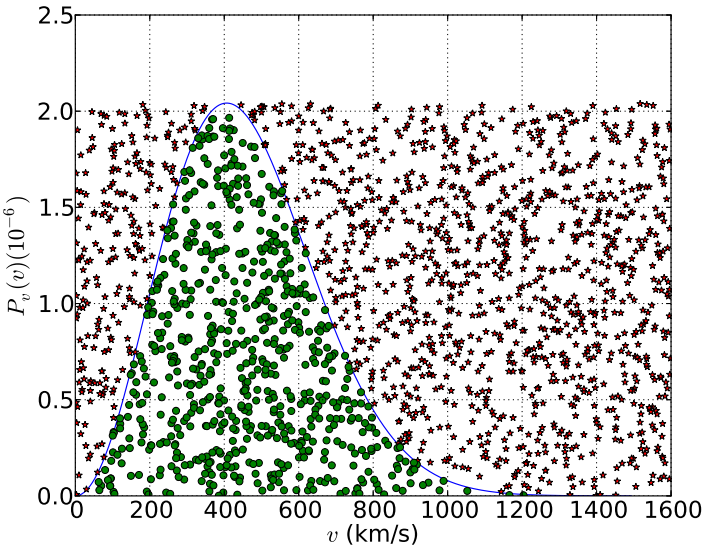
PHYS 240 homework #23 – due May 10 2013, 5:00pm, upload to Canvas

User's choice

Choose either exercise #1 or #2 below to complete (extra credit for doing both).

1. Monte Carlo integration selects random points instead of a regular grid, and is particularly useful when the dimensionality of the integral is large, or the boundaries are complicated. The basic idea is to adopt a simple shape V' that completely encompasses the region V to be integrated. N random points are selected from V', and are 'hits' if they fall within V (see figure below). The ratio of hits to tries is approximately equal to the ratio of integrals over V and V' (where the latter is trivial to calculate).



(a) Choose one of the integrals from the last assignment, and estimate it by Monte Carlo integration. Graph the error as a function of N and comment on its functional dependence.

(b) Next take two 4-dimensional hyperspheres of unit size, whose centers are separated by a distance a. Use Monte Carlo integration to compute the hypervolume of the union of these hypersheres. First verify your program works for $a = 0$ and $a = 2$ (note the hypervolume of one hypersphere is $\pi^2/2$). Then find the results for $a = \frac{1}{2}, 1$, and $\frac{3}{2}$.

2. (a) Write a program that generates a vector of 1024 Gaussian distributed random values, computes the discrete Fourier transform using the FFT, and plots the power spectrum. This time series is an example of **white noise**. Produce at least four plots, each using a different initial seed.

(b) Assemble a time series consisting of white noise plus a sinusoid of amplitude α and a period of 20 data points. Compute the power spectrum for a variety of values of α. In your judgment, what is the minimum value of α for which the sinusoid is distinctly seen in the spectrum?

3. Include any discussion and plots in a report generated in LaTeX and submitted in PDF format. Also submit your Python code separately.