Tech Documentation: Mode Extraction

Increasing energy efficiency of robotic locomotion is a major goal for robotics. Using compliant elements, such as springs, can help achieving this by storing the energy at ground impact and reusing it for take. Based on this idea, Lakatos et al. (2013) derived a Bang Bang controller that excites periodic motions along the direction of an intrinsic mechanical oscillation mode, without requiring model knowledge, but only depending on sensory information. In order to do so, the controller reduces the control of the robotic system with several joints to a one-dimensional controller space. While this transformation strongly simplifies the control, it still allows for optimal energy efficiency of the controlled motion as shown by Stratmann et al (2016).

To drive the motion, the controller requires information about the torques in each of the joints of the system. This torque $\boldsymbol{\tau}$ is a function of the motor coordinate $\boldsymbol{\theta}$ and the joint state $\mathbf{q}$:

$$\boldsymbol{\tau} = \mathrm{k}(\boldsymbol{\theta} - \mathbf{q})$$

As described by Lakatos et al. (2013), the torque is transformed from the higher dimensional joint space, onto a 1D control signal τ_z along modal weights $\boldsymbol{w}$ by

$$\tau_z = \frac{\boldsymbol{w}^T}{||\boldsymbol{w}||}\boldsymbol{\tau}$$

This feedback τ_z is a 1D signal that combines information about both joints. The value is compared to a threshold value ε_T and triggers a control signal θ_z of constant amplitude according to

$$\theta_z = \begin{cases} +\hat{\theta}_z & if\ \tau_z > -\varepsilon_T \\ 0 & if - \varepsilon_T \le \tau_z \le \varepsilon_T \\ -\hat{\theta}_z & if\ \tau_z < \varepsilon_T \end{cases}$$

Using the same weights as introduced for the mapping onto the control space, the signal θ_z can be mapped back onto the joint space

$$\boldsymbol{\theta} = \frac{\boldsymbol{w}^T}{||\boldsymbol{w}||}\theta_z$$

The weights that map the control signal can automatically adapt to match the eigenmode of the mechanical system and continuously excite the system along this oscillation mode. Two possible ways to adapt the weights are implemented in the corresponding NRP experiment in the `set_weight` transfer function. One option is to apply a Principle Component Analysis of the joint angles $\boldsymbol{q}$ to approximate the linearized eigenmode, namely using the Oja Rule [`weight_change=3`]:

$$\dot{\boldsymbol{w}}(\boldsymbol{t}) = \boldsymbol{\gamma}\left(\boldsymbol{w}(\boldsymbol{t})^{\boldsymbol{T}}\, \boldsymbol{q}(\boldsymbol{t})\right) [\boldsymbol{q}(\boldsymbol{t}) - \left(\boldsymbol{w}(\boldsymbol{t})^{\boldsymbol{T}}\, \boldsymbol{q}(\boldsymbol{t})\, \boldsymbol{w}(\boldsymbol{t})\right)]$$

Another option to determine the correct weighting of the motor signal for each joint is the use of the Gradient Descent method [`weight_change=4`].

For further information about the Bang Bang controller, please refer to Lakatos et al. (2013) and Stratmann et al (2016). An overview of the implemented controller taken from Stratmann et al. (2018) is depicted below:

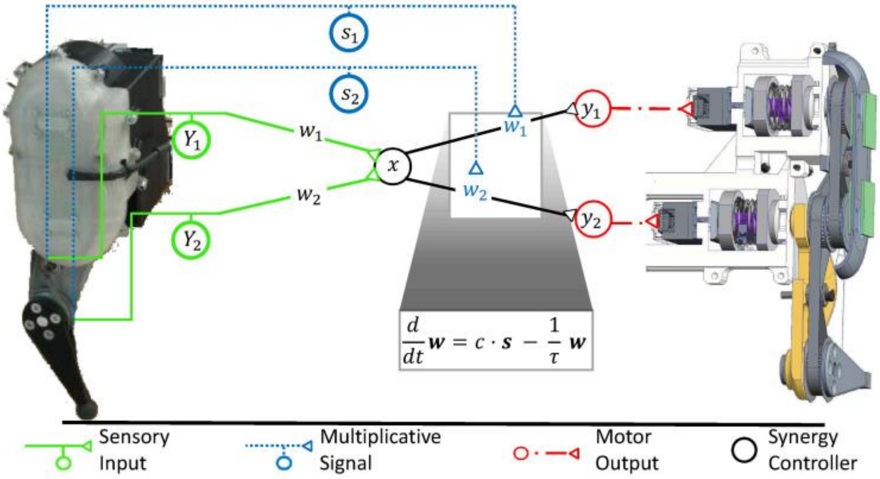
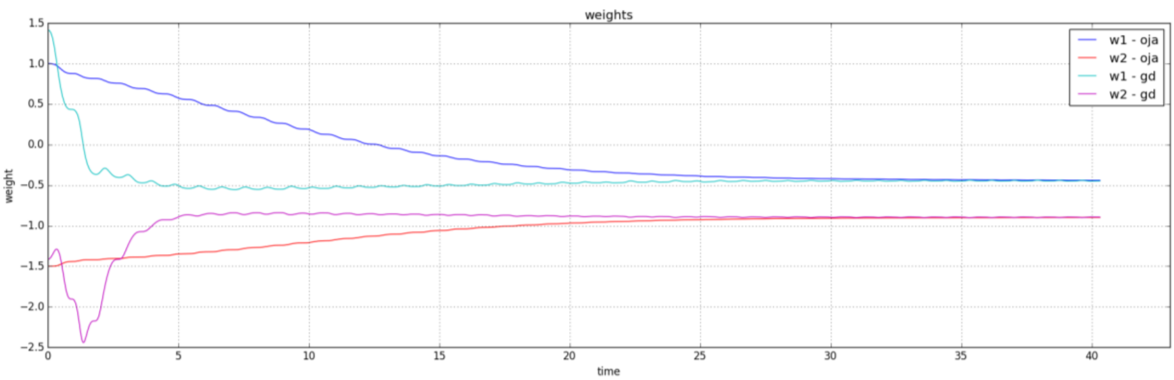


FIGURE 5 | The robotic controller, which was previously developed to maintain a stable, fast, and strong movement, is mathematically equivalent to a synergy controller as illustrated in **Figure 1B**. Sensory input (Y_1, Y_2) signals the movement of individual degrees of freedom of the mechanical system. It is transformed into the low-dimensional synergy space and adjusts the phase and frequency of a synergistic motor signal x. Since the different mechanical degrees of freedom move with high synchrony during fast locomotion, the input weights can be chosen arbitrarily without loss in movement performance (Stratmann et al., 2016). The motor signal x is reversely transformed along the weights (w_1, w_2) to drive the actuators of the same robot. These weights are functionally critical, as the relative forces (y_1, y_2) of different actuators determine, among others, how well the robot can take advantage of its elastic elements to store energy upon ground impact and release it for recoil. In robotic systems, springs typically dominate the elastic properties of the system, as shown on the right hand side of this figure in a cross-sectional illustration of the exemplary robot. As detailed in section 6, the synergy controller can maintain an elastic movement with optimal energy efficiency under changing mechanical conditions. For this purpose, the controller receives sensory inputs s_1 and s_2, which signal the deflections of the degrees of freedom such as joint angles. To adjust to changing mechanical conditions, the controller needs to multiplicatively scale the output transformation weights w_1 and w_2 according to these inputs s_1 and s_2, respectively (Stratmann et al., 2017). A common multiplicative factor c determines the jumping height or distance. To keep the weights bounded, they need to decay exponentially with a time constant τ. The ventral spinal serotonergic system forms a motor feedback loop, as illustrated in **Figure 4**, which functionally resembles the loop of the multiplicative signals presented here.

No matter which method to adjust the weights is chosen (Oja rule: oja, or gradient descent: gd), both converge to the same values as seen in this graph which was produced by running the experiment in the NRP:



Alternatively, the weights can be set to a constant, e.g. to trigger a desired behavior like moving forward.

For investigation purposes, it is also possible to denote the weights by their polar coordinates, such as:

$$\boldsymbol{\theta} = \frac{\boldsymbol{w}^T}{||\boldsymbol{w}||}\theta_z = \begin{pmatrix}\sin(\alpha)\\ \cos(\alpha)\end{pmatrix}\theta_z$$

where α is the angle between the transformation weights $\alpha \in [\pi, 2\pi]$. The constraint on α prevents ambiguity, since a change of α by π corresponds to a change of sign of θ_z. For further details on the use of this weighting, please refer to Stratmann et al (2016).

To adjust the control method to a new mechanical system, different parameter can be tuned:

- threshold (ε_T): threshold that determines when controller signal is triggered
- theta_z : determines the magnitude of the motor signal

When a different compliant system with different spring stiffness should be actuated by this controller, it is important to adjust the value of *k* to correspond to the stiffness defined for each joint in the .sdf-model of the robot, as it is important to calculate the acting torque $\boldsymbol{\tau}$ in each joint. Alternatively, the joint effort can be read out by using a ROS topic to determine this value.

References

Lakatos, D., Görner, M., Petit, F., Dietrich, A., & Albu-Schäffer, A. (2013, November). A modally adaptive control for multi-contact cyclic motions in compliantly actuated robotic systems. In *2013 IEEE/RSJ International Conference on Intelligent Robots and Systems* (pp. 5388-5395). IEEE.

Stratmann, P., Lakatos, D., Özparpucu, M. C., & Albu-Schäffer, A. (2016). Legged elastic multibody systems: adjusting limit cycles to close-to-optimal energy efficiency. *IEEE Robotics and Automation Letters*, *2*(2), 436-443.

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