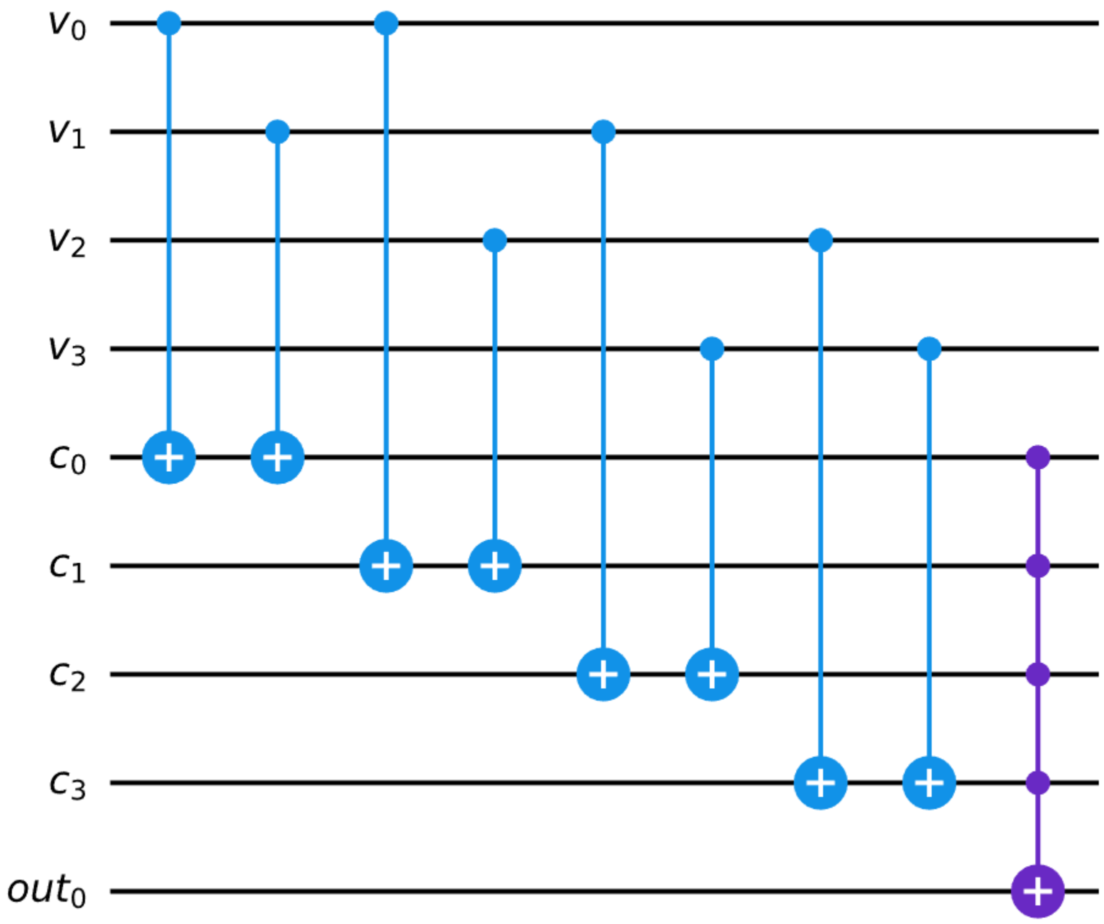
# ADVANCED ORACLES

Solving 1 bit sudoku:



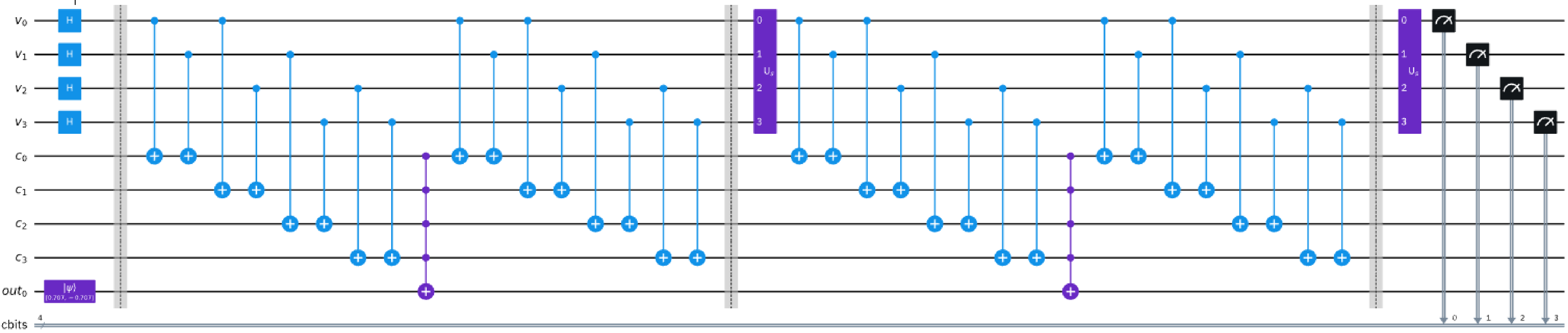

Rules:
Two numbers in the same row or column may not have the same numbers

V0 =/= V1
V0 =/= V2
V1 =/= V3
V2 =/= V3




(Example from: learn.qiskit.org)

# FULL CIRCUIT

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# ENTANGLEMENT

Rotate your first qubit to the 50/50 state
Rotate your second qubit to the 0 state

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}|0_1\rangle + \frac{1}{\sqrt{2}}|1_1\rangle$$

$$|\Psi_2\rangle = |0_2\rangle$$

$$|\Psi_1\rangle \otimes |\Psi_2\rangle = \left(\frac{1}{\sqrt{2}}|0_1\rangle + \frac{1}{\sqrt{2}}|1_1\rangle\right) \otimes |0_2\rangle = \frac{1}{2}|0_1\rangle \otimes |0_2\rangle + \frac{1}{2}|1_1\rangle \otimes |0_2\rangle$$

Apply a cnot gate:

$$CNOT|\Psi_1\rangle|\Psi_2\rangle = \frac{1}{2}|0_1\rangle|0_2\rangle + \frac{1}{2}|1_1\rangle|1_2\rangle$$

Qubits are now entangled

# THANKS FOR LISTENING


Greetings from delft

Contact: tumi.makinwa@tno.nl

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# THANK YOU FOR YOUR TIME

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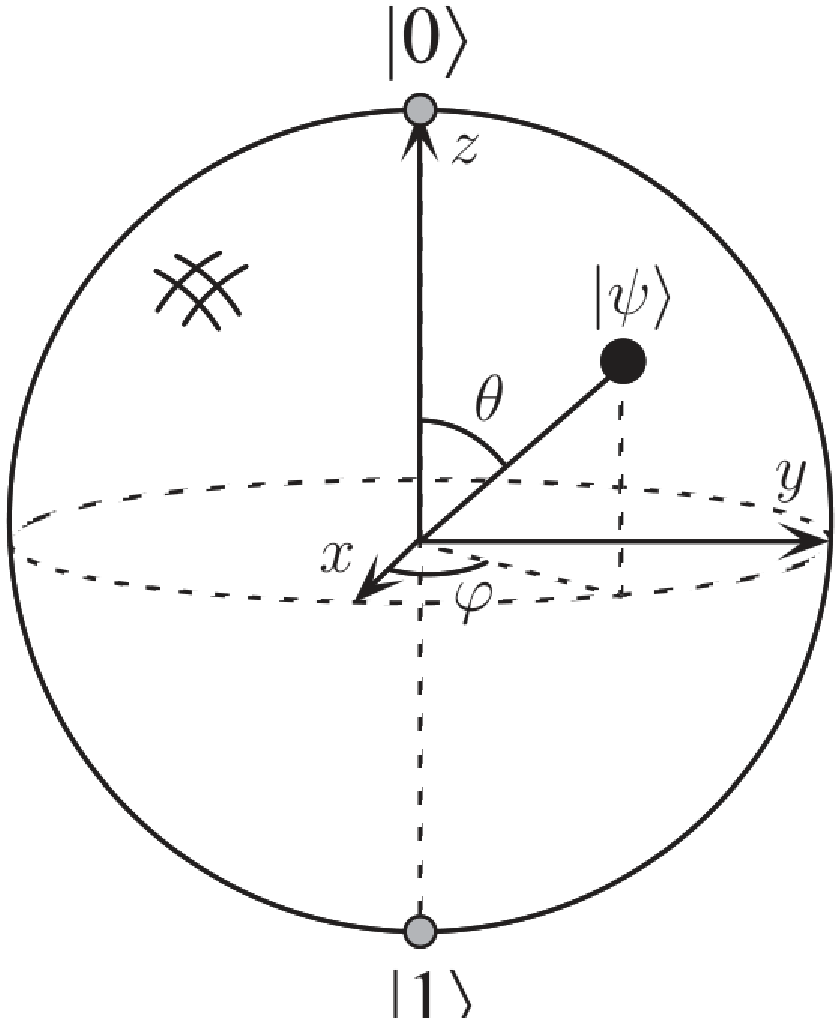
# BREAK TIME

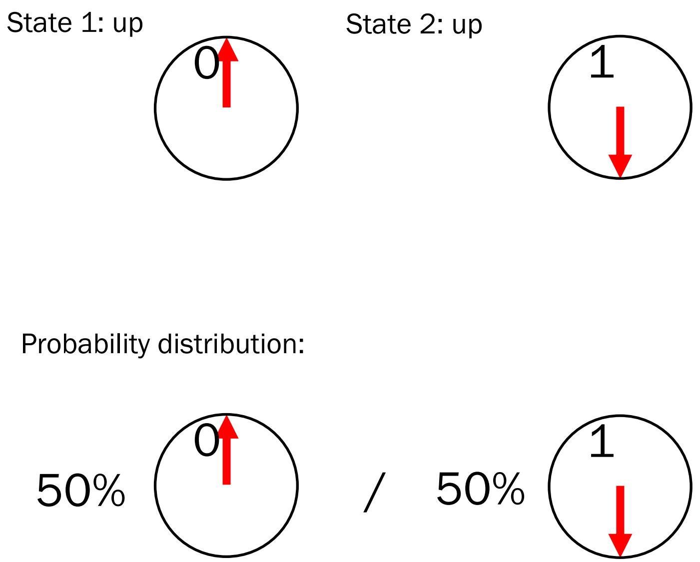
# MATHEMATICAL DESCRIPTION: BLOCH SPHERE

To help understand qubit states and their operations we car
visualize as states and rotations on a sphere:



# QUANTUM SUPERPOSITION

- We nowhave a system of which its probability can get stuck somewhere between the 2 states



# QUANTUM SUPERPOSITION

# FIRST TITLE OF THE SLIDE
# SECOND TITLE OF THE SLIDE

## HEADER

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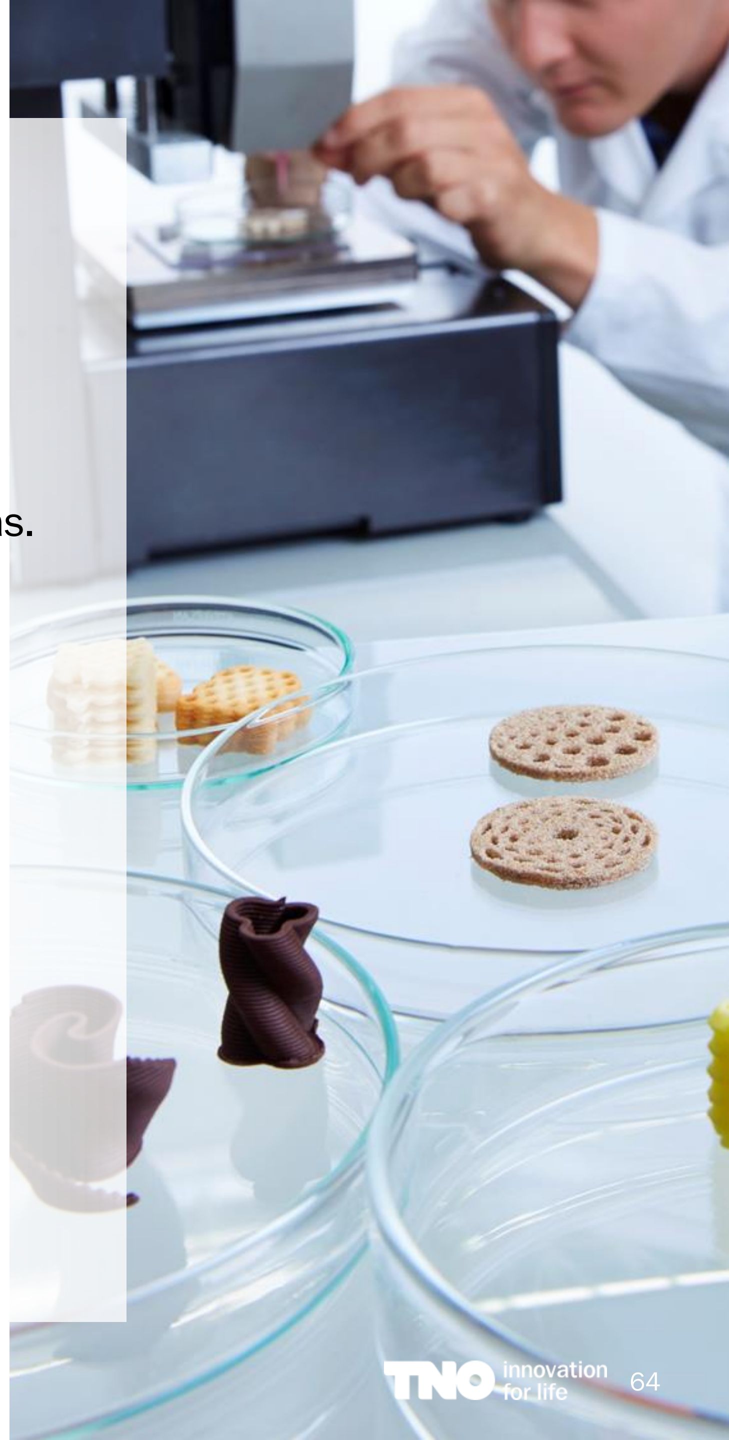
- Fusce est.
- Vivamus a tellus.
- Pellentesque habitant.
- Morbi tristique senectus et.
  - Netus et malesuada fames ac turpis egestas.
  - Proin pharetra nonummy pede. Mauris et orci.



# FIRST TITLE OF THE SLIDE
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- Nunc viverra imperdiet enim.
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- Morbi tristique senectus et.
    - Netus et malesuada fames ac turpis egestas.
    - Proin pharetra nonummy pede. Mauris et orci.

# FIRST TITLE OF THE SLIDE MAX. 2 LINES
## SECOND TITLE OF THE SLIDE

# FIRST TITLE OF THE SLIDE MAX. 2 LINES

## SECOND TITLE OF THE SLIDE

# MATHEMATICAL DESCRIPTION: BLOCH SPHERE

Example 1:
Rotation of pi over purely the Y axis: $Y(\pi)$

$$\left|\Psi(\theta,\varphi)\right\rangle = \cos\left(\frac{\theta}{2}\right)\left|0\right\rangle + e^{i\varphi}\sin\left(\frac{\theta}{2}\right)\left|1\right\rangle$$

$$\left|\Psi(\pi,0)\right\rangle =$$

$$\cos\left(\frac{\pi}{2}\right)\left|0\right\rangle + e^{i0}\sin\left(\frac{\pi}{2}\right)\left|1\right\rangle =$$

$$0*\left|0\right\rangle + 1*1\left|1\right\rangle =$$

$$\left|1\right\rangle$$