PHYS 240 homework #22 – due May 3 2013, 8:00pm, upload to Canvas

Romberg integration

1. Finish the code that we started in class, `int_erf.py`, which should include a function for Romberg integration, and should use this to evaluate the integral:

$$I = \frac{2}{\sqrt{\pi}} \int_0^1 e^{-x^2}\, dx$$

Besides providing the final answer to some reasonable tolerance level, also report the full Romberg table and discuss the relation between number of iterations and the error in the result.

2. Use Romberg integration to numerically evaluate the integrals below:

(a) $\int_0^1 e^x\, dx$ (b) $\int_0^{2\pi} \sin^4(8x)\, dx$ (c) $\int_0^1 \sqrt{x}\, dx$

(d) $\int_0^1 \sqrt{1-x^2}\, dx$ (e) $\int_{-1}^1 P_{10}(x)\, dx$ (f) $\int_{-1}^1 P_{10}^2(x)\, dx$

In each case, evaluate the integral analytically and graph the absolute error for the main diagonal of the Romberg table, $R_{i,i}$, versus i. Show that the error normally decreases with increasing i, but can increase due to round-off. For any integrals that Romberg has difficulties with, discuss why, and find out if there are any `scipy` integrators that do better.

3. Include any discussion and plots in a report generated in LaTeX and submitted in PDF format. Also submit your Python code separately.