Class Attendance
Important Questions
Syllabus & Class Notes
MST-I Result
Assignments
Exam Schedule
MST-I (05-07 Nov 2015)
Dr.(Prof.) Amita Mourya
Contact

Engineering Physics by Dr. Amita Maurya, Peoples University, Bhopal. > Electromagnetics >

Maxwell's equations

Maxwell's first equation or Gauss's law in electrostatics

Statement. It states that the total electric flux φ_E passing through a closed hypothetical surface is equal to $1/\varepsilon_0$ times the net charge enclosed by the surface:

$\Phi_E=\int E.dS=q/\varepsilon_0$

$\int D.dS=q$

where $D=\varepsilon_0 E$= Displacement vector

Let the charge be distributed over a volume V and p be the volume charge density .therefore $q=\int pdV$

Therefore $\int D.dS=\int_v pdV$ (1)

Equation (1) is the **integral form of Maxwell's first equation** or Gauss's law in electrostatics.

Differential form:

Apply Gauss's Divergence theorem to change L.H.S. of equation(1) from surface integral to volume integral

That is $\int D.dS=\int(\nabla.D)dV$

Substituting this equation in equation (1), we get

$\int (\nabla.D)dV=\int_v pdV$

As two volume integrals are equal only if their integrands are equal

Thus, $\nabla.D=p$ (2)

Equation (2) is the **Differential form of Maxwell's first equation.**

Maxwell's second equation or Gauss's law for Magnetism

Statement. It states that the total magnetic flux φ_m emerging through a closed surface is zero.

$$\varphi_m = \int B.dS = 0 \qquad (3)$$

The equation (3) is the **Intergal form of Maxwell's second equation**.

This equation also proves that magnetic monopole does not exist.

Differential Form:

Apply Gauss's Divergence theorem to equation (3)

That is $\int_s B.dS = \int_v (\nabla.B)dV$

As $\int B.dS = 0$

Thus , $\nabla .B = 0 \qquad (4)$

The equation (4) is **differential form of Maxwell's second equation**.

Maxwell's Third equation

Statement.(a) It states that,whenever magnetic flux linked with a circuit changes then induced electromotive force (emf) is set up in the circuit. This induced emf lasts so long as the change in magnetic flux continues.

(b) The magnitude of induced emf is equal to the rate of change of magnetic flux linked with the circuit.

Therefore induced emf $= -\, d\varphi_m/dt$

Where $\varphi_m = \int \mathbf{B.dS}$ (5)

Here negative sign is because of Lenz's law which states that the induced emf set up a current in such a direction that the magnetic effect produced by it opposes the cause producing it.

Also definition of emf states that emf is the closed line integral of the non-conservative electric field generated by the battery.

That is $\text{emf}=\int E.dL$ (6)

Comparing equations (5) and (6), we get

$\int E.dL= -\int_s d/dt\, B.dS$ (7)

Equation (7) is the **integral form of Maxwell's third Equation** or Faraday's law of electromagnetic induction.

Note: You can also read the discussion and derivation of Maxwell first and second equation.

Differential form :

Apply Stoke's theorem to L.H.S. of equations (7) to change line integral to surface integral.

That is $\int E.dL=\int(\nabla \times E).dS$

By substituting above equation in equation(7), we get

$\int_s (\nabla \times E).dS = -\int d/dt(B.dS)$

As two surface integral are equal only when their integrands are equal.

Thus $\nabla \times E= - dB/ dt$ (8)

Equation (8) is the **Differential form of Maxwell's third equation**.

Let us first discuss the Maxwell equations. In this, let us first derive and discuss Maxwell fourth equation:

Maxwell's Fourth Equation or Modified Ampere's Circuital Law

Here the first question arises , **why there was need to modify Ampere's circuital Law?**

To give answer to this question, let us first discuss Ampere's law(without modification)

Statement of Ampere's circuital law (without modification). It states that the line integral of the magnetic field H around any closed path or circuit is equal to the current enclosed by the path.

That is $\int H.dL=I$

Let the current is distributed through the surface with a current density J

Then $I=\int J.dS$

This implies that $\int H.dL=\int J.dS$ (9)

Apply Stoke's theorem to L.H.S. of equation (9) to change line integral to surface integral,

That is $\int H.dL=\int(\nabla \times H).dS$

Substituting above equation in equation(9), we get

$\int(\nabla \times H).dS=\int_s J.dS$

As two surface integrals are equal only if their integrands are equal

Thus , $\nabla \times H=J$ (10)

This is the **differential form of Ampere's circuital Law (without modification) for steady currents.**

Take divergence of equation (10)

$\nabla.(\nabla \times H)= \nabla.J$

As divergene of the curl of a vector is always zero ,therefore

$\nabla.(\nabla \times H)=0$

It means $\nabla.J=0$

Now ,**this is equation of continuity for steady current** but not for time varying fields,**as equation of continuity for time varying fields is**

$\nabla .J= - dp/ dt$

So, there is inconsistency in Ampere's circuital law. This is the reason, that led Maxwell to modify: Ampere's circuital law.

Modification of Ampere's circuital law. Maxwell modified Ampere's law by giving the concept of displacement current D and so the concept of displacement current density J_d for time varying fields.

He concluded that equation (10) for time varying fields should be written as

$\nabla xH=J+j_d$ (11)

By taking divergence of equation(11) , we get

$\nabla .(\nabla xH)= \nabla.J+ \nabla.J_d$

As divergence of the curl of a vector is always zero,therefore

$\nabla .(\nabla x H)=0$

It means, $\nabla .(J+ .J_d)=0$

Or $\nabla. J= - \nabla.J_d$

But from equation of continuity for time varying fields,

$\nabla.J= - d\rho/ dt$

By comparing above two equations of .j ,we get

$\nabla .j_d =d(\nabla .D)/dt$ (12)

Because from maxwells first equation $\nabla .D=\rho$

As the divergence of two vectors is equal only if the vectors are equal.

Thus $J_d = dD/dt$

Substituting above equation in equation (11), we get

$\nabla \times H = J + dD/dt$ (13)

Here ,$dD/dt = J_d$=Displacement current density

J=conduction current density

D= displacement current

The equation(13) is the **Differential form of Maxwell's fourth equation** or Modified Ampere's circuital law.

Intergal form

Taking surface integral of equation (13) on both sides, we get

$\int(\nabla \times H).dS = \int(J + dD/dt).dS$

Apply stoke's therorem to L.H.S. of above equation, we get

$\int(\nabla \times H).dS = \int_l H.dL$

Comparing the above two equations ,we get

$\int H.dL = \int(J + dD/dt).dS$

Statement of modified Ampere's circuital Law. The line integral of the

Magnetic field H around any closed path or circuit is equal to the conductions current plus the time derivative of electric displacement through any surface bounded by the path.

Equation(14) is the **integral form of Maxwell's fourth equation.**

Comments

You do not have permission to add comments.