PHYS 240 homework #11 – due Mar 5 2013, 5:25pm, upload to Canvas

System of springs and matrix inversion

1. Consider the system shown below; the spring constants are $k_1, \ldots, k_4$. The positions of the blocks, relative to the left wall, are x_1, x_2, and x_3. The distance between the walls is L_w, and the unstretched lengths of the springs are $L_1, \ldots, L_4$. The blocks are of negligible width.

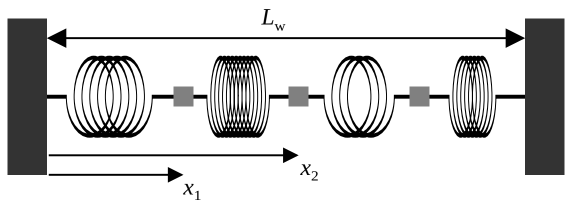

Figure 1: System of blocks coupled by springs anchored between walls.

As discussed in the text, the static solution of this system can be described by this matrix equation:

$$\begin{bmatrix} -k_1 - k_2 & k_2 & 0 \\ k_2 & -k_2 - k_3 & k_3 \\ 0 & k_3 & -k_3 - k_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -k_1 L_1 + k_2 L_2 \\ -k_2 L_2 + k_3 L_3 \\ -k_3 L_3 + k_4 (L_4 - L_\mathrm{w}) \end{bmatrix}$$

Write a program to solve for the rest positions of the masses using matrix inversion. Try your program with the following values:

(a) $\mathbf{k} = [1\ 2\ 3\ 4]$; $\mathbf{L} = [1\ 1\ 1\ 1]$; $L_\mathrm{w} = 4$
(b) $\mathbf{k} = [1\ 2\ 3\ 4]$; $\mathbf{L} = [1\ 1\ 1\ 1]$; $L_\mathrm{w} = 10$
(c) $\mathbf{k} = [1\ 1\ 1\ 1]$; $\mathbf{L} = [2\ 2\ 1\ 1]$; $L_\mathrm{w} = 4$
(d) $\mathbf{k} = [1\ 1\ 1\ 0]$; $\mathbf{L} = [2\ 2\ 1\ 1]$; $L_\mathrm{w} = 4$
(e) $\mathbf{k} = [0\ 1\ 1\ 0]$; $\mathbf{L} = [2\ 2\ 1\ 1]$; $L_\mathrm{w} = 4$

Using general physical arguments, explain the results in each case.

2. The force on the right wall is $F_\mathrm{rw} = -k_4(L_\mathrm{w} - x_3 - L_4)$. Write a program to solve $\mathbf{Kx} = \mathbf{b}$, evaluate F_rw, and plot it as a function of L_w. For the other parameters, select any nontrivial values. Verify numerically that the force on the right wall is

$$F_\mathrm{rw} = -k_0(L_\mathrm{w} - L_0),$$

where $L_0 = L_1 + L_2 + L_3 + L_4$ and

$$\frac{1}{k_0} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \frac{1}{k_4}$$

This is the law of equivalent springs. Since the matrix $\mathbf{K}$ is fixed, it is more efficient to use matrix inverse rather than Gaussian elimination.

3. Include any discussion in a report generated in LaTeX. Also submit your Python code separately.