PHYS 240 homework #7 – due Feb 19 2013, 5:25pm, upload to Canvas

Wilberforce pendulum

1. The Wilberforce pendulum, a popular demonstration device, is illustrated in Figure 1. The pendulum has two modes of oscillation: vertical and torsional motion. The Lagrangian for this system is

$$L = \frac{1}{2}m\left(\frac{dz}{dt}\right)^2 + \frac{1}{2}I\left(\frac{d\theta}{dt}\right)^2 - \frac{1}{2}kz^2 - \frac{1}{2}\delta\theta^2 - \frac{1}{2}\epsilon z\theta$$

where m and I are the mass and rotational inertia of the bob, k and δ are the longitudinal and torsional spring constants, and ϵ is the coupling constant between the modes. Some typical values are $m = 0.5$ kg, $I = 10^{-4}$ kg $\cdot\mathrm{m}^2$, $k = 5$ N/m, $\delta = 10^{-3}$ N $\cdot$ m, and $\epsilon = 10^{-2}$ N. **(a)** Find the equations of motion. **(b)** Write a program to compute $z(t)$ and $\theta(t)$ using fourth-order Runge-Kutta. Try the initial conditions $z(0) = 10$ cm, $\theta(0) = 0$ and $z(0) = 0$, $\theta(0) = 2\pi$. Show that when the longitudinal frequency, $f_z = (2\pi)^{-1}\sqrt{k/m}$, equals the torsional frequency, $f_\theta = (2\pi)^{-1}\sqrt{\delta/I}$, the motion periodically alternates between being purely longitudinal and purely torsional.

2. Include any discussions and plots in a report generated in LaTeX. Also submit your Python code separately.

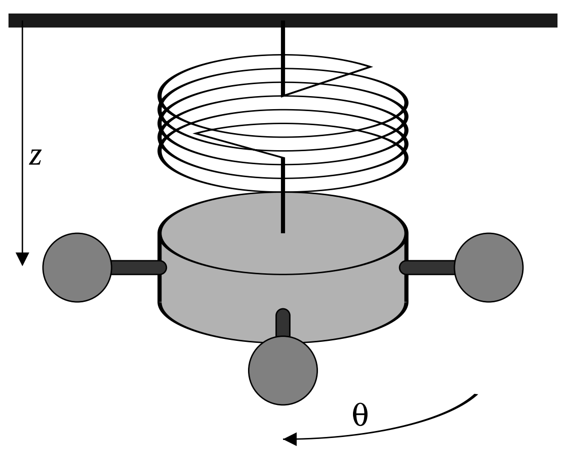

Figure 1: Wilberforce pendulum.