CFA Level I Notes - 2025 Syllabus

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Introduction

These notes are written to match the 2025 syllabus of the Level I CFA exam. If you notice any errors, corrections, or segments which require a more detailed explanation, I will be very grateful.

This set of notes is intended to be fairly comprehensive, and offer a detailed explanation of the majority of topics covered by the syllabus. It does not however touch on all points of the syllabus, and so other sources are recommended to be used. In general, it takes a reasonably broad view of each topic. These notes are also written agnostic of the type of question an exam is likely to ask on each topic.

Examples are given for some topics where calculations are required. I will endeavour to add more over time, but for the examples that are given, these are indicative of the types of calculation that may be required in the exam.

A quick disclaimer about the figures – These are all designed using Matplotlib in Python. These are not always done to scale or plotted using relevant formulas to define some of the curve. I have used a combination of polynomials and exponential functions to produce most of them, based on the ideas they are intended to illustrate. The "final" versions seen here are the result of making an initial guess based on the expected relationships and behaviour, and subsequently varying the input parameters.

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1 Quantitative Methods

1.1 Interest rates and return measurement

- Interest rates measure the time value of money
- Equilibrium interest rates are equivalent to a **required rate of return**, and may also be referred to as a **discount rate**. This can be considered to be the *opportunity cost of current consumption*.
- The **real risk-free rate** is a theoretical construct, and has no embedded risk of inflation or default. This is a *time preference*, and implies the desire to consume in the present as opposed to the future.
- The **nominal risk-free rate** includes an inflation premium.

$$(1 + r_{f,\text{ nominal}}) = (1 + r_{f,\text{ real}}) \times (1 + r_{\text{inflation}}), \tag{1.1}$$

$$r_{f,\text{ nominal}} \simeq r_{f,\text{ real}} + r_{\text{inflation}}. \tag{1.2}$$

- There are several risk premia which may also be added to turn the nominal risk-free interest rate into a nominal interest rate. These may include the following:

Default risk	Risk of payments not being made on time
Liquidity risk	Risk of not being able to sell at fair value if an investment must be sold quickly
Maturity risk	Risk due to capital being tied up for longer, as the prices of longer-term bonds are typically subject to greater volatility

Table 1.1: Risk premia used to convert a real interest rate into a nominal interest rate

- We can view this as an equation in the following way.

$$\begin{aligned} r_{f,\text{ nominal}} \approx r_{f,\text{ real}} &+ \text{inflation premium} + \text{default risk premium} \\ &+ \text{liquidity risk premium} + \text{maturity risk premium}. \end{aligned} \tag{1.3}$$

1.1.1 Holding period return

- The **holding period return** is the return from an investment over any chosen period, and can be expressed as either of the two equivalent formulae below.

$$HPR = \frac{P_1 - P_0 + \sum CF}{P_0}, \tag{1.4}$$

$$HPR = \frac{P_1 + \sum CF}{P_0} - 1. \tag{1.5}$$

  In order to annualise an HPR, this can be done using the following.

$$r_{\text{annualised}} = (1 + HPR)^{\frac{365}{\text{days}}} - 1. \tag{1.6}$$

- Holding period returns may be linked over multiple time periods.
  - The arithmetic mean ignores any compounding effects

$$\overline{x}_{\text{Arithmetic}} = \frac{\sum_{i=1}^{N} x_i}{N}. \tag{1.7}$$

– The geometric mean does account for this, so should be used for compounded returns. If the rate needs to be annualised, this is equivalent to finding the **time-weighted return**.

$$\overline{x}_{\text{Geometric}} = \sqrt[N]{\prod_{i=1}^{N}(1+x_i)} - 1. \tag{1.8}$$

where each r_i is over the same time period.

- The **harmonic mean** is defined as follows

$$\overline{x}_{\text{Harmonic}} = \frac{N}{\sum_{i=1}^{N}\frac{1}{x_i}}. \tag{1.9}$$

In the case that we are considering returns, and we have

$$x_i \equiv (1+r_i),$$

equation 1.9 becomes

$$\overline{r}_{\text{Harmonic}} = \frac{N}{\sum_{i=1}^{N}\frac{1}{(1+r_i)}} - 1. \tag{1.10}$$

- The harmonic mean is used for the average cost per share of stock purchased over time, if each purchase is a constant dollar amount. In this case, each purchase price would be a positive value, and we would use the form of the harmonic mean in equation 1.9
- Of the three means, the arithmetic mean is most sensitive to outliers, and the harmonic mean is least sensitive. Thus, we can say in all cases:

$$\text{Harmonic mean} \leq \text{Geometric mean} \leq \text{Arithmetic mean} \tag{1.11}$$

- Another mathematical relation that is always true is

$$\text{Arithmetic mean} \times \text{Harmonic mean} = [\text{Geometric mean}]^2 \tag{1.12}$$

- Other methods of dealing with outliers include trimming or winsorizing the data, and can be seen in table 1.2.

Trimmed data	Trimmed data simply excludes the top and bottom most extreme values. For example, a 1% trimmed mean would exclude the top 0.5% and bottom 0.5% of values.
Winsorized data	Winsorized data replaces the top and bottom most extreme values with a limit. For instance, 95% winsorized data replaces the top 2.5 percentile values with the 97.5 percentile value, and the bottom 2.5 percentile values with the 2.5 percentile value.

Table 1.2: Winsorised and trimmed data methods used to deal with outliers

1.1.2 Time-weighted and money-weighted rates of return

- The time-weighted rate of return is simply the annualised for individual sub-periods compounded together
- The money weighted rate of return is also known as the following

$$\text{MWRR} \equiv \begin{cases} \text{Internal Rate of Return (IRR) for a portfolio} \\ \text{Yield-to-Maturity (YTM) for a bond} \end{cases} \tag{1.13}$$

- This can be expressed as the rate of return, r, that satisfies

$$\sum_i \frac{CF_i}{(1+r)^i} = 0. \tag{1.14}$$

Learn to do this on the BA II Plus Calculator

EXAMPLE: Suppose at T_0, we buy a share for \$100. At T_1, we buy another share for \$120, and at T_2, we sell both shares for \$130 each. At the end of each period, every share pays a dividend of \$2.

MWRR

Using equation 1.14, we obtain the following.

$$(100) + \frac{(120)+2}{(1+r)^1} + \frac{260+4}{(1+r)^2} = 0.$$

Solving this for r, gives

$$r = 13.86\%,$$

so this is the MWRR for this example.

TWRR

Compounding two holding periods of a year, we obtain the following.

$$HPR_1 = \frac{120+2}{100} - 1 = 22\%,$$
$$HPR_2 = \frac{2\cdot 130 + 2\cdot 2}{2\cdot 120} - 1 = 10\%.$$

Taking the geometric mean,

$$\sqrt[2]{(1+0.22)\times(1+0.10)} - 1 = 15.84\%,$$

so this is the TWRR for this example.

- TWRR is *not* affected by the timing of returns. This is why it is the **preferred measurement of industry**
- MWRR is sensitive to the timing of cashflows. If a manager does have control over the cashflows, then MWRR is an appropriate measure
- If there is an inflow just before a period of poor performance, the MWRR will tend to be **lower** than TWRR

1.1.3 Common measures of return

- Compounding frequency is important to consider. The higher the frequency of compounding, the larger the effective annual rate.

$$PV = \frac{FV_N}{\left(1+\frac{r}{m}\right)^{m\cdot N}}, \tag{1.15}$$

where the variables have the following definitions

EXAMPLE: If you receive a payment of \$1,000 in a year. The stated rate is 6% per annum. What is the PV of this payment assuming (i) semi-annual, (ii) quarterly, (iii) monthly, and (iv) daily compounding?

PV	Present Value of cashflow
FV	Future Value of cashflow
r	Interest rate
m	Number of compounding periods per year
N	Number of years
$m \cdot N$	Number of compounding periods

(i) $PV = \frac{\$1,000}{\left(1+\frac{0.06}{2}\right)^2} = \$942.60$
Effective rate $= 6.09\%$

(ii) $PV = \frac{\$1,000}{\left(1+\frac{0.06}{4}\right)^4} = \$942.18$
Effective rate $= 6.14\%$

(iii) $PV = \frac{\$1,000}{\left(1+\frac{0.06}{12}\right)^{12}} = \$941.90$
Effective rate $= 6.17\%$

(iv) $PV = \frac{\$1,000}{\left(1+\frac{0.06}{365}\right)^{365}} = \$941.77$
Effective rate $= 6.18\%$

In the limit of equation 1.15 where m is taken to infinity, this gives continuous compounding. In this instance, we recover the following.

$$PV = FV \times e^{-r \times T}, \tag{1.16}$$

where r is given by

$$r_{cc} = \ln(1 + HPR). \tag{1.17}$$

EXAMPLE: Consider a security bought for \$100, and sold for \$120 after one year has elapsed. Calculate the continuously compounded rate of return

$$r_{cc} = \ln\left(1 + \frac{120}{100}\right) = 18.232\%.$$

1.2 Major return measures

- There are several return metrics that are commonly used:
  - Gross return – Total return before management and admin fees
  - Net return – Total return after fees have been applied
  - Pretax nominal return – Nominal return before paying taxes
  - Post-tax nominal return – Nominal return after tax liability is deducted
  - Real return – Nominal return adjusted for inflation
  - Leveraged return – Return on the investment relative to cash paid upfront
- The real return may be calculated as follows –

$$1 + r_{\text{real}} = \frac{1 + r_{\text{nominal}}}{1 + r_{\text{inflation}}}. \tag{1.18}$$

V_O	Present Value of cashflow
V_B	Future Value of cashflow
r	Unlevered return
r_B	Interest due on borrowed capital

- The leveraged return is relevant where an investor has borrowed funds as well as committing their own capital to invest.

$$r_{\text{levered}} = \frac{r\left(V_O + V_B\right) - r_B \times V_B}{V_O}, \tag{1.19}$$

where the variables have the following definitions

- A constant growth dividend discount model values a stock by calculating an infinite sum. When evaluated and simplified[1], it yields the following expression.

$$V_0 = \frac{D_1}{k_e - g_c}, \tag{1.20}$$

where the variables have the following definitions

V_0	Stock price today
D_1	Dividend to be received in one year's time
k_e	Required return on equity
g_c	Constant growth in perpetuity

This can be rearranged to give the required growth rate

$$k_e = \frac{D_1}{V_0} + g_c, \tag{1.21}$$

where $\frac{D_1}{V_0}$ is the dividend yield, or the dividend growth rate,

$$g_c = k_e - \frac{D_1}{V_0}. \tag{1.22}$$

1.3 Discounted cash flow, and the time value of money

- The value of any security is the *present value of all future cash flows*. Recalling equation 1.15, we find the **discount factor** to be

$$\text{Discount Factor} = \frac{1}{(1+r)^t} \tag{1.23}$$

where r of course can be replaced with $\frac{r}{m}$ and t with $m \cdot t$.

- This method of valuing a security can be applied to many different financial instruments

EXAMPLE: Consider a zero-coupon bond with 15 years to maturity with a par value of $\$1,000$. Calculate its present value.

$$PV = \frac{\$1,000}{1.04^{15}} = \$555.26$$

[1] See §5.12.3 for a more detailed derivation

This also works for negative yields. If instead, the interest rate offered is -0.5%, calculate the new present value

$$PV = \frac{\$1,000}{0.995^{15}} = \$1,078.09$$

Zero-coupon bond

A zero-coupon bond pays a single cashflow equal to its face value at maturity. The present value of a ZCB can be calculated using equation 1.15, as there is only one cashflow.

Fixed-coupon bond

A fixed-coupon bond requires applying equation 1.15 to all future cashflows, and summing the present value of each of those cashflows. The **coupon rate** is given as a percentage of the face value, and defines the interest paid per period. The yield to maturity however is implied by the price of the bond.

For fixed-coupon bonds, price and yield exhibit an inverse relationship.

Coupon = Yield	Trade at par
Coupon < Yield	Trade below par (discount)
Coupon > Yield	Trade above par (premium)

Table 1.3: Relationships between coupon and yield, and the trading price of fixed income instruments

$$\text{Price} = \underbrace{\text{Coupon} \times \frac{1}{\text{Yield}}\left[1 - \frac{1}{(1+\text{Yield})^n}\right]}_{\text{Same as annuity}} + \frac{\text{notional}}{(1+\text{Yield})^n} \tag{1.24}$$

A perpetuity is an annuity with $n \to \infty$, which yields

$$\text{Price} = \frac{\text{Coupon}}{\text{Yield}} \tag{1.25}$$

Amortizing bond

Similar to a fixed-coupon bond, an amortizing bond makes regular payments each period, but repays its principal over the lifetime of the bond. These are annuity instruments. *Note: set $FV = 0$ when using the calculator.*

Common stock

While common stock conventionally does not have a fixed or guaranteed dividend payment, as dividend payments are a result of management discretion, we can make assumptions on the future dividend payments in order to estimate the value of a stock. In the case of constant growth, equation 1.20 can be used to value the stock. For a multi-stage growth model, different regimes of dividend growth are calculated separately and then combined to give a total price. In all cases, cashflows are discounted using the required rate of return.

$$\text{Assets} = \text{Equity} + \text{Liabilities}. \tag{1.26}$$

Preferred stock

Preferred stock pays a constant dividend. The value of a preferred stock can be derived from equation 1.20 by setting $g = 0$. This is equivalent to a perpetuity instrument.

1.4 Implied returns and cash flow additivity

- Cash flow additivity principle states that the present value of any stream of cash flows is equivalent to the sum of the present values of the individual cashflows. Similarly, any series of cashflows can be split out in any fashion, and the sum of the present values of individual pieces will be equal to the present value of the original cashflow.
- Cash flow additivity also forms a basis for the principle of no-arbitrage. If two otherwise identical series of cashflows have differing prices, investors would buy the lower-priced and sell the higher-priced. This would drive the prices together.
- No arbitrage also applies to forward and spot interest rates, and can be used to calculate rates in the forward market. Tho notation is of a forward rate is f_{ayby} where this denotes the interest rate that will begin in a years and will last for a period of b years. An example of this is
$$(1 + s_{3y})^3 = (1 + s_{3y}) \cdot (1 + f_{1y1y}) \cdot (1 + f_{2y1y}). \tag{1.27}$$
- Forward exchange rates are given by the relative difference in interest rates between two countries,
$$\text{Forward rate} = \text{Spot rate} \times \frac{(1 + r_{price})}{(1 + r_{base})}. \tag{1.28}$$

1.4.1 Valuing common stock

- Valuing common stock is often difficult due to the uncertainty in future cash flows (dividends). This process can be simplified by assuming one of the following
  1. Constant future dividend (This is simply a preferred stock)
     - This can be recovered from equation 1.20 by setting g to 0,
     $$V_0 = \frac{D_1}{k_e}. \tag{1.29}$$
  2. Constant growth rate of dividend
     - This is given by Gordon's growth model, and is shown in equation 1.20, which as we recall is
     $$V_0 = \frac{D_1}{k_e - g_c}.$$
  3. Changing growth rate of dividend
     - While it is difficult to price a constantly changing dividend, we can recycle Gordon's growth model through the use of a multi-stage model. We can then make individual estimates of supernormal dividends, and then calculate a terminal value. The price is then given by
     $$V_0 = PV(\text{Dividends over first } n \text{ years}) + PV(\text{Terminal value}). \tag{1.30}$$
     All dividends here are discounted using the required rate of return.

1.4.2 No-arbitrage option pricing

- An option gives the holder the right to buy (call option) / sell (put option) a security at a specified "strike" price. The holder of a call option price will hope for the price of the underlying to increase, while the holder of a put option will hope for the price of the underlying to fall.

- A binomial tree explores the uncertain future price path of an underlying asset with two scenarios – up or down price movements. The value of an option on the underlying may then be established.

  EXAMPLE: Consider a call option with an exercise price of \$55. The underlying asset is currently trading at \$50, and the expiry of the option is in one year. The two hypothesised scenarios are that the asset will either be trading at \$60 (up-scenario) or \$42 (down-scenario), and the risk free rate for that time period is 3%. Determine the value of the call option.

V_0	V_1	**Option payoff**
\$50	\$60	\$60 − \$55 = \$5
	\$42	\$0 (Let option expire)

  Using no-arbitrage replication, we create a risk-free portfolio with the underlying stock and a short call option, weighted such that the portfolio has the same value in both scenarios. If we write a single call option and balance the portfolio accordingly, the number of share purchased is the hedge ratio.

$$\begin{aligned}\text{Hedge ratio} &= \frac{\text{Total option payoff}}{\text{Up value} - \text{Down value}} \\ &= \frac{\$5}{\$60 - \$42} \\ &= 0.278\end{aligned} \tag{1.31}$$

  so the portfolio should go long 0.278 of the underlying and short a call option.

  In the up-scenario, the value of the portfolio is

$$\text{Total} = \underbrace{\text{Stock}}_{0.278\times\$60} + \underbrace{\text{In-the-money option}}_{-\$5} = \$11.68$$

  In the down-scenario, the value of the portfolio is

$$\text{Total} = \underbrace{\text{Stock}}_{0.278\times\$42} + \underbrace{\text{Out-of-the-money option}}_{\$0} = \$11.68$$

  Thus, the value of the call option may be found by discounting the value of the portfolio back by one period using the risk free rate, since the portfolio bears no risk,

$$PV(\text{Portfolio}) = \frac{\$11.68}{1.03} = \$11.34.$$

  The present value of the portfolio can also be calculated

$$PV(\text{Portfolio}) = \text{Call value, } c_0 - \text{hedge ratio} \times \text{current price of underlying.}$$

  Rearranging this,

$$\begin{aligned}c_0 &= 0.278 \times \$50 - \$11.34 \\ &= \$2.56\end{aligned}$$

1.5 Central tendency and dispersion

- In general, in the world of finance:

$$\text{Central tendency} \Rightarrow \text{Expected return}$$
$$\text{Dispersion} \Rightarrow \text{Risk}$$

- Measure of central tendency include the arithmetic, geometric and harmonic means, median, mode, and trimmed and winsorized means.
- Quantiles give information about the dispersion of a data-set, or in other words, the variability about the central tendency.
- The Interquartile Range is given by the difference between the 3rd and 1st quartiles. The range is the difference the largest and smallest values.
- Mean absolute deviation is the average absolute deviation from the arithmetic mean

$$\text{MAD} = \frac{\sum_{i=1}^{n} |x_i - \overline{x}|}{n}. \tag{1.32}$$

- Variance is given by the mean squared deviation of all values from their mean. Standard deviation is simply the square root of the variance

$$\sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n}, \tag{1.33}$$
$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n}}. \tag{1.34}$$

- Equations 1.33 and 1.34 hold when we have data for an entire population. If we are considering a sample of a population, then the denominator n is replaced with $n-1$ as follows.

$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n-1}, \tag{1.35}$$
$$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n-1}}. \tag{1.36}$$

- The Coefficient of variation is given by

$$CV = \frac{\sigma_x}{\overline{x}}, \tag{1.37}$$

  where a lower CV implies less risk per unit of return
- Target downside deviation has a similar calculation to the sample standard deviation in equation 1.36,

$$S_{target} = \sqrt{\frac{\sum_{x_i<B}^{n} (x_i - B)^2}{n-1}}, \tag{1.38}$$

  where here we sum the downside squared deviation of all values of x lower than some target, B.

1.5.1 Example of dispersion calculations

- Consider the following set of returns

  30% 12% 25% 20% 23%

  The range is calculated as

  $$\text{Range} = 30\% - 12\% = 18\%.$$

  The mean is calculated as

  $$\text{Mean} = \mu = \frac{30\% + 12\% + 25\% + 20\% + 23\%}{5} = 22\%.$$

  The mean absolute deviation is calculated as

  $$\begin{aligned}\text{MAD} &= \frac{|30\% - 22\%| + |12\% - 22\%| + |25\% - 22\%| + |20\% - 22\%| + |23\% - 22\%|}{5} \\ &= \frac{24\%}{5} = 4.8\%\end{aligned}$$

  The population variance is calculated as

  $$\begin{aligned}\sigma^2 &= \frac{|30\% - 22\%|^2 + |12\% - 22\%|^2 + |25\% - 22\%|^2 + |20\% - 22\%|^2 + |23\% - 22\%|^2}{5}, \\ &= \frac{178}{5} = 35.6,\end{aligned}$$

  and the population standard deviation is

  $$\sigma = \sqrt{35.6} = 5.97\%.$$

  The sample variance and standard deviation are calculated

  $$\begin{aligned}s^2 &= \frac{178}{4} = 44.5 \\ s &= \sqrt{44.5} = 6.67\end{aligned}$$

  The coefficient of variation is given by

  $$CV = \frac{\mu}{\sigma} = \frac{22\%}{5.97\%} = 3.69,$$

  and assuming a target return of 24%, the target downside deviation is

  $$S_{\text{target}} = \sqrt{\frac{(24 - 12)^2 + (20 - 24)^2 + (23 - 24)^2}{5 - 1}} = 6.34\%$$

1.6 Skewness and kurtosis

- Skewness measures the degree of symmetry of a distribution. A positive skew implies the right hand tail is much longer, as it being dragged further by extreme positive values. For a positively skewed distribution,

  $$\text{Mode} < \text{Median} < \text{Mean}.$$

  This is demonstrated in figure 1.1.

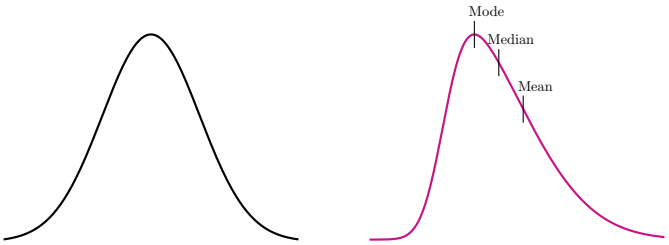


Figure 1.1: Difference between a pure Gaussian (Normal) distribution and one with a positive skew. The mode is at the peak of the distribution, and the median and mean are skewed positively by some large positive outliers.

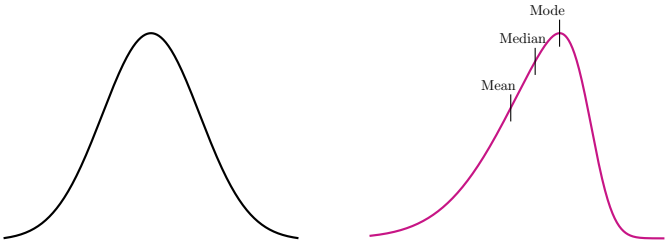


Figure 1.2: Difference between a pure Gaussian (Normal) distribution and one with a positive skew. The mode is at the peak of the distribution, and the median and mean are skewed negatively by some large positive outliers.

The opposite is true for a negatively skewed distribution. Extreme negative values skew the distribution by elongating the left hand tail. In this instance,

$$\text{Mean} < \text{Median} < \text{Mode}.$$

This is demonstrated in figure 1.2.

Off spec: Skewness in a dataset can be quantified as follows:

$$\text{Skew} = \frac{1}{N} \times \sum \frac{(x_i - \overline{x})^3}{{s_x}^3}.$$

- Kurtosis is a measure of how peaked a dataset is, compared to an equivalent Gaussian. A **leptokurtotic** distribution has fatter tails and is more sharply peaked, i.e. is tighter to the mean than a Gaussian. A **platykurtotic** distribution has thinner and is wider around the mean than a Gaussian.

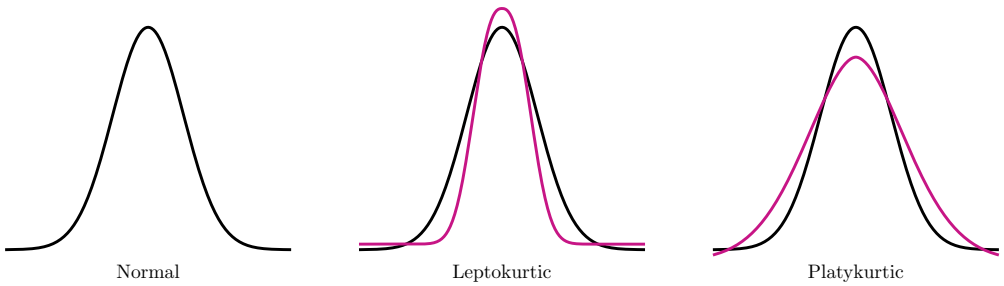


Figure 1.3: Difference between a pure Gaussian, a leptokurtotic, and a platykurtic distribution. Note that these are not exactly to scale, and differences are exaggerated to make the differences more obvious to the reader.

Off spec: Kurtosis in a dataset can be quantified as follows:

$$\text{Kurtosis} = \frac{1}{N} \times \sum \frac{(x_i - \overline{x})^4}{{s_x}^4}.$$

A Gaussian distribution has kurtosis of 3. Excess kurtosis is measured relative to that of a Gaussian. Positive excess kurtosis (or kurtosis > 3) implies a leptokurtic distribution, and negative excess kurtosis (or kurtosis < 3) implies a platykurtic distribution.

1.7 Covariance and correlation

- The sample covariance is defined as

$$S_{x,y} = \frac{\sum (x_i - \overline{x})(y_i - \overline{y})}{n-1}. \tag{1.39}$$

The covariance alone does not give much information about the strength of any linear relationship between two variables. To gauge the strength of the linear relationship between two variables, we must look at the correlation coefficient instead, defined as

$$r_{x,y} = \frac{\text{Cov}(x,y)}{S_x S_y}. \tag{1.40}$$

- The correlation coefficient is bounded between -1 and $+1$. A correlation coefficient of 0 implies no linear relationship.
- Correlation *only* captures linear relationships, and may also pick up spurious correlation, either by random, or due to some other unknown variable.

1.8 Probability Models, Expected Values, and Bayes' Formula

- The expected value of a random variable is a probability weighted average, and is calculated as

$$E(x) = \sum p_i x_i. \tag{1.41}$$

If the probabilities sum to 1, then we may use the population standard deviation. Otherwise, keep using the sample standard deviation.

EXAMPLE:

$P(X_i)$	$R(X_i)$	$E(X_i)$	$[R_i - E(R)]^2$	$P(X_i)\cdot[[R_i - E(R)]^2$
30%	20%	0.06	0.0049	0.00147
50%	12%	0.06	0.0001	0.00005
20%	5%	0.06	0.0064	0.00128
		$E(R) = 0.13$		$\sigma^2 = 0.0028$
				$\sigma = 0.0529$

1.8.1 Bayes' formula

- Probability trees are used to work out conditional probabilities, that is given one event has occurred, what is the probability of a second event occurring.
- Bayes' Formula, defined as

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}. \tag{1.42}$$

In words, this says the probability of event A occurring given event B has already occurred is equal to the probability of B given A multiplied by the probability of A irrespective of B, divided by the probability of B irrespective of A. Alternatively, Bayes' theorem may be expressed as

$$P(A|B) = \frac{P(A \cap B)}{P(A \cap B) + P(A' \cap B)}. \tag{1.43}$$

EXAMPLE: Consider two events, A and B. The probability of A occuring is 0.6. The probability of B occuring is dependent on A. If A has occured, B will occur with a probability of 0.7. If A does not occur, B occurs with a probability of 0.2. Calculate $P(A|B)$.

Recalling equation 1.42

$$\begin{aligned} P(A|B) &= \frac{P(B|A) \times P(A)}{P(B)} \\ &= \frac{0.7 \times 0.6}{0.5} \\ &= 0.84 \end{aligned}$$

1.9 Probability models for portfolio return and risk

- Portfolio expected return is the weighted average of expected returns of the underlying constituents, and is defined mathematically as

$$\mathrm{Cov}(R_i, R_j) = E\left\{\left[R_i - E\left(R_i\right)\right]\left[R_j - E\left(R_j\right)\right]\right\}. \tag{1.44}$$

Sample covariance is calculated the same way as normal, with a divisor of $n - 1$.

Combining variances of two risky assets is done as follows:

$$\mathrm{Var}(R_P) = {\sigma_A}^2 {w_A}^2 + {\sigma_B}^2 {w_B}^2 + 2 w_A w_B \,\mathrm{Cov}(A, B). \tag{1.45}$$

where the last term may be rewritten

$$2 w_A w_B \,\mathrm{Cov}(A, B) \equiv 2 w_A w_B \rho_{A,B} \sigma_A \sigma_B.$$

More generally, equation 1.45 can be expressed as

$$\mathrm{Var}(R_P) = \vec{\sigma}^{\mathrm{T}} \cdot \underline{\underline{w}} \cdot \underline{\underline{\rho}} \cdot \underline{\underline{w}} \cdot \vec{\sigma}. \tag{1.46}$$

EXAMPLE: Consider the following joint-probability function. Calculate the covariance.

$R_A \backslash R_B$	30%	10%	0%	$E(R_B) = 14\%$
20%	0.3	–	–	
12%	–	0.5	–	
5%	–	–	0.2	
$E(R_A) = 13\%$				

$$\begin{aligned} \mathrm{Cov}_{AB} = {} & 0.3 \cdot (0.20 - 0.13)(0.30 - 0.14) \\ & + 0.5 \cdot (0.12 - 0.13)(0.10 - 0.14) \\ & + 0.2 \cdot (0.05 - 0.13)(0.00 - 0.14) = 0.0058 \end{aligned}$$

1.10 Shortfall risk and Roy's safety-first ratio

- Shortfall risk is defined as the probability that a portfolio's return or value will be below a specified target return or value over a specified period. Specifying a minimum level of acceptable return, it is defined

$$\text{RSF} = \frac{E(R_P) - R_L}{\sigma_P}, \tag{1.47}$$

where R_L is the minimum acceptable threshold return. A higher value for the RSF criteria gives a lower probability of shortfall. For a Gaussian distribution, the RSF criteria is equivalent to a z-score. A z-score is the number of standard deviations away from the mean a particular value.

1.11 Lognormal distributions

- We have been making use of Gaussian distributions, but stock prices are bounded at 0. We can however think of the returns on a stock being normally distributed, if the asset's future price is taken to be the continuously compounded return of its current price. This yields the following result

$$P_T = P_0 \times e^{r_0 T}. \tag{1.48}$$

An example of a lognormal distribution plotted on a graph can be seen in figure **??**.

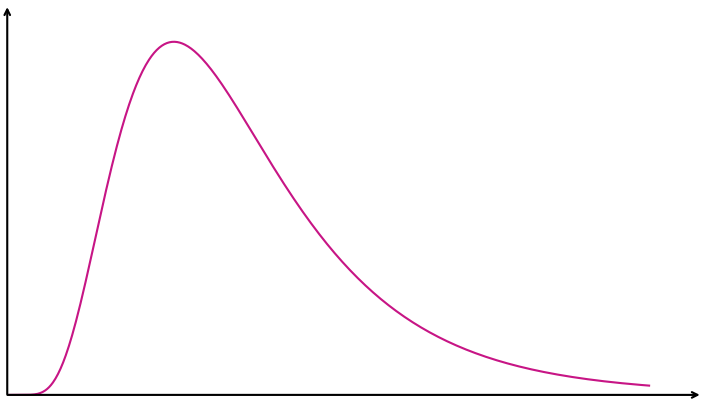


Figure 1.4: Lognormal distribution. This is bounded at $P = 0$, and asymptotically approaches 0. This is a probability density function, so the x-axis is price, and the height of the curve represents the likelihood of the security being priced at any given price.

- Identically distributed returns are stationary, that is to say the mean and variance are constant with respect to time.
- Independently distributed returns are those whereby past returns cannot be used to predict future returns.

1.12 Monte Carlo simulations

- A Monte Carlo simulation is repeated generation of one or more risk factors to generate a distribution of security values. This is used to:
  - value complex securities
  - simulate PnL from a trading strategy
  - estimate VaR
  - simulate pension fund assets and liabilities over time
  - value returns of non-normally distributed assets

among other things. A key benefit of this is that is it *not* dependent on historical data, but it is limited by the accuracy of the assumptions.

- Steps in a Monte Carlo simulation:

  1. Specify probability distributions of input parameters (i.e. mean, variance, skewness, etc.)
  2. Randomly generate values for each of the input parameters
  3. Use these randomly generated values to compute a final value, for example a stock price
  4. Repeat this process many times to build a distribution of predicted values and calculate the mean from this distribution

1.13 The Central Limit Theorem

- The Central Limit Theorem states that for any population with mean $z\mu$, and variance σ^2, as the sample size increases, the distribution of sample means approaches a normal distribution, with mean μ and variance $\frac{\sigma^2}{n}$.
- This sampling distribution is used for both hypothesis testing and confidence intervals.
- If $n > 30$, then the sample distribution of sample means can be considered to be approximately normal.
- The following relations will be useful when we cover hypothesis testing, but for now,
  - If population σ *is* known,
  $$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}, \tag{1.49}$$
  which implies the use of a z distribution.
  - If population σ is *not* known,
  $$s_{\overline{x}} = \frac{s}{\sqrt{n}}, \tag{1.50}$$
  which implies the use of a student's T distribution.

  $\sigma_{\overline{x}}$ and $s_{\overline{x}}$ defines the standard error of the sampling distribution mean. For sufficiently large n, we can assume a z-distribution in any case.
- Sampling when we know the probability in the population of each sample member yields the following:
$$\mu_{\text{population}} - \overline{x} = \text{Sampling error.} \tag{1.51}$$
The sampling error can be reduced by removing any bias, or by increasing the sample size.

1.14 Sampling methods

- Simple (random) sampling is the simplest method, where every member of a population has an equal probability of selection for the sample.
- Systematic sampling picks every n^{th} member of a population to form an approximately random sample
- Non-probability sampling relies on the judgment of the researcher or low-cost / available data. This may however introduce bias, leading to a greater sampling error, due to methods being chosen for convenience over statistical robustness.

- Judgment sampling relies on analyst judgment to pick a representative sample from a population. This is highly susceptible to bias, as sampling choices are at analyst discretion
- Stratified random sampling creates subgroups within a population based on one or more characteristics. Samples are selected from each group in proportion to the size of the subgroup. This ensures a characteristic is appropriately represented in a sample
- Bootstrap resampling is a method for generating data inputs to use in a simulation, and is used with sample data. The steps to carry out bootstrap resampling are as follows.
  1. Start with an observed sample from a population (i.e. historical data)
  2. Repeatedly draw samples of size n, replacing the data after each sample is taken.
  3. Infer population parameters from the sample data (i.e. μ, σ, etc.)
- Jacknife resampling is a less computationally demanding process. Similar to bootstrap resampling above, we take a sample of size n from a larger population. Then from this sample, compute the mean repeatedly, each time, excluding one observation from the sample (and replacing before each subsequent mean is taken). The standard deviation of these sample means can then be used to estimate the standard error.
- Cluster sampling is where the overall population is divided into subsets "clusters", and assumes each cluster is a representation of the wider population.
  - One-stage cluster sampling

    Take random samples of clusters, and include all the data from each cluster in the sample
  - Two-stage cluster sampling

    Take random sampling of each cluster after having defined a sample of clusters

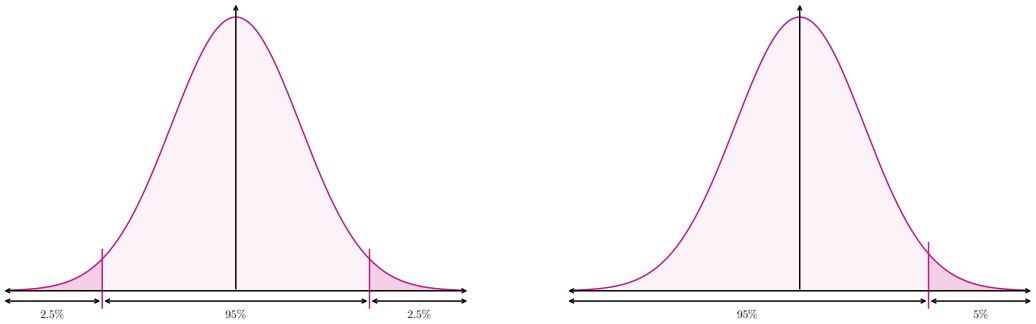
$$\text{Homogeneity} \begin{cases} \text{Stratified sampling } \textbf{within} \text{ groups} \\ \text{Cluster sampling } \textbf{between} \text{ groups} \end{cases}$$

$$\text{Heterogeneity} \begin{cases} \text{Stratified sampling } \textbf{between} \text{ groups} \\ \text{Cluster sampling } \textbf{within} \text{ groups} \end{cases}$$

1.15 Hypothesis Testing Basics

- A hypothesis, used in a hypothesis test is a statement about the value of a population parameter developed to test a theory or belief. The steps involved are
  1. State a hypothesis
  2. Select a test statistic
  3. Specify the level of significance required for the test
  4. State decision rule (reject null hypothesis if test statistic is in tail of distribution)
  5. Collect sample; calculate statistic
  6. Make a decision on the hypothesis
  7. Make a decision based on the test results
- The **null hypothesis**, H_0, states that a value being tested for is either $=$, $<$, or $>$ a hypothesised value. In the case of a strict equality, this requires a two-tailed test. for inequalities, this requires a one-tailed test. The **alternative hypothesis**, H_A is accepted if and only if H_0 is rejected. For example,

$$H_0 : \mu = 0 \qquad H_0 : \mu \leq 0$$
$$H_A : \mu \neq 0 \qquad H_A : \mu > 0$$



(a) Two-tailed hypothesis test (b) One-tailed hypothesis test

Figure 1.5: Example of a two-tailed (figure 1.5a) and one-tailed (figure 1.5b) hypothesis test at 5% significance level. If the test statistic falls above the critical value (or below in the case of the lower bound of a two-tailed test), we should reject the null hypothesis.

- The test statistic is calculated from the sample data. This is then compared to a critical value to test H_0. If the test statistic *exceeds* the critical value, then *reject* H_0.
- Critical values are similar to a confidence interval.
- Type I error

  A type I error occurs upon rejecting a null hypothesis even though it is true.

  The **significance level** is the probability of a type I error.
- Type II error

  A type II error occurs upon failing to reject a null hypothesis if it is false.

  The power of the test is defined as $1 -$ [probability of type II error]. In other words, it is the probability of correctly rejecting the null when it is true.

  This is driven by both the significance level and by the sample size, n. A lower significance increases the likelihood of a type II error, so lowers the power of the test.
- The **p-value** is defined as the smallest level of significance, whereby the null hypothesis can be rejected. This is equivalent to the probability of getting the test statistic by chance if the null is true. If the p-value is given as $0.0214 = 2.14\%$,

  we can reject the null at 5% significance,

  we can reject the null at 3% significance,

  we cannot reject the null at 1% significance.

  So, the test statistic must lie in the tail for the null to be rejected. Defining α as the significance of the test, if

  $$\text{p-value} > \alpha, \text{ fail to reject the null,}$$
  $$\text{p-value} < \alpha, \text{ reject the null.}$$

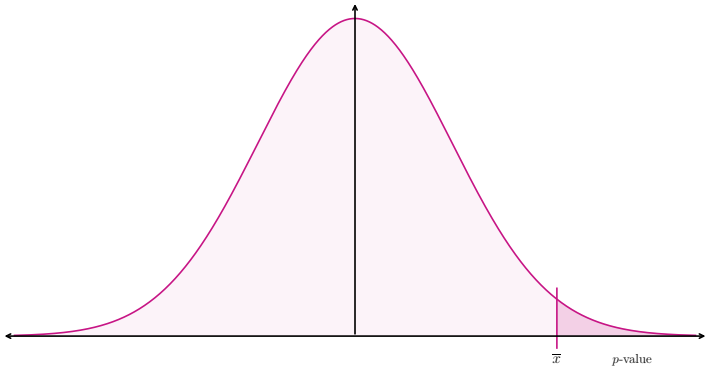


Figure 1.6: The area of the shaded region of the above Gaussian distribution represents the p-value. As written above, it represents the probability of getting the test statistic by chance, under the assumption that the null hypothesis is in fact correct.

1.16 Types of hypothesis test

- We have both parametric and non-parametric tests.
  - A parametric test is based on assumptions about population distributions and / or parameters, such as the mean or variance of a population.
  - A non-parametric test makes no assumption about a distribution and tests things other than parameter values

1.17 Parametric hypothesis tests

1.17.1 Value of a population mean

- For the value of a population mean, we use a z-test if the population variance is known. Otherwise, we use a t-test, and use the sample variance as an estimate for the population variance.

$$\text{z-statistic} = \frac{\overline{x} - \mu_0}{\left(\frac{\sigma}{\sqrt{n}}\right)}, \tag{1.52}$$

where the variables have the following definitions

$\overline{x}$	Sample mean
μ_0	Hypothesised mean
σ	**Population** standard deviation
n	Sample size
$\frac{\sigma}{\sqrt{n}}$	Standard error, as defined in equation 1.49

$$\text{t-statistic} = \frac{\overline{x} - \mu_0}{\left(\frac{s}{\sqrt{n}}\right)}, \tag{1.53}$$

where the variables have the following definitions

- The approximate ranges listed in table 1.4 should be committed to memory for the exam.

EXAMPLE: Consider a set of daily returns. We have 250 observations, and the mean of the sample taken is 0.1%. The sample standard deviation is 0.25%. Carry out a 2-tailed test at the 5% significance level

$\overline{x}$	Sample mean
μ_0	Hypothesised mean
s	**Sample** standard deviation
n	Sample size
$\frac{s}{\sqrt{n}}$	Standard error, as defined in equation 1.50

Confidence interval	SD range
99%	$2.58 \pm \sigma$
95%	$1.98 \pm \sigma$
90%	$1.65 \pm \sigma$
68%	$1.00 \pm \sigma$

Table 1.4: These approximate ranges correspond to confidence intervals for a Gaussian distribution only. For the purposes of hypothesis testing, this is equivalent to a z-statistic test.

- First, we define the null hypothesis and alternative hypothesis.

$$H_0 : \mu = 0,$$
$$H_A : \mu \neq 0.$$

We have a large sample size, so we can use a z-test or t-test equivalently.

$$\text{t-stat} = \frac{\overline{x} - \mu}{\frac{s_{\overline{x}}}{\sqrt{n}}} = \frac{0.001 - 0}{\frac{0.0025}{\sqrt{250}}} = 6.33.$$

From table 1.4, we know that the critical value for a two-tailed 5% significance test is approximately 1.96. This is a two-tailed test, and so we recall the form of figure 1.5a. We can see quite easily that 6.33 falls in the dark-shaded area in the right-hand tail, so we reject the null hypothesis.

This test has a 5% chance of a type I error.

1.17.2 Difference in population means

- To test the equality of two population means, use a t-test. If the samples are independent, use a difference in means test. This requires that the samples are independent, and that they are taken from two normally distributed populations with unknown but equal variances.

Note: Dependent / independent is determined by whether samples are linked

The t-statistic for a difference in means test is given by

$$\text{t-statistic} = \frac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{{s_p}^2}{n_1} + \frac{{s_p}^2}{n_2}}}, \tag{1.54}$$

where ${s_p}^2$ is the pooled variance estimator, defined to be

$${s_p}^2 = \frac{(n_1 - 1){s_1}^2 + (n_2 - 1){s_2}^2}{(n_1 - 1) + (n_2 - 1)}. \tag{1.55}$$

EXAMPLE: Consider the following information.

Abnormal returns of horizontal mergers	$\mu = 1\%$	$\sigma = 1\%$
Abnormal returns of vertical mergers	$\mu = 2.5\%$	$\sigma = 2\%$

Assume the two distributions are independent Gaussians, with 120 degrees of freedom (recall $n_1 + n_2 - 2 =$ d.o.f) We calculate the t-stat according to equation 1.54 to be -5.474.

Determine whether the abnormal returns are the same on average for horizontal and vertical mergers.

We as usual begin by setting out the null and alterntative hypothesis.

$$H_0 : \mu_{\mathrm{H}} - \mu_{\mathrm{V}} = 0,$$
$$H_A : \mu_{\mathrm{H}} - \mu_{\mathrm{V}} \neq 0.$$

Looking up the critical value in a t-stat table, we find that the critical value for a t-test with 120 dof, at 5% significance is ± 1.98. Since the t-stat of this test lies beyond the critical value, we should reject the null hypothesis.

If this is not the case, and the data sets are dependent, then we would use a paired comparison. This is a test of whether the average difference in some mean is significantly different than zero. The t-statistic used in this is defined as

$$\text{t-statistic} = \frac{\overline{d} - \mu_d}{s_{\overline{d}}}, \tag{1.56}$$

where the variables have the following definitions Also worth noting for this is that the degrees

$\overline{d}$	Sample mean difference
μ_d	Hypothesised difference in means
$s_{\overline{d}}$	standard error

of freedom is $n - 1$ for this type of test.

EXAMPLE: Consider the following scenario. We are investigating the betas in an industry before / after deregulation. The betas may have gone up or down. Based on a sample size of 39 (hence dof $= n - 1 = 38$), the t-stat is calculated to be 10.26. At 5% significance, evaluate whether the betas before / after deregulation has changed.

We first define the null and alternative hypothesis,

$$H_0 : \mu_d = 0,$$
$$H_A : \mu_d \neq 0.$$

We can look up the critical value for a t-stat with 38 dof, which is 2.024. Given 10.26 is greater than 2.024, we should reject the null hypothesis.

1.17.3 Value of a population variance

- For a hypothesis test on the value of a population variance, use a chi squared test. This is a two-tailed test, where the test statistic is defined as

$$\chi_{n-1}{}^2 = \frac{(n-1)S^2}{\sigma_0{}^2}, \tag{1.57}$$

The chi-square distribution is shown in figure 1.7 for different degrees of freedom.

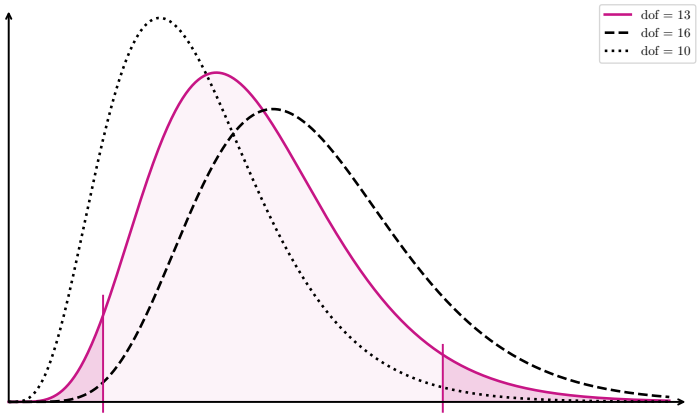


Figure 1.7: The χ^2 distribution for dof $= \{10, 13, 16\}$. As the degrees of freedom reduces, the distribution becomes more sharply peaked at a lower value of the χ^2. For dof $= 13$, the two tails are marked and shaded.

EXAMPLE: Consider a 24-month sample of monthly returns, which have a standard deviation of 3.8%. Conduct a hypothesis test at the 5% significance level, to determine whether the population standard deviation is significantly different from 4%.

We first define the null and alternative hypothesis.

$$H_0 : \sigma_0 = 0.0016,$$
$$H_A : \sigma_0 \neq 0.0016.$$

We look up in a table that the critical value for dof $= 24 - 1 = 23$ at the 5% significance is 38.076. Then, recalling equation 1.57, we calculate the test-statistic as

$$\text{test-statistic} = \frac{(24-1) \cdot 0.038^2}{0.04^2} = 20.76.$$

We see that 20.76 does not exceed the critical value of 30.076, so we do not reject the null in this case.

1.17.4 Equality of two population variances

- To test the equality of two population variances, use an F-test. The test statistic is given by

$$F = \frac{{S_1}^2}{{S_2}^2}. \tag{1.58}$$

Here, we require $S_1 \geq S_2$, and so we can only now look at the upper tail. Our null hypothesis would be that the two variances are equal, and the alternative hypothesis that they are not equal. We must simply construct the test statistic such that it is greater than or equal to 1. When looking up a critical value, the degrees of freedom for each variance is $n - 1$.

EXAMPLE: Consider the following:

31 textile companies have $\sigma = \$4.3$
41 paper companies have $\sigma = \$3.8$

At the 5% significance level, conduct a hypothesis test to determine whether the variance o earnings between the two groups of companies is significantly different.

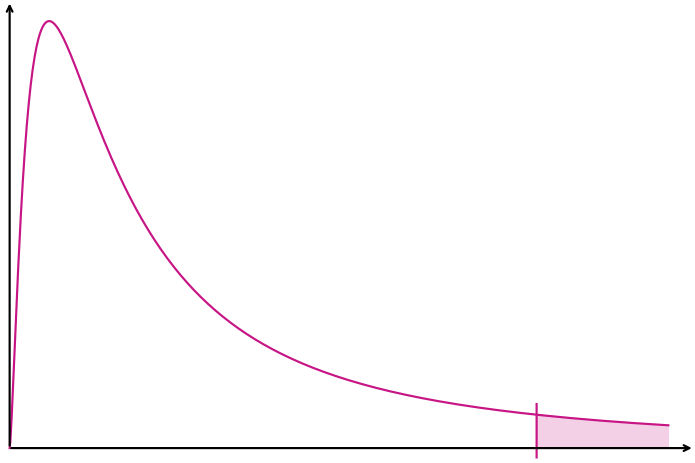


Figure 1.8: The F-stat distribution is, as mentioned before, a one-tailed test by construction. It is only the right-tail that makes up the critical value.

We first define the null and alternative hypothesis.

$$H_0 : {\sigma_1}^2 = \$4.30,$$
$$H_A : {\sigma_2}^2 \neq \$3.80.$$

Then we can calculate the F-stat to be

$$\text{F-stat} = \frac{4.30^2}{3.80^2} = 1.2805.$$

The larger variance dof is 30, and the smaller variance has a dof of 40. Looking up the F-stat in a table, we find that the critical value is 1.94. Therefore, we cannot reject the null.

1.18 Non-parametric hypothesis tests

1.18.1 Correlation between two datasets

- For a test of correlation, the null hypothesis is that the population correlation coefficient is 0, effectively saying the population is independently distributed. The t-statistic for this test is given by

$$\text{t-statistic} = \frac{r\sqrt{n-2}}{1-r^2}, \tag{1.59}$$

where there are $n-2$ degrees of freedom. The Spearman rank correlation test gives a correlation coefficient r that may be used in equation 1.59, to give an indication of whether two data sets are correlated. This correlation coefficient r, is defined as

$$r = 1 - \frac{6\sum {d_i}^2}{n(n^2-1)}, \tag{1.60}$$

where d_i is the difference in rank between a pair of values in the two data sets.

EXAMPLE: Consider the following example

Then, we can use equation 1.60 to evaluate the Spearman rank correlation.

$$r = \frac{6 \cdot \{1+0+1+4\}}{4(4^2-1)} = 0.4.$$

{1}	{2}	Rank 1	Rank 2	d	d_i
100	65	1	2	-1	1
120	80	4	4	0	0
104	71	2	3	-1	1
105	59	3	1	2	4

Then, using equation 1.59 to calculate the t-stat, we find

$$\text{t-stat} = \frac{0.4\sqrt{4-2}}{1-0.4^2} = 0.9524.$$

From here, we can do a hypothesis test as normal.

1.18.2 Independence of two datasets

- This is best illustrated with an example. The expected observation for a pair i and j is

		Dividend yield			
		Low	Medium	High	**Total**
Earnings	Low	28	53	42	**123**
	Medium	42	32	39	**113**
	High	49	25	14	**88**
	Total	**119**	**110**	**95**	**324**

defined as

$$E_{i,j} = \frac{[\text{Total for } i] \times [\text{Total for } j]}{[\text{Total for all columns + rows}]} \tag{1.61}$$

The test statistic is then given by

$$X^2 = \sum_{i}^{\text{rows}} \sum_{j}^{\text{columns}} \frac{(O_{i,j} - E_{i,j})}{E_{i,j}}, \tag{1.62}$$

which for this example is equal to 27.469. The number of degrees of freedom is given by

$$\text{DoF} = [(\text{rows} - 1)(\text{columns} - 1)]. \tag{1.63}$$

1.19 Linear regression basics

Note: Notation convention for this section is as follows:

- Y_i represents an observed value
- $\widehat{Y}$ represents a predicted value
- $\overline{Y}$ represents a mean value

- Simple linear regression explains variation of a dependent variable in terms of the variation in a single variable. Suppose we want to consider the excess return of an index to explain

Dependent	"explained, endogenous, predicted"
Independent	"explanatory, exogenous, predicting"

the variation in excess return on a specific company's common stock, where excess return is defined as

$$\text{Excess return} = R_p - R_f.$$

The line of best fit minimises the sum of squared vertical errors, ("the residuals")

$$SSE = \sum (Y_i - \widehat{Y})^2. \tag{1.64}$$

- A standard linear equation takes the form

$$y = a + bx, \tag{1.65}$$

and introducing an error term, ϵ, which is a random variable, possessing the property $\langle \epsilon \rangle = 0$, to move from the predicted to observe value, we obtain

$$Y_i = \underbrace{b_0 + b_1 X_i}_{\widehat{Y}} + \epsilon. \tag{1.66}$$

so in the process of minimising the SSE defined in equation 1.64, we want to find $\{b_0, b_1\}$ which minimises equation 1.66. We can solve for b_0 and b_1 analytically using the following,

$$\widehat{b}_1 = \frac{\text{Cov}(X, Y)}{{\sigma_X}^2}, \tag{1.67}$$

$$\widehat{b}_0 = \overline{Y} - \widehat{b}_1 \overline{X}. \tag{1.68}$$

The assumptions of linear regression are that

  - Linear relationship is present between variables
  - Variance of error / residual is constant ("Homoskedasticity")
  - Error terms are independently distributed
  - Error terms are normally distributed

Plotting residuals against the independent variable helps to check linearity. Heteroskedasticity is where the variance of the error terms is not constant. Conditional heteroskedasticity is where the error term depends on the independent variable.

- Residuals may also be tested for normality, however with large sample sizes, normality assumptions may be relaxed.

1.20 Analysis of variance (ANOVA) and goodness of fit

- We have already defined the SSE in equation 1.64 to be the sum of squared errors. This is the unexplained variation. Now we can introduce the SSR, or sum of squared regressions, defined as

$$SSR = \sum_i (Y_i - \overline{Y})^2, \tag{1.69}$$

which gives us the explained variation. Combining the SSE and SSR, we obtain the sum of squared totals, SST,

$$SSE + SSR = \sum_i (Y_i - \widehat{Y}_i)^2 + \sum_j (Y_j - \overline{Y})^2, \tag{1.70}$$

$$SST = \sum_i (Y_i - \overline{Y})^2. \tag{1.71}$$

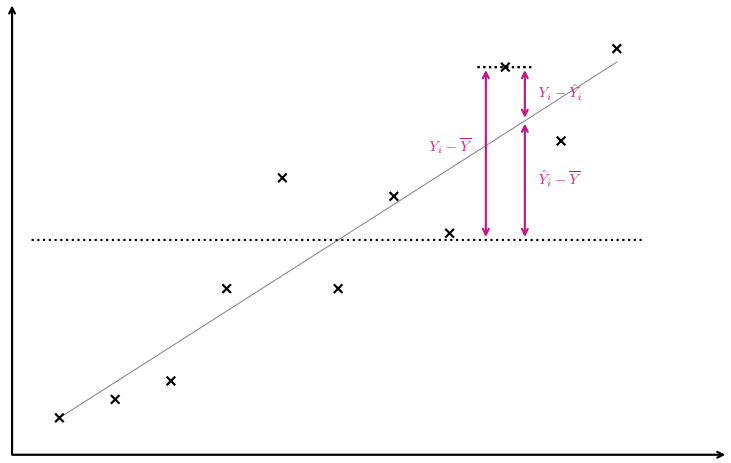


Figure 1.9: A graphical representation of the various ANOVA terms. SST (left), SSE (upper right), and SSR (lower right)

1.20.1 Mean square regression and error

- The mean square regression, MSR, and mean square error, MSE, are defined as

$$MSR = \frac{SSR}{k}, \tag{1.72}$$

$$MSE = \frac{SSE}{n-k-1}, \tag{1.73}$$

  where k is the number of independent variables. MSE is effectively variance around the forecast value.

- We can then define the standard error of the estimate, SEE, as

$$SEE = \sqrt{MSE} \tag{1.74}$$

  Using these, we can construct an anova table, such as in table 1.5, here for the case $k = 1$.

	DoF	Sum of squares	Mean square
Regression (Explained)	1	SSR	$\frac{SSR}{1}$
Error (Unexplained)	$n-2$	SSE	$\frac{SSE}{n-2}$
Total	$n-1$	SST	

Table 1.5: Analysis of variance "ANOVA" table for $k = 1$

1.20.2 The coefficient of determination

- The coefficient of determination, denoted R^2 measures the percentage of the total variation in Y explained by variation in X. It is defined

$$R^2 = \frac{SSR}{SST}. \tag{1.75}$$

  In the case of simple regression, this reduces to

$$R^2 = \text{Corr}_{X,Y}{}^2. \tag{1.76}$$

  A high value of R^2 implies the variation is well modelled, in other words, the model does a good job at explaining changes in the dependent variable.

- The standard error of the estimate measures accuracy of the predicted values from the regression equation, and is defined as

$$SEE = \sqrt{\frac{SSE}{n-2}} = \sqrt{MSE}, \tag{1.77}$$

recalling the expression for the SSE given in equation **??**. A low SEE implies the model is a better fit.

1.20.3 Constructing an F-statistic

- This is a test of whether the independent variables explain variation of the dependent variable.

$$H_0 : \text{slope coefficient} = 0 \qquad \text{(All coefficients are 0)},$$
$$H_A : \text{slope coefficient} \neq 0 \qquad \text{(At least one coefficient is not 0)}.$$

This is a one-tailed test (significance is the right-hand tail probability.

- The F stat is given by

$$F = \frac{\frac{SSR}{k}}{\frac{SSE}{n-k-1}} = \frac{MSR}{MSE}, \tag{1.78}$$

and we can see that the critical F is determined by two degrees of freedom – one in the numerator and one in the denominator.

1.20.4 Regression coefficient t-test

- This is used to determine which variables are significant. The null and alternative hypotheses are as follows

$$H_0 : {b_1}^{\text{Hypothesis}} = 0,$$
$$H_1 : {b_1}^{\text{Hypothesis}} \neq 0,$$

and the t-test statistic is constructed to be

$$\frac{\widehat{b}_1 - \cancel{b_1^{\text{Hypothesis}}}}{S_{\widehat{b}_1}} = \frac{\widehat{b}_1}{S_{\widehat{b}_1}} = \frac{\text{slope}}{\text{standard error}}, \tag{1.79}$$

where the standard error, $s_{\widehat{b}_1}$ is defined

$$s_{\widehat{b}_1} = \frac{SEE}{\sqrt{\sum (X_i - \overline{X})^2}}. \tag{1.80}$$

1.21 Predicted values

- Predicted values of the dependent variable are based upon the estimated regression coefficients

$$\widehat{Y} = \widehat{b}_0 + \widehat{b}_1 X_P. \tag{1.81}$$

- A confidence interval is a prediction interval around a predicted value. For example, the standard error of the estimate is a standard deviation around the forecast value.

- To come up with a confidence interval for predicted Y, we would use the standard error of forecast, s_f, due to joint uncertainty from intercept and slope estimates, and is defined as

$${s_f}^2 = SEE^2 \left[1 + \frac{1}{n} + \frac{(X - \overline{X})^2}{(n-1){s_X}^2}\right]. \tag{1.82}$$

We can see in equation 1.82 that as $n \to \infty$, $s_f \to SEE$. Our confidence interval is then given by

$$\widehat{Y} \pm t_c \times s_f, \tag{1.83}$$

where t_c is two-tailed, with $n-2$ degrees of freedom.

1.22 Functional forms of regression

- When the relationship between S and Y is not linear, fitting a linear model is no longer appropriate. We may however be able to linearise the relationship by transforming one or both of the variables.

lin – lin	Y vs X	$Y = b_0 + b_1 X$
log – lin	$\ln(Y)$ vs X	$\ln(Y) = b_0 + b_1 X$
lin – log	Y vs $\ln(X)$	$Y = b_0 + b_1 \ln(X)$
log – log	$\ln(Y)$ vs $\ln(X)$	$\ln(Y) = b_0 + b_1 \ln(X)$

Table 1.6: Different functional forms and their associated linearised equations

1.23 Introduction to Fintech

- Fintech can be defined as developments in technology applicable to finance

1.23.1 Types of data

- Big Data includes the following

Traditional	Financial Markets
	Company financial statements
	Government statistics
Alternative	Social media
	Website visits
Corporate exhaust	Bank records
	Retail scanner data
Internet of things	Anything on Wi-Fi

Table 1.7: Data categories and some examples

- Data can also be quantified by volume, velocity and variety, which can be seen in the below table

Volume	Grows by order of magnitude
Velocity	Speed of transmission of data (real-time is low-latency)
Variety	Structure in which data is stored

Table 1.8: The three V's of data

- Data structure varies in the following ways:

Structured	Spreadsheets & databases
Semi-structured	Photos & webpage code
Unstructured	Videos

Table 1.9: Different structures of data and some examples

- Data science and data processing involve the extraction, processing, and visualisation of data. Data process involves the following steps:

  1. Capture,
  2. Curation,
  3. Storage,
  4. Search,
  5. Transfer.

  Visualisation of the depends on the type. Word clouds, may be used for more text-based data. Charts are better suited to numbers-based data

- Big data relies on high quality data. This must account for outliers, and so requires processing and organisation of data.
- AI refers to the simulation of human cognition
- A neural network refers to replication of processes similar to those of the human brain
- Machine learning refers to a computer algorithm designed to learn, detect and recognise patterns through either supervised or unsupervised learning. Machine learning requires a lot of data:

  Training dataset : Build algorithm
  Validation dataset : Test prediction ability

  Supervised learning requires the input and output data to be clearly labelled. Unsupervised learning does not require this labelling.

- Over-fitting of data occurs when too complex of a model is created, which identifies spurious patterns, and incorrectly treats noise as true parameters
- Under-fitting of data occurs when parameters are mis-interpreted as noise, and the model fails to identify legitimate patterns

1.23.2 Applications to investment management

Text analytics	Analysis of voice / text Frequency of words . phrases Used for regulatory filings
Natural language processing (speech recognition)	Regulatory compliance Risk modelling Used for research reports
Algorithmic trading	Optimal execution High frequency trading

Table 1.10: Applications of fintech and AI to investment management

2 Economics

2.1 Breakeven, shutdown, and scale

Note: We will cover the definitions of the different market structures in more details in due course.

- Perfect competion
  - Many firms selling identical products
  - Low barriers to entry
  - Firms are price takers
- Imperfect competition (i.e. monopoly)
  - Single price / price discrimination
  - Downard sloping demand curve (Price ↓, Quantity ↑)
- Both maximise profit when

$$\text{Marginal revenue} = \text{Marginal cost}, \tag{2.1}$$

  where marginal revenue and cost are defined as the additional revenue (cost) gained (incurred) upon sale of one additional unit.
- Firms need to consider both short-run and long-run viability
  - The short-run is the period where some factors of production are fixed (i.e. land, labour, capital, entrepreneurship)
  - The lon-run is achieved when all factors are variable and fixed costs are negligible or zero.
- Breakeven is defined to be the point at which

$$\text{Total revenue} = \text{Fixed costs} + \text{Variable costs}. \tag{2.2}$$

  Price is simply defined as

$$\text{Price} + \text{Avg. revenue}. \tag{2.3}$$

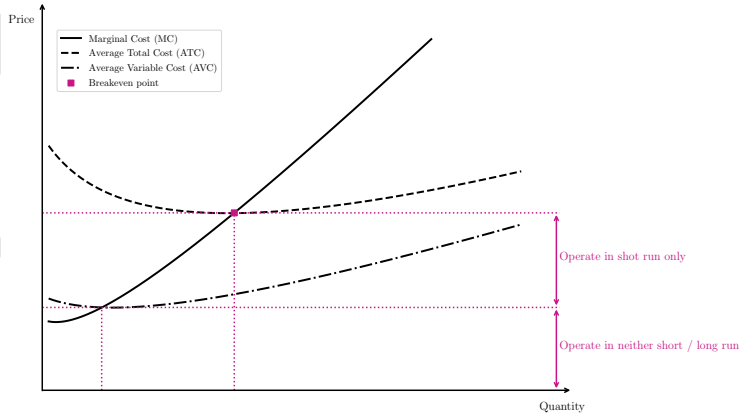


Figure 2.1: Average total cost curve (ATC), average variable cost curve (AVC), and marginal cost curve (MC). Breakeven in the long-run is achieved when average revenue and average cost are equal. In the short run, a firm may survive if revenue exceeds variable costs only, but this is not viable in the long run. If variable costs are not even covered by revenue, then the firm is not viable and should shut down.

If AR < AVC, shutdown in short run
If AR < ATC, shutdown in long run
If AR < ATC, breakeven, continue operating

- Imperfect competition
  - Price here is a variable which can be controlled by the firm. It is also a function of quantity
  - A price searcher will face a downarad sloping demand curve.
- Based on total costs / revenues,

TR = TC	Breakeven
TC >TR > TVC	Continue in short run
TR < TVC	Shutdown

- Monopoly costs, price and revenue:

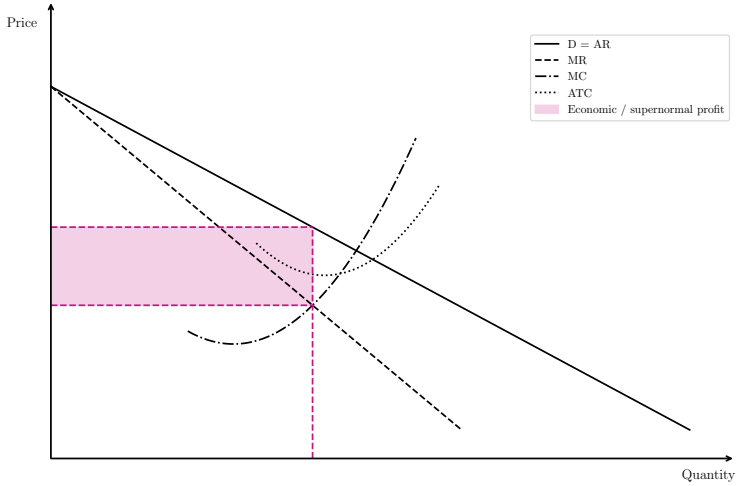


Figure 2.2: A monopoly market structure is characterised by these average total cost (ATC), marginal revenue (MR), and marginal cost (MC) curves. A monopolist is able to choose the quantity they produce, and set a price based on that. The shaded region represents the economic, or "supernormal" profit that a monopolist may benefit from.

- Typically, the total cost will have a type of "local minimum" (not technically correct use of this term but it conveys the idea well). It should therefore seek to produce at this local minimum. This leads to the idea behind economies of scale.
  - The minimum efficient scale is where the average total cost is minimised

2.2 Characteristics of different market structures

- Market structures range from perfect competition to pure monopoly. Determinants of market structure include
  - Number of firms and relative size
  - Product differentiation
  - Bargaining power of firms to set prices
  - Barrier to entry / exit
  - Degree of non-price competition (loyalty schemes)

In all instances, profit is maximised when marginal revenue is equal to marginal cost.

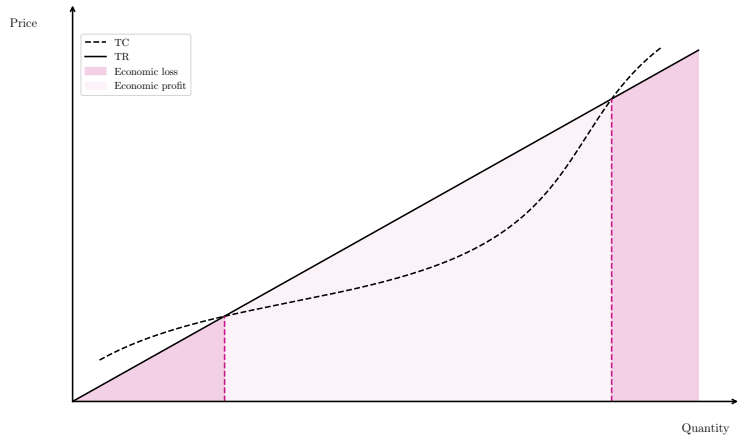


Figure 2.3: A price taker firm on the other hand has a defined total revenue, and must optimise profits by choosing the quanity to produce. For a firm to be profitable, it must operate in the region where total revenue exceeds total costs.

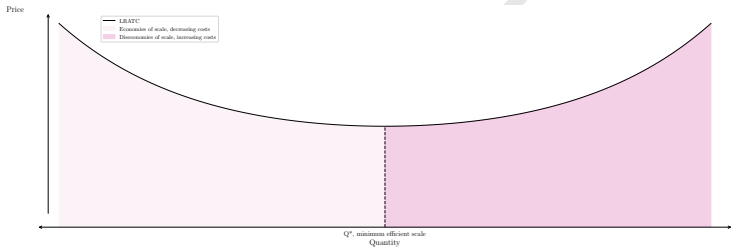


Figure 2.4: The long-run average total cost curve. Either side of the minimum, a firm will experience either economies or diseconomies of scale.

	Perfect Competition	Monopolistic competition	Oligopoly	Monopoly
Number of sellers	Many	Many	Few firms	Single
Barriers to entry	Very low	Low	High	Very High
Nature of substitute products	Very good substitutes	Good substitutes but differentiated	Good substitutes or differentiated	No good substitutes
Nature of competition	Price only	Price / Marketing / Feature	Price / Marketing / Feature	Advertising
Price power	None	Some	Some to significant	Significant

Table 2.1: Comparison of key characteristics of the various market structures

2.2.1 Perfect competition

- Large number of firms
- Each firm is small, relative to the market
- Perfectly elastic demand curve (Price increase implies quantity tends to 0)
- No barriers to entry / exit
- Price determined by costs of entry / exit

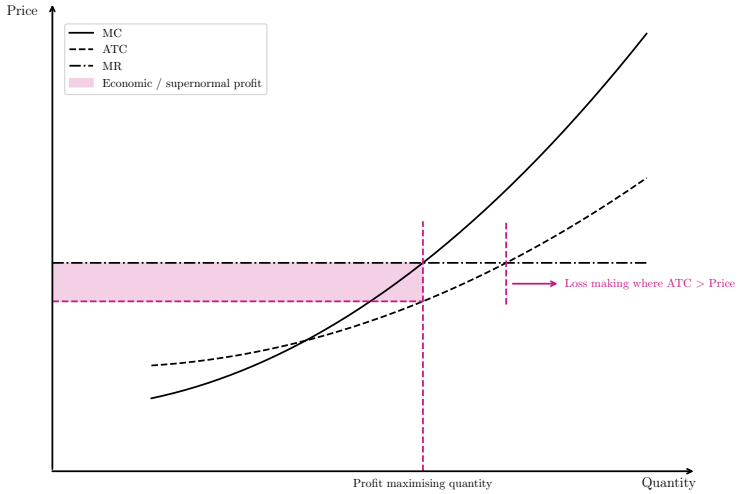
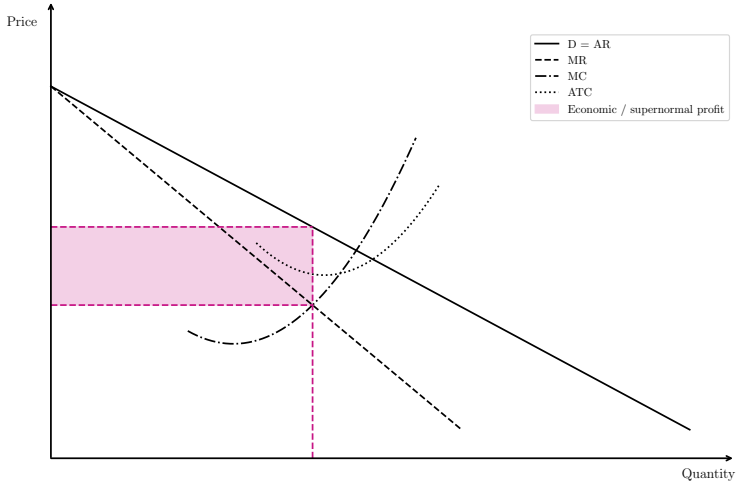


Figure 2.5: Marginal cost (MC), marginal revenue (MR), and average total cost (ATC) curves. A firm in perfect competion should experience zero profit when ATC and price are equal.

2.2.2 Monopolistic competition

- Products are differentiated, and not identical
- Large number of firms, low barriers to entry
- Each firm has a small market share
- Relatively elastic demand curve, downward sloping
- Firms compete on price / quality / marketing
- In the long run, new firms will erode the economic profit, so as a new firm enters, the price will fall



Repeat of Figure 2.2, showing curves of a monopoly market structure.

2.2.3 Oligopoly

- Only a few firms are in the industry. Each firm is interdependent with respect to price / business strategy
- Products may be similar (i.e. oil industry), or different (i.e. automobiles)
- Products are often good substitutes

- Significant barriers to entry

2.2.4 Monopoly

- Firm faces downard-sloping demand curve
- Firm has power to set price
- High barriers to entry
- Control of required resource
- Supported by regulation

2.3 Oligopoly models

- There are four oligopoly models that we will study:
  1. Kinked demand curve model,
  2. Cournot duopoly model,
  3. Stackelberg dominant form model,
  4. Nash equilibrium model.
- In an oligopoly, firms must consider among other things, whether they should collaborate, and whether they are subject to a kinked, or changing, demand curve.

2.3.1 Kinked demand oligopoly

- Competition will not follow a price rise, but will follow a price decrease
- The model suggests a discontinuous marginal revenue curve
- The model does not specify what determines the market price, P_k
- If the MC rises, at Q_k, then firms should not increase the price.

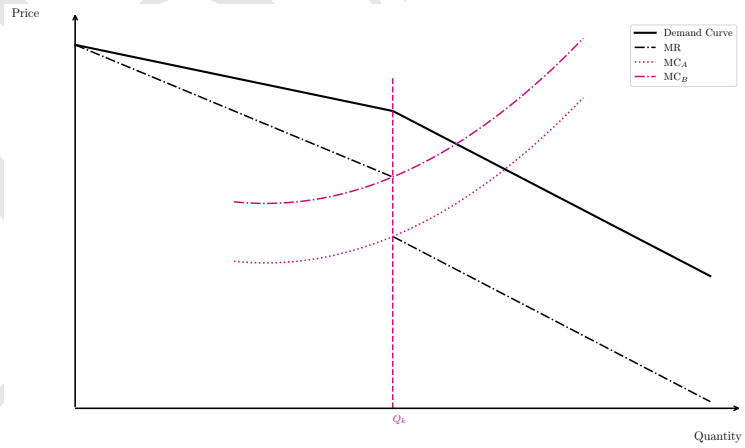


Figure 2.6: Graphical demonstration of kinked demand mechanism. At the kink, the dmand and marginal revenue curves are no longer smooth, and the marginal revenue curve experiences a discontinuity.

2.3.2 Cournot's duopoly model

- Two firms, each with identical MC curves pick their selling prices based on price from the other firm in the previous period.
- The long-run equilibrium is for both firms to sell the same quantity, dividing the market equally at the equilibrium price.
- Each firm assumes the competitor price does not change
- The market price will end up being lower than a pure monopoly, but higher than perfect competition

2.3.3 Stackelberg dominant form model

- This assumes pricing decisions are made sequentially. In this model, a leader "Dominant Firm" (DF) chooses a higher price, and receives a greater proportion of total profits to be made. They receive a first mover advantage. This firm has a significantly large market share, because of greater scale and lower cost structure.
  - Market price is set by the DF, which is taken by other competitive firms (CF)
  - A price decrease by a CF, which increases Q_{CF} in the short run can lead to a price decrease by DF, so the CF reduces output / leaves industry. In the long run, this increases the market share of the DF.

2.3.4 Nash Equilibrium

- A Nash Equilibrium is reached when choices of all firms are such that no other choice makes any firm better off. For example,

		Firm B	
		High price	Low price
Firm A	High price	A profit = 1000 B profit = 600	A profit = 600 B profit = 700
	Low price	A profit = 160 B profit = 0	A profit = 100 B profit = 140

Table 2.2: Nash equilibrium example for two firms, A, and B.

In table 2.2, we can see that the Nash equilibrium is for A to charge a high price, and B to charge a low price. However, an oligopoly profits with collusion. In the above example, if firm A were to pay 200 to B in order to charge a high price, we see that

$$\begin{aligned}\text{A profit} &= 1000 - 200 = 800,\\ \text{B profit} &= 600 + 200 = 800,\end{aligned}$$

so both A and B do better than their Nash equilibrium. More generally, firms can fix industry output at the monopoly quantity and share the profits. If competitors cannot detect cheating in a collusion agreement, a firm can increase their own profits by increasing output beyond the collusion-agreed output. Conditions for collusion success are:

- Few firms
- Homogeneous products
- Similar cost structures

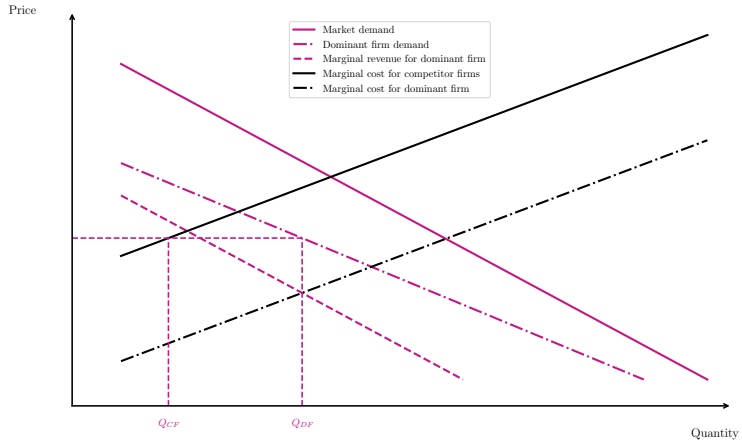


Figure 2.7: Marginal cost (MC) curves for dominant and competitor firms (DF, CF), market demand and dominant firm demand curves, and marginal revenue curve for dominant firms.

  - Retaliation for cheating
  - No external competition

- Collusion vs competition results in:
  - Perfect collusion maximises total profit
  - Perfect competition results in zero economic profit

2.4 Identifying market structures

- We define the price elasticity of demand to be:

$$\frac{\%\Delta Q}{\%\Delta P} = \begin{cases} > 1, \text{ Elastic}, P\downarrow, Q\uparrow, \\ < 1, \text{ Inelastic}, P\downarrow, Q\downarrow\downarrow, \end{cases} \tag{2.4}$$

  noting that the elasticity may change over time.

- We can also use concentration ratios to help identify market structures. Regulators tend to use the % of market share.

2.4.1 N-firm concentration ratio

- We can define the market share of the N largest firms as

$$\text{Market share} = \frac{\text{Firm's sales of N largest firms}}{\text{Total market sales}}. \tag{2.5}$$

  A low ratio implies good competition, however a high ratio suggest an oligopoly.

- This metric however ignores barriers to entry, as well as any effects of mergers

2.4.2 Herfindahl-Hirschman Index

- The Herfindahl-Hirschman Index, or HHI, is defined as

$$\text{HHI} = \sum_{i=1}^{\text{N}} [\text{Market share}]_i^{\,2}, \tag{2.6}$$

  where we sum the squared market share of the N largest firms.

  This is more sensitive to mergers than the N-firm concentration ratio in **??** above, and so is widely used by regulators. However this model also ignores barriers to entry, as well as demand elasticity.

HHI	Level of competition
< 0.1	Highly competitive
$0.1 - 0.18$	Moderately competitive
> 0.18	Uncompetitive

- The HHI also gives us the effective number of firms, calculated as

$$\text{Effective number of firms} = \frac{1}{\text{HHI}}. \tag{2.7}$$

2.5 Business cycles

- Recurrent expansions and contractions in economic activity.
  - The classical cycle is based on real GDP relative to a beginning value
  - The growth cycle refers to changes in the % difference between real GDP and its longer-term trend
  - The growth rate cycle refers to changes in the annualised percentage growth rate from one month to the next

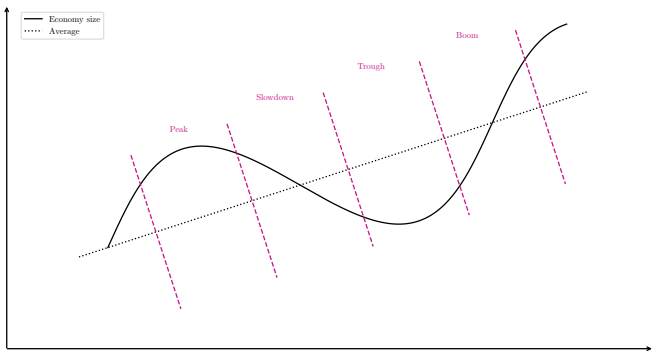


Figure 2.8: Stages of the business cycle. While time is along the x-axis, the progress along this is far from linear. However, this is a reasonable demonstration of the various phases, and conveys the idea well.

2.5.1 Phases of the business cycle

- The phases of a business cycle are
  1. Trough
     - GDP growth rate changes from negative to positive
     - High unemployment rate
     - Increasing use of overtime and temporary workers
     - Spending on consumer durable goods and housing may rise
     - Inflation falls
  2. Expansion
     - GDP growth rate increases
     - Hiring accelerates
     - Investment increases in equipment and construction
     - Inflation rises
     - Imports rise as domestic growth accelerates

3. Peak
   - GDP growth rate decreases, but hiring slows
   - Consumer spending, home construction, and business investments grow at slower rate
   - Inflation rises (with a lag)
4. Contraction
   - Negative GDP growth for two consecutive quarters
   - Hours worked decrease
   - Consumer spending, home construction, and business investments decrease
   - Inflation falls (with a lag)
   - Imports decrease as domestic growth slows

2.5.2 Credit cycles

- Credit cycles refer to cyclical fluctuations in interest rates and the availability of loans. Lenders are typically more willing to lend and offer lower interest rates during expansion and less willing during contractions (thus higher interest rates). Typically these are longer in duration than business cycles.

2.5.3 Indicators of the business cycle

- Inventory to sales ratios Firms would tend to aim for a ratio $\gtrsim 1$

Contraction	Sales $\downarrow$	$\frac{\text{Inventory}}{\text{Sales}} \downarrow$
Expansion	Sales $\uparrow$	$\frac{\text{Inventory}}{\text{Sales}} \uparrow$

- Labour and capital utilisation
  - Firms are slow to hire / lay-off employees, as frequent adjustments are costly. To reduce output, firms will first cut hours, then eliminate overtime, then finally begin lay-offs.
  - At the beginning of a contraction, sales fall, and both labour and capital are used less intensively
  - At the beginning of an expansion, sales increase and both labour and capital are used more intensively.
- Consumer sector activity
  - Spending $\uparrow$ in expansion, $\downarrow$ in contraction.
  - Durable goods are highly cyclical
  - Services are somewhat cyclical
  - Non-durable goods are non-cyclical
- Housing sector
  - Highly cyclical – mortgage rates $\uparrow$, housing $\downarrow$
  - Speculation – purchases based on expected price increases
  - Demographics – household formations, geographic shifts in population density
- External trade sector

Domestic currency appreciates	Imports ↑	exports ↓
Domestic currency depreciates	Imports ↓	exports ↑

  - Imports are determined by domestic incomes, and so is dependent on the domestic business cycle
  - Exports are determined by foreign incomes

- Economic indicators can be split into three types:

  1. Leading – Change direction before peaks / troughs in the business cycle,
  2. Coincident – Change direction around the same time as peaks / troughs in the business cycle,
  3. Lagging – Change direction after expansion / contraction,

  and examples of each can be seen in table 2.3

Leading indicators	Coincident indicators	Lagging indicators
Weekly hours	Non-farm payrolls	Duration of unemployment
New orders	Industrial production	Inventory / sales ratio
Stock prices	Personal income	Loans
Inversion of the yield curve	Manufacturing sales	CPI
Unemployment	Trade sales	Prime rates (excess rate on loans)
Building permits		
Consumer expectations		

Table 2.3: Examples of leading, coincident and lagging indicators of position in the business cycle

2.6 Fiscal and monetary policy

2.6.1 Fiscal policy objectives

- Fiscal policy is defined as governmental use of taxation and spending to influence the level of economic activity and aggregate demand. It also aims to redistribute wealth and income among segments of the population, and allocate resources among economic agents and sectors in the economy.

Surplus	Tax revenue > Government expenditure
Deficit	Tax revenue < Government expenditure
Balanced	Tax revenue = Government expenditure

  The golden rule of fiscal policy is that governments should borrow to invest, not for day-to-day spending.

  - Expansionary fiscal policy involves an increase in spending and decrease in taxation. This increases the deficit and aggregate demand
  - Contractionary fiscal policy involves a decrease in spending and increase in taxation. This decreases the deficit and aggregate demand.

2.6.2 Monetary policy objectives

- Monetary policy is determined by the central bank and aims to impact the quantity of money and credit flowing through the economy.
  - Expansionary monetary policy increases the money and credit supply in the economy. This is done through open market policy to buy bonds, and a lower policy rate respectively.
  - Contractionary monetary policy decreases the money and credit supply in the economy. This is done through open market policy to sell bonds, and a higher policy rate respectively.

  Reserve requirements of banks may also be lowered (or raised) in expansionary (contractionary) policy regimes

2.6.3 Implications of fiscal and monetary policy

- Keynsian economists believe:
  - Discretionary (from the government) fiscal policy can stabilise the economy, moderating aggregate demand to combat recessions and / or inflation
- Monetarists believe:
  - Such effects are temporary and appropriate monetary policy (including the policy rate, open market operations, and the reserve requirement) will dampen economic cycles
- Automatic stabilisers, such as taxes and transfer payments, are non-discretionary, and will increase (decrease) deficits during recession (expansion)
- The debt ratio is defined as

$$\text{Aggregate debt} : \text{GDP}. \tag{2.8}$$

  If a country runs a fiscal deficit, this increase debt and interest. This is evaluated relative to the annual GDP, and gives an indication as to the solvency of a country. If the real interest rate is above (below) the rate of GDP growth, the debt ratio will increase (decrease)
- The size of a fiscal deficit can cause some concern to investors.
  - Higher deficits suggest higher future taxes will be required, and thus a lower GDP for the country
  - If markets lose confidence in the government, investors may not be willing to refinance the debt. Government default and printing money can lead to high inflation
  - Increased government borrowing can lead to crowding out – higher interest rates means fewer private firms borrowing and spending.

  There are however other things to consider about the fiscal deficit.
  - If the debt is held by domestic citizens, the scale of the problem may be overstated
  - If debt is used for capital investment, future gains will ideally cover the repayment.
  - The size of the fiscal deficit may prompt tax reform
  - A fiscal deficit may increase GDP and / or reduce employment if the economy is not at full capacity.

2.7 Fiscal policy tools and implementation

- Fiscal policy spending tools include
  - Transfer payments – benefits to redistribute wealth, i.e. unemployment,
  - Current spending – government purchases of goods / services on a regular basis,
  - Capital spending – government spending on infrastructure / technology to boost future output.
- Government spending may
  - Provide investment in infrastructure and national defence,
  - Provide a minimum standard of living,
  - Provide investment in research and development (VC),
  - Support growth and unemployment targets.
- Fiscal policy revenue tools include
  - Direct taxes levied on income and wealth (income tax, CGT, corporation tax),
  - Indirect taxes levied on goods and services (VAT). These can also be used to moderate consumption of certain good (alcohol, tobacco, etc.).
- The benefits of tax policy include
  - Simple to enforce,
  - Horizontal equality (similar pay ⇒ similar tax),
  - Vertical equality (higher pay ⇒ higher tax),
  - Source of revenue for government spending.
- The benefits of fiscal policy include
  - Potential for fast and efficient implementation,
  - Ability to increase revenue at minimal cost.
- Capital spending is slow to implement.

2.7.1 The fiscal multiplier

- The fiscal multiplier is defined as

$$\text{Fiscal multiplier} = \frac{1}{1 - \text{MPC}(1-t)}, \tag{2.9}$$

  where the variables have the following definitions.

MPC	Marginal propensity to consume – the fraction of income an individual is likely to spend
t	Tax rate

  From this, we can see that government spending has a magnified impact on the economy.

- We also can use the following equation to estimate the impact of fiscal policy on consumption,

$$\text{Fiscal multiplier} \times \text{MPC} \times \text{tax increase} = \text{Decrease in consumption.} \tag{2.10}$$

  From this, we can see that changes in tax have a multiplied effect on aggregate demand.

2.7.2 Ricardian equivalence

- Taxpayer may increase current savings (thereby reducing current consumption) to offset the higher cost of future taxes
- If tax decreases cause taxpayers to anticipate higher future taxes, the resulting decrease in spending reduces the expansionary impact of a tax cut
- If increase in saving is equivalent to a tax decrease, this gives rise to "Ricardian equivalence"

2.7.3 More on fiscal policy

- Discretionary fiscal policy
  - Expansionary fiscal policy occurs when the economy operates below full employment. Thus, in times of recession, spending rises and taxes fall. The inverse is true when contractionary fiscal policy is implemented.
- Fiscal policy limitations
  - Forecasts may be wrong / misinterpreted
  - Fiscal policy implementation may be subject to

Recognition lag	Time taken to recognise problems
Action lag	Time taken to enact change
Impact lag	Time taken for corporations / individuals to act on the policy

Table 2.4: Definitions of the different types of lag affecting fiscal policy implementation.

  - Incorrect policy regimes may be implemented as a result of economic statistics being mis-read
  - The crowding out effect may become more significant, as greater government borrowing tends to increase interest rates, which decreases private investments
  - supply shortages slow economic activity
  - There is a limit to expansionary policy (governments may have deficit ceilings)
  - Fiscal policy cannot address high unemployment and inflation
  - Fiscal policy has limited effect if the economy is at full employment
- Deficit is a natural impact of recession
- The structured budget deficit "cyclically adjusted" assumes full employment, and is used to gauge fiscal policy

2.8 Central bank objectives and tools

- Central banks have several roles
  - Sole supplier of currency,
  - Banker to banks and governments,
  - Regulate banking and payments systems,
  - Lender of last resort – ability to print money,
  - Hold gold and foreign currency reserves,
  - Conduct monetary policy – influence money supply (Independent),

however their primary objective is to control inflation.

- High inflation leads to menu costs (constantly changing prices) and shoe leather costs (value being eroded by inflation)
- Also, some central banks attempt to have
  - Stability in exchange rates with foreign currencies
  - Full employment
  - Sustainable positive growth
  - Moderate long-term interest rates

  The target inflation rate is usually $2-3\%$

2.9 Monetary policy tools

- Policy rate – Interest rate charged to banks on borrowed reserves
  - Increasing policy rate discourages banks from borrowing reserves, thus banks reduce lending
  - Decreasing policy rate tends to increase the amount of lending, and therefore the money supply
  - US Federal Reserve sets a target for the Fed Funds Rate which is for banks to lend short term to each other
  - Repurchase agreements are used to lend money to banks. These are short term loans anywhere from overnight up to 2 weeks. For the UK, the 2 week repo rate is the policy rate

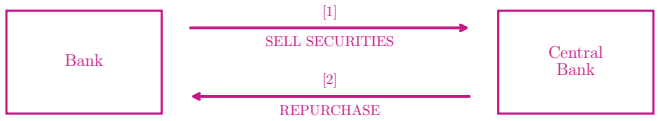


Figure 2.9: In a repurchase agreement, a central bank will by securities from a bank, in exchange for cash. In principle, the bank uses that cash to generate a return and repurchases the securities from the central bank at a set price at a pre-determined future date.

- Open market operations – most commonly used
  - Central bank buys government securities for cash. Reserves, and therefore the money supply increase. Selling securities has the opposite effect, and decreases the money supply
  - Quantitative Easing (Tightening) aims to expand (contract) the economy by putting money in (taking money out) of the system
- Required reserve ratio – seldom changed
  - Reducing the required reserve ratio to be held by banks increases excess reserves and increases the money supply

2.9.1 Monetary policy transmission

- Monetary transmission mechanism has four channels through which changes in policy impact prices and inflation.
- Under contractionary policy, the following occurs.
  1. Policy rate *increases* → Bank's short term lending rate *increases* → Aggregate demand *decreases*
  2. Asset prices *decreases* → Discount rate *increases* → Savings *increase*
  3. Consumer / business expectations decrease expenditure
  4. Domestic currency appreciates
- Monetary policy effects on economy when a central bank is

Open market operation	Buying securities	Selling securities
Bank reserves	*increase*	*decrease*
Interbank lending rates	*decrease*	*decrease*
Short / long term lending rates	*decrease*	*decrease*
Business investment	*increase*	*decrease*
Durable goods spending	*increase*	*decrease*
Domestic currency	*decrease*	*increase*
Exports	*increase*	*decrease*
Aggregate demand	*increase*	*decrease*

Table 2.5: Impact of buying / selling of securities by a central bank on select economic metrics

2.9.2 Monetary policy effects and limitations

- To be effective, central banks should be independent in two dimensions
  - Operational independence — Independent setting of the policy rate
  - Target independence — Independent setting of the inflation target, measurement of inflation, and horizon over which target should be met
- Interest rate targeting is done through increasing (decreasing) money supply growth when interest rates are above (below) targets
- Inflation rate targeting is done through increasing (decreasing) money supply growth when inflation is below (above) the target band
- Central bank targets include exchange rate targeting, a practice commonly used by developing countries to target a currency exchange rate with that of a developed country (the dollar, for instance)
  - If domestic currency falls relative to USD, central bank uses foreign reserves to buy the domestic currency
  - Sell / buy domestic currency when above / below target
  - Central bank does not react to domestic economic conditions
  - Match inflation rates

- Limitations
  1. Expected inflation
     - If consumers believe a decrease in the money supply will be successful, they will expect lower inflation.
     - Long-term bond yields with an inflation premium will fall, tending to increase economic growth. This is the opposite of the intention, which was to slow down the economy.
  2. Monetary policy may be viewed as too extreme
     - Increases probability of recession
     - Reduces long-term interest rates
     - Makes long-term bonds more attractive
  3. Bond Market Vigilantes
     - Believe the central bank is losing grip on inflation. Therefore demand for long-term bonds is reduced, leading to higher yields.
  4. Monetary supply growth may be seen as inflationary
     - Higher future asset prices expected
     - Increases long-term rates
     - Long-term bonds become relatively less attractive
  5. Liquidity trap (occurs if demand for money becomes too elastic)
     - Individuals hold more money, even without an increase in short-term rates
     - Increasing growth of the money supply will not decrease short-term rates (money held in cash)
     - May occur with deflation
  6. Once policy rates are zero, limited further ability to stimulate the economy
     - Quantitative easing was used by central banks to increase the money supply as rates were near zero
     - Large purchases of government bonds / securities to encourage lending and reduce rates
  7. Developing countries do not have a liquid market for their government debt, so open market operations are harder to implement
     - In a rapidly developing economy, it is difficult to determine the policy neutral rate
     - Central banks may lack credibility and independence

$$\text{Taylor rule} \begin{cases} \text{If inflation bigger issue; policy rate} \uparrow, \\ \text{If GDP bigger issue; policy rate} \downarrow. \end{cases}$$

2.10 The interaction of monetary and fiscal policy

- Each may be expansionary or contractionary, and different combinations have differing implications on the economy
  1. Both expansionary
     - Low interest rates, private and public sectors both expand
  2. Both contractionary
     - Lower aggregate demand and GDP, higher interest rates and both public and private sectors contract
  3. Expansionary fiscal, contractionary monetary

- Higher aggregate demand from fiscal policy, with higher interest rates from monetary policy

4. Contractionary fiscal, expansionary monetary
   - Interest rates fall from increased money supply. Consumption and output increase, and private sector grows

		Monetary	
		Contractionary	**Expansionary**
Fiscal	**Contractionary**	Tax ↑, Govt. spending ↓, Policy rate ↑, OMO sell	Tax ↑, Govt. spending ↓, Policy rate ↓, OMO buy
	Expansionary	Tax ↓, Govt. spending ↑, Policy rate ↑, OMO sell	Tax ↓, Govt. spending ↑, Policy rate ↓, OMO buy

Table 2.6: Impact of combined effect of monetary and fiscal policy regimes

2.11 Geopolitics

- Geopolitics can be defined as how geography affects international relations. Geopolitics and geopolitical risk encompasses the interaction of governments (state actors), individuals, companies, and organisations, with respect to economic, financial, and political activities.
  - Governments may be cooperative or non-cooperative based on national interests, with priorities influenced by geophysical resources
  - Cooperation comes through
    1. Trade flows
    2. Capital flows
    3. Exchange of information
    4. Exchange of culture

    and soft power is influence on the above factors without the use of force.
- Countries connected to trade routes tend to be cooperative, whereas land-locked countries tend towards cooperative behaviour with their neighbours
- Globalisation is a long-term trend towards world-wide integration of economic activity and cultures. For business, this results in increased sales and revenues, and decreased costs
- Nationalism (anti-globalisation) is the pursuit of national interests independently of / in competition with other countries.

2.11.1 Non-state actors and globalization

- Examples of international trade organization include
  - World Bank
    - Aim is to combat poverty and empower people.
    - International bank for reconstruction and development
  - IMF
    - Promotion of international monetary cooperation
    - Facilitation of expansion and growth of international trade
    - Promotion of exchange rate stability
    - Establishment of a multilateral payments system

	Globalisation	
	Hegemony Open to global trade, influence State control of key exports	**Multilateralism** Integrated globally Many trading partners Rules standardization
Non-cooperation		**Cooperation**
	Autarky Goal of self-reliance Producing domestically Low external trade / capital flows State ownership of strategic industries	**Bilateralism** Significant cooperation with one other country Limited trade / capital flows with others
	Nationalisation	

Table 2.7: Characteristics of different regimes of joint-globalisation and cooperation

    - Making resources available to members
  - World Trade Organization
    - Replaced the "General agreement on tariffs and trade", previously known as "GATT"
    - Ensures trade flows smoothly and predictably
- Non-state actors include
  - Businesses looking beyond their home country
  - Investors seeking diversification
- Capital flows are driven by
  - Portfolio investment flows (purchase / sale of foreign securities)
  - Foreign direct investment

2.11.2 Geopolitical risk

- Geopolitical risk is defined as the risk of events interrupting peaceful international relations
  - Event risk – Timing known, outcome unknown (i.e. elections)
  - Exogenous risk – Timing / outcome unknown
  - Thematic risk – Known factors having effects over long periods
- Geopolitical risk is encapsulated in the risk premium required by investors, and is quantified by

Probability	Likelihood of occurrence
Magnitude	Size of impact
Velocity	Speed of impact
Black swan risk	Tail risk

Table 2.8: Metrics that define attributes of geopolitical risk.

- Cooperative and globalized countries have lower risk of armed conflict, but higher risk of supply chain disruption.
- Analysis should be focused on high impact risks. This may be affected by the business cycle. One should use scenario analysis to gauge the effects of political risk, and also take care to avoid group think.
- Tools of geopolitics include:
  - National security tools
    - Armed conflict
    - Espionage
    - Bi/multilateral agreements
    - Alliances
  - Economic tools
    - Free trade areas
    - Common markets
    - Economic and monetary unions
  - Financial tools
    - Sanctions
    - Foreign exchange / investment

2.12 International Trade

- Benefits / costs of international trade include
  + Lower cost to consumers of imports
  + Higher employment, wages, profits in exported industries
  + Economies of scale reduce the cost of exports, improve quality
  + Free trade reduces pricing power of domestic monopolies
  - Displacement of workers, lost profits in industries competing with imported goods

Economists believe that the benefits outweigh the costs

- Absolute advantages are for lower cost with respect to resources
- Comparative advantages are for lower opportunity costs to produce
  - The law of comparative advantage: trade makes all countries better off. It allows each country to focus production on goods they can produce efficiently, and then they can trade with other countries for other goods

2.12.1 Trade restrictions

- Economic theory supports trade restrictions for:
  1. Infant industries: Protect a new industry from foreign competition
  2. National security: Ensure domestic production capability

- Economic theory does **not** support trade restrictions for
  1. Protecting domestic jobs – other jobs will be created
  2. Protecting domestic industries – importing means lower prices for consumers
  3. Dumping – selling foreign goods at a loss
- Trade restrictions
  1. Tariff – Government taxes on imported goods
  2. Quota – Limit on level of imports
  3. Export subsidies – Government payments to domestic exporters
  4. Minimum domestic content – Required proportion of product content sourced locally
  5. Voluntary export restraint (VER) – Agreement to limit quantity of goods exported
- Trade restrictions have the following impact on the importing country:
  - Reduce imports
  - Higher prices
  - Decrease consumer surplus
  - Increase domestic quantity supplied
  - Increase producer surplus

  All policies will decrease notional welfare, except quotas and tariffs in a large country, which may end up reducing world prices.

	Domestic consumer	Domestic producer	Domestic government	Foreign exporter
Tariff	Loses	Gains	Gains	Loses
Quota	Loses	Gains	Gains	Gains
VER	Loses	Gains	None	Gains
Export subsidy	Loses	Gains	Loses	—

Table 2.9: Impact of different trade restrictions on parties involved in international trade.

- For quotas, distribution of gains between domestic government and foreign exporter depends on "quota rents" which are collected by the domestic government

2.13 Capital restrictions

- Some countries impose restrictions on the flow of financial capital
  - Outright prohibition of domestic investment by foreigners
  - Punitive taxation on foreign investment
  - Restriction on foreign earning repatriation
- Restrictions decrease economic welfare
  - Short-term benefit for developing countries (by reducing volatile capital inflows and outflows
  - Long-term costs of isolation from global capital markets

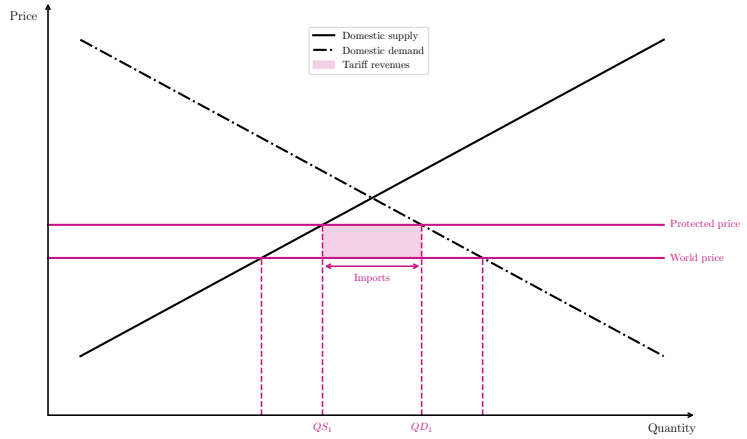


Figure 2.10: Graphical demonstration to show how a deficit in good produced domestically may be made up for by imports. A domestic price can be set, which determines the domestic output, and therefore the level of imports required to meet domestic demand. The central government may then profit from taxation of imports. With free trade, QS_1 and QD_1 reach outward to their lower and upper bounds respectively, and the shaded area collapses to zero, thereby showing no revenue from tariffs (as implied by free trade).

- Objectives of capital restrictions
  - Reduce volatility of domestic asset prices
  - Maintain an exchange rate target (through monetary and fiscal policy)
  - Keep domestic interest rates low
  - Protect strategic industries from foreign ownership

2.13.1 Trading blocs, common markets and economic unions

- Economic welfare is improved by reducing trade restrictions
- Gains from reducing restrictions between members is offset by losses from restrictions imposed on non-member countries
- The different arrangements are as follows
  - Free Trade Area
    - Removes all barriers to trade between member countries
  - Customs Union
    - FTA + common trade restrictions with non-members
  - Common market
    - CU + removes barriers to movement of labour / capital between members
  - Economic union
    - CM + Common institutions and policy
  - Monetary union
    - Economic union + common currency

2.14 The Foreign Exchange Market

2.14.1 Market Participants

- Hedgers
  - Existing FX risk that are eliminated through FX forwards

- Speculators
  - No existing FX risk – trade to earn a profit
- Sell side
  - Market makers (large multinational banks)
- Buy side
  - Corporations
  - Real (own) money accounts (Does not use derivatives)
  - Leveraged accounts (Does use derivatives)

2.14.2 Foreign Exchange Quotations

$$1.416 \underbrace{\text{USD}}_{\text{price}} / \underbrace{\text{EUR}}_{\text{base}}$$

- A US investor buying Euros buys EUR now to be converted back to USD at a later date. They therefore lose if EUR falls or USD rises relative to the other when converting back to USD.

  Hedge 1: Sell EUR forward to fix FX rate

  Hedge 2: Buy USD forward to fix FX rate

  Similar to interest rates, the nominal exchange rate is the quoted rate at any point in time "the spot rate". The Real exchange rate is the nominal adjusted for inflation.

$$\text{Real exchange rate} = \text{Nominal exchange rate} \times \frac{\text{CPI}_{\text{base}}}{\text{CPI}_{\text{price}}} \tag{2.11}$$

  CPI represent the change in price levels in different currencies. In this example, if inflation is higher in Europe, then the purchasing power of USD in the Eurozone falls.

2.14.3 Spot market vs forward market

- Spot exchange rates are exchange rates for immediate delivery (T + 2 settlement)
- Forward rates are agreements to buy / sell a specific amount of foreign currency at an agreed future date

2.14.4 Currency appreciation / depreciation

- We define

$$\%\ \text{change} = \frac{\text{spot}_{\text{end}}}{\text{spot}_{\text{start}}} - 1, \tag{2.12}$$

  EXAMPLE: Consider

$$\text{USD / EUR}_{\text{start}} = 1.42, \qquad \text{USD / EUR}_{\text{end}} = 1.39.$$

  We are asked to calculate the appreciation / depreciation in EUR. To do this, we need the currency of interest as the base, recalling that the quote is for price / base.

  Recalling equation 2.12, we find $\frac{1.39}{1.42} - 1 = -2.11\%$, so EUR has depreciated by 2.11%. If instead we were interested in appreciation / depreciation of USD, that would be given by $\frac{\left(\frac{1}{1.39}\right)}{\left(\frac{1}{1.42}\right)} - 1 = +2.16\%$, so USD has appreciated by 2.16%.

2.14.5 Managing Exchange Rates

- The ideal currency regime has the following properties
  1. Fixed exchange rate – removes any currency uncertainty
  2. Unrestricted capital flows – any purpose / amount allowed
  3. Independent monetary policy – each country has its own targets

  Historically, currencies were backed by gold.
- The IMF has two categories of exchange rate regimes
  - Countries that **do** have their own currency
  - Countries that **do not** have their own currency
    1. Formal dollarisation (using another currency as their own)
    2. Monetary union (using a common currency
- If a country **does** have their own currency, they may follow any of the following policies on their exchange rates:
  1. Currency board arrangement
     - Commitment to fix an exchange rate
  2. Conventional fixed peg (to another currency) ±1%
     - Direct intervention – Buying / selling of currency by the monetary authority to control the exchange rate
     - Indirect intervention – Use of monetary policy / local regulation to control the exchange rate
  3. Pegged exchange rates in a target zone
     - Permitted currency fluctuations
  4. Crawling peg
     - Passive – Adjusts periodically for inflation
     - Active – Adjusts in advance to account for expected future inflation
  5. Crawling bands
     - Width of bands varies over time to allow flexible monetary policy
  6. Managed floating exchange rate "dirty rate"
     - Uses economic indicators such as inflation rates, balance of payments, unemployment data – may be direct or indirect
  7. Independent floating currency
     - Rate determined by the market. Foreign market intervention is only used to slow the rate of change
- Changes in exchange rates impact both imports and exports. The impact on imports and exports is realised more slowly than the impact on capital flows

2.15 Trade deficits and the balance of payments

- Capital flows offset any imbalance between the value of imports to / from another country
- A trade deficit occurs when imports exceed exports. In other words,

$$X - M < 0. \tag{2.13}$$

  In terms of the impact on the balance of payments,

$$X - M \equiv (S - I) + (T - G), \tag{2.14}$$

  where the variables have the following definitions.

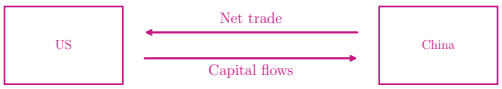


Figure 2.11: Example showing the balance of payments between China and the US. The net impact of all the trade is cash into China and goods into the US

X	Exports
M	Imports
S	Savings
I	Imports
T	Tax
G	Government spending

2.15.1 Cross rates

- The FX rate between two currencies can be calculated via a third common currency. For example, given MXN / USD and AUD / USD, we can work out the MXN / AUD exchange rate via USD. What we would therefore need is

$$\frac{\text{MXN}}{\cancel{\text{USD}}} \times \frac{\cancel{\text{USD}}}{\text{AUD}} = \frac{\text{MXN}}{\text{AUD}}, \tag{2.15}$$

  making sure that any crossing currencies cancel. This is linked to the no arbitrage principal, that any path to convert one currency to another should give the same net result.

2.15.2 No-arbitrage in spot and forward rates

- A country with higher interest rates will see its currency depreciate (trade at a discount in forward markets). The forward rate is given by

$$\text{Forward}_{\text{price/base}} = \text{Spot}_{\text{price/base}} \cdot \frac{(1 + r_{\text{price}})}{(1 + r_{\text{base}})}. \tag{2.16}$$

  The forward premium is then defined as

$$\text{Forward premium} = \frac{\text{Forward}}{\text{Spot}} - 1, \tag{2.17}$$

  where we note that the forward rate must be adjusted for time.

- If the no-arbitrage condition is not satisfied, arbitrageurs will step in until the condition is restored.

- The difference between forward and spot rates may be expressed using (basis) points, $= 0.0001$. This difference is added to the spot rate for discounts, and subtracted from the spot rate for premia. Alternatively, the difference between forward and spot may be given as a relative amount as a %.

3 Features of Corporate Issuers

3.1 Organisational forms of businesses

- The following key questions are used to differentiate different organisational forms of businesses.

Legal entity	Management	Access to capital	Liability	Tax status

1. Sole proprietorship – owned and operated by an individual
   - Sole claim to profits
   - Personally liable for claims against business
   - Profits taxed as personal income [No separate legal entity]
2. General partnership – owned and operated by 2+ individuals
   - Partnership agreement states claims to profits and division of responsibilities for operation of the business
   - General partners are personally liable for any claims
   - Profits are taxed as personal income
3. Limited partnership – general and limited partners own the business
   - General partners operate the business, and are personally liable for any claims
   - Limited partners are only liable for the amount they invest (a "buy-in")
   - Partnership agreement states claim to and division of profits
   - Profits are taxed as income
4. Corporation – legal entity separate from the owners
   - Owners appoint managers to operate the business
   - Owners only are liable for the amount they invest
   - Profits are taxed at the corporate level
   - Dividends distributed are taxed as personal income, and so are subjected to **double taxation**

We also note the following definitions:

$$\text{Effective tax rate} = \frac{\text{Total tax paid}}{\text{Earnings before tax}} \tag{3.1}$$

$$\text{After-tax income} = \text{Net income} \tag{3.2}$$

3.2 Private and public corporations

- Public corporations
  - Shares trade on an organised exchange
  - Minimum designated number of owners
  - Free float is the number of shares not held by insiders, strategic investors, etc., and is commonly expressed as a %.
- A private corporation is one which does not meet any of the above criteria. Private corporations may however become public through:
  1. IPO – Allows for raising of outside funds
  2. Acquisition by a public company / corporation

3. Direct listing of shares – No external funds raised
4. Special purpose acquisition company

Similarly, public corporations may become private through

1. Leveraged buy-out (LBO) – external investors take on a significant portion of debt to buy out all existing shareholders
2. Management buy-out (MBO) – existing managers of the company use debt to buy out all existing shareholders

A private corporation can raise capital in equity through private placement of shares, though this may only be with accredited investors.

3.3 Stakeholders and ESG factors

- Claims of lenders and owners differ, as do their priorities when it comes to running the business and decision making. Debt holders have a legal claim to the principal and interest owed to them by a corporation. They have limited upside (full repayment), but have claim priority over equity holders. Equity holders (owners) have a residual claim to profits (after all other claims are paid). They have a potentially unlimited upside

3.3.1 Impact of leverage on return on equity (ROE)

- A company may take on leverage in order to increase the ROE for its shareholders. Return on equity is defined as

$$\text{ROE} = \frac{\text{Net Income}}{\text{Equity}} \tag{3.3}$$

An example of how this may work can be seen below:

	100% equity	**50% equity 50% debt**
+ Revenue	1000	1000
– Operating Expense	800	800
– Interest @10%	0	50
= Net income	200	150
Equity	1000	500
ROE	$\frac{200}{1000} = 20\%$	$\frac{150}{500} = 30\%$

and so it is clear how the introduction of a debt component increases the return on equity to shareholders. The greater the leverage, the greater the magnification of the ROE.

- In terms of preferences, equity holders would tend to favour increasing growth and taking on more risk in terms of management direction, but this may be opposed by debt holders, or even restricted by debt covenants.
- Corporate governance refers to internal controls and procedures for managing a company.
  - Shareholder theory – Focus on interest of company's owners
  - Stakeholder theory – Focus on interest of stakeholder groups and managing conflicts of interest

The various stakeholders have different priorities. A non-exhaustive list for each can be found in table 3.2.

	Internal	External
Shareholders	Board of directors	Creditors
	Senior managers	Suppliers
	Employees	Customers
		Government / regulator

Table 3.1: The split of stakeholders between those internal and external to the company

Stakeholder	Priorities
Shareholder	Maximise shareholder wealth
Bondholder	Safety – low risk strategy and undertakings
Board of directors	Inside vs independent; supervisory vs management
Employees	Stability, wage, career advancement
Suppliers	Stability, growth, fair trade
Customers	Quality, warranty, reasonable price
Government	Tax, economic growth, compliance

Table 3.2: A table with a non-exhaustive list of priorities for each of the broad groups of stakeholders in a business

- ESG considerations are evaluated by both equity and debt investors.
  - Environmental and social factors are increasingly regulated
  - ESG impact may be material, carrying downside risks
  - Negative externalities (consequences of a company's actions) are regulated
  - Debt investors are relatively less impacted by ESG-related risks. Longer-maturity debt is more likely to be affected
- ESG factors
  - Environmental factors can affect transition work and stranded assets
  - Social factors can affect employee productivity, ability to hire / retain staff, and the company's image
  - Governance factors can result in inadequate internal controls, resulting in shareholder losses

3.4 Corporate governance

- The principal-agent relationship is one between the owners (shareholders) and managers of a business. There is an information asymmetry present – the managers have access to much more information in the process of running the business.
- An agent (senior managers) is hired to act in the interests of the principal (shareholders). However they have competing interests. A director / manager may prefer a lower level of risk to ensure stability, compared to shareholders who would be interested in maximising value.

3.4.1 Stakeholder management

- The relationship with shareholders is maintained through

  - AGM
  - Extraordinary general meetings (for special resolutions)
  - Proxy voting
    * Majority – one vote per share for each board seat
    * Cumulative – votes available to be cast for each shareholder is given by shares × seats. These can be split in any way between the candidates. So for instance, a shareholder may place all their votes for one board candidate. This gives more power to minority investors
  - Activist investors / shareholders
    * Proxy contest
    * Hostile takeovers

- The relationship with creditors is maintained through
  - Bond indentures (agreements) and covenants (terms)
  - Collateral – secured debt
  - Financial institution trustees to monitor compliance with covenants
  - Creditor committees (may be required in the event of bankruptcy)

- Boards of directors include the following committees:
  1. Audit
     - Oversight of financial reporting, implementation of accounting policies
     - Effectiveness of internal controls and internal audit function
     - Recommendation of external auditors / compensation
     - Acting on results of internal / external audits
  2. Nominating / governance committee
     - Oversight of corporate governance code (including board elections)
     - Setting policies for nomination of candidates for board membership
     - Implementing / setting a code of ethics
     - Monitoring changes in laws an regulations
     - Ensuring a firm remains compliant
  3. Remuneration
     - Compensation paid to directors / senior managers
     - Employee benefit plans
     - Should be comprised wholly of independent directors
  4. Other industry-specific committees

- Relationship with employees, suppliers, customers, and government is maintained through
  - Labour laws, employment contracts, unions
  - Employee stock ownership plans (mitigates principal - agent dilemma)
  - Social media
  - Contracts with suppliers (fair, long-term minded)
  - Regulations, governance codes

- Risks of poor management include

  - Exploitation of weaker groups of shareholders
  - Accounting fraud
  - Suboptimal risk taking
  - Related-party transactions
  - Legal and reputational risks
  - Default / bankruptcy

- Benefits of effective management include
  - Higher operational efficiency, thus resulting in higher profits
  - Alignment of interests of all stakeholders
  - Reduction of legal and financial risks

3.5 Liquidity measures and management

- The cash conversion cycle is an important metric for many business. It gives an estimate for how long it takes for cash to be put through the business. It is defined numerically as

$$\text{CCC} = \text{Days of inventory on hand} + \underbrace{\text{Days sales outstanding}}_{\text{Collection period}} - \underbrace{\text{Days payables outstanding}}_{\text{Time to pay suppliers}}. \quad (3.4)$$

  It is obvious that the cash conversion cycle is minimised by carrying low inventory, collecting payment very quickly, and having a long time to pay suppliers. However, each of these factors has their own considerations.
  - If inventory is too low, sales may fall as insufficient inventory is held to cover any potential sales increase
  - If collection period is too short, some potential customers may not be able to buy products
  - If days payable is too long, supplier may charge more.

3.5.1 The effective annual rate

- The effective annual rate (EAR) is relevant when suppliers offer a discount for early repayment. It is defined

$$\text{EAR} = \left(1 + \frac{a}{1-a}\right)^{\frac{365}{c-b}} - 1, \quad (3.5)$$

  where the variables have the following definitions. The notation for this is given as a/b net

a	Discount (provided by supplier)
b	Number of days to pay to avail discount
c	Number of days without discount

  c terms.

  EXAMPLE: If a company is given a 2% discount if invoices are paid within 10 days, and otherwise are given 30 days to pay, is the discount worth taking, given that the cost of borrowing from the bank is 8%?

Equation 3.5 tells us that

$$\text{EAR} = \left(1 + \frac{0.02}{0.98}\right)^{\frac{365}{30-10}} - 1 = 44.6\%.$$

So, the cost of not using the discount is 44.6%. Given the bank loan interest rate is 8%, then the company is better off taking the discount and financing the early purchase with a loan from the bank as they are only paying 8%, instead of 44.6%.

3.5.2 Liquidity sources

- In general, a business will have two broad categories of liquidity sources:

  1. Primary sources
     - Cash, marketable securities on hand
     - Bank loans
     - Cash generated from business
  2. Secondary sources
     - Suspension of dividends
     - Delaying / reducing capital expenditures
     - Selling assets
     - Issuing equity . debt
     - Restructuring debt
     - Bankruptcy

  A company will maintain a cash buffer to cover changes in the CCC (equation 3.4). The cost of liquidity is given by

$$\text{Cost of liquidity} = \frac{\text{Cost of liquidation}}{\text{Fair market value}}. \tag{3.6}$$

- An example of the cost of liquidity is as follows

	Fair market value ($,000)	Liquidation cost (%)
Cash and marketable securities	100	0
Inventory and receivables	200	15
Empty warehouse	300	30

Net Proceed	Liquidation cost
$100 \times (1 - 0) = 100$	0
$200 \times (1 - 0.15) = 170$	30
$300 \times (1 - 0.3) = 210$	90

so, from this we can clearly see the cost of liquidity is given by

$$\text{Cost of liquidity} = \frac{0 + 30 + 90}{100 + 200 + 300} = 20\%$$

- An increase in the CCC reduces liquidity:
  - A drag on liquidity is where inflows lag (increase in DOH and DSO)
  - A pull on liquidity is where outflows accelerate (reduced credit terms)
- Apart from the CCC, we can analyse the working capital as a % of sales relative to industry averages over time.

$$\text{Total working capital} = \text{Current assets} - \text{Current liabilities} \tag{3.7}$$

$$\text{Net working capital} = \text{Current assets (ex. cash and marketable securities)} - \text{Current liabilities (ex. debt)} \tag{3.8}$$

- We can also quantify liquidity using the following ratios

$$\text{Current ratio} = \frac{\text{Current assets}}{\text{Current liabilities}} \quad \text{(short-term)} \tag{3.9}$$

$$\text{Quick ratio} = \frac{\text{Cash + Short-term securities + Receivables}}{\text{Current liabilities}} \quad \text{(Exclude inventory)} \tag{3.10}$$

$$\text{Cash ratio} = \frac{\text{Cash + Short-term securities}}{\text{Current liabilities}} \quad \text{(cash on hand)} \tag{3.11}$$

3.6 Working capital and short-term funding

- A business must allocate enough of their assets to working capital in order to meet operating needs of the business. This includes but is not limited to:
  - Holding sufficient inventory
  - Accounts receivable to extend credit to customers
  - Cash to manage day-to-day fluctuations

  Note that the working capital requirements will be determined by the nature of a specific business
- Working capital and liquidity strategies can vary as shown in table 3.3

Conservative strategies	High working capital as percentage of sales Finance with equity or long-term debt → Greater financial flexibility, lower ROA
Aggressive strategies	Low working capital as percentage of sales Finance with short-term debt → Higher ROA, higher risk of short-term funding gap
Moderate strategies	Fund permanent current assets with equity / long-term debt Fund variable / seasonal current assets with short-term debt

Table 3.3: Table to show characteristics of different working capital and liquidity strategies

- Short-term liquidity sources are affected by
  - Company size – easier for large firms
  - Credit worthiness – easier for mature firms
  - Legal systems – protections for lenders
  - Regulatory concerns – restrictions on debt
  - Underlying assets

  amongst other idiosyncratic factors

3.7 Capital investments and project measures

- Capital investments are those which have a life of greater than one year, or are multi-year projects. There are two types of capital allocation projects

  1. Business maintenance investments
     - Going concern (replacement, cost reduction)
     - Regulatory / compliance projects
  2. Business growth investments
     - Expansion projects
     - Other projects that increase company size / scope

- Capital allocation process is used to determine / select profitable capital allocation projects. It involves the following steps

  1. Generate ideas
  2. Analyse project proposals
  3. Create capital budget for the firm – "capital rationing"
  4. Monitor decisions and conduct a post-audit

3.7.1 Net present value (NPV)

- The net present value (NPV) is defined as

$$NPV = \text{Present value of inflows} - \text{Present value of outflows}. \tag{3.12}$$

  Mathematically, this is written as

$$NPV = \sum_{i=0}^{n} \frac{CF_i}{(1+k)^i}, \tag{3.13}$$

  where k is the cost of capital. The hurdle rate is the risk adjusted discount rate. For all projects, an NPV greater than 0 results in a project being profitable.

  The NPV is the expected change in value of the firm, in current PV dollars from the project. For independent projects, all projects should be accepted where NPV is greater than 0.

3.7.2 Internal rate of return (IRR)

- This is the expected return on a project, in other words the discount rate that results in a PV of 0. Mathematically speaking,

$$PV_{inflows} = PV_{outflows}$$

  If $NPV > 0$, then $IRR >$ cost of capital AND $PV_{inflows} >$ Initial cash outlay.

- Conventional cash flows have **only one** outflow at the beginning.
- For independent projects, IRR and NPV give the same accept / reject decisions.
- For mutually exclusive projects, , the IRR and NPV may differ, based on the timing of cash flows, or different sizes of cash outlay, CF_0
  - IRR assumes CF reinvestment at project's IRR
  - NPV assumes CF reinvestment at cost of capital (more conservative)
- Looking at IRR alone can sometimes cause issues. In some cases, there may be multiple or no IRR that solves the problem. This is not the case when there is only one sign change from cash outflows, to cash inflows. **NPV does not have this problem**. IRR does however provide the relative cushion over the hurdle rate.

3.7.3 Capital allocation principles and real options

- Principles of capital allocation are
  - Decisions should be based on changes in after-tax cash flows
  - Only consider incremental "relevant" cash flows
    - **Do not** consider sunk costs
    - Consider cash opportunity costs
    - Consider externalities – cannibalisation of existing products, etc.
  - Timing of cash flows is important
  - **Do not** consider project-specific financing costs
- Common mistakes can be split into two broad categories
  - Cognitive (calculations)
    - Poor forecasting – allocation of overhead expenses, neglecting competitor response
    - Incorrectly accounting for inflation – Real (nominal) cash flows discounted at real (nominal) rates
    - Not considering the cost of internal funds – retained earnings are **not** free
  - Behavioural
    - Pet projects of senior managers
    - Inertia in setting initial capital budget
    - Basing decisions on EPS and ROE
    - Failure to generate alternative ideas
- Real options are future actions a firm can take if they invest in a project today
  - Timing option – delay investment until more information is available
  - Abandonment option – Stop the project if $PV_{\text{stop}} > PV_{\text{continue}}$
  - Expansion / growth option – Price setting (based on demand); Production flexibility (inputs variety of product)
  - Fundamental option – Project payoffs depend on the price of the underlying.

    $$\text{Project NPV (without option)} > 0 \Rightarrow \text{Accept}$$

    Otherwise, add the option value net of any associated costs and recheck if the present value is greater than zero.

3.8 Return on invested capital

- Return on invested capital (ROIC) is defined as

$$ROIC = \frac{\text{After tax operating profit}}{\text{Average book value of invested capital}}. \tag{3.14}$$

  In this equation, the after tax operating profit is unlevered, and given by $EBIT - T$. The average book value of invested capital is the average debt and equity level, which includes equity, long-term date, and excludes working capital. This is an accounting metric, so ignores the time value of money.

- A firm can be said to be adding value if its $ROIC >$ Required rate of return.

3.9 Capital structure

- Capital structure refers to the debt / equity ratio that comprises a firm
- Proportions of debt and equity are determined by the following factors
  - Internal
    - Industry / company characteristics
    - Debt capacity
    - Corporate tax rate
    - Management preferences, industry norms
  - External
    - Market conditions and business cycle
    - Regulation

3.9.1 Weighted average cost of capital

- The weighted average cost of capital (WACC) is defined as

$$WACC = w_d r_d (1 - t) + w_e r_e, \tag{3.15}$$

where the variables have the following definitions.

w_d, w_e	Weight of debt, equity in the capital structure
r_d, r_e	Required return of debt, equity
t	Tax rate

The $(1 - t)$ term reduces the cost of debt, because debt interest payments are usually made from pre tax earnings, rather than post-tax earnings.

3.9.2 Industry / company characteristics

- Companies with stable, predictable, recurring sales and cash flows are able to take on a higher proportion of debt. This tends to apply to companies with the following characteristics
  - Non-cyclical
  - Low operating leverage (low fixed costs)
  - Subscription-based revenue models
- Companies with high levels of assets available to be offered as collateral may also take on higher proportions of debt. This collateral may come in the form of
  - Tangible assets
  - Liquid assets
  - Fungible assets (easy to substitute)
- During business cycle expansions, debt is more widely available to companies, as well as being at lower cost to them. In addition, high corporate tax rates increase the value of the tax shield from deductibility of paid interest (the $(1 - t)$ term in equation 3.15
- For some firms, capital adequacy regulations may demand a minimum level of equity

3.10 Company Life Cycle Stage

1. Start-up stage
    - Equity only (high risk, little collateral to offer)
    - Convertible debt for high-growth companies
2. Growth stage
    - Revenue and cash flow increasing
    - Mostly equity but some debt. Often, the debt is collateralised with assets
3. Mature stage
    - Risk lower, cash flow significant and stable
    - Debt used widely, both secured and unsecured debt is available at low cost

3.11 Business model features and types

- Customers
    - B2B
    - B2C
    - Government
- Differentiation from competitors
    - Price
    - Quality
    - Innovative solution
- Sales method
    - Direct
    - Through intermediaries (wholesalers / retailers)
    - Alternatives to outright sales
        * Subscription models
        * Licensing and franchising
- Key assets and supplies
    - Expertise
    - Skilled employees
    - Patents
    - Software
    - Supplies
- Pricing strategies
    - Price discrimination
    - Tiered, dynamic, auction pricing
    - Penetration pricing – temporarily low to grow market share
    - Freemium pricing – basic fee, add-ons at cost
    - Hidden revenue pricing – i.e. advertising revenue
    - Bundling
    - Razors and blades
    - Options and add-ons
- Value proposition
    - Customer's perception with respect to competitors
    - **Value chain** – assets of the firm and firm activities that will create value and exploit competitive advantages
- Private label manufacturers
    - Licensing agreements
    - Value-added resellers (customisation)
- Network effects – Increase in network value as it grows
- Crowd sourcing – User input increases value of the product

4 Financial Statement Analysis

4.1 Financial statement roles

- The Financial statement analysis framework consists of the following steps
  1. Objective and context of analysis
  2. Gather data
  3. Process data
  4. Analyse / interpret data
  5. Conclusions and recommendations
  6. Update analysis periodically
- Role of financial reporting
  - Showing performance of a business to investors / creditors by preparing and presenting financial statements
- Role of financial statement analysis
  - Using information in a company's financial statements, alongside other relevant information, in order to make economic decisions on
    - Security valuation
    - Acquisitions
    - Credit worthiness
  - Evaluating a company's past performance and current financial position to form opinions about risk factors and a firm's ability to earn profits and generate future cash flows.
- Standard setting bodies
  - US – Financial Accounting Standards Board (FASB) set out the US Generally Accepted Accounting Principles (**USGAAP**)
  - Internations Accounting Standards Board (IASB) set out the International Financial Reporting Standards (**IFRS**)

4.2 Financial reporting requirements and regulation

- US – SEC
- Members of the EU have their own regulators as well as EU-wide regulations
- International Organisation of Securities Commission (IOSCO)

4.2.1 SEC Filings and forms

- S1 – Registration of securities for public sale
- 10K – Annual report [AUDITED]
- 10Q – Quarterly report
- DEF 14A – Proxy statements; issued to shareholders when a vote is required, for example
  - Board elections
  - Management compensation

  - Stock options
- 8K – Material events
- 144 – Issuance of unregistered stock
- 3,4,5 – Share transactions with corporate insiders

4.2.2 Footnotes and supplementary schedules

- Basis of presentation
- Accounting methods / assumptions
- Further information on amounts in primary statements
- Acquisitions / disposals
- Contingencies
- Legal proceedings
- Stock options, benefit plans
- Significant customers
- Segment data
- Related party transactions
- Post-balance sheet events
- Segment reporting
  - A reportable business or geographic segment is one where *at least 10% of a firm's revenue, income or assets,* ***and*** *totalling 75% of external sales.* For each segment, a firm must report
    - Revenue (internal + external)
    - Profits
    - Assets
    - Liabilities
    - Capex
    - Depreciation
    - Amortisation
    - Other non-cash expenses
    - Income tax expense
    - Share of equity-accounted investment results
- Mangement discussion and analysis (MD&A) "Operating and financial review"
  - Nature of the business
  - Management's objectives
  - Past performance and performance measures used
  - Key relationships, resources, risks
  - Trends in sales and expenses
  - Discussion of critical accounting choices
  - Effects of inflation, price changes, uncertainties on future results

4.2.3 Audit report

- Independent review of a company's financial statements
- Reasonable assurance that the report is free of material errors
- Under US GAAP, must provide opinion on internal controls
- Auditor's opinion
  - Responsibility of **management** to prepare accounts

Unqualified	Unmodified / clean
Qualified	Exceptions to specific parts of accounting principles
Adverse	Statements not presented fairly
Disclaimer of opinion	Unable to form an opinion

  - Properly prepared in accordance with GAAP – provides reasonable assurance (not guarantee) that statements are free of material error
  - Accounting principles and estimates chosen are reasonable
- Key audit matters
  - Highlights accounting choices of greatest significance (i.e. about pensions).
  - Choices requiring judgement / estimates
  - How significant transactions were accounted for
  - Choices that are complex, that the auditor believes to have a significant likelihood of being mis-stated

4.2.4 Choice of accounting standards

- There are differences between USGAAP and IFRS in terms of how some things are treated, and so adjustments are sometimes necessary when comparing two companies to reconcile these differences. More on this later, but examples of where differences arise are
  - Treatment of development costs
  - LIFO vs FIFO inventory valuation
  - Reversal of inventory write-downs
- Reporting standards are also subject to change. This means we must:
  - Monitor new products / transactions
  - Monitor regulator actions

4.2.5 Supplementary sources of information

- Issuer sources
  - Earnings calls
  - Press releases
- Public third-party sources
  - Industry reports
  - Government agency reports
  - Social media
- Proprietary third-party sources
  - Analyst reports
  - Third party consultancies
  - Bloomberg
- Proprietary primary research
  - Commissioned studies
  - Specialist advice

4.3 Revenue recognition

- For both IFRS and USGAAP, revenue is recognised in the period earned, that is, when goods / services are transferred, and when payment is probable. There is a five-step method for revenue recognition:
  1. Identify **contract** with customer

2. Identify **performance obligations** in contracts
3. Determine a total transaction price
4. Allocate transaction price to performance obligations
5. Recognise revenue as / when each obligation is satisfied

- Disclosure requirements are:
  - Contracts with customers, disaggregated into categories
  - Contract-related assets and liabilities
    - Balances and changes
    - Remaining performance obligations
    - Transaction prices allocated to them
    - Significant judgements / changes in judgement
- Progress toward completion of a performance obligation can be measured by either
  - Input % (Fraction of total estimated costs incurred to date
  - Output % (Fraction of measurable milestone)

EXAMPLE: Warehouse built for \$10mn. Estimated construction cost is \$8mn.

- If in first year, the constructor spends \$4mn in costs;

$$\text{Input \%} \Rightarrow \frac{4}{8} = 50\%$$

so \$5mn in revenue realised in Y1.

- If in second year, the constructor spends a further \$2mn in costs.

$$\text{Input \%} \Rightarrow \frac{4+2}{8} = 50\%$$

so \$7.5mn in revenue since beginning, and \$2.5mn recognised in Y2.

EXAMPLE: A travel agent sells a flight for \$10,000. Takes \$1,000 commission, and the rest goes to airline. There is no credit or inventory risk for the travel agent.

- If acting as agent, revenue = \$1,000 commission
- If acting as principal, revenue = \$10,000, expense = \$9,000

  These have the same absolute gross profit, but the gross profit margin for each case is different.

$$\text{Gross Profit Margin} = \frac{\text{Gross Profit}}{\text{Revenue}}$$

  Therefore;
  - If acting as agent, GPM = 100%
  - If acting as principal, GPM = 10%

EXAMPLE: Fast food company franchises its name. They receive a royalty fee of 2%, as well as a licensing fee.

- Revenue disaggregated into
  1. Revenue from company-owned restaurants
  2. Franchies royalty and licensing fees

3. Revenue from suppliers to franchisees (equipment + materials)

EXAMPLE: A software supplier offers customers a choice of:

1. Purchase license, locally install
2. Subscribe to a cloud-based solution
3. This is effectively a contract for a service, and so revenue is recognised over the life of a contract.

- Under purchase of a license, for IFRS, either
  1. Report revenue over the life of the contract
  2. Report revenue at the outset of a contract

  The choice is dependent on access to ongoing updates / enhancements

EXAMPLE: A customer pays for goods ahead of shipping.

- Revenue would typically be deferred, unless **all** of the following criteria are satisfied:
  1. Customer asked for arrangement
  2. Goods identifiable as belonging to the customer
  3. Goods complete and ready for transfer
  4. Goods cannot be redirected to another customer

4.4 Expense recognition

- On an accrual basis, there are three main methods of recognising expenses.
  1. Matching principle – Match costs against associated revenues, i.e. inventory and warranty expense
  2. Capitalisation – Recognise cost of asset on a balance sheet and expense it to the income statement over its life
  3. Period costs – Expenditures that less directly match the timing of revenues

  This has analysis implications on
  - Inventory valuation
  - Warranty expense
  - Depreciation
  - Amortization
  - Doubtful debt provisions
  - Research and development

  and so requires estimates and assumptions that will have a material impact on net income

  EXAMPLE: Matching principle from units perspective

  Assume a firm starts the year with 20 units, buys 90 during the year, and sells 100 during the year.

  Assume the original 20 units cost \$400 in total, and the 90 units purchased during the year were bought at the following prices:

  And given a sales price for the 100 units of \$35 each, this gives a total revenue of \$3,500. The 10 unsold units at the end of the year are comprised of 8 units from purchase 4, and 2 from purchase 3. These had a total cost of

  $$8 \times \$30 + 2 \times \$28 = \$296.$$

  The gross profit can then be calculated to be

	Units	
Sales		100
Beginning inventory	20	
Purchases	90	
Available for sale	110	
Ending inventory (B/S)	(10)	
Cost of goods sold (I/S)		100

Purchase	Units	Price per unit	Total cost
1	20	$22	$440
2	30	$25	$750
3	30	$28	$840
4	10	$30	$300
			$2,330

	$ Amount	
Sales		$3,500
Beginning inventory	$400	
Purchases	$2330	
Available for sale	$2,730	
Ending inventory (B/S)	($296)	
Cost of goods sold (I/S)		$2,434
Gross Profit		$1,066

4.4.1 Capitalising vs expensing

- Costs are capitalised as a balance sheet asset, or expensed in the income statement.
  - Capitalising – spreading an asset's cost over multiple periods, creating a balance sheet asset. This should be done, if benefits extend over multiple periods. The total amount may include additional costs to prepare the asset for use.
  - Expensing – Taking an asset's cost as an expense on the income statement in the current period. This should be done if benefits beyond one period are unlikely / highly uncertain.
    - Subsequent expenditures that provide benefits beyond one year are capitalised
    - Subsequent expenditures that do not provide benefits beyond one year are expensed

EXAMPLE: Costs related to manufacturing expenses

EXAMPLE: Consider some machinery purchased for $12,000. It has an estimated useful life of 4 years, with no salvage value. Depreciation is calculated usinga a straight line method, and is tax-deductible. There are no assets and liabilities except or cash and PP&E. Revenue is $30,000 per year. The operating profit margin (before equipment) is 40%. The tax rate is 30%, with no dividends paid.

For each of these tables, the dark blue columns represent the statement if the asset were to be capitalised, and the light blue if the asset were to be expensed.

Purchase cost		
Freight in	250,000	Capitalise
Taxes		
Installation	10,000	Capitalise
Training	7,500	Expense when incurred
Repair / maintenance	35,000	Expense when incurred
Rebuilding cost	85,000	Capitalise

Income Statement	Y1		Y2		Y3		Y4	
	$	$	$	$	$	$	$	$
Revenue	30,000	30,000	30,000	30,000	30,000	30,000	30,000	30,000
OPM (40%)	12,000	12,000	12,000	12,000	12,000	12,000	12,000	12,000
Depreciation expense	(3,000)	(12,000)	(3,000)	0	(3,000)	0	(3,000)	0
Income before tax	9,000	0	9,000	12,000	9,000	12,000	9,000	12,000
Tax (30%)	(2,700)	0	(2,700)	(3,600)	(2,700)	(3,600)	(2,700)	(3,600)
Net income	6,300	0	6,300	8,400	6,300	8,400	6,300	8,400

Balance Sheet	Y1		Y2		Y3		Y4	
	$	$	$	$	$	$	$	$
Cash	37,300	40,000	46,600	48,400	55,900	56,800	65,200	65,200
PP&E (net)	9,000	0	6,000	0	3,000	0	0	0
Total assets	46,300	40,000	52,600	48,400	58,900	56,800	65,200	65,200
Share capital and APIC	40,000	40,000	40,000	40,000	40,000	40,000	40,000	40,000
Retained earnings	6,300	0	12,600	8,400	18,900	16,800	25,200	25,200
Total equity	46,300	40,000	52,600	48,400	58,900	56,800	65,200	65,200

Cash Flow Statement	Y1		Y2		Y3		Y4	
	$	$	$	$	$	$	$	$
CFO	9,300	0	9,300	8,400	9,300	8,400	9,300	8,400
CFI	(12,000)	0	0	0	0	0	0	0
CFF	40,000	40,000	0	0	0	0	0	0
Change in cash	37,300	40,000	9,300	8,400	9,300	8,400	9,300	8,400
Opening cash	0	0	37,300	40,000	46,600	48,400	55,900	56,800
Closing cash	37,300	40,000	46,600	48,400	55,900	56,800	65,200	65,200

- From this example, we can see the effect that capitalising / expensing has on each of the following:
- The interest expense on funds spent constructing a capital asset is capitalised as part of the following:
  - The asset's value on the balance sheet (self-use)
  - The asset's value in inventory (for sale to others)

  Under IFRS, capitalised interest is reduced by any income on borrowings invested temporarily.

	Capitalise	Expense
Assets and Equity	Higher	Lower
Net Income (Y1)	Higher	Lower
Net Income (Y2+)	Lower	Higher
Income variability	Lower	Higher
ROA, ROE (Y1)	Higher	Lower
ROA, ROE (Y2+)	Lower	Higher
Debt ratio, Debt-to-equity	Lower	Higher
Operating Cash Flow (CFO)	Higher	Lower
Investing Cash Flow (CFI)	Lower	Higher

- We define interest coverage as

$$\text{Interest coverage} = \frac{EBIT}{\text{Interest Expense}} \tag{4.1}$$

EXAMPLE: Consider capitalisation of interest where EBIT = \$160m, the interest expense is \$80m, the interest capitalised is \$20m, and depreciation from the prior year capitalisation is \$10m. Calculate is the interest coverage before / after adjusting for capitalised interest.

$$\text{Before adjustment} \quad \frac{160}{80} = 2$$

$$\text{After adjustment} \quad \frac{160 + 10}{80 + 20} = 1.7$$

If $CFO = \$70\text{m}$, then $CFI = -\$50\text{m}$. This is because interest paid is accounted for in CFO, aside from capitalised interest.

Ignoring tax, what is the impact of the interest capitalisation (\$20m) on CFO and CFI?

\$20m interest was capitalised through CFI. Had this been expensed, CFI would be −\$30m (−\$50m+\$20m), and CFO would be \$50m (\$70m−\$20m). No adjustment to the depreciation is necessary as this is a non-cash charge.

4.4.2 Research and Development

- Internally developed intangibles are expensed as incurred except for R&D, and software development costs. Research involves discovery of new knowledge and understanding. Development costs involve translation of research findings into a plan

IFRS Research expensed, development may be capitalised if the project is technically feasible, resources exist to complete the project, a market exists for the product, and there is an intention to complete and sell the product.

USGAAP Research and development are both expensed

4.4.3 Software

- Software developed for sale

IFRS and USGAAP permit costs to be expensed as incurred, until technological feasibility is established. This requires a judgement call from management.

- Software developed for internal use

IFRS Same treatment as if software were for sale

USGAAP Costs are expensed as incurred, until it is probable that the firm will complete the project and use as intended

4.4.4 Non-recurring items

- These are unusual or infrequent items that are material for a business.
- These are reported pre-tax, before net income from continuing operations
- Items include
  - Gain (loss) from disposal of business segment / assets
  - Gain (loss) from sale of investment in a subsidiary
  - Provisions for environmental remdiation
  - Impairments , write-offs, write-downs, restructuring
  - Integration expense for recently acquired business

  These items should be excluded from forecasts

4.4.5 Discontinued operations

- Operations that management has decided to dispose of but has either not yet done so, or has done so in the current year after generating PnL.
- This is reported net of taxes, after net income from continuing operations.
- Assets, operations, financing activities must be physically and operationally distinct from the firm.
- Again, these items should be excluded from forecasts

4.5 Accounting changes

- A change in accounting policy, for example revenue recognition, requires **retrospective application**
- A change in accounting estimates is treated prospectively, and does not require restatement of prior-period earnings

- Prior period adjustments
  - Correcting errors, changing from inacceptable to acceptable methodology
  - This typically requires retrospective application, and restatement of prior year's earnings
  - The nature and impact of the error / change must be disclosed

- Scope changes
  - Mergers and acquisitions reduce comparability
  - Balance sheet of parent / subsidiary combined at acquisition date

  - Changes in scope are not required for disclosure

- Exchange rates
  - Overseas trade / subsidiaries may operate in foreign currencies
  - Sales and purchases must be converted to the reporting currency
  - Changes in exchange rates do not need to be disclosed

4.6 Earnings per share (EPS)

- Simple vs complex capital structures should be considered when calculating EPS.
- A simple capital structure is one which contains no potentially dilutive securities. In this instance, only basic EPS must be reported.
- A complex capital structure does contain potentially dilutive securities. In this instance, basic and diluted EPS must be reported. Potentially dilutive securities include
  - Stock options
  - Warrants
  - Convertible debt
  - Convertible preferred stock

  any of which may become common stock.

4.6.1 Basic EPS

- Basic EPS is defined

$$\text{Basic EPS} = \frac{\text{Net income} - \text{Preference dividends}}{\text{Weighted avg. \# common stock}} \tag{4.2}$$

  The denominator of equation 4.2 can be affected by various events during the year. This includes:
  - Stock dividends – A 10% stock dividend would increase shares outstanding by 10%
  - Stock split – A 2-for-1 stock split would increase shares outstanding by 100%
  - Stock issue

  In calculating the weighted average shares outstanding, stock dividends and stock splits are applied retroactively to the beginning of the year or issue date of new stock. New stock is weighted by fraction of the year that the new stock was outstanding. So a new issuance of 300 shares on 1st July would increase the weighted average shares outstanding by $300 \times \frac{6}{12}$ since it was outstanding for 6 of 12 months in the year.

  EXAMPLE: Consider a company with shares outstanding at the beginning of the year, and the following events occurring throughout the year.

1-Jan	10,000 shares outstanding
1-Apr	4,000 shares issued
1-Jul	10% stock dividend
1-Sep	3,000 shares repurchased

The company has net income of \$10,000, and pays out \$1,000 in preference dividends and \$1,750 in common dividends.

We first apply the stock dividend retrospectively on the initial shares outstanding and any share-related events before the stock dividend (in this case the share issuance on April 1). We then calculate the weighted average

$$\begin{array}{lllllllll} \text{1-Jan} & 1.1 & \times & 10,000 & = & 11{,}000 & \times \frac{12}{12} & = & 11{,}000 & + \\ \text{1-Apr} & 1.1 & \times & 4,000 & = & 4{,}400 & \times \frac{9}{12} & = & 3{,}300 & + \\ \text{1-Jan} & & & 3,000 & = & 3{,}000 & \times \frac{4}{12} & = & 1{,}000 & - \\ & & & & & & & & \overline{13{,}300} & \end{array}$$

From this, using equation 4.2, we can see

$$\text{Basic EPS} = \frac{10,000 - 1,000}{13,300} = \$0.68$$

4.6.2 Diluted EPS

- A security is considered dilutive if the EPS would decrease upon its conversion to common stock. (Anti-dilutive securities increase the EPS upon conversion)
- Diluted EPS is defined

$$\text{Diluted EPS} = \frac{\begin{array}{r}\text{Net income} - \text{Preferred dividends} \\ + \text{Convertible preferred dividends} \\ + \text{Convertible debt interest}(1-t)\end{array}}{\begin{array}{r}\text{Weighted average shares} \\ + \text{Shares from convertible preferred shares} \\ + \text{Shares from conversion of convertible debt} \\ + \text{Shares issuable from options / warrants}\end{array}} \tag{4.3}$$

We should only include securities that would reduce the EPS below the basic EPS in the calculation. The criteria for this is:

Convertible preference shares	$\frac{\text{Dividends}}{\text{New shares}} < \text{Basic EPS}$
Convertible debt	$\frac{\text{Interest}(1-t)}{\text{New shares}} < \text{Basic EPS}$
Options / warrants	Average price > Exercise price

If any of these conditions are satisfied, then the security can be considered dilutive.

EXAMPLE: Convertible preference stock

Consider a company which has \$4,000,000 available to common shareholders (Net income − Preferred dividends = \$4,000,000) and 2,000,000 ordinary shares outstanding. This company also has \$5,000,000 of 7% convertible preferred stock outstanding all year. Terms of conversion are such that \$10 nominal value of preferred stock can be converted to 1.1 common shares.

$$\begin{aligned}\text{Diluted EPS} &= \frac{\$4,000,000 + (\$5,000000 \times 7\%)}{2,000,000 + \frac{\$5,000,000}{10} \times 1.1} \\ &= \frac{\$4,350,000}{2,550,000} \\ &= \$1.71\end{aligned}$$

Which is less that the basic EPS, so this is a dilutive security. Alternatively, since $\frac{350,000}{500,000} < 2.00$, we can immediately tell that this is a dilutive security.

EXAMPLE: Convertible bonds

Consider a company with 1,000,000 shares outstanding, and \$2,5000,000 available to common shareholders. It is subject to a corporate tax rate of 30%. The company has \$2,000,000 par value of 5% convertible bonds outstanding. \$1,000 par value may be converted to 120 common shares.

$$\text{Diluted EPS} = \frac{\$2,500,000 + \overbrace{\underbrace{0.05 \times \$2,000,000}_{\text{Interest}} \times \underbrace{(1-0.3)}_{\text{After tax}}}^{70,000}}{1,000,000 + \underbrace{\frac{2,000,000}{1,000} \times 120}_{\text{Additional shares}}^{240,000}}$$

$$\text{Diluted EPS} = \$2.07$$

Which is less than the basic EPS so this is a dilutive security. Alternatively, since $\frac{70,000}{240,000} < 2.5$, we can immediately tell that this is a dilutive security.

EXAMPLE: Convertible bonds

Consider a compnay which has \$1,200,000 available to common shareholders. The weighted average number of common stock during the year is 500,000, and the average price of common stock during the year is \$20. This company has 100,000 options outstanding at an exercise price of \$15.

The steps to solve this problem are as follows:

1. Calculate the number of common shares created if options are exercised
2. Calculate cash received from exercise
3. Calculate the number of shares that can be purchase at the average market price with exercise proceeds
4. Calculate net increase in common share outstanding (step 1 − step 2) to give the number of new shares issued

$$\text{Basic EPS} = \frac{\$1,200,000}{500,000} = \$2.40$$

Step 1 is to work out the number of common shares created. We assume that all cash proceeds from option exercise are used to buy back as many shares as possible from the market, and the difference is made up by issuance of new shares. The cash proceeds are $\$100,000 \times 15 = \$1,500,000$, which when taking the average market price of \$20, allows $\frac{\$1,500,000}{\$20} = 75,000$

share to be repurchased with cash proceeds. Given there are 100,000 options outstanding, a further 25,000 shares must be issued to make up the difference. Numerically, this gives

$$\begin{aligned} 100,000 - \frac{100,000 \times \$15}{\$20} &= 100,000 - 75,000 \\ &= 25,000 \end{aligned}$$

The diluted EPS is therefore

$$\text{Diluted EPS} = \frac{\$1,200,000}{500,000 + 25,000} = \$2.29$$

4.7 Vertical common-size Income statements

- Each line of the income statement is calculated as a fraction of the total sales (revenue)

$$\frac{\text{Income statement account}}{\text{Sales } (\equiv \text{Revenue})}$$

The advantages of this are:

- Converts income statement to relative percentages
- Useful for comparing entities of different sizes
- Compare % to the strategy discussed in the MD&A segment
- Allows for time series or cross-sectional use
- Gross and net profit margin are common size ratios

EXAMPLE:

	North Co.		South Co.		
Revenue	75,000,000		3,500,000		
Cost of goods sold	52,000,000	70%	700,000	20%	
Gross profit	22,500,000	30%	2,800,000	80%	Gross profit margin
Admin expense	11,250,000	15%	525,000	15%	
Research expense	3,750,000	5%	700,000	20%	
Operating profit	7,500,000	10%	1,575,000	45%	Operating profit margin

From this, we can see that the gross profit is increased by increasing sales and / or lowering costs. The operating profit is increased purely by lowering expenses.

4.8 Intangible assets and marketable securities

- All assets are classified as tangible or intangible. Intangible assets can be further split into two categories:
  - Identifiable intangibles – These can be acquired singularly, linked to rights, and privilege having a finite benefit period. These are amortized over the useful lifetime
  - Unidentifiable intangibles – These cannot be acquired singularly, and may have indefinite benefit periods, for example goodwill. These are not amortized, and are instead reviewed annually for impairment

- An intangible asset may only be recognised if it can be measured reliably.

IFRS Recognise either at cost or revaluation method (if an active market for the asset exists)
US GAAP Recognise at cost only

This does not account for internally-generated intangibles.

- Typical intangibles include
  - Purchased patents / copyrights
  - Purchased brands / trademarks
  - Direct response advertising
  - Purchased franchise and license costs
  - Computer software development costs
  - Goodwill
- Expensed items include
  - Internally generated brands
  - Start-uup costs
  - Training costs
  - Administrative costs, general overhead
  - Advertising and promotion
  - Relocation costs
  - Redundancy costs
  - Research and development[2]

For IFRS, having a working prototype is sufficient to constitute technical feasibility.

Capitalise	Materials
	Direct labour
	Production labour
Expense	Administrative overhead

4.8.1 Goodwill

- The difference between the acquisition price and fair market value of the acquired firm's net assets is called good will

$$\text{Net assets} = \text{Assets} - \text{Liabilities} \tag{4.4}$$

The additional amount paid represents the amount paid for assets not on the balance sheet. The fair value estimate involves management discretion. Goodwill is not amortised, as it is an unidentifiable intangible asset.

- Impairment indicates that goodwill often results from overpayment to acquire an entity. You should remove the impact of goodwill from any ratios calculated.
  - Remove goodwill from assets
  - Remove any impairment from the income statement
  - Evaluate business acquisitions considering purchase price, net assets, and earnings prospects

4.8.2 Financial instruments (marketable securities)

- Financial instruments and other marketable securities include

[2]Development is capitalised under IFRS, see §4.4.2

  - Stocks
  - Bonds
  - Receivables
  - Notes to receivables
  - Loans to others
  - Derivatives

  They are measured at historical cost, amortised cost or fair value, and attributed to other comprehensive income (OCI)

- Fair value assets use mark-to-market accounting. This covers:
  - Trading / held-for-trading securities
    * Debt held with the intention to sell in the near term
    * Quoted equity

    Unrealised gain / loss on these is put through the income statement
  - Available-for-sale / fair value through OCI securities. This includes debt available for sale.
  - Derivatives (stand-alone or embedded in a non-derivative instrument)
  - Assets with fair value exposures hedged by derivatives

  Dividend incom, interest income, and realised PnL is put through the income statement

- Assets measured at cost or amortised cost include
  - Unlisted instruments
  - Held-to-maturity investments
  - Loans
  - Receivables
  - All other liabilities (bonds, notes payable, trade payables)

- Deferred tax liabilities are a measure of taxable temporary differences between the income tax expense (income statement) and taxes payable (tax return). This is a temporary difference due to timing. For tax purposes, accelerated depreciation is used, whereas straight-line depreciation is used for financial reporting
- Any differences should eventually reverse when all taxes are paid

4.9 Common size balance sheets

- Similar to the common-size income statement, here, all balance sheet accounts are expressed as a percentage of the total assets on the balance sheet. This allows for comparisons over time, as well as cross sectional comparisons.

$$\frac{\text{Balance sheet account}}{\text{Total assets}}$$

	Cash		Current liabilities
+	Accounts receivable	+	Long-term debt
+	Inventory		Total liabilities
+	Plant and equipment	+	Equity
+	Goodwill		**Total liabilities + Equity**
	Total assets		

- From this, we can easily see that the following relation must hold,

$$\text{Assets} = \text{Liabilities} + \text{Equity} \tag{4.5}$$

4.10 Ratios

- Liquidity ratios (short-term debt):

$$\text{Current ratio} = \frac{\text{Current assets}}{\text{Current liabilities}} \tag{4.6}$$

(4.7)

$$\text{Quick ratio} = \frac{\text{Cash + Marketable securities + Receivables}}{\text{Current liabilities}} \tag{4.8}$$

(4.9)

$$\text{Cash ratio} = \frac{\text{Cash + Marketable securities}}{\text{Current liabilities}} \tag{4.10}$$

- Solvency ratios

$$\text{Total debt ratio} = \frac{\text{Total debt}}{\text{Total assets}} \tag{4.11}$$

$$\text{Financial leverage} = \frac{\text{Total assets}}{\text{Total equity}} \tag{4.12}$$

4.11 Cash flow statements

4.11.1 Introduction

- Cash flow is usually split out into three components.
  - Operating cash flow (CFO)
  - Investing cash flow (CFI)
  - Financing cash flow (CFF)

	Operating cash flow CFO	$\longrightarrow$	Current assets, current liabilities
+	Investing cash flow CFI	$\longrightarrow$	Non-current assets
+	Financing cash flow CFF	$\longrightarrow$	Non-current liabilities, equity
	Change in cash balance		
+	Beginning cash balance		
	Ending cash balance		

- Earnings can be considered high quality if $CFO \gtrsim$ Reported earnings. Operating activities relate to current assets and liabilities

Accounts Receivable "T" Account

Amount B / Fwd	10,000	63,000	Cash collections
Sales	68,000	15,000	Amount C/Fwd
	78,000	78,000	

	Increase	Decrease
Assets	Outflow	Inflow
Liabilities and equity	Inflow	Outflow

Table 4.1: Table showing the effect changes in assets, liabilities, and equity has on a cash levels

- Increases and decreases in assets, liabilities, and equity involve the use of cash.

  An increase in receivables or inventory uses cash – cash is spent to buy assets. An increase in payables generates cash – cash is received and must be paid back later.

	Y2		Y1
Revenue (I/S)	2,000,000		1,800,000
Accounts Receivable (B/S)	900,000	$\xleftarrow{+400{,}000}$	500,000
Unearned Revenue (B/S)	1,000,000	$\xleftarrow{+700{,}000}$	300,000

Unearned revenue is revenue that has already been paid, but the service has not yet been provided. This is not recorded as a part of revenue, and is classified as a liability.

$$\begin{aligned}\text{Cash collected} &= 2,000,000 - \underbrace{400,000}_{\text{Asset}} + \underbrace{700,000}_{\text{Liability}} \\ &= 2,300,000\end{aligned}$$

4.11.2 Example balance sheet and income statement

Balance sheet

	T	T–1
Current Assets		
Cash	53,000	11,500
Accounts receivable	10,000	9,000
Inventory	5,000	7,000
Non-current assets		
Land	35,000	40,000
Gross PP&E	69,000	60,000
Accum. Deprec.	(12,000)	(9,000)
Net PP&E	57,000	51,000
Goodwill	10,000	10,000
Total assets	170,000	128,500
Current liabilities		
Accounts payable	9,000	5,000
Wages payable	4,500	8,000
Interest payable	3,500	3,000
Unearned revenue	6,000	2,000
Taxes payable	5,000	4,000
Dividends payable	6,000	1,000
Non-current liabilities		
Bonds	15,000	10,000
Deferred tax liabilities	20,000	15,000
Stockholder's equity		
Common stock	15,000	20,000
Additional paid-in capital	25,000	30,000
Retained earnings	61,000	30,500
Total liabilities and equity	170,000	128,500

Income statement

Sales Revenue		104,000
Expenses		
Cost of goods sold	40,000	
Wages	5,000	
Depreciation	7,000	
Interest	1,000	
Total expenses		53,000
Income from continuing operations		51,000
Gain from sale of land		10,000
Loss on disposal of PP&E		2,000
Pretax income		59,000
Provision for taxes		20,000
Net income		39,000

The company pays a common dividend of 8,500

The company makes a 25,000 investment in assets

4.11.3 Direct method CFO

1. Start with revenue on the income statement
2. Look at balance sheet for any assets / liabilities (typically current) that relate to the income statement item
3. Compute change in the balance sheet asset / liability
4. Adjust income statements for changes
5. Repeat for each line item of income statement
6. Ignore non-cash charges (i.e. depreciation)

Sales	104,000	
Δ Accounts receivable	(1,000)	
Δ Unearned revenue	4,000	
	107,000	Cash collected
Cost of goods sold	(40,000)	
Δ Inventory	2,000	
Δ Accounts payable	4,000	
	(34,000)	Cash paid to suppliers
Operating expense (wages)	(5,000)	
Decrease in salaries payable	(3,500)	
	(8,500)	Cash paid to employees
Interest expense	(1,000)	
Δ Interest payable	500	
	(500)	Cash interest paid
Tax expense	(20,000)	
Δ Tax payable	1,000	
Δ Deferred tax liability	5,000	
	(14,000)	Cash taxes paid

Cash collected from customers	107,000
Cash paid to suppliers	(34,000)
Cash paid to employees	(8,500)
Cash interest paid	(500)
Cash taxes paid	(14,000)
Operating cash flow	**50,000**

4.11.4 Indirect method CFO

1. Start with net income
2. Add back all non-cash charges (i.e. depreciation / amortisation) Subtract gains / add losses on disposal of non-current assets as these are classified under CFI
3. Adjust net income for changes in the relevant balance sheet items, in accordance with table 4.1 in terms of addition and subtraction of changes in assets and liabilities.

	Net income	39,000	
+	Depreciation	7,000	Non-cash charge
−	Gain from sale of land	(10,000)	Part of CFI
+	Loss from disposal of land	2,000	Part of CFI
+	Increase in deferred taxes	5,000	
	Current asset and current liability adjustments		
−	Increase in accounts receivable	(1,000)	
+	Decrease in inventory	2,000	
+	Increase in accounts payable	4,000	
−	Decrease in wages payable	(3,500)	
+	Increase in interest payable	500	
+	Increase in unearned revenue	4,000	
+	Increase in taxes payable	1,000	
	Operating cash flow	**50,000**	

- We can see that the direct and indirect method both lead to the same result for CFO.

- Typical non cash charges that need to be adjusted for

Add back

- Depreciation, depletion, amortisation
- Losses on asset disposal
- Impairments / writedowns
- Losses on early retirement of debt
- Amortisation of bond discounts
- Increases in DTLs / decreases in DTAs
- Losses of equity-associated accounts

Subtract

- Gains on asset disposal
- Gains on early retirement of debt
- Reversals of impairments / writedowns
- Amortisation of bond premiums
- Decreases in DTLs / increases in DTAs

- IFRS and US GAAP allow for both direct and indirect methods to be used for calculation of CFO, but encourage the use of the direct method. If the direct method is used, then the indirect method must be included as part of the disclosures.
  - Most companies report under the indirect method

4.11.5 Indirect to direct method CFO conversion

1. Aggregate all revenues and gains and all expenses and losses
2. Remove all non cash charges and disaggregate the remaining items into direct method categories
3. Convert from accruals to cash flow by adjusting for damages in working capital (customer cash, employee cash, interest cash, etc.)

So, looking at the same example as before, and starting with the income statement

1. Aggregate all revenue and gains, and expenses and losses to derive the net income

			Income statement item
	Revenue and gains	114,000	Revenue, gain from sale of land
−	Expenses and losses	75,000	Expenses, loss on disposal of PP&E, provision for taxes
	Net income	39,000	

2. Remove all non-cash charges and disaggregate the remaining items

$$114,000 - \underbrace{10,000}_{\text{Gain on disposal}} = 104,000$$

$$75,000 - \underbrace{7,000}_{\text{Depreciation}} - \underbrace{2,000}_{\text{Loss on disposal}} - \underbrace{5,000}_{\Delta\text{DTL}} = 61,000$$

	Cost of goods sold	40,000
+	Wages	5,000
−	Interest	1,000
+	Tax payable (provision − ΔDTL)	69,000
		61,000

Cash collected from customers

$$104,000 - \underbrace{1,000}_{\substack{\Delta\text{Accounts} \\ \text{receivable}}} + \underbrace{4,000}_{\substack{\text{Unearned} \\ \text{revenue}}}$$

Cash paid to suppliers

$$- \underbrace{40,000}_{\substack{\Delta\text{Cost of} \\ \text{goods sold}}} + \underbrace{2,000}_{\Delta\text{inventory}} + \underbrace{4,000}_{\substack{\Delta\text{Accounts} \\ \text{payable}}}$$

Cash interest paid

$$-\underbrace{1,000}_{\substack{\text{Income}\\\text{statement}}}+\underbrace{500}_{\substack{\text{Interest}\\\text{payable}}}$$

Cash paid to tax authorities

$$-\underbrace{40,000}_{\text{Provision}}-\underbrace{5,000}_{\substack{\text{Increase}\\\text{in DTL}}}+\underbrace{2,000}_{\text{Tax payable}}$$

4.11.6 CFI

- This covers
  - Purchases of PP&E
  - Proceeds from sale of assets
  - Investments in joint ventures and affiliates
  - Payments for business acquired
  - Purchases / sales of intangibles
  - Purchases / sales of marketable securities
- This excludes
  - Trading securities (Covered by CFO)
  - Cash equivalents (Listed on the balance sheet)

$$CFI = \text{Cash received from asset sales} - \text{Investment in assets} \tag{4.13}$$

$$\text{Gain / loss on disposal} = \text{Cash proceed} - \text{Carrying value at disposal} \tag{4.14}$$

$$\text{Carrying value} = \text{Cost} - \text{Accumulated depreciation} \tag{4.15}$$

- The relevant income statement and balance sheet items are:

	T	T–1
Land	35,000	40,000
Gross PP&E	69,000	60,000
Accum. Deprec.	(12,000)	(9,000)
Net PP&E	57,000	51,000

Depreciation	7,000
Gain from sale of land	10,000
Loss on disposal of PP&E	2,000

The company makes a 25,000 investment in assets.

The calculation for CFI is then

PP&E

	Beginning gross PP&E	60,000
+	PP&E purchased	25,000
−	Gross PP&E sold	
	Ending gross PP&E	69,000

so we can see that the Gross PP&E sold is $60,000 + 25,000 - 69,000 = 16,000$, as all the other components of that equation can be read from the balance sheet.

	Beginning accumulated depreciation	9,000
+	Depreciation expense	7,000
−	Accumulated depreciation on disposal of PP&E	
	Ending accumulated depreciation	12,000

so we can see that the accumulated depreciation on disposal of PP&E is $9,000 + 7,000 - 12,000 = 4,000$, as the other parts can be read from the balance sheet and income statement.

Then, using equation 4.15, we can work out the carrying value of the PP&E,

	Cost	16,000
−	Accumulated depreciation on disposal of PP&E	4,000
	Carrying value	12,000

Equivalently,

	Beginning carrying value	51,000
−	Depreciation expense	(7,000)
+	Additions to PP&E	25,000
−	Carrying value of assets disposed	
	Ending carrying value	57,000

so the carrying value of assets disposed is $51,000 - (7,000) + 25,000 - 57,000 = 12,000$, same as before.

Now, using equation 4.14, the cash proceed from sales is

	Cash proceed	
−	Carrying value at disposal	(12,000)
	Ending gross PP&E	(2,000)

Land (No depreciation)

- We can read off the balance sheet that the gain from sale is 10,000. The carrying value at disposal is the same as the change in land value, which is

$$\Delta\text{Land value} = 25,000 - 40,000 = (5,000)$$

	Cash proceed	
–	Carrying value at disposal	(5,000)
	Gain on sale	10,000

The cash proceeds is given by

The CFI is then given by the sum of all proceeds less the sum of all additions

$$CFI = \sum \text{Proceeds} - \sum \text{Additions} \tag{4.16}$$

$$\underbrace{10,000}_{PP\&E} + \underbrace{15,000}_{Land} - \underbrace{25,000}_{Investment} = 0$$

4.11.7 CFF

- This covers the issuance, purchase, and redemption of
  - Common stock
  - Preferred stock
  - Debt
- Dividend payments may be included here, but fall under CFO under USGAAP

$$\text{Net income} - \text{Dividend declared} = \text{Change in retained earnings} \tag{4.17}$$

$$-\text{Dividend declared} + \text{Change in dividends payable} = \text{Cash dividend paid} \tag{4.18}$$

- The relevant income statement and balance sheet items are:

Balance Sheet

	T	T–1
Dividends payable	6,000	1,000
Bonds payable	15,000	10,000
Stockholder's equity		
Common stock	15,000	20,000
Additional paid-in capital	25,000	30,000
Retained earnings	61,000	30,500

Other data

Net income	39,000
Dividend declared	8,500

- The change in debt is $15,000 - 10,000 = 5,000$

- The change in common stock is $15,000 + 25,000 - 20,000 - 30,000 = (10,000)$
- The change in retained earnings is $61,000 - 30,500 = 30,500$
- The cash divided paid is $\underbrace{(8,500)}_{\text{Declared}} + \underbrace{5,000}_{\text{Still payable}} = (3,500)$

  Now, using equation 4.17,

	Net income	39,000
−	Dividend declared	
	Change in retained earnings	30,500

  we recover the declared dividend of $39,000 - 30,500 = 8,500$ as expected.

4.11.8 Differences between US GAAP and IFRS

- US GAAP and IFRS differ in their treatment of various items and which category of cash flow they are, or can be, attributed to

	USGAAP	IFRS
Interest received	CFO	CFO / CFI
Interest paid	CFO	CFO / CFF
Dividends received	CFO	CFO / CFI
Dividends paid	CFF	CFO / CFF
Taxes paid	CFO	CFO / CFI + CFF
Bank overdraft	CFF	Cash + equiv.

Table 4.2: US GAAP and IFRS treatment for different itmes in the income statement

4.11.9 Cash flow statement analysis

- Questions a cash flow statement analysis should aim to answer
  - Do regular operations cash flow generate enough cash to sustain the business
  - Is enough cash generated to pay off maturing debt
  - Is there a need for additional financing
  - Is the company able to meet unexpected obligations
  - Is the company able to take advantage of new opportunities

- Analyse major sources and uses of cash flow
  - What are the major sources and uses
  - Is CFO sufficient to cover capex
- Analyse CFO
  - What are major determinants of CFO
  - Is CFO higher or lower than net income
  - How consistent is the CFO

- Analyse CFI
  - What is cash being spent on
  - Investing in PP&E
  - What acquisitions have been made
- Analyse CFF
  - How is the company financing – CFI / CFO
  - Capital being raised or repaid
  - Dividends returned to owners
- Common size cash flow statement
  - % of net revenue

    OR
  - Each inflow as % of total inflow
  - Each outflow as % of total outflow

  This can be used to find trends over time

4.11.10 Free cash flow

- Free cash flow is a measure of cash that is available for discretionary use after all capital expenditure has been covered
- Free cash flow to the firm, FCFF, is the cash available to all investors (equity and debt).

$$FCFF = \underbrace{\overbrace{NI}^{\text{Net income}} + \overbrace{NCC}^{\text{Non-cash charges}} - \overbrace{WCInv}^{\text{Working capital investment}}}_{CFO} + \overbrace{\text{Int}(1-t)}^{\text{Net debt expense}} - \overbrace{FCInv}^{\text{Fixed capital investment}} \tag{4.19}$$

- Free cash flow to equity, FCFE, is the cash available for distribution to common shareholders, after all obligations have been satisfied.

$$FCFE = CFO - FCInv + \text{Net debt expense} \tag{4.20}$$

4.11.11 Cash flow performance ratios

4.12 Inventory measurement

- Take the lower of cost or net realisable value. All IFRS firms and most US GAAP firms, except for those using LIFO or retail inventory cost
- The cos includes all costs of bringing inventory to its current location and condition, but excludes
  - Abnormal amounts
  - Storage costs
  - Admin overheads
  - Selling costs
- The net realisable value (NRV) is defined as

$$\text{Net realisable value} = \text{Est. selling price} - \text{Est. cost of completion} - \text{Selling costs} \tag{4.21}$$

  If the net realisable value is less than the cost, then this goes straight to the income statement.

Cash flow to revenue	$\frac{CFO}{\text{Net revenue}}$
Cash return on assets	$\frac{CFO}{\text{Avg. total assets}}$
Cash return on equity	$\frac{CFO}{\text{Avg. equity}}$
Cash to income	$\frac{CFO}{\text{Operating income}}$
Cash flow per share	$\frac{CFO - \text{Preference dividend}}{\text{Net revenue}}$
Debt coverage	$\frac{CFO}{\text{Net revenue}}$
Interest coverage	$\frac{CFO + \text{Interest paid} + \text{Tax paid}}{\text{Interest paid}}$
Reinvestment ratio	$\frac{CFO}{\text{Cash paid for long-term assets}}$
Debt payment	$\frac{CFO}{\text{Cash paid for long-term debt repayment}}$
Dividend payment	$\frac{CFO}{\text{Dividends paid}}$
Investing and financing	$\frac{CFO}{\text{Cash outflows for CFI, CFF}}$

- Reversal of writedowns is allowed under IFRS, but not under US GAAP. Any reversal however is limited to the original loss (in other words, capped at original cost)
- Retail inventory cost methods is the lower of cost or market value.
  - Cost: same as IFRS (US GAAP prohibits reversal of writedown)
  - Market value: current replacement cost, subject to:
    - Upper limit = NRV
    - Lower limit = NRV − profit margin

Example:

	Selling price	225
−	Selling costs	22
	Net realisable value	203

	Original cost	220
−	Replacement cost	197
	Normal profit margin	12

$$\text{Min(Cost, NRV)} = \text{Min}(210, 203) = 203$$

Therefore inventory is written down to 203, recognising a loss of 7.

$$\text{Min(Cost, Market value)} = \text{Min}(210, 197) = 197$$

where market value is the current replacement cost, bounded by the NRV (203) and the NRV less the normal profit margin, $= 203 - 12 = 191$. 197 is in this range.

- If NRV increases to 213 and replacement cost increases to 207

  The original cost of 210 is now the lower of cost and NRV.

  - Under IFRS, the value of inventory may by written up to 210, since 210 is now the minimum of cost and NRV, so the previous loss of 7 is reversed
  - Under US GAAP, there is no reversal, but since the inventory value is at the lower value, and NRV is higher, greater profit margin is then recorded

- Inventory valuation above cost
  - Generally, this is not allowed, but is permitted for producers / dealers of commodity-like products
  - Reported on balance sheet at NRV
  - If active market exists, quoted market price is used. Otherwise, recent market transactions are used
  - Unrealised gains / losses are recognised in the income statement

4.12.1 Inflation impact of FIFO and LIFO

- In an inflationary environment, later acquisitions are made at a higher price

	LIFO	**FIFO**
COGS	Higher	Lower
EBT	Lower	Higher
Taxes	Lower	Higher
NI	Lower	Higher
Inventory	Lower	Higher
Working capital	Lower	Higher
Retained earnings	Lower	Higher
CFO	Higher	Lower

  Lower tax $\Rightarrow$ Higher CFO

- When prices are rising
  - FIFO shows an atificially low value of COGS while LIFO is more useful here
  - LIFO shows an artiicially low value of ending inventory, while FIFO is more useful here.

  If prices are stable, then there is no change

- Effects on ratios (Assuming inflationary environment)
  - Profitability: FIFO > LIFO (Lower COGS implies a higher margin)
  - Liquidity: FIFO > LIFO (Higher ending inventory value)
  - Activity: Inventory turnover under LIFO > FIFO
  - Solvency: LIFO > FIFO (Higher assets means higher equity)

4.12.2 LIFO liquidation

- When goods sold exceed goods replaced;
  - Older (lower) inventory costs are used, and therefore earnings increase
  - Higher earnings not sustainable
- May be intentional (earnings manipulation) or unintentional (drop in demand, strikes, recession)
- Should eliminate liquidation effect by adjusting COGS

EXAMPLE:

[Units]	**T**	**T+1**	**T+2**
Sales	10,000	12,000	16,000
Beginning inventory	0	4,000	7,000
Purchases	14,000	15,000	10,000
Available for sale	14,000	19,000	17,000
Ending inventory	(4,000)	(7,000)	(1,000)
Cost of sales	10,000	12,000	16,000

Prices	**T**	**T+1**	**T+2**
Sales	100	105	110
Purchases	80	84	88

[$ terms]	**T**	**T+1**		**T+2**	
		FIFO	LIFO	FIFO	LIFO
Sales	1,000,000	1,260,000		1,760,000	
Beginning inventory	0	320,000		588,000	572,000
Purchases	1,120,000	1,260,000		880,000	
Available for sale	1,120,000	1,580,000		1,468,000	1,452,000
Ending inventory	(320,000)	(588,000)	(572,000)	(88,000)	(80,000)
Cost of goods sold	800,000	992,000	1,008,000	1,380,000	1,372,000
Gross profit	200,000	268,000	252,000	380,000	388,000

We can see that the LIFO gross profit is higher

4.13 Presentation and disclosures

- Financial statement information

  - Cost of sales
  - Cost flow method (FIFO / LIFO)
  - Carrying value of total inventory, carrying values by appropriate classification
  - Carrying value of inventory reported at fair value less selling costs
  - Write downs / reversals of inventory
  - Assets pledged as collateral

- Inventory analysis
  - Increase in raw materials and work-in-process implies an expected increase in demand
  - Increase in finished goods alone implies expected decrease in demand
  - Finished goods growing faster than sales implies an expected decrease in demand and obsolete / excessive inventory

EXAMPLE:

[$ terms]	**T**	**T+1**	**T+2**
Raw materials	120	207	68
Valuation allowance	−20	−27	−2
Net carrying value	100	180	66
Work in progress	50	95	31
Valuation allowance	0	−5	−1
Net carrying value	50	90	30
Finished goods	403	706	221
Valuation allowance	−53	−76	−17
Net carrying value	350	630	204
Inventory net carrying value	500	900	300

- Inventory turnover is defined as

$$\text{Inventory turnover} = \frac{\text{Cost of sales}}{\text{Avg. inventory}}, \tag{4.22}$$

days of inventory on hand defined as

$$\text{Days of inventory on hand} = \frac{365}{\text{Inventory turnover}}, \tag{4.23}$$

and sales growth is defined as

$$\text{Sales growth} = \frac{\text{Current sales}}{\text{Previous sales}} - 1 \tag{4.24}$$

[$ terms]	T	T+1	T+2
Cost of sales	2,600	4,700	
Inventory	500	900	300

- We can then calculate inventory turnover for the years T and T+1.

$$\text{Inventory turnover}\big|_T = \frac{2,600}{\left[\frac{500+900}{2}\right]} = 3.7$$

$$\text{Inventory turnover}\big|_{T+1} = \frac{4,100}{\left[\frac{900+300}{2}\right]} = 6.8$$

Turnover is higher when costs are higher and when average inventory falls.

	T	T+1
Revenue	5,500	7,500
Cost of sales	2,600	4,100
Gross profit	2,900	3,400

If sales for T = 5,300, then sales growth is

$$\text{T+1}: \frac{5,500}{5,300} - 1 = 4\% \qquad \text{T+2}: \frac{7,500}{5,500} - 1 = 36\%$$

Gross profit margin is therefore

$$\text{T+1}: \frac{2,900}{5,500} - 1 = 53\% \qquad \text{T+2}: \frac{3,400}{7,500} - 1 = 45\%$$

With the current ratio defined in equation 4.6 as $\frac{\text{Current assets}}{\text{Current liabilities}}$, the quick ratio defined in equation 4.8 as $\frac{\text{Current assets} - \text{Inventory}}{\text{Current liabilities}}$, and the cash ratio defined in equation 4.10 as $\frac{\text{Cash} + \text{Marketable securities}}{\text{Current liabilities}}$;

	T	T+1
Cash	1,250	2,675
Trade receivables	1,520	3,020
Inventory	900	300
Inventory	3,670	5,995
Current liabilities	866	1,505
Current ratio	$\frac{3,670}{866} = 4.23$	$\frac{5,995}{1,505} = 3.98$
Quick ratio	$\frac{3,670 - 900}{866} = 3.13$	$\frac{5,995 - 300}{1,505} = 3.78$
Cash ratio	$\frac{1,250}{866} = 1.44$	$\frac{2,675}{1,505} = 1.78$

4.13.1 Intangible long-lived assets

- Intangible assets lack physical substance. Recall from §4.8 that identifiable intangible assets can be separated from, and controlled by the firm. They are expected to provide probable future benefits and their cost can be reliably measured.
- Unidentifiable intangible assets cannot be separated from the firm (i.e. goodwill)
  - Finite-lived intangibles are amortised
  - Indefinite intangibles are tested for impairment
  - Internally developed intangibles are expensed as incurred, except for R&D (§4.4.2)and software development costs. Research costs involve the discovery of new knowledge and understanding, whereas development costs are a translation of research findings into a plan.

    Under IFRS, research may be expensed, but development may be capitalised. Under US GAAP, both research and development are expensed.
- Purchased intangibles are recorded at cost. For a group of assets, the price is disaggregated based on fair value
- For intangibles obtained in a business acquisition identifiable net assets are recorded at fair value, and the difference between purchase value and fair value of identifiable assets ($A - L$) reported as goodwill
- Recall from §4.4.3 on software that for:
  - Software developed for sale, under IFRS and US GAAP, costs are expensed as incurred until technological feasibility is established
  - Software developed for internal use, IFRS has the same treatment as above, but US GAAP expenses costs as incurred until it is probable the firm will complete the project and use the software as intended.

4.14 Impairment and de-recognition

- Impairment is defined as an unanticipated decline in the carrying value of an asset. This is expensed in the income statement
- IFRS
  - Annually assess for indications of impairment
  - Asset is impaired when the book value (carrying value in the balance sheet) is greater than the recoverable amount, which is defined

$$\text{Recoverable amount} = \text{Max}(\text{Fair value} - \text{Selling cost}, \text{Value in use}), \tag{4.25}$$

$$\text{Value in use} = \text{PV of future cashflows}. \tag{4.26}$$

    If there is an impairment, the asset is written-down to the recoverable amount, and a loss is recognised in the income statement.
  - Loss reversal is allowed up to a maximum of the original loss
- US GAAP
  - Assess assets for impairment only when there is an indication that the book value may not be recoverable through future use

* Impairment when:

$$\text{Book value} > \text{Est. undiscounted future cash flows}$$

* Loss recognition: If the asset is impaired, asset is written down to fair value (or if unknown, the value of the discounted future cash flows. Recognise this loss in the income statement

– Loss reversal is prohibited for assets held for use.

EXAMPLE: Consider the following as information about an asset

Original cost	900,000	
Accumulated depreciation	100,000	
Expected future cash flows	795,000	(undiscounted)
Fair value	790,000	
Value in use	785,000	(discounted)
Selling costs	30,000	

* Under IFRS:

$$\text{Book value} = 900,000 - 100,000 = 800,000$$

$$\text{Recoverable amount} = \text{Max}\begin{cases} \text{Fair value} - \text{Selling costs} = 790,000 - 30,000 = 760,000 \\ \text{Value in use} = 785,000 \end{cases}$$

$$\begin{aligned} \text{Impairment} &= 800,000 - 785,000 \\ &= 15,000 \end{aligned}$$

* Under US GAAP
  · Book value = 800,000, and undiscounted future cash flow = 795,000. Since the book value is higher, there is an ipairment. So, the asset is written down to its fair value of 790,000.

$$\begin{aligned} \text{Impairment loss} &= 800,000 - 790,000 \\ &- 10,000 \end{aligned}$$

4.14.1 Impact of impairment

– Balance sheet
  * Decreases assets (Lower net book value)
  * Decreases equity (Impairment charge)
– Income statement
  * Decreases current net income (Impairment charge)
  * Increases future net income (Lower depreciation)
– Cash flow
  * Unaffected (Impairment is a non-cash charge)
– Disclosure
  * MD&A, Footnotes
– Fixed assets and turnover ratios
  * Increase (Lower assets, therefore smaller denominator)
– Debt-to-equity ratio

    - Increase (Lower equity, as equity = A − L)
  - Current-year ROA, ROE $\left(\frac{NI}{A}, \frac{NI}{E}\right)$
    - Decrease (% fall in NI > % fall in A, E)
  - Future ROA, ROE
    - Increase (Lower A, E, higher NI since reduced depreciation)

- Analysis of impairment
  - Past earnings overstated due to insufficient depreciation
  - Management has control of timing / size of impairment loss
  - Impairments involve judgement – these estimates will have a material impact on accounts
- Impairment of long-lived assets
  - Assets held for sale (IFRS, USGAAP): Tested for impairment when transferred from held for use to held for sale. Depreciation expense is no longer recognised. The asset is impaired if book value is greater than the net realisable value, defined as

$$\text{Net realisable value} = \text{Fair value} - \text{Selling costs}. \tag{4.27}$$

    If the asset is impaired, write-down to NRV. Both IFRS and USGAAP allow loss reversal up to the original loss

4.14.2 Derecognition of long-lived assets

$$\text{Proceeds} - \text{Carrying value} = \text{Gain / loss},$$

where proceeds are attributed to CFI and carrying value is the cost and accumulated depreciation removed from the balance sheet. The gain / loss refers to an accounting gain / loss taken to the income statement

  - When asset is sold / exchanged
    - Carrying value removed from balance sheet
    - Cash or new asset added to balance sheet (part-exchange)
    - Gain / loss reported on income statement
    - Cash proceeds in CFI
  - When asset is abandoned
    - Carrying value removed from balance sheet
    - Losses reported on income statement

$$\text{Proceeds} = \begin{cases} 0 \text{ if abandoned} \\ \text{Fair value if exchanged} \end{cases}$$

    - Discussed in MD&A and / or footnotes
  - An asset is classified as held-for-sale once the sales process commences. Take the lower of carrying value or NRV at this point.

  - A spin-off constitutes the transfer of assets that comprise an entire or subsidiary into a new legal entity

4.14.3 Long-term asset disclosure

- IFRS requires the statement of:
  - Carrying value of each asset class (plant, land, machinery etc.) is defined as

$$\text{Carrying value} = \text{Cost} - \text{Accumulated depreciation} \tag{4.28}$$

  - Either the accumulated depreciation, or amortization and and the depreciation rate
  - Title restrictions and assets pledged as collateral
  - For impaired assets, the loss amount, location in income statement (I / S), and the circumstance
  - For revalued assets, state the revaluation date, how future value is determined, carrying value using historical cost model, and state the revaluation surplus in the OCI (Other Comprehensive Income)
- US GAAP requires the statement of
  - Depreciation expense and depreciation methods
  - By major asset class (same as before) the balance and the accumulated depreciation
  - Intangibles as listed plus an estimate of amortization for the next 5 years
  - For impaired assets, the loss amount, where it falls in the income statement, and circumstances (same as IFRS), how future value is determined, and a descriptions of the asset
- Depreciation and amortisation under IFRS
  - May appear on face f income statement if using the nature of expense approach. Not on face however if using function of expense method. Instead it will appear under COGS of SG&A
  - For indirect statement of cash flow – Depreciation and amortisation are non-cash charges
  - For direct statement of cash flow – Does not appear in the CFO computation

  Under US GAAP, reconciliation of direct / indirect methods is required in the footnotes

4.14.4 Using footnote disclosures

- Analysts can use financial statement disclosures to estimate the average age of fixed assets and the average depreciable life of fixed assets. This is used to identify old and insufficient assets, alongside any potential need for significant investment. Fixed asset turnover, Total useful life, and average age are defined as

$$\text{Fixed asset turnover} = \frac{\text{Revenue}}{\text{Avg. fixed assets}} \tag{4.29}$$

$$\text{Total useful life} = \frac{\text{Historical cost}}{\text{Annual depreciation}} \tag{4.30}$$

$$\text{Average age} = \frac{\text{Accumulated depreciation}}{\text{Annual depreciation}} \tag{4.31}$$

$$\frac{\text{Total useful life} - \text{Average age}}{\text{Remaining useful life}} = \frac{\text{Carrying value of net PP\&E}}{\text{Annual depreciation}} \tag{4.32}$$

EXAMPLE: Consider a company with gross PP&E of $3,000,000$, accumulated depreciation of $1,000,000$, and straight-line annual depreciation of $500,000$. We can then calculate

$$\text{Avg. age} = \frac{1,000,000}{500,000} = 2 \text{ years}$$
$$\text{Useful life} = \frac{3,000,000}{500,000} = 6 \text{ years}$$
$$\text{Remaining life} = 6 - 2 = 4 \text{ years}$$

4.15 Leases

- A contract must
  1. Refer to a specific asset
  2. Give the lessee the economic benefits of that asset during the contract
  3. Give the lessee rights over how to use that asset
- Advantages of financing through lease as opposed to purchase include
  - Typically lower cost of financing
  - Little / no up-front payment
  - Lower risk of obsolescence
- IFRS and US GAAP have two classification of leases. These are
  - **Finance lease** – Benefits and risks of ownership are substantially transferred to the lessee
  - **Operating lease** – Refers to all other long-term leases
  - Leases less than one year (IFRS, US GAAP), or with a value $< \$5,000$ (IFRS only) are exempt and payments are reported as expenses

4.15.1 Lessee accounting

- A lease is classified as a finance lease if any of the following criteria are satisfied
  - Transfers ownership to lessee
  - Lessee has the option to buy and expects to exercise it
  - Lease is for most of the assets useful life
  - PV of lease payments $\geq$ Fair value of the asset
  - Lessor has no alternative use for the asset
- Under IFRS, financing and operating lease treatment is identical:
  - Recognise a right-of-use asset "ROU" equal to the PV of the lease payments on the balance sheet (discount at the lease or borrowing rate)
  - Recognise a lease liability of equal amount on the balance sheet
  - Equity is therefore unchanged at lease inception
  - Straight-line amortisation of the ROU asset is shown on the balance sheet
  - Amortisation of the ROU asset and interest component of the lease payment is shown on the income statement

  - Principal component of the lease payments reduce balance sheet liability and reported under CFF
  - Asset and liability vary over the life of the lease but reconcile by the end of the term

- Under USGAAP
  - Finance lease treatment is identical to IFRS
  - Operating lease treatment is the same as a finance lease, except the ROU asset is amortised by the decrease in lease liability in each period

$$\text{Lease payment} = \text{Total expense (Amortisation + Interest)} \tag{4.33}$$

    The ROU asset and liability are therefore the same at all points over the life of the asset

- Cash flow statement[3] for finance leases (and operating leases under IFRS)
  - Principal payments fall under CFF
  - Interest payments fall under CFO for US GAAP, and CFO or CFF under IFRS
- Cash flow statement for operating leases
  - The total payment is classified under CFO

EXAMPLE: Financing lease – Consider a company which leases a machine for 4 years. At the end of the contract, the machine is returned to the lessor. There are annual payments of $\$10,000$, and the implicit interest is 5% (used for ROU asset straight-line amortisation)

$$N = 4 \qquad I/Y = 5\% \qquad PV = \qquad PMT = 10,000 \qquad FV = 0$$

Then, using CPT PV, the calculator gives a PV of $35,460$ for the ROU asset, the same liability as at the start of the lease. The ROU asset is amortised across the four-year lifespan, giving $\frac{35,460}{4} = 8,865$ / year. Interest repayments fall under CFO for US GAAP, and CFO or CFF for IFRS.

	Beginning liability	Interest expense (5%)	Lease payment	Principal repayment (CFF)	Ending liability	BV of ROU asset
	(A)	(B)	(C)	(D)	(E)	
		$0.05 \times (A)$		$(C) - (B)$	$(A) - (D)$	
Y1	35,460	1,773	10,000	8,227	27,233	26,595
Y2	27,233	1,362	10,000	8,638	18,595	17,730
Y3	18,595	930	10,000	9,070	9,525	8,865
Y4	9,525	475	10,000	9,525	0	0

If we were to take the same example, but for an operating lease under US GAAP, the only thing that would change is that the book value of the ROU asset would match the ending liability for every year. With regards to cash flow, the full repayment would be attributed to CFO

[3] See table 4.2 for more information

	Finance lease		US GAAP Operating lease	
Balance sheet ROU asset	Lower	Amortisation > Decrease in liability	Higher	
Balance sheet liabilities	Same		Same	
Income statement earnings (early)	Lower	(10,638, Y1)	Higher	(10,000, Y1)
Income statement earnings (late)	Higher	(9,975, Y3)	Lower	(10,000, Y3)
EBIT	Higher	(only amortisation goes through)	Lower	(10,000)
Interest expense	Higher		Lower	(—, all is part of the lease expense)
Operating cash flow (CFO)	Higher	(Interest only so smaller outflow)	Lower	(Lease part)
Financing cash flow (CFF)	Lower	(Principal so larger outflow)	Higher	—

4.15.2 Lessee disclosures

- Disclose the carrying amount of ROU asset by class of underlying asset
- Cash out flows relating to lease
- Interest expense included in the income statement from the lease liability
- Depreciation of ROU (amortisation) by asset class
- Expenses related to variable lease payments not included in lease liabilities
- Additions to ROU assets
- Maturity analysis of lease liabilities and current / non-current split
- Expenses relating to low-value and short-term leases
- Quantitative and qualitative information regarding the nature of leasing activities, future cash out flows, restrictions and covenants, sale and leaseback

4.15.3 Lessor accounting

- From the lessor perspective, finance and operating lease classification is the same.
- For a finance lease, under IFRS and US GAAP:
  - Remove the leased asset from the balance sheet and replace it with a **lease receivable asset**

$$\text{Lease receivable asset} = \underbrace{\text{PV of lease payments}}_{\text{ROU asset+Lease liability}} \tag{4.34}$$

  - The book value of the lease receivable asset is recognised as a profit or loss
  - Interest component of the lease payment is recognised as interest income
  - The principal component of the lease payment reduces the value of the lease receivable asset.
  - The entire lease payment is a CFO inflow

  - If manufacturing / dealing the leased equipment is the main business operation, then this is treated as a sales-type lease.

$$\text{Sales proceeds} = \text{Revenue line}$$
$$\text{CV of asset} = \text{Cost of sale}$$

  - If the lessor is a financing company, it is treated as a direct financing lease
  - No gain / loss upon initiation of the lease is recognised on the income statement. The gain / loss is deferred and recognised over the life of the lease as an interest income / expense

- For an operating lease, under IFRS and US GAAP:
  - Leased asset remains on balance sheet
  - Lease payments are treated as income
  - Depreciation and other lease costs are expenses
  - Entire payment is a CFO inflow

EXAMPLE: Same as before (4.15.1). We are also told that the current carrying value in inventory of the asset is $30,000$, and that it has a residual value of $2,000$.

- The revenue recognised is the PV of the lease payments, which is $35,460$ as calculated before. This gives the **deemed proceeds** of the transaction
- The cost of sale is given by

$$\begin{aligned}\text{Cost of sale} &= \text{Carrying value} - \text{PV of residual value} \\ &= 30,000 - \underbrace{\frac{2,000}{(1+0.05)^4}}_{1,645} \\ &= 28,355\end{aligned}$$

The gross profit is therefore

$$\begin{aligned}\text{Gross profit} &= \text{Revenue} - \text{Cost of sale} \\ &= 35,460 - 28,355 \\ &= 7,105 \text{ gain}\end{aligned}$$

The asset is removed from the inventory, and the lease receivable is is defined as

$$\begin{aligned}\text{Lease receivable} &= \text{PV of lease payment} + \text{PV of residual / salvage value} \\ &= 35,460 + 1,645 \\ &= 37,105, \text{ “Net investment in lease”}\end{aligned}$$

	Beginning lease receivable	Interest income	Lease payment received	Principal repayment	Ending lease receivable
	(A)	(B) $(A) \times 0.05$	(C)	(D) $(C) - (D)$	(E) $(D) - (A)$
Y1	37,105	1,855	10,000	8,145	28,960
Y2	28,960	1,448	10,000	8,552	20,408
Y3	20,408	1,020	10,000	8,980	11,428
Y4	11,428	572	10,000	9,428	2,000

- If instead this were to be an operating lease, again using the same example

  The asset remains on the balance sheet (within the PP&E line item). The yearly depreciation is given by

$$\begin{aligned}\text{Annual depreciation} &= \frac{\text{Current CV in inv.} - \text{Residual value}}{\text{Length of lease}} \\ &= \frac{30,000 - 2,000}{4} \\ &= \frac{28,000}{4} = 7,000\end{aligned}$$

	Depreciation	Net PP&E	Lease revenue	Net I / S impact
Y1	7,000	23,000	10,000	3,000
Y2	7,000	16,000	10,000	3,000
Y3	7,000	9,000	10,000	3,000
Y4	7,000	2,000	10,000	3,000

4.15.4 Lessor disclosures

- Finance leases
  - Profit / loss is realised upon derecognition of asset
  - Finance income relates to the lease receivable asset
  - Income relates to the variable lease payments
  - Significant changes in the net investment of the lease
  - Maturity analysis of the lease payments received
  - Reconciliation of the un-discounted lease payments to net investment in the lease
- Operating leases
  - Lease income 00 split out for variable lease payments
  - Maturity analysis of lease payments receivable – minimum for each of the next 5 years, aggregated beyond
  - Underlying asset in balance sheet must comply with IAS 16, 36 disclosures

4.16 Deferred compensation and associated disclosures

- Deferred compensation can refer to pension schemes (both defined benefit and defined contribution) as well as share-based compensation

4.16.1 Defined contribution plan reporting

- Income statement
  - Pension expense, equal to the employer contribution
- Balance sheet
  - No future obligation to report as liability
  - If paid, decrease in cash. If not paid, increase in current liability

4.16.2 DC pension disclosures

- Annual employer contribution disclosed

4.16.3 Defined benefit plan reporting

- Balance sheet
  - Funded status is defined by whether

$$\text{Asset} > \text{Liability} \longrightarrow \text{Net pension asset}$$
$$\text{Asset} < \text{Liability} \longrightarrow \text{Net pension liability}$$

- The estimated plan liability is based on
  - Salaries
  - Employee turnover
  - Average age
  - Life expectancy
  - Discount rate
- A defined benefit plan under IFRS
  - A change in funded status on the income statement or other comprehensive income comprises
    1. Service cost (I / S) – PV of additional benefits (i.e. an extra year worked), including any changes tto past service costs under changes in plan terms
    2. Net interest expense / income (I / S) – Net pension asset / liability multiplied by the discount rate
    3. Remeasurements (OCI) – Actuarial gains . losses and differences (changes in estimates) between actual and expected return on plan assets
- A defined benefit plan under US GAAP
  - A change in funded status on the income statement or other comprehensive income comprises
    1. Service cost (I / S) – Current period
    2. Interest expense / income (I / S)
    3. Expected return on plan assets (I / S)
    4. Past service cost (OCI)
    5. Actuarial gains / losses (OCI)
- Manufacturing companies allocate a pension expense based on
  - Inventory and cost of goods sold for employees who provide direct labour to production
  - Salary / administrative expense for other employees.
  - Pension expense details are disclosed in the notes

4.16.4 DB pension disclosures

- IAS 19 objectives
  - Explain characteristics and risks
  - Identify amounts in financial statements
  - Describe how plan affects amounts, timing, and uncertainties relating to future cash flows
- Minimum required disclosures
  - Nature of plan, governance, regulatory framework, risk exposures
  - Reconciliation of beginning / ending value for funded status, PV of DBO and plan assets
  - Sensitivity analysis for key actuarial assumptions
  - Composition of plan assets by asset type
  - Expected employer contributions for next period and beyond
  - Maturity profile of DBO

4.16.5 Share-based compensation reporting

- The purpose of share-based compensation is to align interests of managers and shareholders (mitigating the principal-agent dilemma)
- Share-based compensation results in no cash out flows
- Share-based compensation dilutes the proportional ownership of existing shareholders, thereby reducing the EPS
- Share-base compensation reporting:
  - Estimate the fair value of share-based compensation at grant date
  - Expense this to the income statement over the vesting period
    1. Stock grants – Awarded outright, with restrictions, or contingent on performance

$$\text{Fair value} = \text{Share price on grant date}$$

    2. Performance shares – Dependent on meeting a set performance target (restricted stock units)
    3. Stock options – Option to invest in company's stock at exercise price at a future date. If the option exercised, company issues new shares. Option valuation models are used to compute the fair value
    4. Stock-based appreciation rights (SAR) – Generates cash flow for holders linked to stock performance. Payoffs are similar to stock options, and results in cash out flows for the company when the stock performs well. Non-exchange traded firms may use a version of this called phantom stock.

4.16.6 Share-based compensation disclosures

- Nature of plan, key details such as grant date, vesting date, service period, and settlement characteristics
- How fair value was determined
- Effect of share-based transactions on the income statement and balance sheet

4.17 Tax treatment

4.17.1 Tax return definitions

- Taxable income – Amount of income subject to taxes
- Taxes payable – Actual tax liability for the current period
- Income tax paid – Actual cash flow for taxes
- Tax loss carry-forward – Current net taxable loss available to reduce taxes in future years. May result in deferred tax assets
- Tax base – Net amount of asset / liability used for tax reporting

4.17.2 Financial reporting definitions

- Accounting profit – Pretax financial income, earnings before loss
- Income tax expense – Tax payable + Δ Deferred tax liability - Δ Deferred tax asset
- Deferred tax liability – Balance sheet item when taxes payable is less than the income tax expense due to temporary differences
- Deferred tax assets – Balance sheet item when taxes payable is greater than the income tax expense due to temporary differences
- Valuation allowance – Reserve against deferred tax assets that may not reverse in the future
- Carrying value – Balance sheet value of asset or liability

4.17.3 Difference between accounting profit and taxable income

Financial Accounting		Tax reporting	
Revenue	10,000	Revenue	10,000
Accrual-based costs	(5,000)	Tax allowable costs	(8,000)
Pre-tax income	5,000)	Taxable	2,000
Tax @ 30%	(1,500)	Tax payable @ 30%	(600)
	3,500		1,400

The difference between the accounting tax and cash-tax paid is given by

$$\underbrace{\text{Income tax expense}}_{\text{Accounting}} = \underbrace{\text{Taxes payable}}_{\text{Tax}} + \Delta\text{Deferred tax} \tag{4.35}$$

- Both DTL and DTA are presented on the balance sheet. Under IFRS, the DTL / DTA is always non-current. Under US GAAP, it is split into current and non-current
- Temporary (timing) differences
  - Differences between the balance sheet carrying value and the tax base of an asset can be temporary or permanent
    - Temporary – Same total passing through the I / S and tax return over time, but different individual periods
    - Permanent – Differences that will not reverse in the future

- Examples can include
  - Revenues and expenses recognised in different periods for accounts and tax
  - Difference in carrying value of asset an liability (i.e. depreciation methods)
  - Tax loss carry forward (DTA)
  - Gains an losses calculated differently for tax and financial statement

4.17.4 DTLs and DTAs

- For a DTL

  $$\text{Tax deduction} > \text{Accounting expense}$$

  Therefore taxable income is less than the pre-tax profit, and so the tax payable is less than the income tax expense

- A DTL arises if revenue is recognised in income statement before the tax return. Expenses are tax deductible before income statement recognition

- For a DTA

  $$\text{Tax deduction} < \text{Accounting expense}$$

  Therefore taxable income is greater than the pre-tax profit, and so the tax payable is greater than the income tax expense

- A DTA arises if revenue is recognised in income statement after the tax return. Therefore the expenses are tax deductible after the income statement recognition. A DTA also comes from post-employment benefits, unearned revenue, warranty expenses, and tax loss carry forwards

- Differences may also arise from a difference in depreciation method. For example, taxes use a double declining method, and accounting uses a straight line method

4.17.5 Taxable and deductible temporary differences

- If difference do not revert, then there is no deferred tax.
  - Tax exempt income, non-deductible expenses
  - Tax credits from some expenditures

  The result of this is that the effective tax rate is not equal to the statutory tax rate

Balance sheet	Carrying value vs Tax base	DTA / DTL
Asset	CV > TB	DTL
Asset	CV < TB	DTA
Liability	CV > TB	DTA
Liability	CV < TB	DTL

- Tax expense is the the income statement tax expense / provision

  $$\text{Tax expense} = \text{Tax payable} + \Delta\text{DTL} - \Delta\text{DTA} \tag{4.36}$$

  Changes in the tax rate can also impact the DTL / DTA. If the tax rate falls:

  - DTL ↓, therefore the income tax expense ↓

– DTA ↓, therefore the income tax expense ↑

according to equation 4.36

EXAMPLE: Assuming a statutory tax rate of 30%;

		Tax Return			Income Statement		
		Y1	Y2	Y3	Y1	Y2	Y3
	Revenue	100,000	120,000	130,000	95,000	114,000	123,500
–	Cost of sales	28,500	34,200	37,050	28,000	34,200	37,050
–	Other expenses	19,950	23,940	25,935	19,950	23,940	25,935
–	Depreciation	10,000	10,000	10,000	8,000	8,000	8,000
–	Warranty costs	2,000	5,000	8,000	0	0	0
–	Interest expense	10,000	12,000	12,500	10,000	12,000	12,500
	Taxable income / EBT	29,550	34,860	36,515	28,550	35,860	40,015
	Tax payable / Tax expense	8,865	10,458	10,955	8,565	10,758	12,005

Table 4.3: Example to show the difference between tax calculation of the tax return compared to the income statement

From table 4.3, we can see that the cost of sales, other expenses, and interest expense are all the same, so there is no need to look further at these.

- Depreciation
  - Assume acquisition of 40,000 PP&E at the start of Y1 with no residual value. Straight-line depreciation over 4 years for tax purposes and 3 years for the accounts. Eventually, 40,000 will go through both, but different amounts in each intervening year.
  - For tax purposes – Allowable depreciation > Income statement depreciation, so there is lower tax intially, and thus a DTL is observed

    One could also consider the tax base and carrying value rather than depreciation expenses

	Y1	Y2	Y3	Y4	Y5
Carrying value	32,000	24,000	16,000	8,000	0
Tax base	30,000	20,000	10,000	0	0
Timing difference	2,000	4,000	6,000	8,000	0
DTL at 30%	600	1,200	1,800	2,400	0
ΔDTL	+600	+600	+600	+600	−2,400

- Revenue and warranty cost differences
  - Income statement revenue – net of any returns / allowance, i.e. estimated warranty provisions, warranty liability shown on balance sheet
  - Tax return – Warranty costs can only be used to save tax when an actual expenditure is incurred

    Looking at the relevant lines from table 4.3,

Tax Return						
	Y1	Y2	Y3	Y1	Y2	Y3
Revenue	100,000	120,000	130,000	95,000	114,000	123,500
Warranty costs	2,000	5,000	8,000	0	0	0
	Actually incurred					

The liability for each of the years is given by the difference in tax return revenue and income statement revenue. So, for Y1, we have $100,000 - 95,000 = 5,000$ and so on. This gives us the warranty provision which can be compared to the warranty expenditure in order to find any DTA / DTL.

	Y1	Y2	Y3
Warranty provision	5,000	6,000	6,500
Warranty expenditure	2,000	5,000	8,000

In words, this gives

$$\begin{aligned}\text{Balance sheet warranty liability} =& \text{Original liability} \\ &+ \text{Increase in warranty provision from sales} \\ &- \text{Warranty expenditure}\end{aligned} \tag{4.37}$$

	Y1	Y2	Y3
Warranty liability	0	3,000	4,000
Warranty provision	5,000	6,000	6,500
Warranty expenditure	(2,000)	(5,000)	(8,000)
Warranty liability c/f	3,000	4,000	2,500

We can compare the carrying value of this to the tax base.

$$\begin{aligned}\text{Tax base of balance sheet liability} = \text{Carrying value} - \text{Amount of liability...} \\ \text{...that will pass through future returns}\end{aligned} \tag{4.38}$$

	Y1	Y2	Y3
Carrying value	3,000	4,000	2,500
Tax base	0	0	0
Timing difference	3,000	4,000	2,500
DTA (30%)	900	1,200	750
ΔDTA	+900	+300	−450

Combining the treatment of the depreciation and warranty cost, we can now convert from tax payable to tax expense using equation 4.36

		Y1	Y2	Y3
	Tax payable	8,865	10,458	10,955
+	Δ DTL	600	600	600
−	Δ DTA	(900)	(300)	450
	Tax expense	8,565	10,758	12,005

4.17.6 Realizability of DTLs / DTAs

- A valuation allowance reduces a DTA. This is based on the likelihood of realisation.

IFRS Only show net figure

US GAAP Full DTA shown, offset by valuation allowance

Increasing the valuation allowance decreases the income (VA↑, DTA↓, Tax expense↑, Income↓)

- A DTL should be treated as a liability if expected to reverse. If it is not expected to reverse, reduce the DTL and increase the equity.

4.17.7 Tax rate reconciliation

- Required deferred tax disclosures
  - DTL / DTA – any valuation allowance and net change in valuation allowance
  - Unrecognised deferred tax liability of undistributed earnings of subsidiaries and joint ventures
  - Current-year tax effect of each type of temporary difference
  - Components of income tax expense
  - Tax loss carry-forwards and credits
  - Reconciliation between effective and statutory tax rate
- Analysis
  - Be aware of differences in the reconciliation
  - Cumulative differences from impairments, post-retirement benefits
  - Restructuring charges may create a DTA
  - Unrealised gains are DTL – they are not taxed until realised
  - Decreasing valuation allowance on a DTA is a good thing, as it suggests future taxable income will be higher

EXAMPLE:

	Y1	Y2	Y3
Statutory rate	35%	35%	35%
State income taxes	2.1%	2.2%	2.3%
Benefits and foreign operations	(6.5%)	(6.3%)	(2.7%)
Tax rate changes	0.0%	0.0%	0.0%
Capital gains on asset sales	0.0%	(3.0%)	0.0%
Special items	(1.6%)	8.7%	2.5%
Other, net	0.8%	0.7%	(1.4%)
Effective tax rate	29.8%	37.3%	33.7%

Table 4.4: An example of what tax reconciliation may show

4.18 Reporting quality

- The quality of financial reporting is high if
  - Reporting is compliant with IFRS / US GAAP
  - Information is relevant, neutral, complete, free from errors, and decision useful
  - Statements accurately represent the economic reality of activity of the business
- Earnings quality is high if
  - Earnings are sustainable
  - Earnings provide adequate return to investors
- Biased accounting choices may be
  - Aggressive choices – Increase current period earnings, financial position
  - Conservative choices – Decrease current period earnings, financial position

  Conservative bias may also result from accounting standards themselves
- Management may smooth earnings by making conservative choices when earnings are high and aggressive choices when earnings are low. This introduces bias through the focus of reports
- Conditions for low quality reporting
  1. Motivation
     - Meet / exceed benchmark EPS
     - Increase compensation, reputation
     - Drive up stock price
     - Avoid violation of debt covenants (highly levered, unprofitability)
     - Improve view of companies from investors, analysts, customers
  2. Opportunity
     - Weak internal controls
     - Inadequate board oversight
     - Range of acceptable treatments in GAAP
     - Minimal consequences
  3. Rationalisation
- Mechanisms to monitor quality include
  - Government regulation
    - Security registration
    - Audits, disclosure requirements
    - Management responsibility
    - Enforcement
  - Auditors
    - Opinion on financial reporting
    - US only – assessment of internal controls
  - Private contracts may have loan covenants, specific methods to calculate accounting measures, financial figures for return on investment

- Non-GAAP presentation
  - Companies may present pro-forma accounts to influence expectation

  IFRS Non-IFRS measures must be defined, explained, and reconciled

  US GAAP Non-GAAP measures may not be displayed more prominently. GAAP compliant measures should still be disclosed, alongside necessary explanation and reconciliation

4.18.1 Accounting choices and estimates

- Revenue recognition choices
  - Shipping terms – recognise at shipping point or destination
  - Discounts to increase orders in the current period
  - Delay shipments to defer revenue to a later period
  - Increase shipments to distributors
  - Bill and hold transactions – recognise revenues for goods not yet shipped
- Management of accruals
  - Allowance for bad deb, warranty expense
- Depreciation method
  - Straight-line vs accelerated
- Depreciation estimates
  - Economic life, salvage value
- Valuation allowance
  - Contra account to DTA
- Inventory cost flow assumptions
  - Prices ↑, so FIFO COGS < WAC COGS < $\underbrace{\text{LIFO COGS}}_{\text{Only for US GAAP}}$
- Capitalisation vs expensing
  - Defers expense to future periods
- Impairments
  - Delaying recognition of impairment chargees
- Related party transactions
  - Can move earnings in / out of firm
- Managing operating cash flow
  - Capitalisation – outflow through CFI
  - Expensing – outflow through CFO
  - Stretching payables – CFO ↑
  - Capitalising construction interest, then depreciate going forwards
  - Under IFRS
    - Interest / dividends <u>paid</u> ⇒ CFF / CFO
    - Interest / dividends <u>received</u> ⇒ CFI / CFO

4.19 Warning signs

- Indicate if more analysis required, determine if there is a business purpose, or if statements are being manipulated
- If multiple warning signs are present, do not invest
- Revenue recognition warnings
  - Growth not in line with peers
  - Change in revenue recognition method
  - Bill and hold transactions item Changes in rebate estimates
  - Receivables turnover $\left(\frac{\text{Revenue}}{\text{Avg. receivables}}\right)$, total asset turnover$\left(\frac{\text{Revenue}}{\text{Total avg. assets}}\right)$ decreasing over time
  - Non-operating or one-time items included in revenue
- Inventory warning signs
  - Inventory turnover ratio $\left(\frac{\text{COGS}}{\text{Avg. inventory}}\right)$ declining over time
  - Decrease in inventory units under LIFO – Results in unsustainably low COGS
- Capitalisation and cash flow warning signs
  - Capitalisation of costs not capitalised by peers
  - Ratio of CFO : Net income is less than 1, or declining over time
- Other signs
  - Depreciation methods, useful lives, salvage values out of line with peers
  - Fourth quarter earning surprises
  - Significant related-party transactions
  - Recurring "Non-recurring" expenses
  - Lack of transparency / disclosure
  - Any emphasis on non-GAAP earning figures
  - Numerous acquisitions – Fair value of net assets is subjective

4.20 Financial ratios

- Use of ratios
  - Project future earnings / cash flows
  - Evaluate firm's flexibility
  - Assess management's performance
  - Evaluate changes in firm and / or industry
  - Compare firm with competition
  - Cross-sectional analysis
  - Time-series analysis
- Limitations
  - Should not be used in isolation
  - Different accounting treatments may have been used

  - Difficult for multi-industry companies
  - Target / comparable ratios difficult to find

- Vertical common-size statements

$$\frac{\text{Income statement account}}{\text{Sales}}$$
$$\frac{\text{Balance sheet account}}{\text{Total assets}}$$

- Horizontal common-size statements
  - Each line shown relative to some baseline
- Graphs
  - Facilitate comparisons over time
  - Communicate conclusions
- Categories of ratios
  - Activity – Efficiency of operations
  - Liquidity – Ability to meet short-term obligations
  - Solvency – Ability to meet long-term obligations
  - Profitability – Ability to generate a profit from sales
- If using items from only one of the income statement or balance sheet, use values from the current income statement or balance sheet as relevant.
- If using a combination, use values from the current income statement, and average value of the balance sheet item $\left(\frac{\text{Beginning+Ending}}{2}\right)$

4.20.1 Activity ratios

$$\text{Receivables turnover} = \frac{\text{Revenue}}{\text{Avg. receivables}}$$

$$\text{Days of sales outstanding (DSO)} = \frac{365}{\text{Receivables turnover}}$$

- Days taken to pay by customers – compare to the credit terms extended by suppliers

$$\text{Inventory turnover} = \frac{\text{COGS}}{\text{Avg. inventory}}$$

- High inventory turnover can imply effective management, or that the company does not hold enough inventory

$$\text{Days of inventory on hand (DOH)} = \frac{365}{\text{Inventory turnover}}$$

$$\text{Payables turnover} = \frac{\text{COGS}}{\text{Avg. trade payables}}$$

- High payables turnover implies the amount owed to suppliers is relatively low. This could be due to the company no taking advantage of credit terms, or from the company benefiting from a prompt payment discount

$$\text{Number of days of payables} = \frac{365}{\text{Payables turnover}}$$

- Days taken to pay suppliers. If low, it could imply a short-term cash flow issue, or also a prompt payment discount

$$\text{Fixed asset turnover} = \frac{\text{Revenue}}{\text{Avg. net fixed assets*}}$$

*Net of accumulated depreciation

- Gives an indication of how well assets generate revenue

$$\text{Total asset turnover} = \frac{\text{Revenue}}{\text{Avg total assets}}$$

- Low turnover implies an inefficient use of assets, but this is also impacted by the age of assets

$$\text{Working capital turnover} = \frac{\text{Revenue}}{\text{Average working capital}}$$

- Where average working capital is given by current assets less current liabilities

4.20.2 Liquidity ratios

$$\text{Current ratio} = \frac{\text{Current assets}}{\text{Current liabilities}}$$

$$\begin{aligned}\text{Quick ratio} &= \frac{\text{Current assets} - \text{Inventory}}{\text{Current liabilities}} \\ &= \frac{\text{Cash} + \text{Receivables} + \text{Short-term marketable securities}}{\text{Current liabilities}}\end{aligned}$$

$$\text{Cash ratio} = \frac{\text{Cash} + \text{Short-term marketable securities}}{\text{Current liabilities}}$$

- For all of these, a higher measure is better

$$\text{Defensive interval} = \frac{\text{Cash} + \text{Receivables} + \text{Short-term marketable securities}}{\text{Daily cash expenditure*}}$$

*This covers goods, SG&A, R&D, etc.

- This metric defines the number of days of spending covered by liquid assets

$$\begin{aligned}\text{Cash conversion cycle (CCC)} = {} & \text{Days of inventory on hand} \\ & + \text{Days sales outstanding} - \text{Days payables outstanding}\end{aligned}$$

- A low cash conversion cycle is better

4.20.3 Solvency ratios

$$\text{Debt-to-assets ratio} = \frac{\text{Total debt}}{\text{Total assets}}$$

$$\text{Debt-to-capital ratio} = \frac{\text{Total debt}}{\underbrace{\text{Total debt} + \text{Total equity}}_{\text{Capital structure}}}$$

$$\text{Debt-to-equity ratio} = \frac{\text{Total debt}}{\text{Total shareholder equity}}$$

$$\text{Financial leverage ratio} = \frac{\text{Avg. total assets}}{\text{Avg. total equity}} = \frac{A}{A - L}$$

$$\text{Interest coverage} = \frac{\text{EBIT}}{\text{Interest payments}}$$

$$\text{Debt-to-EBITDA} = \frac{\text{Total debt}}{\text{EBITDA}}$$

- Indicates the number of years required to pay off debt

$$\text{Fixed charge coverage} = \frac{\text{EBIT} + \text{Lease payments}}{\text{Interest payments} + \text{Lease payments}}$$

4.20.4 Profitability ratios

$$\begin{aligned}\text{Gross profit margin} &= \frac{\text{Gross profit}}{\text{Revenue}} \\ &= \frac{\text{Revenue} - \text{COGS}}{\text{Revenue}}\end{aligned}$$

$$\begin{aligned}\text{Operating profit margin} &= \frac{\text{Operating income}}{\text{Revenue}} \\ &= \frac{\text{Gross profit} - \text{Operating costs}}{\text{Revenue}} \\ &\approx \frac{\text{EBIT}}{\text{Revenue}}\end{aligned}$$

- EBIT also contains some non-operating items, such as dividends and capital gain / loss

$$\text{Pretax margin} = \frac{\text{EBT (after interest}}{\text{Revenue}}$$

$$\text{Net income margin} = \frac{\text{Net income}}{\text{Revenue}}$$

$$\text{Return on assets (ROA)} = \frac{\text{Net income}}{\text{Avg. total assets}}$$

$$\underbrace{\text{Return on assets}}_{\text{Alternative}} = \frac{\text{Net income} + (\text{Interest expense})(1 - \text{tax rate})}{\text{Avg. total assets}}$$

$$\text{Operating ROA} = \frac{\text{Operating income}}{\text{Avg. total assets}}$$

$$\text{Return on total capital} = \frac{\text{EBIT}}{\text{Shot + long-term debt and equity}}$$

$$\text{Return on invested capital} = \frac{\text{After-tax operating profit}}{\text{Avg. invested capital}}$$

- After tax operating profit = Net income + Post-tax income expense
- Avg. invested capital = Avg. book value of equity and debt

$$\text{Return on equity (ROE)} = \frac{\text{Net income}}{\text{Avg. total equity}}$$

$$\text{Return on common equity} = \frac{\text{Net income} - \text{Preferred dividend}}{\text{Avg. common equity}}$$

4.20.5 Industry specific financial ratios

Services / consulting	Sales per employee
Retail	Growth in same-store sales Sales per square foot
Hotel industry	Average daily rate Occupancy rate
Subscription services	Average revenue per user
Financial services	Capital adequacy Value at risk (VaR) Reserve requirements Liquid asset requirement $\text{Net interest margin} = \frac{\text{Interest income}}{\text{Interest earning assets}}$

4.20.6 Examples

EXAMPLE:

Balance sheet	**T**	**T-1**
Cash + equivalents	105	95
Accounts receivable	205	195
Inventories	310	290
Current assets	620	580
Gross PP&E	1,800	1,700
Accumulated depreciation	360	340
Net PP&E	1,440	1,360
Non-current assets	1,440	1,360
Total assets	2,060	1,940
Accounts payable	110	90
Short-term debt	160	140
Current potion of long-term debt	55	45
Current liabilities	325	275
Long-term debt	610	690
Deferred tax	105	95
Common stock	700	700
Retained earnings	320	180
Total equity	1,020	880
Total liabilities and equity	2,060	1,940

Current year I / S	**T**
Sales revenue	4,000
Cost of goods sold	3,000
Gross profit	1,000
Operating expenses	650
Operating profit	350
Interest expense	50
Pretax income	300
Taxes	100
Net income	200

- We can calculate the current ratio

$$\text{Current ratio} = \frac{\text{CA}}{\text{CL}} = \frac{620}{325} = 1.9$$

- We can calculate the total asset turnover as

$$\text{Total asset turnover} = \frac{\text{Revenue}}{\text{Avg. total assets}} = \frac{4,000}{\frac{(2,060+1,940)}{2}} = 2.0$$

- The net profit margin is

$$\text{Net profit margin} = \frac{\text{Net income}}{\text{Revenue}} = \frac{200}{4,000} = 5\%$$

- The return on common equity is

$$\begin{aligned}\text{Return on common equity} &= \frac{\text{Net income} - \text{Preference dividends}}{\text{Avg. common equity}} \\ &= \frac{200 - 0}{\frac{(1,020+880)}{2}} = 21.1\%\end{aligned}$$

- The debt-to-equity ratio is

$$\begin{aligned}\text{Debt-to-equity ratio} &= \frac{\text{Total debt}}{\text{Total shareholder equity}} \\ &= \frac{160 + 55 + 610}{1,020} = 80.9\%\end{aligned}$$

EXAMPLE: What conclusions can be drawn from the following?

	T	T-1	T-2
Current ratio	2.0	1.5	1.2
Quick ratio	0.5	0.8	1.0
DOH	60	50	30
DSO	20	30	40

- Over time, we can see that the current ratio is rising while the quick ratio is falling. This implies either inventory is rising, or other current assets are falling in aggregate.
- When looking at days of inventory on hand, we see that this is rising. This tells us that inventory is rising, rather than low cash.
- Looking at days sales outstanding, this tells us that cash is being collected faster
- A logical conclusion from this is that inventory is accumulating, and collections are accelerated in order to make up for poor inventory management

EXAMPLE: What conclusions can be drawn from the following?

	T	T-1	Industry avg.
Current ratio	1.9	2.1	1.5
Total asset turnover	2.0	2.3	2.4
Net profit margin	5.0%	5.8%	6.5%
Return on equity	21.1%	24.1%	19.8%
Debt-to-equity	80.9%	99.4%	35.7%

- Conclusions:
  - Liquidity – Higher than average, but lower than last year
  - Activity – Lower than last year average
  - Profitability – Below industry average
  - Solvency – More leverage than average

4.21 Dupont analysis

- Dupont analysis is a way of splitting up the return on equity calculation into smaller pieces

$$\text{Return on equity} = \underbrace{\frac{\text{Net income}}{\text{Equity}}}_{\substack{\frac{\text{Net income}}{\text{Total assets}} \times \frac{\text{Total assets}}{\text{Equity}} \\ \text{ROA} \quad \text{Financial leverage ratio} \\ \frac{\text{Net income}}{\text{Revenue}} \times \frac{\text{Revenue}}{\text{Total assets}} \\ \text{Net profit margin} \quad \text{Asset turnover}}} \tag{4.39}$$

In its constituent components, the three-part Dupont analysis looks like this:

$$\text{ROE} = \underbrace{\frac{\text{Net income}}{\text{Revenue}}}_{\text{Net profit margin}} \times \underbrace{\frac{\text{Revenue}}{\text{Total assets}}}_{\text{Asset turnover}} \times \underbrace{\frac{\text{Total assets}}{\text{Equity}}}_{\text{Leverage}}$$

EXAMPLE: What conclusions can be drawn from the following?

	T	T-1	T-2	
ROE	17.4%	18.0%	18.1%	
Net profit margin	5.34%	6.37%	7.05%	↓
Asset turnover	1.171	1.207	1.326	↓
Leverage	2.778	2.339	1.933	↑

- Increasing leverage offsets decline in margin and asset turnover

4.21.1 Dupont system extended (5 part)

- The return on equity calculation can be split up further as follows

$$\text{ROE} = \underbrace{\frac{\text{Net income}}{\text{EBT}}}_{\text{Tax burden}} \times \underbrace{\frac{\text{EBT}}{\text{EBIT}}}_{\text{Interest burden}} \times \underbrace{\frac{\text{EBIT}}{\text{Revenue}}}_{\text{EBIT margin*}} \times \frac{\text{Revenue}}{\text{Total assets}} \times \frac{\text{Total assets}}{\text{Equity}} \tag{4.40}$$

EXAMPLE: What conclusions can be drawn from the following?

	Company A	Company B
Revenue	500	900
EBIT	35	100
Interest	5	0
EBT	30	100
Taxes	10	40
Net income	20	60
Avg. assets	250	300
Avg. equity	150	250
ROE	13.3%	24.0%
Tax burden	0.667	0.600
Interest burden	0.857	1.000
EBIT margin	0.070	0.111
Asset turnover	2.000	3.000
Leverage	1.667	1.200

- From this, and using the Dupont analysis breakdown in the lower half of the table, we can see that Company A hs a higher tax burden, higher interest burden, lower asset turnover, and lower EBIT margin, but is more leveraged than Company B

4.22 Financial statement modelling

- Business risk
  - Use coefficient of variation for size-adjusted measures. This will aid analysis of assessing relative and absolute degrees of risk faced by a firm

$$\text{CV Sales} = \frac{\text{Std. dev. sales}}{\text{Mean sales}} \tag{4.41}$$

$$\text{CV Operating income} = \frac{\text{Std. dev. operating income}}{\text{Mean operating income}} \tag{4.42}$$

$$\text{CV Net income} = \frac{\text{Std. dev. net income}}{\text{Mean net income}} \tag{4.43}$$

- Model building

  - Common size statements and ratios can be used to model / forecast results
    - Expected relationships
    - Earnings model
    - Revenue-driven models
  - Sensitivity of analysis
  - Scenario analysis
  - Simulation

- Financial statement modelling for pro-forma financial statements
  - Estimate sales and COGS
  - Estimate SG&A and financing costs
  - Estimate tax expense and cash taxes
  - Estimate balance sheet items that flow from income statement
  - Use depreciation and CapEx to estimate capital expenditure and net PP&E on balance sheet
  - Form pro-forma balance sheet and income statement
  - Prepare cash flow statement

- Biases in forecasting

Overconfidence	Faith in estimates	Scenario analysis, critique
	Underestimating error	Past success in forecasting
Illusion of control	Using complex models'	Use only variables with known forecasting power
	"Expert opinion"	Relevance of opinion
Conservatism (anchoring)	Resistance to incorporating new information	Use simple models
		Audit of forecasting errors
Representativeness bias	Classifying by past known classifications	Consider inside / outside views
	Base rate prioritises generic information over specifics	
Confirmation bias	Looks for agreeable opinions	Look for diverse opinions

4.23 Porter's five force analysis

- Porter's five forces are
  1. Threat of substitute products
     - If threat high, low pricing power
  2. Intensity of rivalry
     - If competition high, pricing power is low.
     - Pricing power low when
       - Industry concentration low

    - Fixed cost, exit barriers high
    - Industry growth slow
    - Products not differentiated
3. Bargaining power of suppliers
    - If supplier bargaining power high, prospect for earning growth lower
4. Bargaining power of consumers
    - If consumer bargaining power high, pricing power low
5. Threat of new entrants
    - If threat high, low pricing power, low prospect for earnings growth

4.24 Input cost price inflation

- Firms with commodity-type inputs may hedge their exposure
- Vertically-integrated firms are less affected firms are less affected by input cost price inflation
- Analyst determines whether price increase passed on to customer
- Effect of price increase depends on elasticity of demand and actions of rivals
- Forecast horizon
    - May be based on expected holding period
    - Must include mid-cycle for cyclical firms
    - If recent material events (acquisitions, merger, restructuring) have occurred, then the horizon should allow for the proper manifestation of these events
    - Methods for valuation could include
        - Multiples approach
        - DCF

EXAMPLE:

[$ terms]		T
Revenue	1000 @ $100	100,000
COGS	1000 @ $40	40,000
Gross profit		60,000
SG&A		30,000
Operating profit		30,000
Gross profit margin		60.0%
Operating profit margin		30.0%

- Now we can consider the following scenarios

Price rise of 5%, entire rise passed on to customer, no change in units sold

[$ terms]		T
Revenue	1000 @ $105	105,000
COGS	1000 @ $45	45,000
Gross profit		60,000
SG&A		30,000
Operating profit		30,000
Gross profit margin		57.1%
Operating profit margin		28.6%

Price rise of 5%, entire rise passed on to customer, units sold decreases by 5%

[$ terms]		T
Revenue	950 @ $105	99,750
COGS	950 @ $45	42,750
Gross profit		57,000
SG&A		30,000
Operating profit		27,000
Gross profit margin		57.1%
Operating profit margin		27.1%

Price rise of 5%, entire rise passed on to customer, units sold decreases by 10%

[$ terms]		T
Revenue	900 @ $105	94,500
COGS	900 @ $45	40,500
Gross profit		54,000
SG&A		30,000
Operating profit		24,000
Gross profit margin		57.1%
Operating profit margin		25.4%

5 Equity

5.1 Markets, assets and intermediaries

- Financial system functions
  - Allow entyities to borrow, save, issue equity, manage risk, exchange assets and use information
  - Determine the returns that equate savings and borrowin
  - Allocate capital efficiently
- An investor expects to earn the equilibrium (fair) return over time.
- An information trader expects to earn positive risk-adjusted retunr (i.e. active management)
- A hedger takes on a position to offset existing risk
- Classification of assets
  - Financial vs. real assets
  - Debt vs equity securities
  - Public vs private securities
  - Physical derivatives vs financial derivatives (refers to the type of underlying security / asset behind the contract)
- Classification of markets
  - Spot markets vs futures markets
  - Primary vs secondary markets
  - Call markets (trading at specific times) vs continuous markets
  - Money markets (debt $<$ 1 year) vs capital markets)
  - Traditional markets (debt / equity) vs alternative investment markets (real estate / commodity)
- Types of assets
  - Equities
  - Fixed income
  - Commodities
  - Real assets
  - Currencies
  - Pooled investments
    - ETFs
    - Mutual funds
    - Asset backed securities
    - Hedge funds
  - Contracts
    - Forwards
    - Futures
    - Swaps
    - Options
    - Insurance
- Financial intermediary roles
  - Brokers / exchangers
    - Connect buyers and sellers
  - Dealers
    - Hold inventory, match buyers / sellers at different points in time
  - Arbitrageurs
    - Transact in same security at the same time at different prices
  - Securitisers, depository institutions

    - Sell interests in a diversified pool of assets
  - Insurance companies
    - Manage a diversified pool of risks
  - Clearing houses
    - Reduce counterparty risk and promote market integrity

5.1.1 Positions and leverage

- An investor may enter into a long position by
  - Purshasing stock
  - Buying a call option
  - Selling a put option
  - Taking a long position in a future / forward contract
- An investor may enter into a short position by
  - Selling short stock
  - Selling a call option
  - Buying a put option
  - Taking a short position in a future / forward contract
- An investor may enter into a levered position by
  - Borrowing part of the purchase price, or posting less than the asset value through the use of futures

5.1.2 Short selling

- Selling short involves two steps
  1. Borrow stock and sell
  2. Later, repurchase the stock and return to the original lender

$$\text{Profit} = \text{Selling price} - \text{Repurchase price} - \text{Interest / Commission} \tag{5.1}$$

Short seller must pay all dividend to the lender of the security. A short seller is also required to deposit margin / collateral. The objective of a short seller is to capitalise on the fall of an asset price.

5.1.3 Buying stock on margin

- Borrowing a portion of the purchase price
- Broker holds stock as collateral

$$\text{Leverage ratio} = \frac{1}{\text{Initial margin}}, \tag{5.2}$$

$$\text{Initial margin requirement} = \text{Minimum equity percentage } \underline{\text{at time of}} \text{ purchase}, \tag{5.3}$$

$$\text{The maintenance margin} = \text{Minimum equity percentage } \underline{\text{after}} \text{ purchase}, \tag{5.4}$$

$$\text{Equity percentage} = \frac{\text{Stock value} - \text{Loan}}{\text{Stock value}}. \tag{5.5}$$

EXAMPLE:

Suppose an investor buys 1,000 shares with 40% margin at \$100 per share. The margin loan incurs interest of 4% per year. The stock pays an annbual dividend of \$2 per share, and there is a commission of \$0.05 charged per share on purchase / sale. If the investor sells the stock one year later at a price of \$110, calculate the leverage ratio and return on margin position.

SOLUTION:

The leverage ratio is calculated as

$$\text{Leverage ratio} = \frac{1}{\text{Initial margin}} = \frac{1}{0.4} = 2.5,$$

so the leverage ratio is 2.5. We can see trivially that the return on the stock is 10%, so with this leveraged position, the return on equity investment is $2.5 \times 10\% = 25\%$.

Now thinking about the return on margin position,

$$\begin{aligned}
\text{Investor equity} &= 0.4 \times 1,000 \times \$100 = \$40,000,\\
\text{Commission on purchase} &= 1,000 \times \$0.05 = \$50,\\
\text{Cash investment} &= \$40,050,
\end{aligned}$$

$$\begin{aligned}
\text{Dividend} &= 1,000 \times \$2 = \$2,000,\\
\text{Interest on loan} &= 0.04 \times \$60,000 = \$2,400,\\
\text{Sale proceeds} &= 1,000 \times \$110 = \$110,000,\\
\text{Commission on sale} &= 1,000 \times \$0.05 = \$50,
\end{aligned}$$

$$\begin{aligned}
\text{Net proceeds} &= \$110,000 + \underbrace{\$2,000}_{\text{Dividend}} - \underbrace{\$60,000}_{\text{Principal}} - \underbrace{\$2,400}_{\text{Interest}} - \$50,\\
&= \$49,550,
\end{aligned}$$

$$\text{Return on margin position} = \frac{\$49,550}{\$40,000} - 1 = 23.72\%.$$

- Margin call – if the value of an investor's equity in a position falls below the maintenance margin, the investor must either deposit cash or marginable securities, or close out the position.

$$\text{Trigger price} = P_0 \times \frac{1 - \text{Initial margin}}{1 - \text{Maintenance margin}} \tag{5.6}$$

$$\text{Maintenance margin} < \text{Initial margin}$$

5.1.4 Order execution and validity

- Trading instructions
  - Execution – How to trade
  - Market order
    - Immediate execution at the best available price

    - Useful when trading based on information
    - Can execute at an unfavourable price
  - Limit order
    - Buy at limit price or lower, sell at limit price or higher
    - Avoid price execution uncertainty
    - Order may not end up being filled
  - Stop order
    - Trade at a trigger price which activates a market order

      A trader who owns a stock trading at $55.00 may enter a stop sell order at $51.50

      A trader who is short a stock trading at $55.00 may enter a stop buy order at $58.50

      A technician who believes a stock price reaching $55.00 is indicative of a further upward move may enter a stop buy order at $55.00

- Order execution
  - Validity – When to trade
    - Good until cancelled
    - Immediate or cancelled
    - Day order
  - Bid-ask spread: A broker / dealer buys at bid price and sells at the ask price
    - Bid-ask spread ≡ Dealer profit
    - Best bid = Highest bid
    - Best ask = Lowest ask
    - Best bid / ask = "Make the market"

5.1.5 Primary and secondary capital markets

- Primary markets involve the sale of newly-issued stocks and bonds. This includes
  - Underwritten offer – Investment bank guarantees security sale
  - Best efforts – Investment bank acts as broker
  - Private placement – Sell directly to qualified investors
  - Shelf registration – Issue securities over time
  - Dividend reinvestment plan – Issue new shares to investors who reinvest dividends
  - Rights offering – Sell new shares to existing shareholders
- Secondary markets are how a security trades after its initial offering. Secondary markets provide liquidity and information about value to investors

5.1.6 Market structures

- Markets may be
  - Quote-driven – Investors trade with dealers, who act as market makers for less-liquid securities
  - Order-driven – A set of rules is used to match buyers and sellers, for example a price-time hierarchy
  - Brokered market – Brokers find a counterparty for trades
- Call vs continuous markets

  - Call markets are those where securities trade at specific times. All bids / asks are accumulated, then a price is set which clears the market. This is used in smaller markets and to open major markets.
  - Continuous markets are those where trades may occur at any time during market hours. Price is set by auction or by dealer bid-ask spreads.

- A well-functioning financial system is on which is
  - Complete – Assets and contracts are available
  - Operationally efficient – Low transaction costs
  - Informationally efficient – Prices reflect fundamental / intrinsic value
  - Financial intermediaries facilitate transactions

- Objectives of market regulation
  - Protect unsophisticated investors
  - Require minimum standards of competency
  - Prevent insider trading
  - Require common financial reporting standards
  - Require minimum levels of capital

5.2 Indices

- Security market indices represent the value / performance of an asseet class, security market, or market segment over time. The calculated price is based on the underlying constituents that make up an index.

- Indices may be a price-return or total return index

$$\text{Price return} = \frac{\text{End price} - \text{Beginning price}}{\text{Beginning price}} \tag{5.7}$$

$$\text{Total return} = \frac{\text{End price} + \overbrace{\text{Cash flows}}^{\text{Dividend / coupon}} - \text{Beginning price}}{\text{Beginning price}} \tag{5.8}$$

- Index construction should consider the following carefully:
  - What markets does the index represent
  - Which securities should be included
  - What weighting method should be used
  - Rebalancing frequency / rules

5.2.1 Index weighting methods

- Price-weighted index is defined as

$$\text{Index price} = \frac{\sum \text{Stock prices}}{\#\text{ stocks in the index, adjusted for splits}}. \tag{5.9}$$

  In order to match the performance of the index, one should buy an equal number of shares in the index

  A % change in a highly-priced stock has the largest impact on the index price.

The DJIA and Nikkei are examples of indices which are price-weighted

- A market-cap weighted index is defined as one where the price of each stock is weighted by the market-capitalisation of that firm, where market-cap is defined

$$\text{Market-capitalisation} = \text{Number of shares} \times \text{Share price}. \tag{5.10}$$

Firms with larger market-caps have greater influence over the price of the index

The S&P 500, and FTSE 100 are examples of indices which are market-cap weighted

- A market-float weighted index is defined as one where the number of shares is equal to the investable shares. In other words, this excludes shares held by controlling investors and those held by governments / corporations. A free-float index is used when shares not available to foreign investors are excluded

- An equal-weighted index is defined such that the same weight is given to the performance of each stock. In order to match the return of the index, one should make an equal investment as a dollar-amount in each stock. The index return is equivalent to the average holding period return on each of the underlying constituents

EXAMPLE: Consider an index with three constituents. Before any split, the stock prices are

Stock	Price
A	\$10
B	\$20
C	\$90

If stock C then splits 2-for-1, the price after the split is $\frac{\$90}{2} = \$45$. Calculate the new divisor of the index. The original index price is calculated using equation 5.9 –

$$\text{Price} = \frac{\$10 + \$20 + \$40}{3} = \$40. \tag{5.11}$$

After the split, the index price should remain unchanged.

$$\begin{aligned} \$40 &= \frac{\$10 + \$20 + \$45}{x} \\ x &= \frac{\$40}{\$10 + \$20 + \$45} \\ x &= 1.875 \end{aligned}$$

5.2.2 Comparison of index weighting schemes

- A price-weighted index places more weight on highly-priced stocks. Stock splits change all weights (and the divisor)
- An equal-weighted index places more weight on small-cap stocks and less on large cap. Thus, a portfolio tracking that index requires more rebalancing to ensure it stays tracking the index
- A float-adjusted index more closely matches the investable shares proportion
- A fundamental-weighted index has a value tilt, with a weighting scheme constructed according to that fundamental value

5.2.3 Uses and types of indices

- An index is used for the following
  - Reflection of market sentiment
  - Performance benchmark
  - Measure of market return
  - Calculate beta
  - Calculate expected and risk-adjusted returns
  - Model portfolio for index funds
- Equity indices
  - Broad market (various weighting schemes)
  - Multi-market (market-cap weighted)
  - Style (Large / mid / small cap, value vs growth)
  - Sector
- Fixed income indices
  - Index construction rules may take into account
    - Maturity
    - Country
    - Sector
    - Issuer type
    - Region
    - Collateral
  - Construction issues include
    - High turnover
    - Lack of price data
    - Illiquidity
- Alternative asset indices
  - Commodity indices
    - Index of futures contracts, so performance may differ from that of the underlying commodity ("basis risk")
    - Wide variety of commodity weighting schemes
  - Real estate indices
    - Appraisals, repeat sales, REITs
  - Hedge fund indices
    - Self-selection and survivorship bias artificially increase returns of an index, relative to the industry

Index	Constituents	Number of Constituents	Weighting	Other notes
DJIA	Large US stocks	30 constituents	Price-weighted	Chosen by WSJ editors
Nikkei	Large JPY stocks	225 constituents	Modified-price weighting	Adjusted for high-priced shares
TOPIX	All Tokyo stock exchange listings	Variable	Market-cap, adjusted for float	Contains 93% of the Jpy market, including many small / illiquid stocks
MSCI All Country World Index	23 developed and 24 emerging markets	Variable	Market-cap, float-adjusted	

5.3 Market efficiency

- Market efficiency refers to informational efficiency – how quickly market prices reflect available information about securities. Prices are considered to be efficient if investors cannot use information to earn positive risk-adjusted returns in the long run
- The **market value** of a security is the current trading price on exchange
- The **intrinsic value** of a security is the price a "rational investor" would be willing to pay
- Inefficiencies in markets lead these two values to differ from one another. Active strategies may seek to capitalise on these difference to earn positive risk-adjusted returns
- Factors affecting market efficiency include
  - Number of market participants
  - Availability of information
  - Impediments to trading
  - Transactions and information cost

5.3.1 The Efficient Market Hypothesis

- The Efficient Market Hypothesis proposes three forms of market efficiency.

Weak form efficient	Past information priced in	Market information
Semi-strong form efficient	Public information priced in	Public information
Strong form efficient	Private information priced in	Private information

  As markets move from weak to strong form, additional information is priced in, so strong form efficient implies market and public and private information is all priced in.

- A portfolio manager's role in efficient markets is
  - Establish portfolio risk / return objectives
  - Construct a well-diversified portfolio
  - Asset allocation based on the risk / return objectives
  - Tax minimisation

5.3.2 Market anomalise

- Anomalies are observed market inefficiencies. These may include
  - Time series anomalies
    - Calendar effects – January effect, turn of month, day of week (−ve Mondays), weekend (+ve Fridays), festive seasons
    - Overreaction effects – Prices inflated after good news, and depressed after bad news
    - Momentum effects – High short-term returns continue in following periods – may be a rational reaction
  - Cross-sectional anomalies
    - Size effect – Small-cap stocks outperform large-cap stocks (This is sensitive to time period)
    - Value effect – Low P/E, low market-to-book, high-dividend yielding stocks tend to outperform (This disappears with the Fama French model)
  - Other anomalies
    - Slow adjustments to earnings surprises
    - IPOs, initial overreaction, long-term underperformance
- Evidence for the existence of these anomalies includes
  - Evidence contingent on methodology
  - Trading strategies may not be profitable when including transaction costs
  - Some strategies cease to work over time
  - Portfolio management should not be based on anomalies with no economic basis

5.3.3 Behavioural finance

- While models assume investors are rational, investors may behave in ways that are not rational
- Investors may have cognitive biases
  - Loss aversion – Investors dislike losses more that they like equal-sized gains
  - Overconfidence – Investors overestimate their ability to value securities
  - Gambler's fallacy – Recent results affect estimates of future probabilities
  - Information cascades – Herd behaviour of uninformed investors mimicking others' activitities

  Despite this irrational behaviour, markets may still be efficient

5.4 Types of equity investments

5.4.1 Ordinary / common stock

- For ordinary / common stock;
  - Dividends are variable – no obligation for issuing firm to pay dividends
  - Common share holders have a residual claim to firm assets
  - Common shareholders vote for board members. Also see §3.4.1 on this
  - Different classes of shares may have differing voting rights

EXAMPLE: Suppose a shareholder holds 100 shares, and a board has three positions available.

- Under statutory voting, the shareholder may give up to 100 votes to candidates for each of the three available positions
- Under cumulative voting, the shareholder has $3 \times 100 = 300$ votes that can be split among all candidates standing for election in any way they choose. So they can give a maximum of 300 votes to a single individual.

5.4.2 Preferred stock

- Characteristtics of preferred shares that make them like common stock include
  - Dividend payments are not an obligation
  - No maturity date on these shares
- Characteristics of preferred shares that make them like debt securities include
  - Fixed payment
  - Usually no voting rights
  - Does not participate in high profits
- Cumulative preferred stock must receive all unpaid dividends before common shareholders received dividends
- Participating preferred shares receive an additional dividend payment if the firm does well
- Convertible preferred stock may be converted to common stock at a defined conversion ratio. The preferred dividend is paid before any dividend to common shareholders, but an investor can benefit from girm growth by converting to common shareholders. However, preferred stock is less risky than common stock
- Callable preferred stock allows the firm to buy back the preferred stock at a pre-determined price
- Putable preferred stock allows the shareholder to sell the preferred stock back to the company at a put price

5.4.3 Private equity

- Private equity firms have lower reporting requirement and fewer required disclosures, and tend to have greater focus on the long term. There is greater return potential upon public offering.
- These firms are less liquid, and its is less simple for them to raise capital
- Private equity investments may be
  - Venture capital – Provides financing for early stages of firm development
  - Leveraged buyout – Use debt to buy all outstanding stock
  - Management buyout – Management-led LBO
  - Private investment in public equity (PIPE – public firm raises equity capital in private placement

5.5 Foreign equities and equity risk

- The disadvantages of direct investment in a foreign exchange include
  - Investment and return must be made in a foreign currency
  - Often there is less liquidity and transparency
  - Exchange regulations / procedures may differ
- Instead of direct investment in foreign equity, this can be done through the use of depository receipts
  - Shares are deposited with a bank
  - Claims to the deposited shares are then traded like local stock on local exchanges, and crucially in the local currency too
  - This means accounting standards / market procedures are identical to those of a local company
- There are two types of depository receipt:
  - Sponsored depository receipt
    - Firm involved in issue
    - Same voting / dividend rights as shareholders
    - Greater reporting requirements
  - Unsponsored depository receipt
    - A depository (bank) buys shares in the foreign market
    - Bank retains voting rights on the underlying shares
- An American depository receipt (ADR) is denominated in USD, and traded on US exchanges
- A Global depository receipt (GDR) is issued outside of the US

5.6 Characteristics of equity

- Components of return
  - Dividends (compounding of reinvested dividends)
  - Capital gain . loss
  - Share buyback
  - FX gain / loss
- Risk characteristics of equity
  - Preferred stock is less risky than common stock due to
    - Fixed dividend payment
    - Dividend distribution before common stock
    - Claim to par value if firm liquidates (but after debtholders)

5.7 Equity issuance

- Equity issuance may serve any of the following purposes
  - Provides funds to buy productive assets which increase shareholder wealth
  - Cn be used to buy other companies, or for employee incentive compensation
  - Decreases reliance on debt financing

5.7.1 Book and market value of equity

- The book value and market value are defined

$$\text{Book value} = \text{Net assets from balance sheet}, \tag{5.12}$$
$$\text{Market value} = \text{Reflection of investor expectations}. \tag{5.13}$$

- Net assets is defined in §4.8.1, equation 4.4 as

$$\text{Net assets} = \text{Assets} - \text{Liabilities},$$

  so gives the **book value of equity**.

- The market value of equity is a reflection of investor expectations, regarding risk and future cash flows

  EXAMPLE:

	T	T+1
Total shareholder equity	18,503	17,143
Net income available to common	3,526	3,056
Stock price	\$16.80	\$15.30
Shares outstanding	3,710	2,790

$$\text{ROE} = \frac{\text{Net income}}{\text{Avg. book value}} = \frac{3,526}{\left(\frac{17,143+18,503}{2}\right)} = 19.78\%$$
$$\text{Market value of equity} = \$16.80 \times 3,710 = \$62,328$$
$$\text{Book value per share} = \frac{18,503}{3,710} = \$4.99$$
$$\text{P/B ratio} = \frac{\$16.80}{\$4.99} = 3.37$$

- The return on equity (ROE) is defined as

$$\text{ROE} = \frac{\text{Net income} - \text{Pref. dividends}}{\text{Avg. equity}}, \tag{5.14}$$

  and is the return generated on equity capital

- The cost of equity is the minimum rate of return required by investors

5.8 Company research reports

- Initial reports on a company are likely to include
  1. Front matter (including targets)
  2. Recommendations and rationale
  3. Company description (business model+ strategy)
  4. Industry overview and competitive positioning
  5. Financial analysis

6. Valuation
7. ESG + other risk factors

- Subsequent reports will likely include

1. Front matter
2. Recommendations
3. New information analysis
4. Valuation
5. Risks

5.8.1 Company business model

- The business model should highligh the key drivers of financial results. This should consider
  - Products / services offered
  - Cutomers
  - Sales channels
  - Pricing and payment terms
  - Reliance on key suppliers and other contacts
- The types of information used include
  - Company information
  - Publically available third-party information
  - Proprietary third-party information
  - Proprietary research

5.8.2 Revenue, profitability and capital

- Revenue drivers can be can be analysed top down or bottom up.
  - Bottom up analysis considers
    - Sales volume
    - Divisional performance
    - Geographic variables (market share, GDP growth)
    - Cannibalisation
  - Top-down analysis condisers
    - Market size
    - Market share
- Pricing power is defined as the extent to which a company can set the selling price without negatively impacting sales volume
  - It is determined by market structure and competitive position (see §2.2 for more information)
  - Highly competitive markets imply firms are price-takers with comparatively little pricing power. This means returns are close to the cost of capital
  - Less competitive structures (monopoly, oligopoloy, monopolistic competition) imply higher pricing power

5.8.3 Operating profitability

- There are three forms of operating cost analysis:
  1. Relationship with output (fixed / variable costs)
  2. Nature (work in process, utilities, promotion)
  3. Function (selling, advertising, travel, taxation)

5.8.4 Fixed and variable costs

- The operating profit is defined

$$\text{Operating profit} = [Q \times (P - VC)] - FC, \tag{5.15}$$

where the variables have the following definitions.

Q	Quantity sold
P	Price
VC	Variable costs that change with output
FC	Fixed costs that do not change with output

- The contribution margin, CM per unit is defined

$$\text{Contribution margin} = P - VC. \tag{5.16}$$

If $CM > 0$, then each unit sold sufficiently covers the variable cost and contributes to covering fixed costs.

- If Q is sufficiently large, then the firm makes an operating profit
- Operating leverage rises as a company's fixed costs rise relative to variable costs

$$\text{Degree of operating leverage} = \frac{\%\Delta\text{Operating profit}}{\%\Delta\text{Sales}} \tag{5.17}$$

5.8.5 Operating cost classifcations

- Operating cost classifcations include
  - Gross profit is defined

$$\text{Gross profit} = \text{Revenue} - \text{Cost of sales} \tag{5.18}$$

  - EBITDA is defined

$$\text{EBITDA} = \text{Revenue} - \text{Cost os sales} - \text{Operating expenses} \tag{5.19}$$

  - EBIT is defined

$$\begin{aligned}\text{EBIT} &= \text{EBITDA} - \text{Depreciation} - \text{Amortisation} \\ &= \text{Operating profit}\end{aligned} \tag{5.20}$$

5.8.6 Operating profitability

- All companies in the same industry tend to own the same types of revenues and incur similar costs
  - Competition between companies influence industry profitability
- Economies of scale
  - Greater output at lower average cost (Fixed cost spread out over more quantity produced)
- Economies of scope
  - Increases in divisions / product lines enable cost sharing

5.8.7 Working capital

- Long cash conversion cycle requires greater esxternal financing
- Higher accounts receivable and inventory lengthens the cycle
- Higher accounts payable shortens cycle

- Positive net working capital can be financed internally
- Negative net working capital financed externally (i.e. from suppliers)

- Source of capital include
  - Cash flows from operations
  - Proceeds from debt / share issuance
  - Proceeds from asset sales
- Uses of capital include
  - Asset purchases
  - Debt repayment
  - Dividend payment
  - Share repurchases

5.8.8 Capital investments and structures

- Capital investments are evaluated on whether it will generate the required rate of return (weighted average cost of capital, see §3.9.1, equation 3.15
- Capital structure is evaluated on whether opportunities exceed risks
- A key measure of capital structure risk is the degree of financial leverage (DFL),

$$\text{Degree of financial leverage} = \frac{\%\Delta\text{Net income}}{\%\Delta\text{Operating profit}} \tag{5.21}$$

5.9 Industry analysis

- The purpose of industry analysis is to
  - Determine the long-run expected rate of return for an industry
  - Enable future projections of profitability
  - Assess a firms relative position to its peers
  - Come up with more accurate financial forecasts
  - Discern attractive investments
- The steps involved are
  1. Industry classification [subjective]
  2. Industry survey (size / growth / profitability)
  3. Industry structure (Porter's five forces)
  4. External influences (PESTLE analysis)
  5. Competitive analysis

5.10 Industry classification

- Commercial classification groups companies by products / services offered. Examples of classification schemes include
  - GICS – General Industry Classification Standard
  - ICB – Industry Classification Benchmark
  - TRBC – The Refinitiv Business Classification
- The 11 common sectors / industries are
  - Energy
  - Financials
  - Basic materials
  - Technology
  - Industrials
  - Communications
  - Consumer discretionary
  - Consumer staples
  - Utilities
  - Real estate
  - Healthcare
- Issues with classifying companies may include
  - Is the bucketing system too wide / narrow
  - Does a company's product offerings span multiple categories
  - What about geographic classification
  - Do the products / services offered change over time
- Other classification schemes may include
  - Defensive / cyclical classification
  - Financial measures (market cap., valuation, profitability)
  - ESG classification

5.10.1 Industry survey

- An industry survey may include analysis on

  - Industry size
  - Growth characteristics
  - Profitability
  - Market share trends

- Industry size / growth rate
  - Industry size is calculated as the product's annual total sales
  - Industry growth calculation can be arithmetically or geometrically calculated
  - Growth indsutries ahve considerable remaining growth potential
  - Business cycle sensitivity
  - Style box allows for grouping of growth rate and sensitivity
- Industry profitability and market share
  - Profitability should be assessed using return on invested capital (difficult for private companies)
  - The goal is to determine long-term trends
  - The N-firm concentration ratio (§2.4.1, equation 2.5) or the Herfindahl-Hirschmann Index (§2.4.2, equation 2.6) may be used to measure concentration
- Industry structure and competitive positioning
  - Porter's five forces, introduced in §4.23 are
    1. Threat of substitute products
    2. Intensity of rivalry
    3. Bargaining power of suppliers
    4. Bargaining power os customers
    5. Threat of new entrants

    and are used to determine the intensity of industry competition
- PESTLE analysis of external factors affecting an industry cover
  - Political factors
  - Economic factors
  - Social factors
  - Technological factors
  - Legal factors
  - Environmental factors
- Competitive strategy and position
  - Effective strategies achieve consistent and positive economic proftis over the long run. Strategies include
    * Cost leadership – Low production costs, low prices, profit through volume)
    * Differentiation – Distinction with respect to type, quality, delivery
    * Focus – Target a niche market

5.11 Forecasting in comapny analysis

- The principles of forecasting include
  1. Drivers of financial statement lines
  2. Individual financial statement lines
  3. Summary measures (Net income, total equity, etc.)
  4. Ad hoc objects (Regulatory changes, lawsuits, etc.)
- Forecasting approaches include the use of
  - Historical results
  - Historical base rate and convergence
  - Management guidance
  - Discretionary forecasting

  The forecast horizon should be at least half a business cycle for cyclical industries
- Top down analysis starts with expectations about a macroeconomic variable (i.e. GDP growth). The expected relationship between that variable and company sales is then modeled. Alternatively, one could use market growth and market share to forecast sales
- Bottom up analysis starts with individual company attributes. Revenue drivers include
  - Average selling price / volumes
  - Product line / segment revenues
  - Capacity-based measures (Number of locations, sales per location, etc.)
  - Return-based measures (Interest income based on loan balances, etc.)
- Hybrid combines top / bottom approach
- Non-recurring items should not be included in forecasts. Both visible and non-visible should be identified and quantified
  - Visible – Large special orders, foreign exchange gains
  - Invisible – Requires greater insight to identify

5.11.1 Forecasting operating expenses

- Cost of goods sold (COGS) is linked to revenue. It is estimated as a % of revenue

$$\text{Forecast COGS} = \text{Historical COGS} \times \frac{\text{Future revenue estimate}}{\text{Historical revenue}} \tag{5.22}$$

$$\text{Forecast COGS} \equiv [1 - \text{Gross margin}] \times \text{Future revenue estimate} \tag{5.23}$$

  We would expect the gross margin to increase alongside market share

  EXAMPLE: Consider a company where the current COGS is 20% of sales. Input costs double, and the cost can be passed on to customers in full, and assume the volume is constant.

- SG&A expenses are less sensitive to changes in sale volume due to the fixed cost element
  - Fixed elements should be modelled using a fixed growth rate + inflation
  - Variable elements should be directly related to the sales volume

Current	Sales = 100	COGS = 20	Gross profit = 80
Future	Sales = 120	COGS = 40	Gross profit = 80
Current	$\frac{\text{COGS}}{\text{Sales}} = 20\%$	Gross profit margin = 80%	
	↓ **Increase**	↓ **Decrease**	
Future	$\frac{\text{COGS}}{\text{Sales}} = 33.3\%$	Gross profit margin = 67.7%	

Estimate	**Effect on cash**
Accounts receivable decrease	Increase
Accounts payable decrease	Decrease
Inventory decrease	Increase

- When forecasting working capital,
- When forecasting accounts receivable, forecasted DSO and related measures are as follows:

$$\text{DSO} = \frac{\text{Accounts receivable}}{\left(\frac{\text{Revenue}}{365}\right)} = \frac{365}{\text{Receivables turnover}}$$

$$\text{Receivables turnover} = \frac{\text{Revenue}}{\text{Avg. receivables}}$$

$$\text{Accounts receivable} = \frac{\text{DSO}}{\left(\frac{365}{\text{Revenue}}\right)}$$

- When forecasting inventory, forecasted DOH and related measures are as follows:

$$\text{DSO} = \frac{\text{Inventory}}{\left(\frac{\text{COGS}}{365}\right)} = \frac{365}{\text{Inventory turnover}}$$

$$\text{Inventory turnover} = \frac{\text{Annual COGS}}{\text{Avg. inventory}}$$

$$\text{Inventory} = \text{DOH} \times \frac{\text{COGS}}{365}$$

- When forecasting accounts payable, forecasted days payable outstanding (DPO) and related measures are as follows:

$$\text{DPO} = \frac{\text{Accounts payable}}{\left(\frac{\text{COGS}}{365}\right)} = \frac{365}{\text{Payables turnover}}$$

$$\text{Payables turnover} = \frac{\text{Purchases}}{\text{Accounts payable}}$$

$$\text{Accounts payable} = \text{DPO} \times \frac{\text{COGS}}{365}$$

5.11.2 Forecasting capital investments and structure

- Forecasting the capital requirements for a firm requires
  - The cash flow statement for additions and disposals
  - The income statement for depreciation
- Historical depreciation will increase by the relevant inflation rate. Replacement asset costs increase with inflation
- Forecasting the value of future asset purchases is subjective and requires knowledge of management growth strategies
- Forecasting capital structure requires analysis of leverage ratios, target structure and borrowings

5.11.3 Scenario analysis

- Any single point estiate underlying a forecast is unlikely to be sufficient. Analysts should construct multiple alternative assumptions which could affect net income
- The sensitivity of net income to these changes is then examined. Net income will likely be affected by changes in assumptions regarding
  - Economic environment
  - Competition
  - Technological changes
  - Cannibalisation of existing revenues by new products

5.12 Security valuation

- The difference between the market value and estimated value of a security can give an indication as to whether it is overvalue or undervalued. For this valuation to form a profitable strategy, the security must be misvalued and converge towards the future intrinsic value

Market price $<$ Estimated value	Undervalued
Market price $>$ Estimated value	Overvalued

The market price is likely to be correct for a security followed by many analysts.

5.12.1 Types of equity valuation models

- Discounted cash flow (DCF) models
  - Estimated value is the PV of either
    - Future cash distributed to shareholders ("Dividend discount" models)
    - Future cash available to shareholders ("Free cash flow to equity" models
- Multiplier models
  - Price multiplier – Ratio of stock price to earnings, sales, book value, or cash flow
  - Enterprise value multiplier – Ratio of enterprise value (equity + debt) to sales or EBITDA
- Asset-based models

$$\text{Equity value} = \text{Total asset value} - \text{Liabilities} - \text{Preferred stock values} \tag{5.24}$$

This usually undervalues any going concerns

5.12.2 Type of dividend

- Cash dividend – payment to shareholders in cash
  - Regular dividend
  - Extra "special" dividend
- Stock dividend – Paymetn to shareholders in shares of stock
- Stock split – Proportionate increase in shares outstanding
- Reverse stock split – Proportionate decrease in shares outstanding

The last three do not change the total value of shares outstanding in the market (market cap)

- Share repurchases are an alternative to a cash dividend as a way to distribute cash to shareholders
  - Tax advantage to shareholders as this materialises as a capital gain, not ordinary income
  - Suggests management feel that shares are undervalued in the market
  - Offsets dilution from exercise of stock options
- Dividend payment chronology
  1. Declaration date – Date dividend is announced to the market
  2. Ex-dividend date – First day stock trades without divided included in price
  3. Holder of record date – Date shareholders must own stock to receive dividend payment
  4. Payment date – Date dividend is paid out to shareholders

5.12.3 Dividend discount models

- Valuing common stock
  - For a 1-year holding period

$$V_0 = \frac{D_1}{(1+k_e)} + \frac{P_1}{(1+k_e)} \tag{5.25}$$

where the variables have the following definitions.

D_1	Dividend paid in a years time
P_1	Sale price after D_1 paid
k_e	Required rate of return on common equity

  - More generally

$$V_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+k_e)^t} + \frac{P_\infty}{(1+k_e)^\infty}, \tag{5.26}$$

$$V_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+k_e)^t}. \tag{5.27}$$

The second term can be assumed to fall to zero, under the assumption that growth rate in price is below the return on common equity.

- Equation 5.27 is the most general form of the dividend discount method of stock valuation. To make this more easily calculable, we can assume a constant growth rate of dividend. A more accurate approach is required for companies of differing maturity.
  - A 2-stage dividend discount model is appropriate for firms with a high current growth that will fall to a stable rate
  - A 3-stage dividend discount model is appropriate for young firms still in the high growth phase

- First, let us consider equation 5.27 for the case of a preferred stock, which has a constant dividend, paid to perpetuity. Therefore, $D_t = D_p \; \forall \; t$, and taking k_p to be the required rate of return on preferred equity,

$$\begin{aligned} V_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+k_p)^t} &= \frac{D_p}{1+k_p} \times \frac{1}{1-\frac{1}{1+k_p}} \\ &= \frac{D_p}{\cancel{1+k_p}} \times \frac{\cancel{1+k_p}}{\cancel{1}+k_p-\cancel{1}} \\ &= \frac{D_p}{k_p} \end{aligned} \tag{5.28}$$

- Now taking equation 5.27 and assuming a constant dividend growth, that is $D_t = D_0(1+g_c)^t$, we find

$$\begin{aligned} V_0 = \sum_{t=1}^{\infty} \frac{D_0(1+g_c)^t}{(1+k_e)^t} &= D_0 \frac{1+g_c}{1+k_e} \times \frac{1}{1-\frac{1+g_c}{1+k_e}}, \\ &= \frac{D_0(1+g_c}{\cancel{1+k_e}} \times \frac{\cancel{1+k_e}}{\cancel{1}+k_e-(\cancel{1}+g_c)}, \\ &= \frac{D_0(1+g_c)}{k_e-g_c}, \\ &= \frac{D_1}{k_e-g_c}, \end{aligned} \tag{5.29}$$

  which matches the result stated in equation 1.20. We can also see that in the case where $g_c = 0$, which is the case for a preferred stock, this reduces to equation 5.28

5.12.4 Relative valuation measures

- The following are common price multiples based on comparables across financial statement
  - Price / Earnings
  - Price / Cash flow
  - Price / Sales
  - Price / Book value
- Advantages of price multiples include the fact that they are widely used, readily available, easy to calculated, and can be used for cross-sectional or time-series analysis. They are also associated with equity returns.
- Multiples may be historical $\left(\frac{P_0}{E_0}\right)$ or forward looking $\left(\frac{P_0}{E_1}\right)$, where E_1 is the forecasted earnings.
- To calculate P / E based on fundamentals, we start with equation 5.29,

$$P_0 = \frac{D_1}{k-g},$$

  we can divide through by the forecasted earnings, E_1 to give the leading P / E ratio, which is

$$\frac{P_0}{E_1} = \frac{\frac{D_1}{E_1}}{k_e-g_c}, \tag{5.30}$$

  and $\frac{D_1}{E_1}$ is the payout ratio.

- All else being equal, $\frac{P_0}{E_1}$ will be higher if either the dividend growth rate or dividend payout ratio is higher, or the required return on equity is lower (however still requiring $k > g$) Also note

$$g = \text{ROE} \times (1 - \text{payout ratio}). \tag{5.31}$$

This relationship commonly appears in example questions and is worth memorising

EXAMPLE: Interpretation of P / E

	Company	Industry Avg.
Dividend payout ratio	25%	16%
Sales growth	7.5%	3.9%
Total debt-to-equity	113%	68%

A higher dividend payment implies a higher P / E.

A higher sales growth implies a higher dividend growth, which implies a higher P / E

A higher debt implies higher risk, so a higher required return, so a lower P / E

- Based on the law of one price, two comparable assets should sell for the same multiple. Therefore if one company has a lower multiple, the stock is undervalued

EXAMPLE: Calculation of price multiples

		T	T-1	T-2
(BV)	Total shareholder equity	$55,600,000	$54,100,000	$52,600,000
(S)	Net revenue	$77,300,000	$73,600,000	$70,800,000
(E)	Net income	$3,200,000	$1,100,000	$400,000
(CF)	CFO	$17,900,000	$15,200,000	$12,200,000
	Stock price	$11.40	$14.40	$12.05
	Shares outstanding	4,476,000	3,994,000	3,823,000

Calculate P / E, P / BV, P / S, P / CF for the company

In order to do this, we must first convert each company to a per-share basis, and then calculate the relevant price multiples.

	T	T-1	T-2	Industry Avg.
P / BV	0.9	1.1	0.9	3.6
P / S	0.7	0.8	0.7	1.4
P / E	16.1	51.4	120.5	8.6
P / CF	2.9	3.8	3.8	2.9

Comparing the company to the industry averages, P / BV, P / S, P / CF is all less than industry average, which implies the firm is undervalued. The P / E difference to the industry average warrants further investigation due to how different it is to the other multiples.

5.12.5 Enterprise value multiple

- The enterprise value (EV) is defined

$$\begin{aligned}\text{Enterprise value} &= \text{Market value of common stock} \\ &\quad + \text{Market value of debt} - \text{Cash and short-term investments},\end{aligned} \tag{5.32}$$

and gives the market value of the firm. The ratio $\frac{EV}{EBITDA}$ represents the total earnings to both debt and equity.

- This metric is of particular use when firms have differing capital structures and / or earnings are negative

EXAMPLE:

Share price	$\$40.00$
Shares outstanding	$200,000$
MV long-term debt	$\$600,000$
BV long-term debt	$\$900,000$
BV Total debt + liabilities	$\$2,100,000$
Cash + marketable securities	$\$250,000$
EBITDA	$\$1,000,000$

$$\begin{aligned}EV &= \underbrace{\$40 \times 200,000}_{\text{MV equity}} + \underbrace{\$600,000}_{\text{MV LT debt}} + \underbrace{(\$2,100,000 - \$900,000)}_{\text{BV ST debt [assume close to MV]}} \\ &= \$9.800.000 - \underbrace{\$250,000}_{\text{Cash not invested}} \\ &= \$9,550,000\end{aligned}$$

The $\frac{EV}{EBITDA} = 9.6\times$. This should be compared to the industry average.

5.12.6 Asset-based models

- Equity = Market or fair value of the net assets (equation 4.4)
- Asset book values should be adjusted to market value
- Asset based valuation provides a floor value of the assets

EXAMPLE: Consider a company with $2,000$ shares outstanding, and where the market value of net assets is $1.2\times$ the book value

Cash	$\$10,000$	Accounts payable	$\$5,000$
Accounts receivable	$\$20,000$	Notes payable	$\$30,000$
Inventories	$\$50,000$	Term loans	$\$45,000$
Net fixed assets	$\$120,000$	Common equity	$\$120,000$
Total assets	$\$200,000$	Total liabilities + equity	$\$200,000$

Assuming the market value equals the book value for liabilities and short-term assets, calculate the net assets per share

$$\begin{aligned}\text{MV assets} &= \$10,000 + \$20,000 + \$60,000 + 1.2 \times \$120,000 \\ &= \$224,000\end{aligned}$$

$$\begin{aligned}\text{MV liabilities} &= \$5,000 + \$30,000 + \$45,000 \\ &= \$80,000\end{aligned}$$

$$\begin{aligned}\text{Adjusted equity value} &= \$224,000 - \$80,000 \\ &= \$144,000\end{aligned}$$

$$\text{Adjusted equity value per share} = \$72$$

5.12.7 Comparison of valuation models

- Advantages / disadvantages of PV models include
  - \+ Theoretically sound
  - \+ Widely accepted
  - − Inputs are estiamted
  - − Sensitive to input values (k_e, g_c)
- Advantages of multiplier models include
  - \+ Widely used, long-term link to stock returns
  - \+ Easily calculated, readily available
  - \+ Good for identifying attractive companies
  - \+ Good for time series / cross-sectional analysis
  - − Differences in accounting methods
  - − Variable when company is cyclical
- Asset-based models
  - \+ Can provide a floor value
  - \+ Useful for firms with mainly short-term assets
  - \+ Useful if a firm is about to undergo liquidation
  - − Ongoing firm value may be greater than asset value
  - − Fair value of assets may be difficult to estimate (Made harder with intangibles and inflation estimates)
- The chocie of valuation model should ultimately
  - − Be based on available inputs
  - − Be based on intended use
  - − Consider multiple methods
  - − Consider uncertainty in inputs
  - − Consider uncertainty in appropriateness

  and we should remember that complexity in a model does not necessarily make the model better

6 Fixed Income

6.1 Fixed income instrument features

- Major fixed-ncome instruments include loans and bonds.
  - Loans are privte, non-tradable agreements between a borrower and a lender
  - Bonds are standardized, tradable securities in which investors lend capital to the issuer of the bond.
    - The issuer promises to pay the principal borrowed plus some amount of interest "coupon"
  - The coupon is often a fixed percentage of the face "par" value and is paid periodically
- Bond issuers include
  - Corporations
  - Sovereign governments
  - Non-sovereign governments (local governments, munis)
  - Quasi-government entities (i.e. GNMA → MBS securities)
  - Supranational entities (i.e. European Investment Bank
  - Special purpose entities (ABS)
- Credit risk from a bond issuer is quantified by its credit rating

Credit Risk	**AAA** ⟶ **BBB–**	**BB** ⟶ **D**
	Investment Grade	*High Yield*

  and the credit rating is liable to change over time.
- Basic features of most bonds include
  - Maturity
  - Principal / Par value /Face value / Nominal
  - Coupon rate (Annual %)
  - Coupon frequency (Annual / Semi-annual)
  - Zero coupon bond (Pays no interest / coupons, sold at a discount, all interest comes as capital gain)
  - Floating rate notes (Coupon at a variable market rate (MRR + margin))
  - Seniority (In issuer bankruptcy, senior debt ranks before junior "subordinated")
  - Contingency provisions "embedded options"
    - Callable bond – Issuer holds right to call bond early at a fixed call price
    - Putable bond – Investor holds right to sell bond back to issuer at a fixed price

6.1.1 Bond yields and returns

- Yields
  - While fixed-coupon bonds pay a fixed rate of interest, bond yields may fluctuate, affecting bond prices
  - Bonds have an inverse price / yield relationship

  - Yields are reflective of the credit risk of an issuer. The credit spread is the spread over the risk-free rate.

- The components of bond returns include
  - Coupons – Typically fixed by terms of the bond
  - Reinvestment interest – Associated with reinvestment risk
  - Capital gain / loss – Price risk

    Reinvestment risk and price risk offset each other

- Bond indetures are the legal contract between the issuer and the bondholder. It defines the obligations of, and restrictions imposed on the issuer

- Sources of repayment include
  - Sovereign bonds are repaid from taxes on economic activity and / or the ability to create new currency
  - Local government bonds are repaid from local government taxes or revenue from operational infrastructure
  - Secured bonds are repaid from the issuers operating cash flow, with the added security of a legal claim "lien" on a specific collateral
  - Unsecured bonds have no added security

6.1.2 Bond covenants

- Bond covenants are specific requirements that the issuer must filfill within the bond indenture

- Affirmative covenants specify requirements the issuer must fulfill;
  - Provide timely reports to bondholders
  - Bond holder's right to redeem at par / premium in a merger
  - Cross-default provision – Any issuer defaults also apply to this bond
  - "Pari Passu" clause ensures the bond continues to have a senior claim

- Negative covenants place restrictions on the issuer so that the risk of default does not increase
  - Entering into sale / leaseback agreements
  - Pledges of collateral
  - Negative pledge clause – issuance of more senior debt
  - Incurrence test – Additional borrowings, share repurchases, dividend payment permissible contingent on financial ratios meeting a threshold

6.1.3 Fixed income cash flows and types

- Bullet structure
  - Principal repaid in a single payment at maturity. Coupons are merely interest payments

- Partially amortising
  - Periodic payments include interst + principal with a balloon payment at the end of the term

- Fully amortising

  - Equal payments each period, including interest and principal that fully repay the loan over the lifetime of the bond

- Sinking fund
  - Bonds are retired or redeemed early on scheduled dates. Lower credit risk, higher reinvestment risk associated with these.

- Waterfall structure
  - Used for MBS / ABS securities, this tends to abide by the following structure

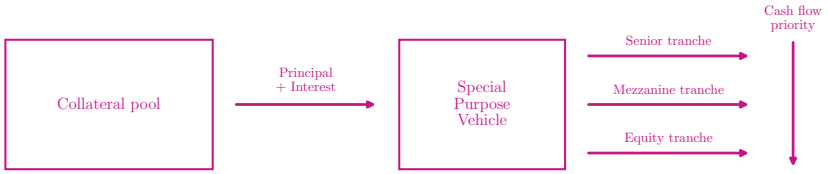


Figure 6.1: Waterfall structure of payments from a collateral pool through the tranches in order of seniority

- Floating rate notes
  - FRNs pay periodic interest based on a market reference rate, the "MRR" plus a fixed margin
  - Most FRNs pay quarterly coupons, and use a 90-day MRR
  - It is important to adjust the annual MRR / margin for quarterly payments as follows

$$\text{Quarterly payment} = \frac{\text{MRR} + \text{Margin}}{4} \tag{6.1}$$

The following coupon structures are less common, but worth knowing about.

- Step-up coupon bonds
  - Structured so that the coupon rate increases over time according to a pre-determines schedule. This protects against rising rates

- Leveraged loans
  - Coupon rate increases if credit quality of issuer decreases (i.e. If the total $^{\text{Debt}}/_{\text{EBITDA}}$ increases)

- Credit-linked note
  - Coupon rate increases if the credit rating of the issuer deteriorates

- Payment-in-kind bond "PIK"
  - Allows the issuer to pay coupon payments by increasing the principal owed
  - Firms issue PIK bonds in anticipation of cash flow problems
  - A PIK is indicative of ahigh level of existing leverage / debt service

- Index-linked bonds
  - Coupon payments / principal values are based on a specific published index, such as inflation linked bonds
  - Capital-indexed bonds pay a constant coupon rate, but the principal sum is linked to inflation (TIPS)

  - Interest-indexed bonds have a coupon payment adjusted for inflation

- Green bonds
  - The coupon rate is increased if environmental targets are not met by the issuer
- Deferred coupon bonds
  - Regular payments start at a future date after issuance
- Zero coupon bonds
  - Sold at a discount, redeemed at par. This minimises the reinvestment rate risk

6.1.4 Fixed income contingency provisions

- A contingency provision in a contract describes an action that may be taken if an event "contingency" occurs
  - Embedded options are contingency provisions in bond indentures. They give rights to either the bond issuer (callable) or holder (putable)
- Callable bonds give the issuer the right to redeem all / part of the bond issue at a fixed price.
  - Investors require a higher yield as compensation. If rates decrease, the issuer can recall the bonds and refinance at a lower rate. This brings additional risk to the investor
- Putable bonds give the bond holder the right to sell the bond back to the issuer at a fixed price
  - The embedded put provides a price floor to the bond holder

Straight bond	Callable bond	Putable bond
5% yield	7% yield	3% yield
—	Highest yield	Lowest yield
—	Lowest price	Highest price

- Convertible bonds give the bond holder the right to exchange the bond for a specific number of common shares

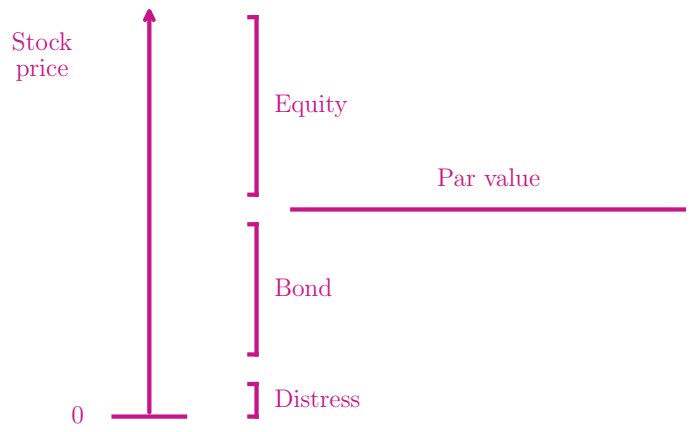


Figure 6.2: Diagram showing convertible bond behaviour based on stock price

Convertible bonds will have the following defined:

  - Conversion ratio – The number of shares gained on conversion
  - Conversion value – The market value of shares gained on conversion
  - Conversion price – The par value per share at which the bond may be converted

6.1.5 Warrants

- Warrants are an alternative way to give obndholders an opportunity for additional returns
  - Attaching warrants to straight bonds gives the holder the right ot buy common shares at a fixed price
  - Warrants can be detached from the bond issue and traded separately
- Contingent convertible bonds convert from debt to equity if a specific event occurs

6.1.6 Domestic and foreign bonds

- Domestic bonds are those issued by an issuer in its home coutry and domestic currency
- Foreign bonds are those with a foreign issuer, trade in domestic currency, and are used to raise capital in the broader market
- A national bond market includes the trading of both types of bond issues

- Eurobonds
  - Eurobonds are sold by an international syndicate and issued simultaneously to investors in many countries
  - They are issued outside the jurisdiction of a single country, and issued in a currency other than the issuer's domestic urrency
  - Eurobonds can reach a large investor pool, and are used to avoid regulation, no tax witholding, for example issuance of USD denominated bonds without registering with the SEC, so cannot be traded there
- Global bonds
  - Eurobonds that trade in a domestic bond market
- International bonds
  - Foreign bonds / global bonds / Eurobonds
- Sukuk bonds
  - Sharia-compliant bonds, following Islamic law. They have restrictions on interest payments and use of proceeds

6.1.7 Taxation of bond income

- Interest income paid to bondholders is often taxed as ordinary income at the same rate as wage / salary
  - Municipal bonds – interest income issued by municipal governments is often exempt from tax
- Capital gains / losses may occur when bonds are sold redeemed. CGT rates are often lower.
  - Original-issue discount bonds may generate income tax liabilities, such as zero coupon bonds

6.2 Fixed income and trading

6.2.1 Fixed income classifications

- The types of issuer may include government, corporations, special purpose entites, or others

S&P / Fitch	**AAA**	$\longrightarrow$	**BBB−**	**BB**	$\longrightarrow$	**D**
Moody's	**Aaa**	$\longrightarrow$	**Baa3**	**Ba1**	$\longrightarrow$	**C**
	Investment Grade			*High Yield*		

- Maturity of bonds is broadly in three categories
  - Money market – < 1 year
  - Intermediate term $1 - 10$ years
  - Long-term 10+ years

	Short	**Intermediate**	**Long**
	< 1 year	$1 - 10$ years	10+ years
Default-risk free	Treasury bills	Treasury notes	Treasury bond
Investment grade	Repo agreements Commercial paper ABCP (Asset-backed commercial paper)	Unsecured corporate bonds ABS	MBS
High yield		Secured corporate bonds Leveraged loans	

Table 6.1: The liquidity-return trade-off for bonds and debt-instruments broken down by maturity and risk classification. A higher yield is demanded by lower-quality issuers and longer-term issues

6.2.2 Investment grade funding

- Commercial paper is issued by investment-grade companies to fund short term working capital requirements
- Intermediate-term debt is used to fund medium-term investment and permanent working capital
- Long-term debt funding is used to fund capital investment in fixed assets
- Short and medium-term issues may be covered by a bank syndicate

6.3 Fixed income indices

- Corporations tend to have multiple bond issues as opposed to a handful of shareclasses (preferred / common). The issues vary based on maturity and coupon payment which will reflect the financial position of economic position at the time of issue
- Bonds mature and need to be replaced over time, leading to higher turnover in indices
- Bonds are issued across multiple sectors. Changes in debt issuance trends (maturity, credit quality, etc.) affect bond indices over time

	Short	Intermediate	Long
		⟵ Financial intermediaries ⟶	
Default-risk free		⟵ Central banks ⟶	↑
	Money market funds		Pension funds
Investment grade	Corporate issuers	Bond funds / ETFs	Insurance companies
		Asset managers	↓
High yield		Hedge funds	
		Distressed debt funds	

Table 6.2: Typical investor positioning across risk brackets and maturity

- Aggregate indices contain a broad selection of bonds. Narrower-focus indices focus on geography, credit quality, sector, and maturity
- Bond indices may also include ESG factors in their construction. This tends to screen out particular industries

Primary markets

Primary markets are for the sale of newly-issued bonds

- A public offering is registered with regulators for sale ot the public
- A private placement is not registered for public sale and is only sold to selected investors
- A debut issuer is one issuing bonds for the first time, typically to replace bank loans in its capital structure. Shelf registration with a regulator via a master prospectus is used for frequent bond issuance

Financial intermediaries

- Investment banks arrange the sale of new issues, and may underwrite the issue. Typically, the intermediary will carry out a roadshow before the issue
  - An underwritten offering is a bond price guarantee ofered by the intermediary
  - A best efforts agreement carries no guarantee, but the intermediary charges commission

Secondary markets

Secondary markets are for trading of previously-issued bonds

- Most trading in the secondary market is OTC by dealers, who post bid / ask quotes
- Bid-ask spread varies across bonds, based on liquidity
- Bonds with greater liquidity tend to be on-the-run bonds (most-recent issues), developed market bonds, and higher-quality corporate bonds

Distressed debt

- Distressed debt refers to bonds from issuers that themselves are in financial distress
- Some investors are restricted from holding distressed debt
- Typically, distressed debt trades well below par, but investors may be attracted to distressed bonds due to the high yield that comes from the risk associated with them

6.4 Fixed income markets for corporate issuers

6.4.1 Non-financial corporations

- Non-financial companies usually raise external funds for investment in short-term assets via loan financing or security based financing
  - Loan financing – Credit lines, securted loans
  - Security-based financing – Commercial paper (IG, $<$ 1yr)

External loan financing

- Uncommitted line of credit
  - Least costly as interest only paid on borrowings
  - Least reliable as banks may refuse to honour the line of credit
- Committed line of credit
  - Formal written agreement, so more reliable for the borrower
  - Up-front fees, risk of non-renewal
- Revolving (operating) line of credit
  - Revolvers are for longer-term loant, may contain restrictive covenants and similar up-front fees

Secured loans

- Secured (asset-backed) loans are backed by some form of collateral. Receivables can act as collateral for loans at a discount to face value. The discount size is indicative of the credit risk associated with the issuer / loan

Commercial paper

- Short-term, investment grade, unsecured debt security
  - Interest cost is lower than a bank loan
  - Maturity $\lesssim$ 3 months
  - Used to fund working capital "Bridge funding"
- Rollover risk
  - Liquidity risk if the CP cannot be issued / rolled over
- Eurocommercial paper is the international equivalent

6.4.2 Financial corporations

- Commercial and retail deposits are a major source of funding
  - Checking accounts – Demand deposits
  - Operational deposits – Cash management for large customers
  - Savings deposits – Specified maturity / interest rate
  - Certificate of deposit – Specified maturity $<$ 1 yr
  - Negotiable CDs – Can be sold before maturity
  - Interbank funds and repos – Banks lending to each other

    Excess funds are held in money market and capital market securities

Central bank fund market

- Banks must satistfy a reserve requirement
- Banks with excess funds may lend at the central bank funds rate "Interbank market"
- Central bank acts as the lender of last resort, providing liquidity

Asset-backed commercial paper

- ABCP is a short-term form of an ABS
  - Financial institution transfers short-term loans made by the bank to a special purpose entity in exchange for cash
  - The SPE sells ABCP to investors with a backup credit liquidity line provided by the bank
  - Investors have purchased a liquid short-term note with interest and principal payments from a loan portfolio

Assets off balance sheet $\Rightarrow$ Lower reserves required

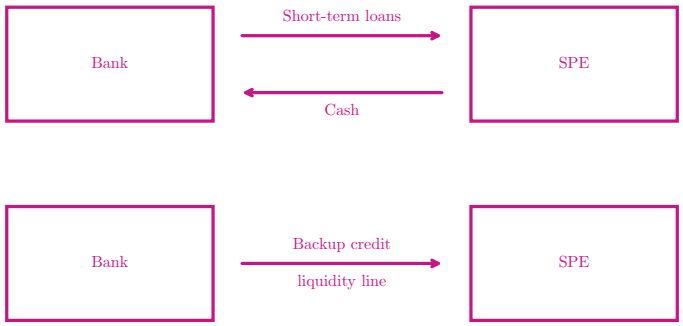


Figure 6.3: ABS payment structures. The SPE then sells securitised instruments on to investors.

Repurchase agreements "Repos"

- A repo is an agreement to sell a security to a counterparty, and buy it back later at a higher price
  - Repos are used for short-term funding
  - Repo rate is annualised interest implied by the buy / sell price
  - Securities sold are collateral for the loan and an initial margin of excess collateral is required
  - Overnight (one-day) / term (otherwise) repos
- Repo collateral is usually high-quality, liquid, sovereign bonds. Special collateral may contain hard-to-source, illiqid collateral. The repo rate may be lower "discounted"
- The contractual terms are agreed under a master repurchase agreement In this arrangement, the borrower is usually looking for short-term funding, and posts the security as collateral for the loan. The lender is usually an entity with excess liquidity, and benefits from this by earning the repo rate on the loan amount. The lender also obtains collateral which reduces the risk of the loan
- A tri-party repo involves a third party which holds both cash and security.

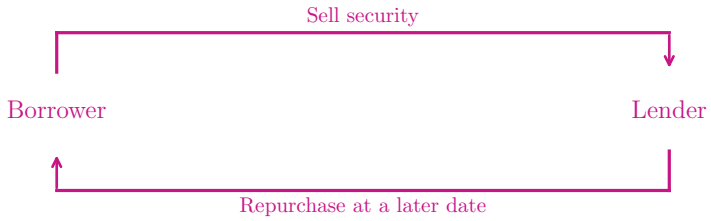


Figure 6.4: Repo payment structure. The borrower enters into a repo agreement, and the lender a reverse-repo agreement. The borrower initially sells a security, with the intention of repurchasing it at a later date.

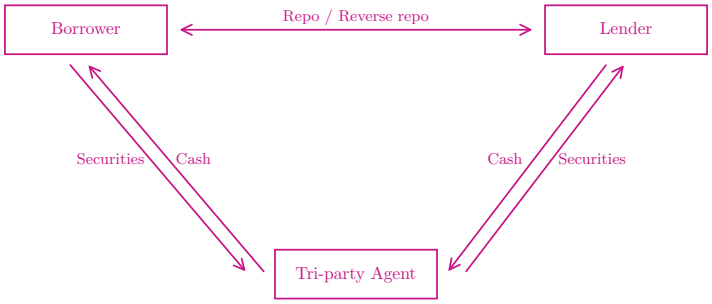


Figure 6.5: Tri-party repo payment structure. The tri-party agent serves as a custodian, holding cash and security for both the borrower and lender in this agreement, thereby shifting risk for both borrower and lender to the custodian instead of each other.

EXAMPLE: Consider a firm selling a $\$1,000,000$ market vale bond, and repurchasing it 90 days later at a repo rate of 2% and initial margin of 3%.

$$\begin{aligned}\text{Purchase price (Loan amount)} &= \frac{\text{Market value of securities}}{1+\text{Initial margin}} \\ &= \frac{\$1,000,000}{1.03} \\ &= \$970,874\end{aligned}$$

$$\begin{aligned}\text{Repurchase price} &= \text{Loan amount} \times \left[1+\text{Repo rate} \times \frac{\text{Days}}{360}\right] \\ &= \$970,874 \times [1+0.02 \times 90360] \\ &= \$975,728 \qquad [\text{Principal} + \text{Interest}]\end{aligned}$$

$$\text{Haircut} = \frac{\text{Market value} - \text{Loan amount}}{\text{Market value}} = 2.91\%$$

- Variation margin may be required should the market value of the collateral fall. In this case, the repo lender will ask the borrower for additional collateral.

  EXAMPLE: continued from before, suppose that after 30 days the market value of the bond has fallen to $\$990,000$. Calculate the variation margin.

  To find the adjusted loan amount,

$$\begin{aligned}\text{Adjusted loan} &= \$970,874 \times \left[1+0.02 \times \frac{30}{360}\right] \\ &= \$972,492, \text{ amount owed after 30 days}\end{aligned}$$

We can then apply the haircut, by multiplying this adjusted amount by 1 + the initial margin, to get

$$\text{Adjusted loan amount} = \$972,492 \times 1.03 = \$1,001,667$$

The variation margin is then given by this adjusted loan amount less the new market value of collateral

$$\begin{aligned}\text{Variation margin} &= \text{Adjusted loan amount} - \text{MV of collateral} \\ &= \$1,001,667 - \$990,000 \\ &= \$11,667\end{aligned}$$

so extra $11,667 of collateral required The variation margin may be negative, in the event that the market value of the collateral rises, in which case the borrower may request the release of part of the collateral.

6.4.3 Repo applications

- The main uses of repo agreements are
  - Financial institutions use repos to finance trading positions
  - Lenders, such as mutual funds and pension funds, earn the repo rate on assets
  - Central banks use repos to enact monetary policy
  - Short sellers (hedge funds), use repos to borrow securities
  - Reverse repo if motivation is to borrow a security
  - Special trade if a security is scare / hard to source (negative repo rate)
- Factors affecting the repo rate; The repo rate is:
  - High when short-term rates are high
  - Low when credit quality of collateral is high
  - High when term is longer
  - Low when collateral is hard to source
  - High if repo is undercollateralised
  - High if the collateral is not delivered (unsecured)
- Repos are a source of debt financing. Overuse can lead to financial distress or insolvency. Risk include
  - Default risk – Borrower of cash fails to repay at end of repo
  - Collateral risk – Value of collateral falls in the event of default
  - Margining risk – Relating to calculation / payment of margin
  - Legal risk – Contracts not able to be legally enforced
  - Netting and selttlement risk – Netting of cashflows across contracts
- Tri-party repos can help mitigate some of these risks
  - A third party intermediary acts as an agent to arrange / administer repo transactions
  - Credit risk is not reduced
  - Cost efficiencies are improved, providing easier access to capital
  - Valuation / safekeeping of assets is done by custodian
- Bilateral repos have no intermediary

Investment grade	High yield
Risk of rating downgrade is dominant risk	Risk of default is dominant risk
Narrow credit spreads	Wide credit spreads
Less variation in yield across maturities	Greater variation in yield across maturities
Few covenants	Many covenants
	Collateral required

6.4.4 Investment-grade versus high yield issues

- Rollover risk is lower for IG issues due to standardisation across multiple maturities. There are fewer maturity options for HY issuers
- The ability to repay earlier is more common with HY issues. HY may use leveraged loans / callable debt that contains prepayment options
- HY returns are more uncertain and equity-like. IG returns have lower uncertainty, and behave as traditional bonds

6.5 Fixed income markets for government issuers

6.5.1 Sovereign government debt

- National governments issue bonds to raise funds for spending on public gods / services, and investment in public infrastructure. Typically, they have have the following attributes
  - High credit rating (backed by taxes)
  - Largest debt issuers
  - Assessing ability to pay comes from assessment of the economic balance sheet (forward-looking), and particular focus on cahs transactions in place of accruals

Developed and emerging market issuers

- Developed markets
  - Stable, diversified economies with consistent and transparent fiscal policy
- Faster growing, less stable, more concentrated economies, and less-stable tax revenues. There is greater reliance on dominant national industry / commodities
- Debt management policy
  - Developed market debt – Denominated in a reserve
  - Emerging market debt – May be borrowed domestically or externally (Local / hard currency issues)
- Ricardian equivalence comes from
  - Taxpayers expecting government debt to be offset by future higher tax. Therefore the government should be indifferent about collecting tax as opposed to raising debt.
    - Short-term borrowing avoids term premiums and reduces costs, but introduces rollover risk
    - In practice, governments diversify debtmaturities and issue debt at regular intervals

6.5.2 Non-sovereign government debt

- Issued by state / provinces / counties / SPEs
  - Local and regional authorities may issue general obligation bonds which are backed by local tax-raising powers
  - Quasi-government bonds are issued by government agencies for specific purposes (i.e. GNMA)

6.5.3 Supranational bonds

- Issued to promote international trade, and set up by multiple sovereign governments
- High credit quality since they are backed by sovereigns

6.5.4 Public auctions

- Sovereign issuers use regular public auctions to issue government debt securities
  - Non-competitive bids are allocated first and are guaranteed to have their allocation met
  - Competitive bids are ranked in order of highest price and allocated top down
  - Cut-off yield is the yield of lowest price competitive bid that receives an allocation
- In a single price auction, all investors pay at the cut-off price / yield, irrespective of the bid made
- In a multiplce-price auction successful bidders pay the price that they bid
- To minimise volatility, government issuers will choose a single-price auction. Lower volatility means a successful auction is more likely
- Primary dealers are designated financial institutions. They are required to make competitive bids in auctions, and submite bids on behalf of third parties. They also act as ounterparties to the central bank for open market operations
- Once issued, sovereign debt trades in quote-driven OTC dealer markets. Trading is most active for on-the-run bonds

6.6 Fixed income bond valuations: Prices and yields

- Bonds are typically valued using DCF valuation, where the price is given by the sum of the present value of all future cash flows. Recalling equation 1.15,

$$PV = \frac{FV_N}{\left(1 + \frac{r}{m}\right)^{m \cdot N}}.$$

  For a straight bond, FV_N is equal to the coupon payment for all N, and m defines the coupon frequency.

- Sensitivity of bond price to changes in yield come from

$$\begin{array}{l} \text{Longer maturity} \\ \text{Lower coupon} \\ \text{Lower initial yield} \end{array} \quad \Longrightarrow \quad \text{Higher sensitivity}$$

Coupon = Yield	Trades at par
Coupon < Yield	Trades below par
Coupon > Yield	Trades above par

The components of return include coupon, reinvestment interest, and any capital gain / loss incurred upon purchase / sale of the asset

6.6.1 Flat price, full price, and accrued interest

- Bonds accrue coupon interest between payment dates which increases the value of the bond

Bond price at last coupon payment date (No accrued interest)

Flat price "Clean" = Full price − Accrued interest

Full price "Dirty" = Includes accrued interest

(6.2)

$$\text{Full price} = \text{Flat price} + \text{Accrued interest} \tag{6.3}$$

EXAMPLE: Consider a 5% semi-annual bond making coupon payments onf June 15 and December 15, with a yield to maturity of 4%. There are four coupons remaining, when the bond is purchased on August 21

Date	Cash flow	
...	...	↑
Jun-15	2.5	Past
Dec 15	2.5	Future
Jun-15	2.5	↓
Dec-15	2.5	
Jun-15	102.5	

June	+15	days
July	+31	days
August	+21	days
	67	days accrued

June	+15	days
July	+31	days
August	+31	days
September	+30	days
October	+31	days
November	+30	days
December	+31	days
	183	days between coupons

Alternatively we could assume 30 days / month, 360 days / year, which is known as "30 / 360" as opposed to the exact calculation done above, which is called "Actual / Actual"

At next coupon payment,

$$N = 4 \qquad I/Y = 2.5\% \qquad PV = \qquad PMT = 2.5 \qquad FV = 100$$

Then, using CPT PV, the calculator gives a PV of \$101.904 for value of the bond at the next coupon payment. The accrued interest is given by

$$\text{Accrued interest} = \$2.5 \times \frac{67}{183} = 0.915,$$

and the full price is given by

$$\text{Full price} = \$101.904 \times (1.02)^{\frac{67}{183}} = \$102.646,$$

therefore the flat price is given

$$\text{Flat price} = \$102.646 - \$0.915 = \$101.731$$

6.6.2 Price-yield relationship of bonds

- The price and yield of bonds exhibit an inverse relationship, while it can be approximated to a linear relationship locally, convexity attempts to correct for the non-linearity in the relationship

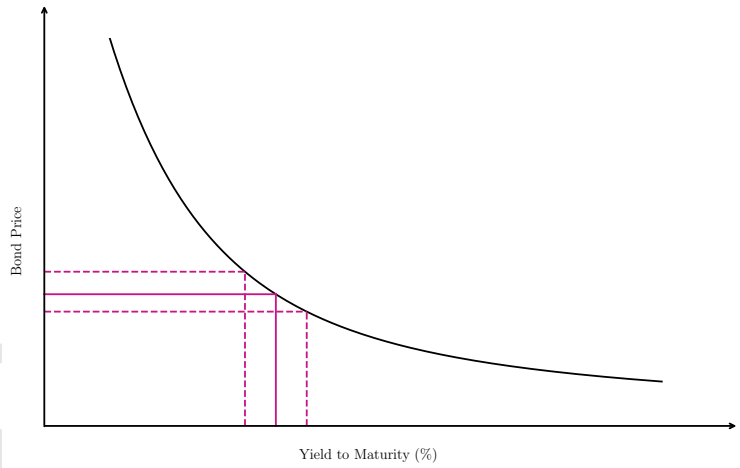


Figure 6.6: Price-yield relationship of bonds. The linear approximation overestimates the price decrease when yields rise, and underestimates the price rise when yields fall

- At maturity, bonds are redeemed at par value. As time passes, bonds "pull to par", assuming no further changes in the yield from time of purchase.

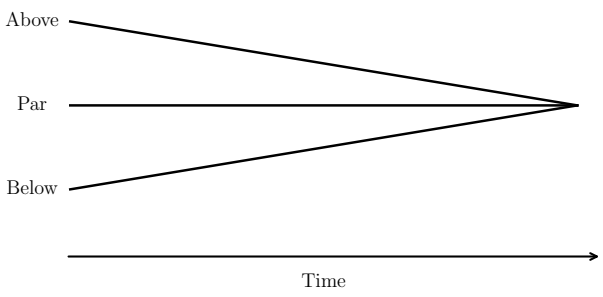


Figure 6.7: Pull-to-par effect on bond prices. If a bond is initially purchased below par, then the bond exhibits capital gain throughout the holding period. If a bond is initially purchased above par, then it returns additional income to the investor

- We can use matrix pricing in order to estimate the price or yield-to-maturity for illiquid bonds. By matching bond features to traded bonds as closely as possible (Credit quality, maturity, coupon), we can estimate the required YTM of an illiquid bond

  EXAMPLE: Suppose we are given the following, and asked to recreate the attributes of a 3 year A+ rated bond paying 4% annually.

Rating	Maturity	Yield to maturity
A+	2yr	$YTM = 4.3\%$
A+	5yr	$YTM = 5.1\%$
A+	5yr	$YTM = 5.3\%$

  We can do this by interpolation. Looking first at the second two bonds given, we can take the arithmetic mean of the YTM to give an average of 5.2% for a 5-year A+ bond. We then take a simple weighted average of the 2-year bond and the average 5-year bond to replicate a 3-year bond, which can be done as follows;

$$\text{Years:} \qquad \frac{2}{3} \cdot 2 + \frac{1}{3} \cdot 5 = 3$$
$$\implies \frac{2}{3} \cdot 4.3\% + \frac{1}{3} \cdot 5.2\% = 4.6\%$$

  Then, we can use equation 1.15 as before to calculate the price,

$$N = 3 \qquad I/Y = 4.6\% \qquad PV = \qquad PMT = 4 \qquad FV = 100$$

  Then, using CPT PV, the calculator gives a PV of \$98.35 for value of the bond at the next coupon payment.

- Matrix pricing and interpolation can also be used to estimate spreads over the risk free rate for newly issued corporate bonds.

  EXAMPLE: Estimate the spread for a newly issued A-rated bond with a maturity of 6 years given the following

Maturity	Yield to maturity
4yr Tsy bond	$YTM = 1.48\%$
6yr Tsy bond	$YTM = 2.15\%$
5yr A-rated corp bond	$YTM = 2.64\%$

  Again, using simple interpolation, the YTM of a 5yr treasury is given by

$$\frac{1.48\% + 2.15\%}{2} = 1.815\%.$$

  Then, comparing this to the corporate bond given, the estimated 5 year spread is

$$2.64\% - 1.815\% = 0.825\%.$$

  Assuming the spread is constant with respect to time to maturity, applying this to the 6 year treasury YTM, we recover

$$2.15\% + 0.825\% = 2.975\%$$

6.7 Yield and yield-spread measures

- The yield to maturity, YTM, is simply the IRR of the bond (see §3.7.2 for more on the IRR, and §1.1.2 for how this is used in a whole-portfolio context)
- The greater the periodicity of coupon payments, the more compoundin periods, and the greater the effective annual yield.

$$\text{Annual yield} = \left(1 + \frac{YTM}{n}\right)^n - 1, \tag{6.4}$$

where n is the number of compounding periods per year

- It may be required to directly compare bond yields when the periodicity of coupon payment is different

EXAMPLE: Consider a semi-annual bond with a YTM of 4%. What yield should be used to compare this to a quarterly or annual bond with the same quoted YTM? The effective annual yield of the semi-annual bond is

$$\left(1 + \frac{0.04}{2}\right)^2 - 1 = 4.04\%.$$

The quarterly yield is given by

$$\left(1 + \frac{0.04}{2}\right)^2 - 1 = 0.995\%,$$

and so the quoted annual rate on a quarterly basis is

$$4 \times 0.995\% = 3.98\%.$$

The effective annual yield of the semi-annual bond is

$$\left(1 + \frac{0.04}{2}\right)^2 - 1 = 3.98\%.$$

Alternatively, this result could be reached by decomposing the effective annual yield by rearranging equation 6.4 in terms of the YTM,

$$\underbrace{\left((1 + 0.404)^{\frac{1}{4}} - 1\right)}_{0.995\%} \times 4 = 3.98\%$$

6.7.1 Street convention versus true yield

- Bond yields calculated using the stated coupon payment dates follow street convention
  - Coupon payments are made on the first business day following the scheduled payment date, if the scheduled date is a weekend / holiday
- The yield calculated due to actual coupon payment dates is known as the true yield

  In general, the true yield is slightly below street convention

6.7.2 Daycount conventions

- Corporate bonds tend to use 30 / 160, whereas government bonds tend to use actual / actual. In order to compare corporate government yields, restate corporate yields using the actual / actual convention by multiplying the yield by $\frac{365}{360}$

6.7.3 Yield conventions

- The current yield only considers one source of return – the annual interest income "Income / running yield",

$$\text{Current yield} = \frac{\text{Annual cash coupon payment}}{\text{Bond price}}. \tag{6.5}$$

  This ignores capital gains and any reinvestment income.

- The simple yield takes the discount / premium into account by assuming linear declining of discount / premium until par value is redeemed at maturity, much in the same way as shown in figure 6.7,

$$\text{Current yield} = \frac{\text{Annual cash coupon payment}}{\text{Bond price}}. \tag{6.6}$$

- For callable bonds, yields are not quite as simple.
  - For a callable bond, the investor's yield will depend on if / when the bond is called. The yield-to-call can be calculated for each possible call date and price.
  - The yield-to-worst is the lowest of the various yields-to-call or the yield-to-maturity.

    An issuer is likely to exercise the call option if rates fall

EXAMPLE: Consider a 5yr semi-annual bond paying a 6% coupon, trading at \$102 on 1-Jan-2024. The bond may be called at \$102 on / after 1-Jan-2027, or \$101 on / after 1-Jan-2028. Calculate the yield-to-worst of this bond

YTM:

$$N = 10 \qquad I/Y = \qquad PV = 102 \qquad PMT = 3 \qquad FV = 100$$

Then, using CPT I/Y, the calculator gives an I/Y of 2.768%, so an annual stated yield of $2 \times 2.768\% = 5.54\%$

Yield to first call:

$$N = 6 \qquad I/Y = \qquad PV = 102 \qquad PMT = 3 \qquad FV = 102$$

Then, using CPT I/Y, the calculator gives an I/Y of 2.941%, so an annual stated yield of $2 \times 2.941\% = 5.88\%$

Yield to second call:

$$N = 10 \qquad I/Y = \qquad PV = 102 \qquad PMT = 3 \qquad FV = 101$$

Then, using CPT I/Y, the calculator gives an I/Y of 2.768%, so an annual stated yield of $2 \times 2.830\% = 5.66\%$

The yield-to-worst is then the minimum of these three, so is 5.54%, which is the same as the yield-to-maturity

6.7.4 Option-adjusted yield

- The option-adjusted yield / option-adjusted spread removes the effect of an embedded option to allow for direct comparison of yield with a straight bond

$$\text{Callable bond value} = \text{Straight vond value} - \text{Call option value} \tag{6.7}$$

Callable bond		**Straight bond**		**Putable bond**
Issuer owns option		No option		Bond-holder owns option
Lower price	$\xrightarrow[\text{OAS}]{\text{Yield falls}}$	Straight price	$\xleftarrow[\text{OAS}]{\text{Yield rises}}$	*Higher price*

6.7.5 Yield spread

- The yield spread / benchmark spread is the difference in yield between a corporate bond and a benchmark security (typically a treasury bond of matching maturity). The benchmark bond should have a similar maturity and be an on-the-run bond (actively traded / liquid). Use interpolation / matrix methods if necessary.

6.7.6 G-spread, I-spread, and Z-spread

- The G-spread is defined as the excess yield demanded by an investor holding a corporate bond over some benchmark yield,

$$\text{G-spread} = \text{Corporate bond yield} - \text{Interpolated benchmark bond yield} \tag{6.8}$$

- The I-spread is defined as the excess return over the interbank MRR used in swap contracts. It is used primarily for bonds denominated in Euros
- The zero-volatility spread, or z-spread, is defined as the spread which when added to each spot rate of the benchmark curve, produces the market price of the bond. It contains the required yield demanded for taking on:
  - Credit risk
  - Liquidity risk
  - Tax risk
  - Optionality risk

  The z-spread is found by trial and error, or by numerical methods as opposed to analytically

EXAMPLE: Consider a 3yr, 8% semi-annual corporate bond priced at 103.165. The 1yr and 4yr treasury yields are 3% and 5% respectively.

For the corporate bond,

$$N = 6 \qquad I/Y = \qquad PV = -103.165 \qquad PMT = 4 \qquad FV = 100$$

Then, using CPT I/Y, the calculator gives an I/Y of 3.4078%, so an annual stated yield of $2 \times 3.4078\% = 6.81\%$, so this is the YTM of the corporate bond.

Interpolating the treasury bond yields to construct a synthetic 3yr treasury bond,

$$\frac{1}{3} \cdot 3\% + \frac{2}{3} \cdot 5\% = 4.33\%.$$

Thus, the G-spread is given as

$$\text{G-spread} = 6.81\% - 4.33\% = 249 \text{ bps}$$

EXAMPLE: Consider a 3yr 9% annual coupon corporate bond trading at 89.464. The YTM is 13.5% and the YTM of a 3yr treasury bond is 12%. The 1yr, 2yr, and 3yr treasury yields are given as 4%, 8.167%, and 12.377% respectively The G-spread (≡ Yield spread) can simply be calculated as the the difference between the YTM of the corporate bond and treasury bond,

$$\text{G-spread} = 13.5\% - 12\% = 1.5\%.$$

The z-spread can be calculated using equation 1.15 in its fully-expanded form,

$$\frac{9}{(1+0.04+z)} + \frac{9}{(1+0.08167+z)^2} + \frac{109}{(1+0.12377+z)^3} = 89.464$$

and solving analytically for z. There are multiple ways this can be done, but I would suggest the use of a python script, and implementation of a fixed-point iteration method, or Newton-Raphson iteration. In this instance, $z = 0.01667 = 166.7$ bps

6.7.7 Option-adjusted spread (OAS)

- If we want to value the associated optionality contained within the yield, we can use the OAS

$$\begin{aligned} \text{Option value} &= \text{z-spread} - OAS, \\ OAS &= \text{z-spread} - \text{Option value}. \end{aligned} \tag{6.9}$$

The OAS applies to the government spot curve.

- Comparing the z-spread and OAS,

	z-spread	OAS
Credit risk	✓	✓
Liquidity risk	✓	✓
Tax risk	✓	✓
Optionality	✓	×

6.8 Floating rate note yields

- Floating rate note (FRN) values tend to be more stable because the coupon rate is reset periodically

$$\text{FRN coupon} = \text{MRR} + \text{Fixed margin} \tag{6.10}$$

  - The MRR is reset using the current MRR and paid at the end of the period. Interest is paid in arrears
  - The fixed margin is determined by the credit quality of the issuer, as well as liquidity / tax treatment

$$\text{Quoted margin} = \text{Fixed margin} \tag{6.11}$$

6.8.1 Quoted margin and discount margin

- The fixed margin is defined in the bond indentures. The discount margin however reflects the credit risk of the issuer, and is variable. The yield of the bond is variable, and is given by

$$\text{Yield} = \text{MRR} + \text{Discount margin}. \tag{6.12}$$

$QM = DM$	Coupon=Yield	Trades at par
$QM > DM$	Coupon>Yield	Trades above par
$QM < DM$	Coupon<Yield	Trades below par

This discount margin is incorporated into the YTM of the bonds.

At issue, the quoted margin and discount margin are the same,

$$\text{Quoted margin} = \text{Discount margin}.$$

For the purposes of any calculations using the calculator, the payment and interest per period are defined as

$$PMT = MRR + DM \qquad I/Y = MRR + DM$$

EXAMPLE: Consider a semi-annual bond with a quoted margin of 120 bps, which is to be paid on top of the 180 day MRR. On reset date, with 5 years to maturity, the $MRR = 3\%$ (annualised), and the $DM = 1.5\%$. Given also that the par value of the bond is $\$100,000$, compute the price of this bond.

$$N = 10 \qquad I/Y = \frac{3\% + 1.5\%}{2} = 2.25\% \qquad PV = \qquad PMT = \frac{3\% + 1.2\%}{2} = 2.1\% \qquad FV = 100$$

Then, using CPT PMT, the calculator gives an answer of $PV = \$98,670$

6.9 Money market instruments

- A money market instrument is one which has a maturity of under a year. There are different conventions as to how the yield of such an instrument may be quoted.

6.9.1 Add-on yield

- The add-on yield is defined

$$\text{Quoted add-on yield} = \text{Holding period yield} \times \frac{365}{\text{Days to maturity}}. \tag{6.13}$$

EXAMPLE: Consdier a 100-day bank CD with annualised add-on yield of 1.5% (based on 365 day year. Calculate the purchase price of a $\$1,000$ investment into this CD security

$$1.5\% \times \frac{100}{365} = 0.41\%,$$

$$\$1,000 \times (1 + 0.0041) = \$1,004.01,$$

so an investor would receive $\$1,004.10$ after making an initial deposit of $\$1,000$.

6.9.2 Add-on yield

- The discount yield is defined

$$\text{Quoted discount yield} = \text{Actual discount} \times \frac{360}{\text{Days to maturity}}. \tag{6.14}$$

This is an annualised current discount from the face value received at maturity

EXAMPLE: Consider a 180 day T-bill quoted at a discount yield of 2.2% annualised. Calculate the price of the T-bill which has a face value of \$989.

$$2.2\% \times \frac{180}{360} = 1.1\%$$

$$\$1,000 \times (1 - 0.011) = \$989$$

For this, we can also compute the holding period yield,

$$\text{Holding period yield} = \text{Holding period return} \tag{6.15}$$

The holding period yield is then

$$\text{Holding period yield} = \frac{\$1,000}{\$989} - 1 = 1.11\%,$$

which is higher than the discount yield

- Converting yields to different conventions may also be necessary. The following examples are examples of this

EXAMPLE: Consider a \$1,000 face value, 90day T-bill priced with an annualised discount of 1.2%. Calculate the marekt price and the annualised add-on yield based on a 365-day year

$$\begin{aligned}\text{90-day discount} &= \$1,000 \times 1.2\% \times \frac{90}{360} \\ &= \$3\end{aligned}$$

$$\begin{aligned}\text{Current market price} &= \$1,000 - \$3 \\ &= \$997\end{aligned}$$

$$\text{90-day add-on yield} = \frac{\$3}{\$997} = 0.3009\%$$

$$0.3009\% \times 36590 = 1.2203\%$$

EXAMPLE: Consider a \$1,000,000 negotiable CD with 120 dys to maturity, quoted with an add-on yield of 1.4% based on a 365-day year. Calculate the payment at maturity and its bond-equivalent yield.

$$\$1,000,000 \times \left(1 + 1.4\% \times \frac{120}{365}\right) = \$1,004,602.74$$

$$\text{Bond-equivalent yield} = \text{Add-on yield} = 1.4\%$$

EXAMPLE: Consider a bank deposit for 100 days that is quoted with an add-on yield of 1.4%, based on a 360-day year. Calculate the bond-equivalent yield, and the yield on a semi-annual basis. The bond-equivalent yield is

$$\text{Bond-equivalent yield} = 1.4\% \times \frac{365}{360} = 1.5208\%.$$

The 100-day holding period yield is then

$$\text{100-day HPY} = 1.5\% \times \frac{100}{360} = 0.4167\%.$$

Using this to calculate the effective annual yield, we get

$$\text{Effective annual yield} = (1 + 0.004167\%)^{\frac{365}{100}} - 1 = 1.5294\%.$$

The semi-annual yield is therefore

$$\text{Semi-annual yield} = (1 + 1.5294\%)^{\frac{1}{2}} - 1 = 0.7618\%.$$

The semi-annual bond basis is therefore

$$\text{Semi-annual bond basis} = 0.7618\% \times 2 = 1.5236\%$$

6.10 Term structure of interest rates

6.10.1 Spot rates

- A spot rates is an interest rate starting today for a specific period. These can be used as a discount rate for future cashflows.
- Zero-coupon bonds may be used to infer a spot rate. Recalling equation 1.15, we can re-write it slightly to allow the interest rate to be variable, and thus give

$$PV = \sum \frac{CF_i}{(1 + \text{Spot}_i)^i} \tag{6.16}$$

EXAMPLE: Consider a 3-year 5% annual coupon bond, whete the 1-year, 2-year, and 3-year spot rates are 3%, 4% and 5% respectively. Calculate both the value of the bond and the YTM of the bond. The value of the bond can be calculated as

$$PV = \frac{5}{(1.03)} + \frac{5}{(1.04)^2} + \frac{105}{(1.05)^3} = 100.18.$$

Using this, the YTM of the bond is therefore

$$N = 3 \qquad I/Y = \qquad PV = -103.165 \qquad PMT = 5 \qquad FV = 100$$

Then, using CPT I/Y, the calculator gives an I/Y of 4.93%.

6.10.2 Par yields

- Par yields are defined as the coupon rate that a hypothetical bond at each maturity would need to offer, in order to be priced at par. [Normally solved through trial and error, or other numerical method]. Given a known set of spot rates on the yield curve, it can be calculated

$$100 = \frac{x}{(1 + s_1)} + \frac{x}{(1 + s_2)^2} + \cdots + \frac{100 + x}{(1 + s_n)^n}, \tag{6.17}$$

where $\{s_i\}$ are known and we are solving for x.

6.10.3 Forward rates

- Forward rates are for borrowing / ending for a specific period of time, starting at a defined future date. The nomenclature is

$$f_{3y2y} = \text{Rate for a 2-year loan starting 3 years from today.}$$

Using the no-arbitrage principle, combining spot rates and forward rates should make no difference, for example

$$\begin{aligned}(1+s_3)^3 &= (1+s_1)(1+f_{1y1y})(1+f_{2y1y}),\\ &= (1+s_2)^2(1+f_{2y1y}),\\ &= (1+s_1)(1+f_{1y2y})^2.\end{aligned}$$

6.10.4 Spot rate yield curves

- The spot curve is a plot of spot rates of a particular issuer (such as the US Treasury) against maturity. Typically, we would expect an upward sloping yield curve, where one receives a higher yield for longer-maturity instruments. Under certain economic conditions, the yield curve may invert, and so the spot rate for longer maturity bonds is lower.

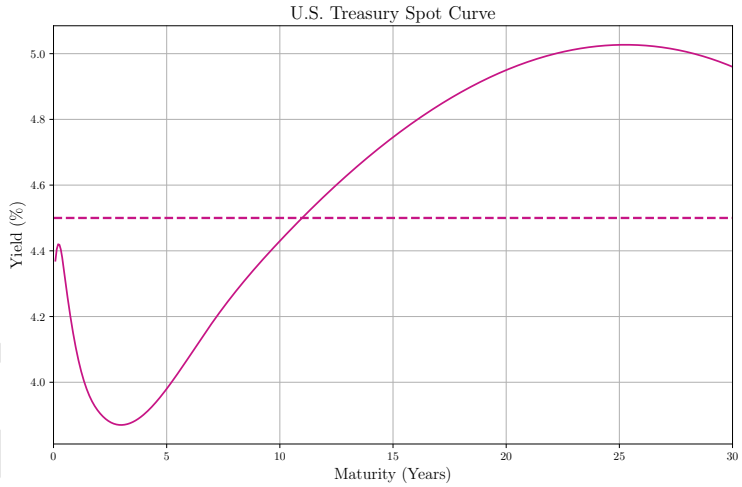


Figure 6.8: The US Treasury yield curve as of 24-July-2025. the early part of the curve is inverted, before becoming upward sloping. Yields are quoted on a semi-annual basis. Source: Federal Reserve Economic Data, https://fred.stlouisfed.org/, accessed 27-July-2025.

- For coupon bonds, the yield curve shows the YTM for a similar type of actively-traded coupon bonds at various maturities.
  - Yields must be estimated from bond prices due to illiquidity, so on-the-run bonds are typically used
  - Gaps in the curve may exist due to insufficient on-the-run securities of a particular maturity existing
  - Tax distortions are caused by bonds trading above / below par
- The par-bond yield curve is the yield curve of par yields for various maturities. This avoids the practical issues when using coupon bond yields (constructed from spot curves)
- The forward yield curve gives the forward rates for bonds or money market securities for annual periods in the future
  - Forward rates drive spot rates, which drive par yields
  - Forward rates are typically quoted on a semi-annual basis

6.11 Interest rate risk and return

- The sources of return on a fixed income instrument are

  1. Coupon and prinicpal payments
  2. Interest from reinvested coupons over the holding period
  3. Any capital gain / loss

$$\text{Yield per annum} = \left[\frac{\text{End price} + \text{Coupons} + \text{Interest}}{\text{Beginning price}}\right]^{\frac{1}{N}} - 1 \tag{6.18}$$

$$N = \text{Time horizon} \qquad I/Y = \qquad PV = \text{Beginning price}$$
$$PMT = 0 \qquad FV = \text{End price} + \text{coupons} + \text{interest}$$

  - An investor who holds a fixed-rate bond to maturity will earn an annualised return equal to the YTM of the bond when purchased if the YTM is unchanged over the life of the bond
  - An investor who sells the bond before maturity will earn a rate of return equal to the YTM at purchase if the YTM has not changed since purchase
  - If the market YTM increases between purchase and the first coupon being paid, an investor holding the bond to maturity will earna a higher realized return than the original YTM when purchased
  - If the market YTM increases between purchase and the first coupon being paid, an investor holding the bond for a short period will earn a lower realised return than the original YTM when purchased

  $\Rightarrow$ Over longer periods, reinvestment interest becomes more significant. In order to offset the price / reinvestment risk, an investor should match Macaulay Duration to investment horizon

6.11.1 Horizon yield

- Horizon yield is the compound annual return over the investment horizon

  EXAMPLE: Consider a 6% annual-pay 3yr bond, purchased as 7% YRM and held to maturity. Calculate the compound annual return.

  First we calculate the beginning price

  $$N = 3 \qquad I/Y = 7 \qquad PV = \qquad PMT = 6 \qquad FV = 100$$

  so the price is 97.376.

  Now calculating the FV of the coupons and reinvested interest

  $$N = 3 \qquad I/Y = 7 \qquad PV = 0 \qquad PMT = -6 \qquad FV =$$

  gives the FV of the coupons + any reinvestment income as 19.289. There are three payments of 6, so the interest income is 1.289

  The overall return is therefore

  $$N = 3 \qquad I/Y = \qquad PV = 97.376 \qquad PMT = 0 \qquad FV = 100 + 19.289$$

  which gives $I/Y = 7$, as expected.

6.11.2 Carrying value

- The carrying value of the bond is defined as the value at some point after purchase, assuming the original yield has not change.
  - Pull-to-par as time goes by, (see figure 6.7) on a constant yield-price trajectory, assuming YTM is unchanged.
  - Balance sheet value is reported at carrying value. Capital gain / loss measured at the carrying value

EXAMPLE: Consider an investor who has purchased a 20yr bond, paying a 5% semi-annual coupon, bought at a YTM of 6%. The investor sells it in 5 years for 91.40.

The carrying value in 5 years is

$$N = 30 \qquad I/Y = 3 \qquad PV = \qquad PMT = 2.5 \qquad FV = 100$$

giving a PV of 90.20. The capital gain / loss is therefore

$$91.40 - 90.20 = 1.20$$

capital gain per 100 face value owned.

EXAMPLE: Consider a 3yr 6% annual bond with a YTM of 7% bought for 97.376. The bond is sold after 2 years when the YTM is still 7%. Determine the carrying value and annualised return

The carrying value is given by

$$N = 1 \qquad I/Y = 7 \qquad PV = \qquad PMT = 6 \qquad FV = 100$$

giving $PV = 99.065$.

The coupon and reinvested interest income is given by

$$N = 2 \qquad I/Y = 7 \qquad PV = 0 \qquad PMT = -6 \qquad FV =$$

$FV = 12.420$, of which 12 is from coupon payments, and the remaining 0.420 is from reinvestment income.

The overall return is given by

$$N = 2 \qquad I/Y = \qquad PV = 97.376 \qquad PMT = 0 \qquad FV = 112.420$$

so I/Y is 7%, as expected, since the YTM is unchanged.

EXAMPLE: Consider a 3yr 6% annual bond with a YTM of 7% bought for 97.376. The investor holds the bond for a period of 1 year, and the YTM rises to 8% before the first coupon

The coupon payments and reinvestment interest income is given by

$$N = 3 \qquad I/Y = 8 \qquad PV = 0 \qquad PMT = 6 \qquad FV =$$

$FV = 19.478$.

The overall return is therefore

$$N = 3 \qquad I/Y = \qquad PV = 97.376 \qquad PMT = 0 \qquad FV = 119.478$$

which gives an $I/Y = 7.06\%$. This is higher that the original 7% since the bond is held to maturity.

EXAMPLE: Consider a 3yr 6% annual bond with a YTM of 7% bought for 97.376. The investor holds the bond to maturity, and the YTM rises to 8% before the first coupon

The carrying value is given by

$$N = 2 \qquad I/Y = 8 \qquad PV = \qquad PMT = 6 \qquad FV = 100$$

giving $PV = 96.433$.

The coupon and reinvested interest income is simply 6, the value of the coupon payment after one year, as this has no time to accrue any interest.

The overall return is given by

$$N = 1 \qquad I/Y = \qquad PV = 97.376 \qquad PMT = 0 \qquad FV = 96.433 + 6$$

so I/Y is 5.19%, lower than the original YTM, as there is no time for the reinvestment rate to offset the capital loss incurred.

Over short time-horizons, the price risk dominates

EXAMPLE: Consider a 3yr 6% annual bond with a YTM of 7% bought for 97.376. The investor holds the bond to maturity, and the interest rate falls to 6% before the first coupon is paid. The coupon payments and reinvestment interest income is given by

$$N = 3 \qquad I/Y = 6 \qquad PV = 0 \qquad PMT = 6 \qquad FV =$$

$FV = 19.102$.

The overall return is therefore

$$N = 3 \qquad I/Y = \qquad PV = 97.376 \qquad PMT = 0 \qquad FV = 119.102$$

which gives an $I/Y = 6.94\%$. This is lower that the original 7% since the bond is held to maturity.

Over long horizons, the reinvestment rate risk dominates

- In summary

Short horizon	Price risk dominates
Long horizon	Reinvestment risk dominates (No price risk if held to maturity)

6.11.3 Balancing price and reinvestment risk

- Price and reinvestment risk must be considered when investing in fixed income securities. Holding a bond to maturity eliminates all price risk, but reinvestment risk becomes significant.
- The price risk and reinvestment risk perfectly offset when the Macaulay duration matches the investment horizon exactly. The Macaulay duration is defined

$$\text{Macaulay duration} = \frac{\sum \frac{CF_i}{(1+r)^{t_i}} \times t_i}{\sum \frac{CF_i}{(1+r)^{t_i}}}. \tag{6.19}$$

In other words, it is the weighted-average time to receive future cash flows.

- The duration gap is defined

$$\text{Duration gap} = \text{Macaulay duration} - \text{Investment horizon}. \tag{6.20}$$

A positive duration gap is dominated by price risk, and a negative duration gap by reinvestment risk.

- Making reference to sensitivity to interest rates,

Low yields	Steeper curve on price-yield curve
Low coupon	Wait longer to receive cash flows
Long maturity	Higher duration

- Consider a 5yr 11% annual coupon bond, priced at 86.59 with a YTM of 15%. Calculate the Macaulay duration of this bond.

Using equation 6.19, we find

$$\frac{1\times\frac{1}{1.15}+2\times\frac{11}{1.15^2}+3\times\frac{11}{1.15^3}+4\times\frac{11}{1.15^4}+5\times\frac{111}{1.15^5}}{\frac{1}{1.15}+\frac{11}{1.15^2}+\frac{11}{1.15^3}+\frac{11}{1.15^4}+\frac{111}{1.15^5}}=\frac{348.99}{86.59}=4.03 \text{ years}$$

6.11.4 Modified duration

- The modified duration is defined

$$\text{Modified Duration} = \frac{\text{Macaulay Duration}}{1+\text{YTM}}, \tag{6.21}$$

where Macaulay duration is defined in equation 6.19. Generally, the YTM in the denominator should be the interest per period, so in the case of a semi-annual coupon bond, it should be replaced with $\frac{\text{YTM}}{2}$.

- Mathematically, the Modified Duration can be thought of as the derivative of price with respect to yield. If we recall equation 1.15, and sum over all future cash flows, we get

$$PV = \sum \frac{CF_i}{(1+\text{YTM})^{t_i}}. \tag{6.22}$$

For simplicity, we take $t_i = i,\ \forall\ t_i$. Taking the derivative, we get

$$\begin{aligned}
\frac{\partial\, PV}{\partial\, \text{YTM}} &= \sum \frac{\partial}{\partial\, \text{YTM}}\left[\frac{CF_i}{(1+\text{YTM})^i}\right], \\
&= \sum \frac{CF_i}{(1+\text{YTM})^{i+1}} \cdot -i \cdot \underbrace{\frac{\partial\, PV}{\partial\, \text{YTM}}\left[(1+\text{YTM})\right]}_{=1}, \\
&= -\sum \frac{i\cdot CF_i}{(1+\text{YTM})^{i+1}}, \\
&= -\frac{1}{(1+\text{YTM})}\sum \frac{i\cdot CF_i}{(1+\text{YTM})^i}, \\
&= -\frac{\left[\sum \frac{i\cdot CF_i}{(1+\text{YTM})^i}\right]}{(1+\text{YTM})}, \\
&= -\frac{\text{Macaulay Duration}}{(1+\text{YTM})}.
\end{aligned} \tag{6.23}$$

The tangent to a price-yield curve will be negative at all points, but is always expressed as a positive number, so the sign is just convention.

- The modified duration is just a linear approximation as to how the price varies when the yield changes., so we can approximate

$$\text{Price(YTM)} \approx \text{Price}|_{\text{YTM}_0} - [\text{Mod. Dur.}] \times \Delta\text{YTM} \tag{6.24}$$

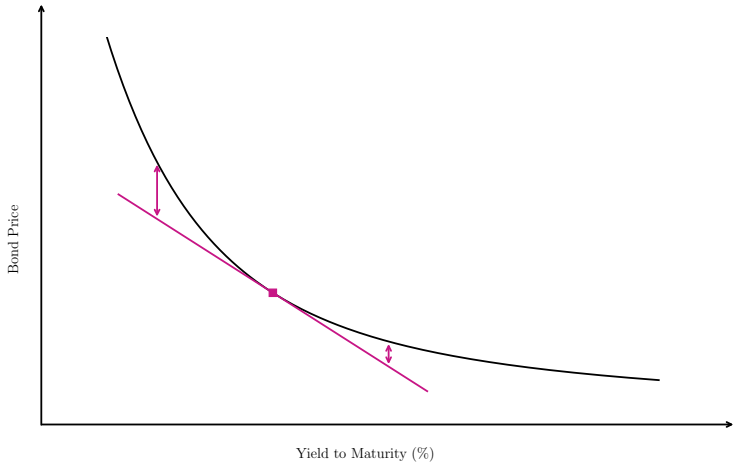


Figure 6.9: Price-yield curve for a bond with a tangent line drawn modified duration, and arrows marking the difference between the linear approximation and the actual price-yield curve.

Figure 6.9 clearly demonstrates that the linear approximation provided by modified duration underestimates the price after any change. If yields were to fall, the linear approximation underestimates the appeciation in price, and if yields were to rise, the linear approximation overestimates the fall in price,

$$\text{Estimated price} < \text{True price.}$$

Larger changes in yield lead to a worse estimate.

EXAMPLE: Consider a 5yt 11% annual coupon bond. With a YTM of 15%, it is priced at 86.59, and has a Mod Dur of 3.5. The expected change in yield is +50 bps. Calculate the change in price.

$$\begin{aligned}\Delta\text{Price} &= -3.5 \times 0.5\% \\ &= -1.75\%\end{aligned}$$

$$86.59 \times (1 - 0.0175) = 85.075$$

Using a full repricing method, the PV is 85.092, which is higher than the estimate above.

6.11.5 Approximate modified duration

- Modified duration is an exact calculation, and may be computationally intensive. Instead, we can use approximate modified duration. This is an approximation using the average of the price rise /fall from V_+ and V_- to give an estimate of the gradient. This is demonstrated in Figure 6.10.

$$\text{Approx. Mod. Dur.} = \frac{V_- - V_+}{2\Delta\text{YTM}} \cdot \frac{1}{V_0}, \tag{6.25}$$

where:

V_0 is the current price

V_- is the price at $\text{YTM} - \Delta\text{YTM}$

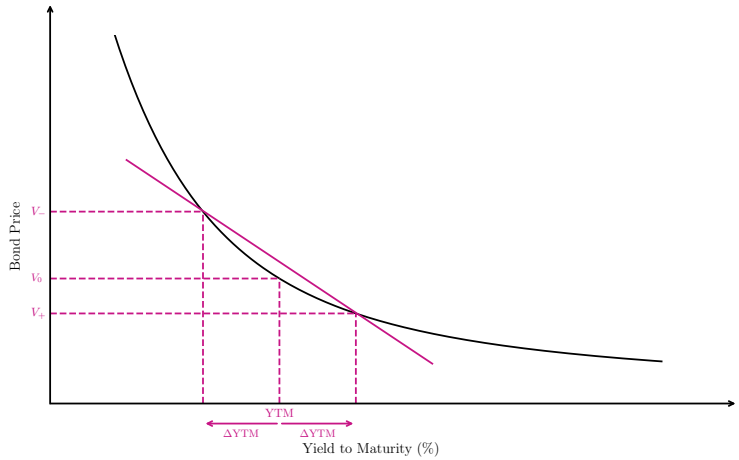


Figure 6.10: Price-yield relationship with approximate modified duration shown.

V_+ is the price at YTM + ΔYTM

EXAMPLE: Consider a 5yr 11% annual bond priced at 86.57. $V_+ = 85.092$ and $V_- = 88.127$. ΔYTM = 50bps. Calculate the approximate modified duration.

Using equation 6.25,

$$\text{Approx. Mod. Dur.} = \frac{88.127 - 85.092}{2 \times 0.005} \cdot \frac{1}{86.57} = 3.505$$

6.11.6 Money duration

- So far, all of these duration measures refer to a relative, (%) price change of the bond. The dollar deviation incorporates money values rather than relative values into price / yield change estimates from the (approximate) modified duration.

$$\begin{aligned} \%\text{ price change} &= -\text{Mod. Dur.} \times \Delta\text{YTM}, \\ \text{Money duration} &= -\text{Mod Dur.} \times \Delta\text{YTM} \times \text{Bond price}. \end{aligned} \tag{6.26}$$

  This gives the approximate change in absolute value for a given change in yield.

- As before, this is more accurate for $\Delta\text{YTM} \ll 1$

- This is used to determine the price value of a basis point (PVBP, DV01, BPV) (having meanings "price value of a basis point", "dollar value 01", "basis point value"). These all are equivalent and refer to the price change in $ terms of a 1bp change in yield.

  EXAMPLE: Consider a bond with Mod. Dur. = 7.42. It has a full price of 101.32 per 100 face value, and a par value of 2,000,000. Calculate the impact of a 25bp increase in YTM on the market value. Recalling equation 6.21,

$$\text{Mod. Dur.} = \frac{\text{Macaulay Duration}}{1 + \text{YTM per period}}.$$

  The market value of the bond is given by

$$\text{Market value} = 2{,}000{,}000 \times \frac{101.32}{100} = 2{,}026{,}400,$$

so the fall in market value is therefore

$$\begin{aligned}\Delta\text{Mkt. val.} &= -\text{Mod. Dur.} \times \Delta\text{Yield} \times \text{Mkt. val.},\\ &= -7.42 \times 0.0025 \times 2,026,400,\\ &= -37,589.72.\end{aligned}$$

In reality, as shown by figure 6.9, this is an overestimate of the fall in value.

EXAMPLE: Consider a 20yr annual-paying straight bond, priced at 101.39, with a par value of $1,000,000$. Calculate the DV01 effect on the full par value of the bond

The current YTM is given by

$$N = 20 \qquad I/Y = \qquad PV = -101.39 \qquad PMT = 6 \qquad FV = 100$$

so I/Y is 5.88%.

A 1 bp move either way gives a new YTM of 5.87% in the downward move, and 5.89% in the upward move.

First calculating V_-,

$$N = 20 \qquad I/Y = 5.87 \qquad PV = \qquad PMT = 6 \qquad FV = 100$$

so V_- is 101.507.

Now calculating V_+,

$$N = 20 \qquad I/Y = 5.87 \qquad PV = \qquad PMT = 6 \qquad FV = 100$$

so V_+ is 101.273.

The price change per 100 par value is therefore

$$\frac{101.507 - 101.273}{2} = 0.1117$$

per 100 par value, which for the full $1,000,000$ is a price change of $1,170$.

6.11.7 Convexity

- Modified duration is a linear approximation of the price / yield relationship. As figure 6.9 shows, the accuracy of the approximation decreases as you move further from the point of expansion.
- We can improve this approximation by adding a second order term to the Taylor expansion given by equation 6.24, which would take the form

$$\text{Price(YTM)} \approx \text{Price}|_{\text{YTM}_0} - [\text{Mod. Dur.}] \times \Delta\text{YTM} + \frac{1}{2}\text{Cvxty.} \times (\Delta\text{YTM})^2 \times P_0. \quad (6.27)$$

- Recalling the derivation for modified duration from equation 6.23, we can extend this to calculate the convexity exactly by taking the second derivative of price with respect to YTM.

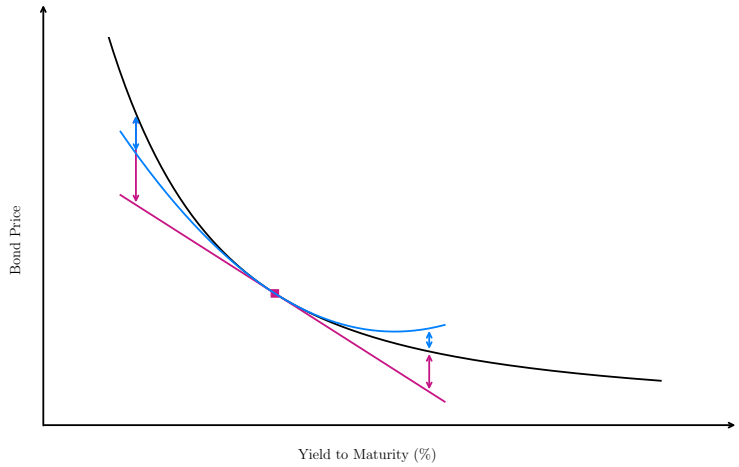


Figure 6.11: The convexity approximation for a bond is a closer approximation to the true yield-price relationship, and we can see clearly that the approximation is a better match than just the linear approximation

$$\begin{aligned}
\frac{\partial^2 PV}{\partial \text{ YTM}^2} &= \sum \frac{\partial^2}{\partial \text{ YTM}^2}\left[\frac{CF_i}{(1+\text{YTM})^i}\right], \\
&= -\sum \frac{\partial}{\partial \text{ YTM}}\left[\frac{i \cdot CF_i}{(1+\text{YTM})^{i+1}}\right], \\
&= -\sum -(i+1)\cdot \frac{i \cdot CF_i}{(1+\text{YTM})^{i+2}} \cdot \underbrace{\frac{\partial}{\partial \text{ YTM}}\left[(1+\text{YTM})\right]}_{=1} \\
&= \sum \frac{i(i+1)\cdot CF_i}{(1+\text{YTM})^{i+2}} \\
&= \frac{\sum \frac{i(i+1)\cdot CF_i}{(1+\text{YTM})^i}}{(1+\text{YTM})^2}
\end{aligned} \tag{6.28}$$

The convexity of a single cash flow at period i is given by

$$\text{Convexity}|_i = \frac{i(i+1)}{(1+\text{YTM})^2}.$$

For a coupon-paying bond, the convexity is simply the weighted-average convexity of individual cashflows. as can be seen above.

- Again, similar to duration, we can calculate an approximate convexity, using a similar method detailed in equation 6.10.

$$\begin{aligned}
\text{Approximate convexity} &= \frac{V_- + V_+ - 2V_0}{(\Delta\text{YTM})^2 \cdot V_0} \\
&= \left[\frac{\frac{V_- - V_0}{\Delta\text{YTM}} - \frac{V_+ - V_0}{\Delta\text{YTM}}}{\Delta\text{YTM}} \cdot \frac{1}{V_0}\right]
\end{aligned}$$

- Convexity is impacted by the same factors affecting duration, such as a long maturity, a low coupon rate, and a low YTM.
- If duration is equal between bonds, the one with cash flows dispersed over a greater time will have a greater convexity

$$\%\Delta\text{Price} = -\text{Mod. Dur.} \times \Delta\text{YTM} + \frac{1}{2} \times \text{Convexity} \times (\Delta\text{YTM})^2 \tag{6.29}$$

- In the same vein as duration, money convexity converts relative changes to monetary price changes.

$$\Delta\text{Bond price} = -\text{Money Dur.} \times \text{YTM} + \frac{1}{2} \times \text{Money cvxty.} \times (\Delta\text{YTM})^2 \tag{6.30}$$

EXAMPLE: Consdier a 5yr 11% coupon bond with a YTM of 15% and price of 86.59138. It has Mod. Dur = 3.5 and Cvx = 16.9. Estimate the new price with $\Delta\text{YTM} = -50\text{bps}$

$$\begin{aligned}\%\Delta P &= -3.5 \times (-0.005) + \frac{1}{2} \times 16.9 \times (0.005)^2 \\ &= 1.75\% + 0.0211\% \\ &= 1.7711\%\end{aligned}$$

The new price is therefore

$$\begin{aligned}\text{New price} &= 86.59138 \times 1.017711, \\ &= 88.125.\end{aligned}$$

Using full reval, the new price would be calculated 88.127.

Now, taking the full par value of the bond to be $10,000,000$,

$$\begin{aligned}\text{Money duration} &= \text{Mod. Dur.} \times \text{Price} \\ &= 3.5 \times 0.8659138 \times 10,000,000 \\ &= 30,306,983\end{aligned}$$

$$\begin{aligned}\text{Money convexity} &= \text{Convexity} \times \text{Price} \\ &= 16.9 \times 0.8659138 \times 10,000,000 \\ &= 145,339,432\end{aligned}$$

$$\begin{aligned}\text{Duration effect} &= -\text{Money duration} \times \Delta\text{YTM} \\ &= -30,306,983 \times -0.005 \\ &= 151,934,92\end{aligned}$$

$$\begin{aligned}\text{Convexity effect} &= \frac{1}{2} \times \text{Money convexity} \times (\Delta\text{YTM})^2 \\ &= \frac{1}{2} \times 146,339,432 \times 0.005^2 \\ &= 1,829.25\end{aligned}$$

$$\text{Total change} = 153,364.17$$

6.11.8 Portfolio duration and convexity

- There are two apporaches to aggregate several bonds in a portfolio
  - Single calculation of portfolio duration adn convexity, based upon aggregate cash flows of all bonds in the portfolio
  - Weighted average of bond deviation / convexity by market value. This assumes a parallel shift in YTM.

6.12 Curve-based and empirical fixed income risk measures

- Recalling the following measures, these are all easily applicable and intuitive for straight bonds, that is bonds which have no embedded optionality in them.

Macaulay Duration	Weighted average time to receive cash flows
Modified Duration	$\frac{\text{Macaulay}}{1+\text{Yield}}$ = First order taylor expansion
Approximate modified duration	Two-point gradient formula
Money duration	Monetary impact of duration
DV01	Dollar value of a 1bp move

Table 6.3: Table showing duration metrics applicable to straight bonds

- If instead we are interested in bonds with embedded options, such as callable and putable bonds, these have uncertain future cash flows
  - A fall in rates impacts a callable bond as this benefits the issuer
  - A rise in rates impacts a putable bond as this benefits the investor "Floor price"

  These are referred to as contingent cash flows, as they only occur if a particular scenario occurs.
- The reason these are appropriate risk measures for straight bonds is that the YTM is well-defined, and all cash flows are certain, due to the absence of any options. The Macaulay duration (equation 6.19) and Modified Duration (equation 6.21) are both yield-based risk measures.
- For bonds with embedded options, we need to use curve-based risk measures

6.12.1 Effective duration and effective convexity

- Effective duration is a measure of interest rate sensitivity for bonds with embedded options. This is a curve-based statistic. It is similar to the modified duration, but instead of a shock to a specific point on the yield curve, it involves a shock across the curve, impacting all maturity yields. A similar shock may be applied to define effective convexity.

$$\text{Effective duration} = \frac{V_- - V_+}{2V_0 \cdot \Delta\text{Curve}} \tag{6.31}$$

$$\text{Effective duration} = \frac{V_- + V_+ - 2V_0}{V_0 \cdot [\Delta\text{Curve}]^2} \tag{6.32}$$

- Using these in a Taylor expansion of the price,

$$\%\Delta\text{Expected price change} = -\text{Eff. Dur.} \times \Delta\text{Curve} + \frac{1}{2}\text{Eff. Cvx.} \times [\Delta\text{Curve}]^2$$

  For a straight bond, Mod. Dur. $\equiv$ Eff. Dur., since ΔCurve $\equiv \Delta$YTM.

6.12.2 Price-yield relationship for callable bonds

- A callable bond is one where the issuer holds the right to call in the bond at a specified price.

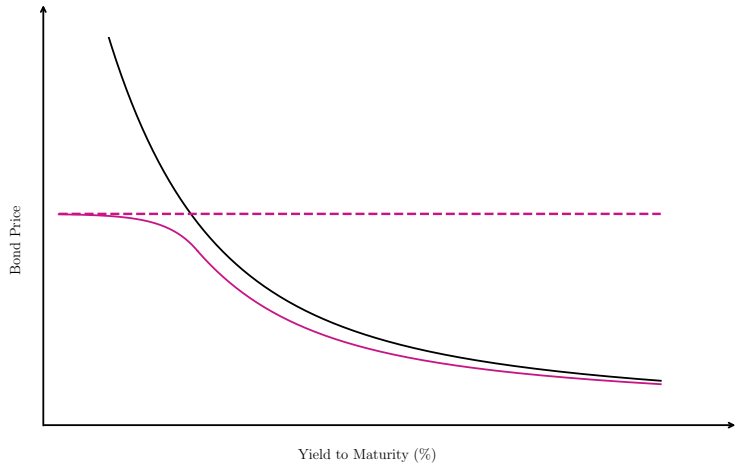


Figure 6.12: The price-yield relationship of a callable bond is shown in purple, compared to that of a straight bond. The price ceiling is also shown. For yields lower than the point of inflection, the curve exhibits negative convexity

- If yields fall, the price of a straight bond would rise, but the price of the callable bond is capped at the call price. This causes negative convexity in the price-yield curve at low yields for the bond. Because of this, investors tend to require a higher yield, or lower price to compensate them for this additional risk.
- A callable bond is preferable to investors during stable period. In times of volatility, an investor is more likely to sell callables.
- Also note that MBS securities have an embedded short call option

6.12.3 Price-yield relationship for putable bonds

- A putable bond is one where the bond holder holds the right to sell the bond back to the issuer at a specified price.
- If yields rise, the price of the bond is floored at the put price. The pt option becomes more valuable as the probability of it being exercised becomes higher.
- Opposite to callable bonds, they show excess convexity at high yields, so a larger rise in yields results in a lesser fall in price.

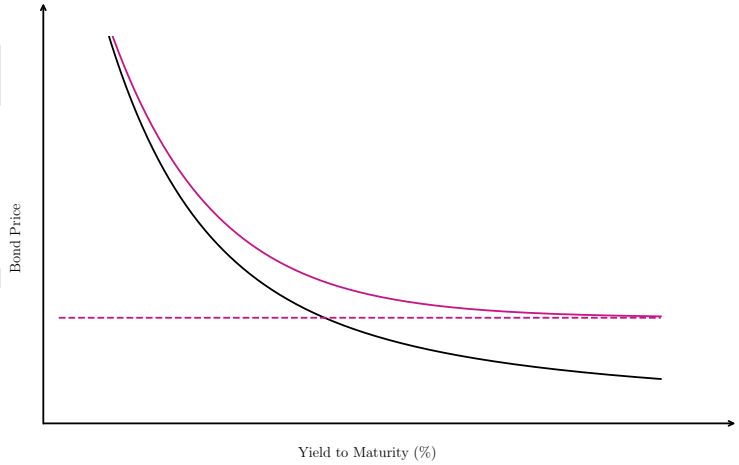


Figure 6.13: The price-yield relationship of a putable bond is shown in purple, compared to that of a straight bond. The price floor is also shown. For high yields, the price asymptotically approaches the price floor.

6.13 Key-rate duration

- The key-rate duration (KRD) measures the impact of non-parallel shifts in the benchmark yield curve. Up until this point, we have only been considering parallel shifts, but it is perfectly conceivable that the yield curve may move in different manners.
- Examples of the various types of shifts in the yield curve are shown in figure 6.14.

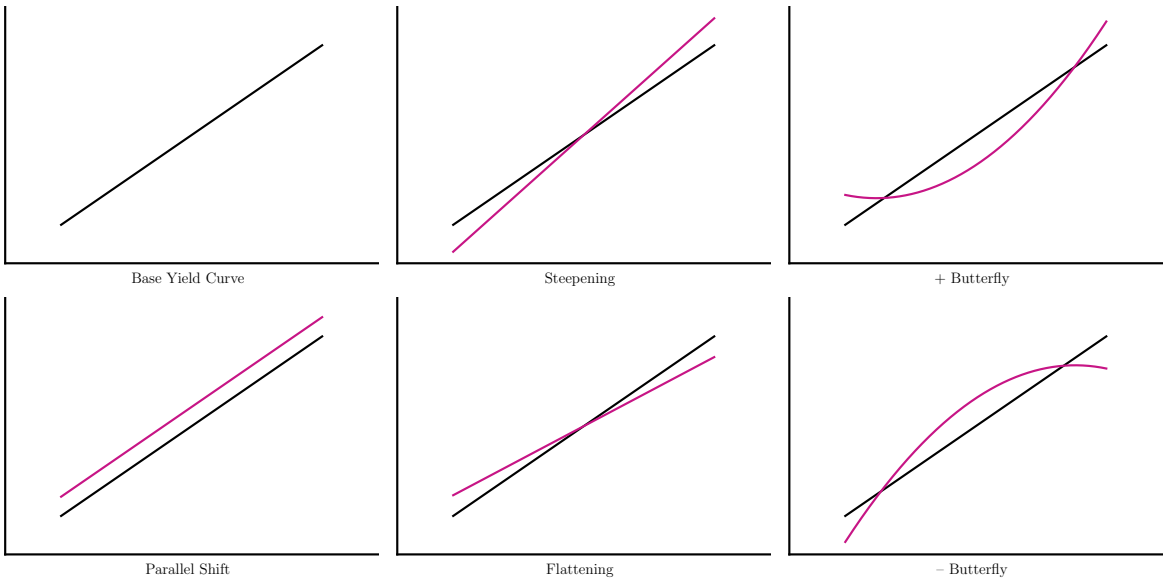


Figure 6.14: Different changes to the yield curve. For each of these, the x-axis shows maturity and y-axis shows yield. We see the base curve in black in each of the charts, and possible types of shift from the base curve.

- KRD is defined as the sensitivity of the value of a portfolio to changes in the benchmark yield of a specific maturity, holding alll other yields constant.
- KRDs can be calculated by using the following steps:
  1. Calculate the Macaulay duration
  2. Use this to determine the Modified Duration
  3. KRD is then given by the weight of exposure to that point multiplied by the modified duration at that point,
  $$\text{KRD}_i = \text{Mod. Dur.} \times w_i. \tag{6.33}$$
  4. Any relative change in price is given by the product of KRD and change in yield
  $$\%\Delta\text{Price} = -\text{KRD} \times \Delta\text{Yield} \tag{6.34}$$
- Each maturity has its own KRD.

$$\text{Effective duration} = \sum \text{KRD}_i \tag{6.35}$$

Shaping risk is the effect of a non-parallele shift in the yield curve of a bond portfolio. The effect of this non-parallel shift is quantified by KRD.

EXAMPLE: Consider an equal-weighted portfolio invested in two zero-coupon bonds (ZCB). The first has 3 years to maturity and has a YTM of 5%. The other has 10 years to maturity, and has a YTM of 6%. Both have an annual compounding period. What is the performance of the portfolio if the 5yr yield rises 50bps and 10yr yield falls by 25bps.

Bond	Macaulay	Mod. Dur.	KRD
5yr	5yrs	$\frac{5}{1.05} = 4.672$	$0.5 \times 4.762 = 2.381$
10yr	10yrs	$\frac{10}{1.06} = 9.434$	$0.5 \times 9.434 = 4.717$

Then, using this to work out the price impact,

$$\begin{aligned}\%\Delta\text{Price} &= \underbrace{-2.381 \times 0.0050}_{-0.0119} + \underbrace{-4.717 \times -0.0025}_{+0.0118} \\ &= -1.19\% + 1.18\% \\ &= -0.01\%\end{aligned}$$

6.13.1 Empirical and analytical duration

- Analytical measures are based on mathematical analysis. This includes Macaulay duration, Mod Dur, approx. Dur., and eff. Dur.
- Empirical measures are based on estimates using historical moves of benchmark yield changes and bond price changes. This is useful for riskier bonds, when benchmark yields and credit spreads are decorrelated

6.14 Credit risk

- Credit risk can largely be broken up into four main pieces.
  - Probability of default
  - Loss given default / recovery rate
  - Exposure
  - Liquidity risk
- Bottom-up credit analysis focuses on the risk of coupon / principal sums not being paid by the issuer

Capacity	Borrower's ability to make payments on time
Character	Borrower's commitment to debt obligations
Capital	Capital available to borrower to reduce reliance on debt financing
Collateral	Value of assets pledged to lender as security against loans
Covenants	Legal terms agreed by all parties

Table 6.4: Factors likely to be used by an analyst when conducting bottom-up credit analysis on an issuer

- Top-down credit analysis focuses on the macro environment of the issuer

Conditions	General economic conditions affectng ability to make payments
Country	Geopolitical environment, legal and political systems that apply to the debt
Currency	Movements in exchange rates affecting a borrower's ability to service foreign-denominated debt

Table 6.5: Factors likely to be used by an analyst when conducting top-down credit analysis on an issuer

- Sources of repayment

  - Sources of repayment are dependent not only on the nature of the borrower, but also the specific terms of the bond issue
  - Secured corporate issues are backed by the operating cash flows and investments of the issuer, as well as the fact that any cash flows from collateral assets are pledged as security
  - Unsecured corporate issues are backed by the operating cash flows and investments of the issuer
  - Secondary sources include asset sales, divestiture of subsidiaries, or additional debt / equity issuance

- Sovereign debt
  - Sovereign debt is generally backed by tax revenue, tariffs, and other fees
  - Additional debt issuance and sale of public assets (privatisation) are other ways of raising money
  - Sovereign credit risk factors include poor economic conditions, political uncertainty, fiscal deficits, and high debt levels

- Illiquidity and solvency
  - Deault can result from a debt issuer being insolvent, or having insufficient liquidity
    * Insolvency is reached when the value of any assets is less than liabilities
    * Illiquidity is when there is insufficient cash to meet obligations

- A cross-default clause protects investors by stating that any default on one bond casues a default on all issues. Pari-Passu ensures bonds of equal rank are treated equally

6.14.1 Measuring credit risk

- Measuring credit risk involves assessing the expected loss from a debt investment in the event of issuer default.

$$\text{Expected loss} = \underbrace{\text{Probability of default}}_{\text{Annualised basis}} \times \text{Loss given default} \tag{6.36}$$

$$\text{Credit spread} \approx \text{Probability of default} \times \text{Loss given default (\%)} \tag{6.37}$$

- In order to estimate the probability of default, an analyst may look at any of the following

EBIT Margin	High EBIT margin implies lower risk
Interest coverage ratio, $\frac{\text{EBIT}}{\text{Interest}}$	High coverage implies the issuer is likely to be able to make required interest payments
Leverage multiples $\left(\text{e.g.}\frac{\text{Debt}}{\text{EBITDA}}\right)$	Low leverage multiples suggest a comparatively low debt burden on the company
CFO	A high cash flow to net debt ratio suggest the debt payments are easily serviceable

Detriorating financial strength impacts the probability of default

- In order to estimate the loss given default, the debt seniority (senior / junior / subordinated) and whether the debt is backed by collateral are useful indicators.

Senior secured debt has a lower loss given default

EXAMPLE: Consider a 4% coupon bond trading at par. The issuer has a probability of default of 3%, and a recovery rate of 75%. A government security of similar maturity is trading with YTM = 2.5%.

Since it is trading at par,

$$\text{Corp YTM} = 4\%.$$

Using equation 6.37, the spread demanded by an issuer is approximately

$$\text{Required spread} = 0.03 \times (1 - 0.75) = 0.03 \times 0.25 = 0.75\%.$$

The inferred spread from comparing the YTM of the government bond and the corporate bond is $4\% - 2.5\% = 1.5\%$. As this is greater than the required spread, this does adequately compensate the investor.

6.14.2 Credit ratings

- Credit rating agencies assign forward-looking ratings to both issuers and specific bond issues, based on qualitative and quantitative credit risk factors
- Investors use these ratings to compare credit-worthiness of bonds. Changes in ratings provide a broad overview of changing market conditions
- Credit migration risk is the risk of a credit downgrade

Investment Grade		**High Yield / Junk**	
Moody's	S&P, Fitch	Moody's	S&P, Fitch
Aaa	AAA	Ba1	BB+
Aa1	AA+	Ba2	BB
Aa2	AA	Ba3	BB
Aa3	AA−	B1	B+
A1	A+	B2	B
A2	A	B3	B−
A3	A−	Caa1	CCC+
Baa1	BBB+	Caa2	CCC
Baa2	BBB	Caa3	CCC−
Baa3	BBB−	C, C	C, D

Table 6.6: Table showing the different rating schemes used by the three main rating agencies. The lower the rating, the more equity-like the behaviour of a bond is observed to be.

- Using credit ratings is not an infallible method to assess a particular issuer or bond issue.
  - Ratings lag market pricing – Spreads change much faster than ratings
  - Risks may be difficultto assess (i.e. litigation, natural disasters)

  Additional due diligence should be employed.
- Credit rating agencies give a rating to both issuers as well as specific bond issues
  - Issuer – Corporate family rating (CFR)

  - Bond issue – Corporate credit rating (CCR)

  Differences may arise between ratings of CFR and CCR. This is called notching, but is a practice less common for investment-grade rated firms.

6.14.3 Credit spread risk

- Credit spread risk is the risk of yield spreads widening and bond price falling. As default is unlikely to occur suddenly, credit spread risk is a primary concern for investors
- Drivers include
  - Macroeconomic factors (economic contraction)
  - Issuer-specific factors (ligiation, natural disasters)
  - Market trading factors (market crisis, i.e. GFC)

Macroeconomic factors

- Credit cycles are strongly correlated to the economic cycle
  - Economic expansions – Credit curves fall and steepen, as the near-term probability of default falls
  - Economic contractions – Credit curves rise and flatten, and the HY curve may invert as the near-term probability of default rises
  - High-yield spreads are more sensitive to changes in the economic conditions, with a wider dispersion of yield spreads across issuers
  - Flight to quality – In a crisis, investors sell risky assets and buy safe assets. Bid-ask spread widen more for HY than IG bonds typically

Issuer-specific factors

- Issuer-specific factors have a significant impact on yield spread level and volatility. The financial performance of the issuer has a significant impact on credit rating and yield spread of the debt.
- Comparisons may be drawn by comparing an issuer's yield spread to the average yield spread with a similar credit rating.
- Greater difficulty in servicing the debt will bring a higher yield

Market factors

- This is linked to transaction costs of trading a bond
  - Bid-ask spread[4] – Wider spread implies a higher t-cost, and so a higher market liquidity risk
  - Larger issuers are those with more debt outstanding, and have more actively traded debt
  - Market stress and crisis may impact both of these

[4] An investor would sell at bid and buy at ask. As such, Bid > Ask, always.

- In summary;

$$\text{Mid-price} = \frac{\text{Bid} + \text{Offer}}{2}$$
$$\text{Liquidity spread} = \text{Yield}|_{\text{Bid}} - \text{Yield}|_{\text{Ask}}$$
$$\text{Credit spread} = \text{Yield spread over bmk} - \text{Liquidity spread}$$

EXAMPLE: Consider a 10yr 5% annual coupon bond, with a bid / offer of 99.5 100.5. The benchmark 10yr yield is 3%. Decompose the spread into credit and liquidity

From equation 6.14.3,

$$\text{Mid-price} = \frac{100.5 + 99.5}{2}.$$

Then, calculating liquidity spread, the bid yield is

$$N = 10 \qquad I/Y = \qquad PV = -99.5 \qquad PMT = 6 \qquad FV = 100$$

giving $I/Y = 5.065$.

The ask yield is

$$N = 10 \qquad I/Y = \qquad PV = -100.5 \qquad PMT = 6 \qquad FV = 100$$

giving $I/Y = 4.935$.

Using equation 6.14.3, we find

$$\text{Liquidity spread} = 5.065\% - 4.935\% = 0.130\%.$$

The $\text{yield}|_{\text{mid}} = 5\%$, since it is trading at par. The yield spread is therefore

$$\text{Yield spread} = 5\% - 3\% = 2\%,$$

and the credit spread is therefore

$$\text{Credit spread} = 2\% - 0.130\% = 1.87\%$$

6.15 Credit analysis for government issuers

- The term government issuers encompasses sovereign issuers and non-sovereign issuers, such as agencies, government sector bonds, and supranationals
- Sovereign government debt
  - The ability to service debt comes from the ability to tax economic activity in its jurisdiction.
  - Credit assessment is based on the factors which provide stable economic growth with low inflation
  - Qualitatitive and quantitative factors are relevant ot establish credit worthiness
- The term non-sovereign government debt includes
  - Agencies
    - Quasi-government entities, backed by law

    - Implicit government support, established for a specific purpose
  - Government sector banks
    - Issuing bonds for specific projects
  - Supranational issuers
    - World bank, IMF
    - Projects to alleviate poverty, encourage growth
  - Regional governments (States → Municipal bonds) issue
    - General obligation bonds – These are unsecured instruments, and are backed by instruments
    - Revenue bonds – These are covered by revenue from specific projects (i.e. tolls). Credit analysis of these is similar to corporate bonds. The focus should be on cash flows and debt coverage ratios.

- Qualitative factors include
  - Institutions and policy factors
    - "Capacity" – Economic stability
    - "Character" – Willingness to repay
  - Fiscal flexibility factors
    - Ability to increase taxes / reduce spending to ensure debt payments can be made
  - Monetary effectiveness factors
    - Ability of the central bank to vary the money supply and interest rates to encourage stable growth
    - Independence and credibility of the central bank
  - Economic flexibility factors
    - Growth trends
    - Income per capita
    - Diversity of income
  - External status factors
    - Status of local currency in international markets
    - Countries with reserve currencies are widely held for foreign reserves at central banks

- Quantitative factors include
  - Fiscal strength
    - Low debt burden ratios, such as

      $$\text{Debt} : \text{GDP}, \qquad \text{Debt} : \text{Revenue}, \qquad \text{Interest} : \text{Revenue}.$$

  - Economic growth and stability
    - High real GDP growth
    - High GDP per capita
    - Large real economy size
    - Low GDP growth volatility
  - External stability

    - High foreign currency reserves to GDP and to debt
    - Low debt to GDP
    - Over-reliance on a single commodity

6.16 Credit analysis for corporate issuers

- Similar to governments, analysis includes both qualitative and quantitative factors.
- Qualitative factors include
    - Business model – Business risk
    - Comptetitive landscape – Expected changes
    - Deviations in revenue – Issuer specific, industry specific, external
    - Covenants – Rights to issue further dbt, past actions of management
    - Accounting policies – Capitalising vs expensing, off-balance sheet expenses, changing auditors
- Quantitative factors include
    - Estimating future cash flows
        - Factors driving probability of default and loss given default
        - Expected changes over the economic cycle
        - Top-down, bottom-up, hybrid analysis

Top-down	**Bottom-up**
Industry size	Issuer-specifc assets
Market share	Liabilities
External shocks	Cash flows

- Factors that indicate higher-quality issuers include
    - Strong operating profits, recurring revenues
    - Low levels of leverage, less reliance of debt on capital structure
    - High coverage of debt service payments from periodic income
    - High levels of liquidity to meet short-term debt payments
- Financial ratios used in credit analysis include
    - EBITDA (Operating income + depreciation + amortisation)
        - This does not adjust for capital expenditures or changes in working capital. Cash needed for these uses is not available to debt holders
    - CFO (Net cash paid / received in continuing operations)
        - Net income + non-cash charges − Increase in working capital
        - Disclosed in the cash flow statement
    - FFO (Funds from operation)
        - Net income + depreciation + amortisation + deferred tax + non-cash
        - CFO excluding the change in working capital
    - FCF (Free cash flow)
        - CFO − fixed asset expenditure + net interest expense
        - Represents discretionary cash flow of the company

    - Could be paid to providers of finance after all obligations met
  - RCF (Retained cash flow)
    - CFO − dividends paid

- Ratios for corporate credit analysis include

Type	Name	Calculation
Profitability	EBIT margin	$\frac{\text{EBIT}}{\text{Revenue}}$
Coverage	EBIT to interest expense	$\frac{\text{EBIT}}{\text{Interest expense}}$
Leverage	Debt to EBITDA	$\frac{\text{Debt}}{\text{EBITDA}}$
Leverage	RCF to net debt	$\frac{\text{RCF}}{(\text{Debt} - \text{Cash \& marketable securities})}$

Table 6.7: Table listing commonly used ratios for corporate credit analysis

EXAMPLE: Consider the following two companies. Calculate the relevant ratios to determine what conclusions may be drawn about the two companies.

	ABC corp.	**DEF corp.**
Revenue	2,200,000	11,000,000
Depr. and Amort.	220,000	900,000
EBIT	550,000	2,250,000
CFO	300,000	850,000
Interest expense	40,000	160,000
Total debt	1,900,000	2,700,000
Cash + Marketable sec.	500,000	1,000,000
Dividends	30,000	200,000

Calculating the relevant ratios, we find

	ABC corp.	**DEF corp.**
EBIT Margin	**0.25**	0.205
EBIT : Interest expense	13.75	**14.06**
Debt : EBITDA	2.47	**0.857**
RCF : Net debt	0.193	**0.38**

Looking at this, we can conclude that while ABC corp. is more profitable, DEF corp is less reliant on debt

6.16.1 Priority of claims

- In the event of default, each class of debt is ranked equally. The value of any remaining assets could deteriorate from loss of cusstomers / employees, as well as any legal costs.
- In order of highest to lowest seniority, the ranking of debt with regards to priority of claims over assets is given in table 6.8

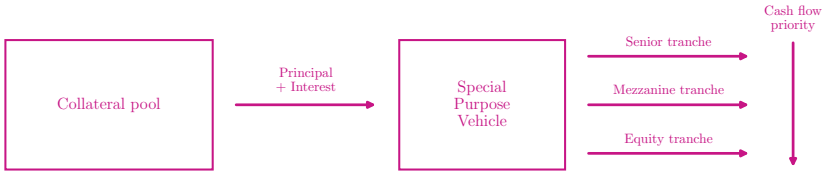
Most senior	First lien on a specific asset
	Second lien "Senior secured"
	Junior secured
	Senior unsecured
	Senior subordinated
	Subordinated
	Junior subordinated
	[Equity preferred]
Least senior	*[Equity common]*

Table 6.8: Ranking of different tiers of debt with regards to priority of claims over any assets

- Structural subordination occurs when both a parent company and subsidiary have outstanding debt. In theory, any cash from the subsidiary may be swept up by the parent company to service its own debt. This is called upstreaming. Covenants may however be put in place to restrict this, in which case subsidiary bonds would have priority claim on the subsidiary cash.

6.17 Fixed income securitisation

- The securitisation process involves the following steps:
  1. Bank makes loans to customers ("originates" the loan)
  2. Loans are pooled and sold to a special purpose entity / vehicle ("collateral pool")
  3. SPE issues fixed income securities supported by the cash flows from the collateral
- Securities are created from the underlying loan cash flows, and then sold on to investors. The loan pool serves as collateral for investors. We recall figure 6.1 which demonstrates the csecuritisation process and the split into different tranches.



Repeat of Figure 6.1: Waterfall structure of payments from a collateral pool through the tranches in order of seniority

- In effect, the lender sells cash flows to the SPE, in order to boost their own liquidity. The process of securitisation connects owners of capital with those that require capital, and removes the originating bank from the process.
- The SPE is independent from any financial troubles of the lender.
- Different tranches of instrument allow for the risk level to be chosen. The equity tranche usually offers no fixed coupon payment, so the price behaves more like an equity than a bond.
- Benefits to the lender include
  - Improvement in liquidity, through selling illiquid loans for cash.
  - Risk is removed from the balance sheet

  - Lower capital requires (as lower risk-weighted assets)
  - Allows for increased business activity and profitability. The originator receives cash, which is used to make more loans

- Benefits to investors include
  - Tailored risk / return profile of tranche to meet requirements
  - Allows access to returns from the collateral pool, without needing specialised resources and expertise in loan origination / servicing
  - More liquid as a security than the underlying collateral

- Benefits to economies and markets include
  - Provides liquidity – securitisation improves liquidity in financial markets
  - Improved market efficiency (investor sets prices for market equilibrium)
  - Lower financing costs (Originators receive cash in return for selling the loans)
  - Lower leverage for originators

- Risks to investors in ABS
  - Cash flows from collateral to ABS are uncertain, due to variation and uncertainty in timing and size of cash flows
  - Credit risk of collateral is passed on from the originator to the ABS investor

- The trustee
  - The trustee is appointed to overee the safekeeping of collateral owned by the SPE. This is a "disinterested trustee", since there is no other interest in the structure. The SPE is "bankruptcy-remote" from the originator.

- ABS investors only have claims against ABS collateral, and not on any assets of the originator. Important documents include
  - Purchase agreement (Collateral sold to SPE)
  - Prospectus (Terms of securitisation)

  EXAMPLE: A motor company sells cars on retail installment plans. They are the originator of loans used by customers to finance their car purchases. Via a subsidiary, the motor company services the loans (responsible for payment taking and reposession).

  Currently there are $50,000$ loans totalling $\$1,000,000,000$ which it wants to remove from the balance sheet. This acts as a source of funding / liquidity. In order to do this, they sell the loan to and SPE (in this case, an auto loan trust) for \$1B. This makes them "bankruptcy-remote".

  The SPE sells ABS to investors. The loan portfolio is the collateral which supports the ABS. Borrower cash flows are the source of funds used.

ABS

- Covered bonds are senior debt obligations (similar to ABS). They are:
  - Typically mortgage loans (Issuer required to meet a particular cash flow schedule)
  - Segregated from other issuer assets in a "cover pool"

  - On the balance sheet, so no SPE is created. Assets remain on the balance sheet of issuer and need capital reserves

- To mitigate credit risk,
  - Over collateralisation (collateral is worth more than the loan itself)
  - Dual recourse (Investors also have claim to issuer assets as well as the cover pool, in the event that the cover pool ceases to be sufficient)
  - Mortgage LTV limits (Upper loan-to-value limits increase the collateral in the event of default
  - Monitory (Typically through a third-party)
- Covered bond provisions, in the event of issuer default
  - Hard bullet-covered bonds are in default if the issuer fails to make a scheduled payment. (Acceleration in payments to covered bond holders)
  - Soft bullet-covered bonds are those which may postpone default and payment acceleration for up to 1 year
  - A conditional pass-through covered bond becomes a pass-through bond at maturity if any payments remain
- Credit enhancement structures for ABS securities include
  - Overcollateralisation (Value of collateral > Value of ABS)
  - Excess spread builds up reserves in an ABS structure by earning a higher coupon than is actually sold to investors
  - Tranching into senior / mezzanine / equity

EXAMPLE: Consider the following setup of an SPE

	Tranche	Face value ($)	Interest rate
A	Senior notes	$300,000,000$	MRR + 0.5%
B	Subordinated	$80,000,000$	MRR + 1.5%
C	Subordinated	$30,000,000$	Variable
		$410,000,000$	

In this structure, C will be the first tranche to absorb any losses. A has $\$110,000,000$ of protection, so bears the lowest credit risk of any of the tranches.

Non-mortgage ABS

- Business loans, accounts receivable, car loans, credit card loans
  - Credit card receivables are backed by credit card debt
  - Solar ABS are used to finance installation of solar panels on property

The ABS may be amortising or non-amortising, depending on the prospectus

Credit card, Solar ABS

- Cash flows, interest, principal, membership, late fees
- Non-amortising (principal paid at borrower's discretion
- Lockout period (interest-only) applies to principal payments. This prevents early repayment of the loan principal amount. This allows cash from investors to be used to buy additional receivables
- For solar ABS;
  - This involves the use of ESG objectives, and may come in the form of both secured and unsecured loans
  - Typically, these are made to individuals with high credit scores
  - These often involve over collateralisation and excess spread

6.18 Credit debt obligation instruments

- CDOs, or credit debt obligations, are structured securities issued by an SPE for which the collateral offered is a pool of debt obligations
  - CBOs (bond) are backed by corporate and EM debt
  - CLOs (loan) are backed by a portfolio of leveraged bank loans
- CDOs have a collateral manager, who dynamically buys and sells securities in the collateral pool to generate sufficient cash to make promised payments to investors
- CDOs are issued in subordinated tranches, much in the same way as shown in figure 6.1

6.18.1 Types of CLO

- Cash flow CLO (static)
  - Generated from cash flows of underlying collateral
- Market value CLO
  - Generated from trading market value of underlying collateral
- Synthetic CLO
  - Generated through credit derivative contracts
  - SPE sells credit insurance (CDS, Credit Default Swap), and earns the premiums, which are then paid to investors
  - No collateral pool
  - Usually, investor funds are put into treasuries
- CLO collateral
  - Coverage of payment obligations
  - Over collateralisation
  - Diversification in collateral pool
  - Credit quality limits

6.19 MBS securities

- The borrower has the right to repay the loan early. In effect, they are long a call option. The borrower may repay faster or slower, depending on their individual circumstances
- The investor has no control. In effect they are short a call option, so will demand a higher yield to compensate them for that risk. Prepayment speed impacts the investor
- Prepayment risk
  - Prepayments are repayments made in excess of the schedule for amortising loans
    - Prepayment speeds: Uncertain – MBS investors may be repaid faster or slower
    - Contraction risk: Prepayments faster than expected (Occurs when rates fall)
    - Extension risk: Prepayments slower than expected (Occurs when rates rise)

Contraction risk	Extension risk
Rates fall	Rates rise
Repayments rise	Repayments fall
Pool contracts down, weighted average maturity falls	Pool contracts stable, weighted average maturity rises
(Money in-hand, to be reinvested at a time of lower yields)	(Slower repayments so investor cannot capitalise on the higher yields available)

- Time tranching may be used to balance extension / contraction risk
  - This reapportions contraction / extension risk in an MBS structure. The SPE issues different bond classes with different maturities are issued.

Shorter maturity	Longer maturity
Ealier prepayment	Later prepayment
Extension risk lower	Contraction risk, reinvestment risk lower
Contraction risk, reinvestment risk higher	Extension risk higher

6.19.1 Residential mortgage loans (RMBS)

- Residential property posted as collateral. This is generally more diversified and carries lower risk than CMBS (Commercial MBS)
- If the borrower defaults, the lender has a legal claim to the collateral.
  - Takes possession of the property. Foreclosure means they can sell the property to recover the debt
- LTV is the % of collateral value loaned to the borrower. It is a measure of default risk

$$\text{Low LTV} \Rightarrow \text{High borrower equity} \Rightarrow \text{Low risk}$$
$$\text{High LTV} \Rightarrow \text{Low borrower equity} \Rightarrow \text{High risk}$$

- Debt-to income ratio (DTI) is defined

$$\text{Debt-to-income} = \frac{\text{Monthly debt payments}}{\text{Monthly gross income}}. \tag{6.38}$$

Prime loans tend to have a high LTV and low DTI

EXAMPLE: Consider a borrower who wishes to take out a $300,000$ mortgage on a property valued at $400,000$. The annual interest rate is 6%, repaid monthly over 25 years. The borrower has a pretax gros income of $80,000$.

The LTV is

$$\text{LTV} = \frac{300,000}{400,000} = 75\%.$$

The monthly payment can be calculated as

$$N = 25 \times 12 \qquad I/Y = \frac{6\%}{12} \qquad PV = -300,000 \qquad PMT = \qquad FV = 0$$

which gives a monthly payment of $1,932.90$. Using this, the DTI is

$$\text{DTI} = \frac{1,932.90}{\left(\frac{80,000}{12}\right)} = 29\%$$

- Agency RMBS
  - These are guaranteed by government / government-sponsored enterprises
    - GNMA backed by US government
    - FNMA, FHLMC, SLM backed by GSE
    - High minimum underwriting standards required to qualify as collateral

    The government guarantee reduces the credit risk associated with these securities
- Non-agency RMBS
  - These are issued by private entities, banks, and have no governmental guarantee.
  - Credit enhancement through external insurance, letters of credit, tranching, and private guarantee
  - The GFC caused losses ot non-agency RMBS backed by subprime mortgage collateral
- Features of mortgages
  - Prepayment penalty – Additional payments to lenders if the principal is repaid early
  - Non-recourse loans only have specified property as collateral
  - Recourse loans give a claim to other assets owned by the borrower if foreclosure does not match the full outstanding debt repayment
  - Negative equity if the mortgage balance exceeds the property value
- Mortgage pass-through securities
  - These represent a claim on the cash flows from a pool of mortgages (Net administration)
    - Weighted average maturity and weighted average coupon are weighted by the outstanding principal balance

{Mortgages} $\longrightarrow$ **Pool** $\longrightarrow$ **{Investors}**

Interest rate (%)	Beginning balance	Current balance	Original term (months)	Time to maturity (months)
2.6	100,000	90	240	210
1.0	200,000	72	300	100
5.4	300,000	247	360	280

EXAMPLE: Consider the following.

Calculate the weighted average maturity and weighted average coupon.

The total balance is

$$90 + 72 + 247 = 409$$

The WAM is calculated as

$$\text{WAM} = 210 \times \frac{90}{409} + 100 \times \frac{72}{409} + 280 \times \frac{247}{409} = 233 \text{ months}$$

The WAC is calculated as

$$\text{WAC} = 2.6 \times \frac{90}{409} + 1.0 \times \frac{72}{409} + 5.4 \times \frac{247}{409} = 4\%$$

6.19.2 Collateralised mortgage obligations

- A CMO is a security that is collateralised by pass-through MBS and pools of mortgages
- Each CMO has multiple bond tranches with diffeent exposure to prepayment risk
  - Institutional investors have different tolerances for rpepayment risk
  - Contraction risk and extension risk exposures can be minimised
  - CMOs partition cash flows from RMBS to better match investor preferences
- Sequential pay CMOs are those which pay principal payments to tranches in a specific order
  - Highe priority tranche is shorter, and so is protected from extension risk
  - Low priority tranche is longer and so is protected from contraction risk
- Other CMO structures include
  - Z-tranches (accrual / accretion bonds)
    - No-interest paid for a specific period
  - Prinicipal-only securites [Rates ↓, Value ↑]
    - Pay only principal from collateral. Faster payments imply a higher return
  - Interest-only securities [Rates ↑, Value ↑]
    - Pay only interest from collateral. Slower payments imply a higher return
    - These exhibit negative convexity since rates and value move in the same direction
  - Floating rate tranches
    - Coupon linked to variable market reference rate

    - Inverse floaters are possible (Coupon = x% − MRR)
  - Residual tranche
    - Equity tranche (most junior tranche available)
  - Planned amortisation class tranches (PAC)
    - Pay predictable level of payments to investors to protect them from both extension and contraction risk
    - A "support tranche" receives prepayments to protect CMO investors from acceleration of payments

6.19.3 Commercial mortgage-backed securities

- CMBS securities include apartments, industrial property and office buildings, amongst other holdings
  - Typically there are fewer mortgages in the collateral pool, as each property involves larger loan sizes than RMBS
  - Commercial mortgages are paid by real estate investers who rely on income from tenants to provide cash flows to service the loans

$$\text{Weighted-average mortgage proceeds} = \text{WAC} \tag{6.39}$$

- There is a greater focus on credit risk
  - Income generated from property pays the debt. The credit risk is calculated based on the property, not the issuer themselves
- Debt service coverage ratio is defined as

$$\text{DSCR} = \frac{\text{Net operating income}}{\text{Debt service}}, \tag{6.40}$$

  and LTV is defined in a similar manner to before, as

$$\text{LTV} = \frac{\text{Current mortgage amount}}{\text{Current appraised value}}, \tag{6.41}$$

  where we use the appraised value due to lack of real-time data on property prices
- Call protections may be implemented, which restrict early return of principal.
- Loan-level protection includes
  - Prepayment lockout – Borrower cannot repay the loan within a given time frame
  - Prepayment penalty points – Penalty fee on principal repayments
  - Defeasance – Borrower buys government securities which are sufficient ot make the scheduled loan repayments. This allows the borrower to reomvoe lenders' lien if sold
- Balloon payment
  - Commerical mortgages are not fully amortised, so some principal may remain
  - Ballon risk is the risk fo the borrower being unable to arrange finance to make the balloon payment, leading to borrower default. In this event, a workout period may be agreed with the lender. This introduces extension risk.

7 Derivatives

7.1 Instruments and market features

- Derivatives are securities that derive value from an underlying, typically a price or interest rate.
- Examples of the underlying include
  - Equity / Equity indices
  - Bond / Bond indices, interest rates
  - Hard & soft commodities
  - Credit / Credit indices
- Derivative markets can either be:
  - OTC markets – Formal / informal networks
    - Dealers (market makers) trade with users and among themselves
    - Securities are customisable, and as a result are less liquid and less transparent than exchange-traded derivatives. As a result, this tends to incur higher trading costs.
    - Many OTC markets are required to have a CCP (novation) and collateral deposits, thereby reducing counterparty risk.
  - Exchange traded derivatives – Formal networks
    - Market makers post buy / sell prices, and enter into offsetting trades with users
    - Contracts are standardised (Delivery date / Quantity / Underlying / Delivery obligations)
    - Central clearing is present (Collateral deposits / Mark-to-market / Novation by exchange (less CP risk))
    - Standardised securities give greater liquidity and lower transaction costs
    - Clearing and settlement processes are efficient

7.2 Forward and futures contracts

7.2.1 Forward contracts

- Forwards are customised, so there tends to be no active secondary market for forward contracts.
- Forward contracts specify a specific asset, and a specific expiry date upon which delivery of the asset occurs
  - The long party gains if the asset price at delivery exceeds the forward price
  - The short party gains if the asset price at delivery is less than the forward price

  Long (short) position buys (sells) underlying
- Forward contract settlement involves two types
  - Delivery
    - Short delivers underlying to long in exchange for cash payment of the forward price
  - Cash settlement
    - Negative side of he contract pays the positive side, where this is determined by the difference between forward contract-specified price and current market price
- An owner of shares can hedge their position with derivatives

    - To hedge: If long the underlying, an investor should hedge this by going short on a forward contract
  - An investor with no position can speculate on price movements using derivatives

    EXAMPLE: Consider a forward contract where the long party agrees to buy 100 shares of ABC corp. from the short on November 15 at a price of \$30 per share (set at incpetion of contract)

    IF: Deliverable contract; 100 shares transferred in return for a $\$3,000$ payment.

    IF: Cash-settled contract;

$$\begin{aligned}\text{Long receives}&: (\text{Spot} - 30) \times 100,\\ \text{Short pays}&: (\text{Spot} - 30) \times 100.\end{aligned}$$

    If the spot price at settlement is \$35, the net effect of this payment is a \$500 gain to the buyer and \$500 loss to the seller.

7.2.2 Futures contracts

- Futures contracts bear many similarities to forward contracts, however a key difference is that the contracts are standardised
- Futures contracts trade on an exchange, thus providing an active secondary market. Exchange-traded contracts require a margin deposit, and CCP clearing means there is no risk of CP default.
- Characteristics of futures contracts
  - Quantity / quality of the underlying must be specified, alongside a delivery date / time and location
- The exchange will specify
  - Minimum price fluctuation "tick". (Precision to whic hthe price is measured)
    - Tick size measure in unit of price
    - Tick value measured in USD
  - Daily price limit
  - Clearing house must act as CCP
  - Margin posted and marked-to-market daily
  - Margin is collateral (NOT a loan)
- Initial margin – deposited at inception of contract
- Maintenance margin – minimum margin that triggers a margin call. When the posted margin falls below the maintenance margin, variation margin must be deposited to make up the difference
- Settlement price – Average of trades during closing period (30 sec – 2 min) used to calculate the required margin
- Spot price – Price of underlying asset for immediate delivery
  - Future price tends to spot price as time progresses. At expiration, settlement price and spot price are identical

Price limits

- Exchanges place a limit on how much the contract price is permitted to change each day. Exchange members are prohibited from trading at prices outside these limits.
- Some exchanges have circuit breakers instead

Marking to market

- Marking to market is the concept of adjusting the margin balance daily for daily variation in the futures price
- After adjusting margin balance for daily gain / lss, the futures price and settlement price are equivalent

  EXAMPLE: Consider a futures contract to buy 5,000 bu. wheat at \$10 per bu. The initial margin is \$2,500, and the maintenance margin is \$2,000.

  Day 2: The settlement price at the end of day 2 is \$9.95.

  This represents the new futures contract price. The new margin is therefore

  $$\text{New margin} = 2,500 - 5,000(10 - 9.95) = 2,250 > \text{Maintenance margin.}$$

  This is above the maintenance margin, so no variation margin is required.

  Day 3: The settlement price at the end of day 3 is \$9.85.

  This represents the new futures contract price. The new margin is therefore

  $$\text{New margin} = 2,250 - 5,000(9.95 - 9.85) - 1,750 < \text{Maintenance margin.}$$

  This is below the maintenance margin, so the buyer of the contract is required to deposit \$750 of variation margin to make the value of the contract back up to \$2,500.

7.2.3 Swap agreements

- For a notional amount, each party makes periodic payments based on an interest rate, or on the performance of an index / bond / portfolio / comodity
- Payments are typically <u>netted</u> (principal exchanged for securities)
  - This may or may not require margin (Today, margin requirements are becoming more common, but are not strictly necessary)
  - This may have multiple settlement dates
- These are custom instruments, equivalent to a series of individual forward contracts

  EXAMPLE: Consider a swap agreement witha notional principal of \$10 mn. The floating rate is a 90-day SOFR, and the annualised fixed rate is 2%. The Tenor of the swap agreement is 2 years, and settlement is quarterly. Payments are netted.

	T_0	T_{90}	T_{180}	$\cdots$
Fixed		$\frac{2\%}{4}$	$\frac{2\%}{4}$	$\cdots$
Floating		$\frac{\text{90-day SOFR}_{T_0}}{4}$	$\frac{\text{90-day SOFR}_{T_{90}}}{4}$	$\cdots$

  At T_i, we know the 90-day SOFR rate which is to be settled at T_{i+90}. The difference between the two payments is calculated, and a single payment is made to settle the difference.

7.2.4 Credit default swaps (CDS)

- In a CDS, the buyer of the protection makes periodic payments ("Coupons" ≡ Insurance premiums). The protection seller only pays out of pocket if a credit event (i.e. default) occurs
- Changes in probability of default or loss given default increases the swap fixed payment (and spread of the underlying)
- CDS is used to hedge or take on credit risk
- The buyer is short credit risk.

7.3 Options

7.3.1 Option basics

- An option buyer (owner, long position) pays the premium on an option to purchase the right to exercise an option at a future date.
- An option seller (writer, short position) is obliged to give / take receipt of an asset for that fixed price only if the owner exercises the option.
  - The owner of a call option holds the right to buy ("call from the market") an underlying at a strike price. The writer therefore must deliver the asset to the option owner at expiry, if exercised.
  - The owner of a put option holds the right to sell an underlying asset ata a strike price. The writer must therefore purchase the asset from the option owner at expiry, if exercised.
- European options are exercisable only at expiration
- American options are exercisable at any time until expiration

7.3.2 Call options

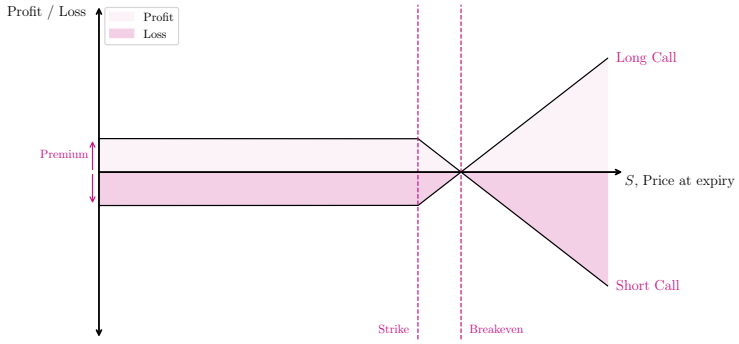


Figure 7.1: Profit / Loss chart for a typical call option. The owner will exercise the option if the asset value at expiration exceeds the strike price. Breakeven is at $X + C$.

$S < X$	Option not exercised	Buyer loses full premium
$X < S < X + C$	Option exercised	Buyer realises smaller loss than premium
$S > X + C$	Option exercised	Buyer realises pure gain

Table 7.1: Profit / Loss criteria for a typical call option

7.3.3 Put options

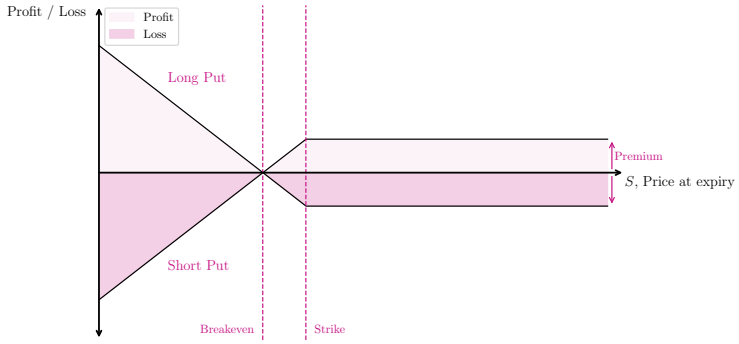


Figure 7.2: Profit / Loss chart for a typical put option. The owner will exercise the option if the asset value at expiration is below the strike price. Breakeven is at $X - P$.

$S > X$	Option not exercised	Buyer loses full premium
$X > S > X - P$	Option exercised	Buyer realises smaller loss than premium
$S < X - P$	Option exercised	Buyer realises pure gain

Table 7.2: Profit / Loss criteria for a typical put option

7.3.4 Forward commitments and contingent claims

- Futures, forward contracts, and swaps are forward commitments. They carry with them an obligation to fulfill the terms of the contract
- Options and credit derivatives are contingent claims. The obligation of one party depends on an event (exercise by option holder, default of an issuer, etc.)

7.4 Benefits, risks, issuer and investor uses

Benefits

- Transfer / manage risk
- Easier to get a short position (compared to a short sale
- Lower transaction costs than cash market
- Less cash required, so greater degree of leverage
- Greater liquidity (Higher traded volume than spot markets)
- Gives information on
  - Expected volatility
  - Estimates of future price / interest rates (spot vs forward)

$$\underbrace{\text{Price}}_{\text{Premium}} = \text{Price}(\text{Time}, \text{Spot}, \text{Strike}, R_f, \text{Vol.}) \tag{7.1}$$

Risks

- Basis risk
  - Underlying mismatch with hedged risk (Does derivative match instrument trying to hedge)
  - Mismatch of expiration debt and date of hedged transaction
- Liquidity risk
  - Mismatch of derivative cash flows with those of existing risk to be hedged (i.e. variation margin calls)
- Counterparty credit risk
  - Depends on derivative position and margin requirements
- Systemic risks
  - Excessive speculation may have an adverse impact on financial markets. (Comes from leverage / contagion)

Uses by corporate issuers

- Used to reduce duration risk of fixed-rate debt with floating-rate payer swap.

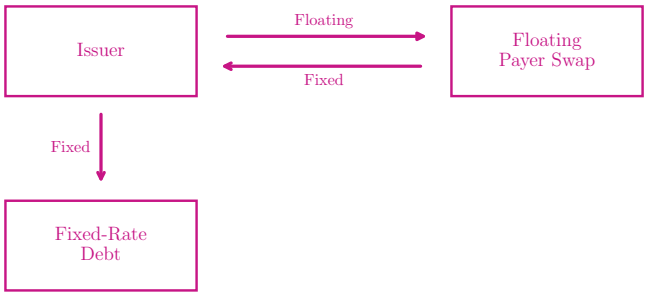


Figure 7.3: Swap agreement used to protect against duration risk

- An airline can hedge risk for fuel costs by buying jet fuel futures. The airline goes short fuel and long fuel futures.
- An international corporation can hedge uncertainty about future payments and receipts in a foreign currency with forwards / futures
- Hedge accounting uses gains an losses on derivatives to offset the effects of changing asset and liability values
- Currency forward to reduce uncertainty about the value of foreign currency payment / receipt
- Fair value hedges are those such as a gold miner's inventory hedged by selling forward contracts in gold
- A floating rate payer swap may be used to offset changes in the balance sheet value of fixed-rate bond liability
- A net value hedge is used to hedge the value of a foreign company's subsidiary equity on a paret's balance sheet wih currency forwards

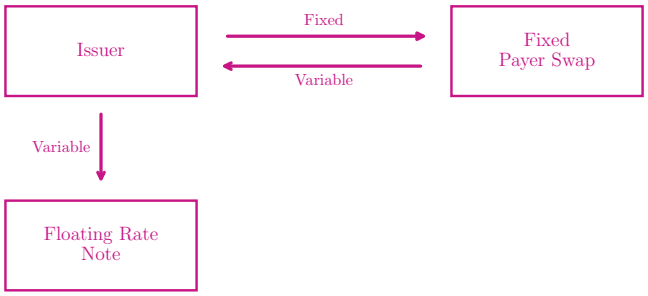


Figure 7.4: Swap agreement used for hedge accounting to reduce uncertainty about future floating-rate interest payments

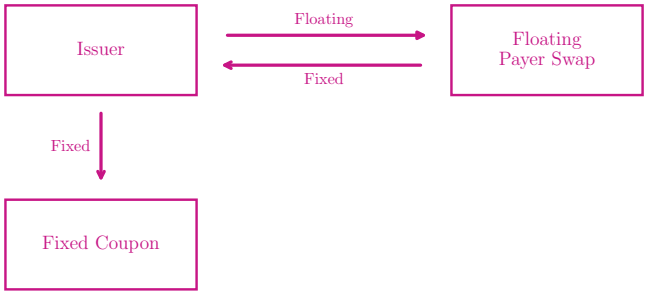


Figure 7.5: Swap agreement used to to manage changes in the balance sheet value of assets / liabilities

Uses by investors

- Speculation of price by buying futures / forward contracts
- Increase (decrease) of duration in a portfolio by buying (selling) a fixed-rate swap. The fixed rate swap has negative duration since the floating rate is less sensitive to interest rate moves
- Altering risk of an equity portfolio
    1. Buy index forward to increase risk exposure
    2. Sell index forward to decrease risk exposure
    3. Buy index puts to limit downside "Protected put"
    4. Buy index calls ot leverage upside

7.5 Arbitrage, replication, and cost of carry

Arbitrage

- Arbitrage is a risk-free strategy. For two assets that have the same future payoffs, regardless of future events, but are available for different prices, buyng the lower priced asset and simultaneously selling the higher-priced asset gives a "riskless arbitrage profit"
- The action of arbitrageurs pushes the price difference to zero

Replication

- We can replicate a derivative by creating a portfolio that has future payoffs identical to that of the derivative.

EXAMPLE: Consider a long forward contract to buy shares in ABC. corp at 31.50 in 1 year, with the current trading price of ABC being 30. Compare this to borrowing 30 at a risk free rate, and holding the physical stock for a year. (Assume no dividend)

We are not told the risk free rate, but can infer an estimate given the spot and future price of the asset.

$$S_0 = \frac{S_{\text{future}}}{(1+R_f)^T} \qquad \Rightarrow \qquad R_f = \frac{31.50}{30} - 1 = 5\%$$

The initial cost of each is zero. The payoff at time T is $S_T - 31.50$. So 31.50 is the no-arbitrage 1-year forward price, $F_0(T)$, of an ABC share when $R_f = 5\%$. Therefore,

$$F_0(T) = S_0(1+R_f)^T.$$

If $F_0(T) = 32$, the forward price is greater than the arbitrage-free price. Therefore an investor should sell the forward contract "short contract" and borrow to buy the underlying. This results in a riskless gain. "Cash and carry arbitrage"

If $F_0(T) = 31$, the forward price is less than the arbitrage-free price. Therefore an investor should buy the forward contract "long contract" and short the underlying, investing at the risk free rate. This results in a riskless gain. "Reverse cash and carry arbitrage"

Benefits and costs

- The benefits and costs of holding the underlying must be factored into the forward price of an asset. the Benefits include any monetary benefits (cash flows, dividends, etc.) and non-monetary benefits (convenience yield). Costs include storage and insureance costs, which are mostly monetary costs. The inclusion of the risk free rate is the opportunity cost of holding the asset.

$$\begin{aligned} F_0(T) &= \{s_0 - PV_0(\text{Benefits}) + PV_0(\text{Costs})\} + (1+R_f)^T \\ &= S_0(1+R_f)^T - FV(\text{Benefits}) + FV(\text{Costs}) \end{aligned} \tag{7.2}$$

  Equation 7.2 shows us that an increase in the present value of any benefits lowers the price of the futures contract, and an increase in the present value of any costs raises the price of the futures contract.

- Note that equation 7.2 uses discrete compounding periods. If the compounding is continuous,

$$FV = Se^{rT} \qquad \left[e^{rT} = \lim_{n\to\infty}\left[1+\frac{rT}{n}\right]^n\right], \tag{7.3}$$

  and therefore

$$F_0(T) = s_0 \cdot e^{(R_f+c-i)} \tag{7.4}$$

  where c is the cot, and i is the benefit.

7.6 Forward exchange rates

- We recall from §2.15.2, equation 2.16, that

$$\frac{\text{Forward}(A)}{\text{Forward}(B)} = \frac{(1+{R_f}^A)^T}{(1+{R_f}^B)^T} \cdot \frac{\text{Spot}(A)}{\text{Spot}(B)} \text{ for discrete compounding} \tag{7.5}$$

$$= e^{{R_f}^A - {R_f}^B} \cdot \frac{\text{Spot}(A)}{\text{Spot}(B)} \text{ for continuous compounding} \tag{7.6}$$

  Higher domestic interest rates lead to depreciation of the domestic currency.

7.7 Pricing and valuation of forward contracts

- Recall equation 7.2 under the assumption that there are no monetary or non-monetary benefits / costs associated with a given underlying.

$$\begin{aligned} F_0(T) &= \{s_0 - PV_0(\text{Benefits}) + PV_0(\text{Costs})\} + (1+R_f)^T \\ &= S_0(1+R_f)^T \end{aligned} \tag{7.7}$$

The no-arbitrage price is the forward price that ensures the forward has a value of zero at initiation of the contract.

At time t,

$$V_t(T) = \underbrace{S_t}_{\text{Current price}} - \underbrace{\frac{F_0(T)}{(1+R_f)^{T-t}}}_{PV \text{ of forward contract}}, \tag{7.8}$$

and at settlement, when $t = T$,

$$V_{t=T}(T) = S_t - F_0(T), \tag{7.9}$$

More generally, we can write

$$V_t(T) = S_t - PV_t(F_0(T)). \tag{7.10}$$

EXAMPLE: Consider a long position ina one-year forward contract, with a price of 35. The risk free rate is 3%. After 9 months, the spot price of the underlying is 36.. What is the present value of the forward contract.

Using equation 7.10,

$$V_{t=9 \text{ months}}(1 \text{ Yr}) = 36 - \frac{35}{(1.03)^{0.25}}, \tag{7.11}$$

$$= 1.26. \tag{7.12}$$

7.8 Forward rate agreements (FRA)

- This covers how forward prices are determined for interest rate-based products
- By CFA exam convention, a party that is "long FRA" will pay a fixed rate and receive a floating (underlying MRR).
- At settlement, the difference between the fixed and floating is paid. So if the $MRR >$ Fixed, the long receives the difference, and if the $MRR <$ Fixed, then the long pays the difference.
- Replicating an FRA can be done as follows.
- Generally, a company will tend to borrow using bank finance at a floating rate.

  EXAMPLE: Consider a company borrowing \$10 mn for 6 months, with the loan commencing in 3 months time. In order to fix the interest payments they will make on this loan, they enter into a long position of an FRA.

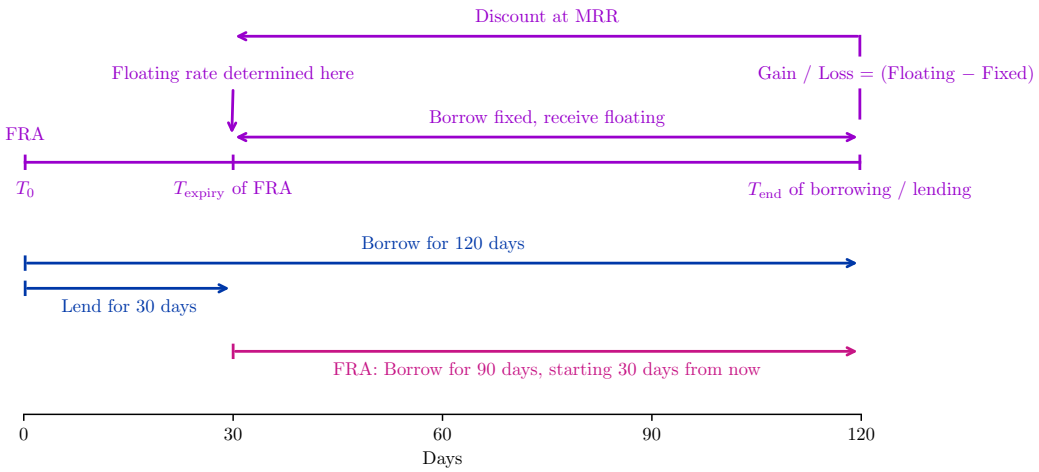


Figure 7.6: Mechanism showing the contracts and agreements required to recreate an FRA

Calculating the no-arbitrage forward rate for a 6m MRR beginning 3m from today, if the spot 3m rate is 1%, and the spot 9m rate is 1.2% (both annualised)

$$\begin{aligned} 1 + 0.012 \times \frac{9}{12} &= \left(1 + 0.01 \times \frac{3}{12}\right)\left(1 + F_{3,6} \times \left(\frac{6}{12}\right)\right) \\ \Rightarrow F_{3,6} &= 0.01297 \\ &= 1.297\% \end{aligned}$$

where the usual notation for forward rates is used.

- FRA payoffs for the long party involve receipt of $(MRR - \text{Fixed})$, discounted by the MRR from teh end to the start of the borrowing period, where the fixed rate is set by the FRA.
- An FRA may be used for
  - Company expecting to borrow in the future can fix borrowing costs with a pay-fixed position in an FRA.
  - Company expecting to lend in the future can fix the lending rate with a pay-floating position in an FRA.

7.9 Pricing and valuation of futures contracts

Notation: Forward contracts are denoted with capital F. Futures contracts are typically denoted with a lower case f.

- Much in the same way as forward contracts, as in equation 7.2, the price at initiation is defined by
$$f_0(T) = \{s_0 - PV_0(\text{Benefits}) + PV_0(\text{Costs})\} + (1 + R_f)^T \tag{7.13}$$
- After initiation, the price of a forward does not change. The value changes as the asset price varies, but the price remains unchanged. The price of a futures contract however does change. The value changes as asset price changes due to mark-to-market cash flows. The value returns to zero on a daily basis as gains / losses are settled.
- The MTM value is given by
$$\text{MTM value} = \Delta\text{Settlement} \tag{7.14}$$

EXAMPLE: Consider a long futures contract on gold at $1,870 / oz, for 100 oz.

Day 0	Price = settlement price = 1,870	
Day 1	Settlement price = 1,875	MTM value = 5 ×100 = 500
	500 addition to margin	
	New futures price = 1,875	MTM value = 0
Day 2	Settlement price = 1,855	MTM value = −20 × 100 = −200
	2,000 deduction from margin	
	New futures price = 1,855	MTM value = 0

7.10 Forward vs futures prices

- Because futures have daily MTM cash flows, if interest rates are positively correlated with the underlying asset value, a long futures position is preferred to a forward with no cash flows.

$$\text{Long u/l } \uparrow \Rightarrow \text{Futures price } \uparrow,$$

  therefore any profits from the margin calls can be reinvested at the higher interest rate. As there is a higher rate when lending, as compared to borrowing, the long position in this contract is preferable.

- It is worth noting that this works in theory, but in practice, there are no significant price / value idfferences

- Short term interest rate futures
  - Based on deposit at end of contract
  - IMM index convention
  - Price = 100 − annualised forward rate. As the price falls, the forward rate increases
- Long position interest rate futures increase in value when the forward rate falls
- Long FRA (paying fixed) gains when the floating rate rises. In order to hedge borrowing costs, an investor shoul go long on an FRA and short an interest rate future.

  EXAMPLE: Consider a long futures contract for 1mn on a 6-month MRR priced at 97.50

  The price is given by

$$\text{Price} = (1 - \underbrace{\text{Annualised MRR}}_{2.5\%}) \times 100.$$

  Each basis point change in the MRR changes the payoff by

$$0.0001 \times \frac{6}{12} \times 1,000,000 = 50.$$

  This is a linear payoff.

  If the MRR is 2.44% at settlement, the futures price is $100 - 2.44 = 97.56$. The long party receives payments of

$$(2.50\% - 2.44\%) \times \frac{6}{12} \times 1,000,000 = 300.$$

  Therefore 300 is the additional interest required to be paid on a six month deposit.

If we consider an equivalent FRA (Fixed rate of 2.5%),

- At settlement, with MRR = 2.51%, the payment to the long party is

$$\frac{50}{\left(1+\frac{0.0251}{2}\right)} = 49.3803$$

- At settlement, with MRR = 2.49%, the payment that the long party must make is

$$\frac{50}{\left(1+\frac{0.0249}{2}\right)} = 49.3852$$

These are the present values of the pay-offs at the end of each borrowing / lending period. The asymmetry in the payoffs comes from convexity, which as we can see here works against the long position.

7.10.1 Convexity of forward payoffs

- Gain from interest rate decrease is larger than the loss from an increase.
- Similar to bond convexity, forward convexity favours the investor
- Payoff difference is smaller for short-dated FRAs

7.11 Pricing and valuation of interest rate swaps

- A fixed-rate swap payer pays a fixed rate and receives MRR × some notional on each payment date
- Each payment is equivalent ot an FRA at the swap fixed rate

$$\Rightarrow \text{Swap} \equiv \{\text{FRAs at swap (fixed) rate}\}$$

- At initiation, the swap has zero value, but the individual FRAs may have non-zero values.
- If we consider a one-year quarterly pay fixed-swap agremement,

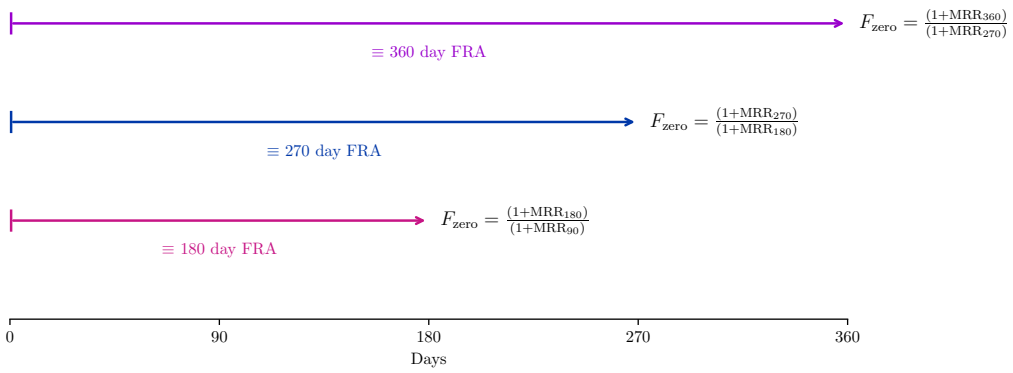


Figure 7.7: Diagram showing the mechanism and agreements involved in a quarterly pay fixed swap agreement, with a life of 1 year. The MRRs are all add-on rates. The MRR at $T = i$ determines the payment to be made at $T = i+90$

The zero-arbitrage F_0s are not all the same, but for the purposes of the swap, we can assume they are. The values may be positive or negative.

- The swap price is the fixed rate. At initiation, the swap value is zero, as the following relationship is true:

$$PV(\text{Fixed payments}) = PV(\text{Floating payments}).$$

We can also see that by combining different contracts,

$$\text{Pay-floating swap} + \text{Fixed-rate debt} \Rightarrow \text{Floating-rate debt}.$$

A pay-floating swap loses value when the forward rate curve expectations shift upward.

$$\text{IR expectations } \uparrow \Rightarrow \text{Forwar rates } \uparrow .$$

For a pay-floating, the floating payments increase, but the fixed are unchanged, so the value of the contract falls. The opposite is true for a pay-fixed agreement.

- A swap can be priced given the set of spot rates, $\{S_1, S_2, S_3, \cdots, S_n\}$. Using these spot rates, the set of forward rates, $\{F_{0,1}, F_{1,1}, F_{2,1}, \cdots, F_{n,1}\}$ can be inferred.

$$F_{0,1} = S_1, \quad F_{1,1} = \frac{(1+S_2)^2}{(1+S_1)} - 1, \quad F_{2,1} = \frac{(1+S_3)^3}{(1+S_2)^2} - 1, \quad \cdots, \quad F_{n,1} = \frac{(1+S_n)^n}{(1+S_{n-1})^{n-1}} - 1.$$

The present value of the floating rate payments is therefore given by

$$PV(\text{Floating}) = \frac{F_{0,1}}{(1+S_1)} + \frac{F_{1,1}}{(1+s_2)^2} + \frac{F_{2,1}}{(1+S_3)^3} + \cdots \frac{F_{n,1}}{(1+S_n)^n} \tag{7.15}$$

The present value of the fixed payments must be equal to the present value of the floating payments at inception of the contract.

$$PV(\text{Fixed}) = \frac{\text{Fixed}}{(1+S_1)} + \frac{\text{Fixed}}{(1+s_2)^2} + \frac{\text{Fixed}}{(1+S_3)^3} + \cdots \frac{\text{Fixed}}{(1+S_n)^n} \tag{7.16}$$

EXAMPLE: Consider an annual-pay agreement, where the 1-year, 2-year, and 3-year spot rates are 1.2%, 1.3% and 1.4% respectively. What is the value of the fixed payment required for a zero-value swap.

We can see easily calculate the forward rates using the principal of no-arbitrage, which gives forward rates of

$$F_{0,1} = 1.2\%, \qquad F_{1,1} = 1.4001\%, \qquad F_{2,1} = 1.6003\%.$$

The PV of the expected floating rate payments is therefore

$$PV(\text{Floating}) = \frac{0.012}{1.012} + \frac{0.014001}{1.013^2} + \frac{0.016003}{1.014^3} = 0.040859.$$

By construction, this gives us

$$PV(\text{Floating}) = \frac{\text{Fixed}}{1.012} + \frac{\text{Fixed}}{1.013^2} + \frac{\text{Fixed}}{1.014^3} = 0.040859.$$
$$\Rightarrow \text{Fixed} = 0.0139815, \text{ for a zero-value swap}$$

- The price of the swap is determined by the fixed rate which satisfies $PV(\text{Fixed}) = PV(\text{Floating})$.
- The value of a swap is given by

$$PV(\text{Remaining floating}) - PV(\text{Remaining fixed}). \tag{7.17}$$

An increase in expected MRR increases the value to the fixed-rate payer.

7.12 Pricing and valuation of options

Intrinsic value

- For a European option, this is the amount by which an option is in the money.

European call:	$\text{Max}\,(0, [S - X])$	In the money if $S > X$
European put:	$\text{Max}\,(0, [X - S])$	In the money if $S < X$

  The intrinsic value is floored at zero. By construction, it cannot be negative.

Moneyness

- The option premium is defined as

$$\text{Option premium} = \text{Intrinsic value} + \text{Time value}. \tag{7.18}$$

  The time value for an option is also floored at zero, and tends to zero over time.

Forwards vs contingent claims

- Forward and future commitments have
  - Zero value at initiation $F_0(T) = 0$, $f_0(T) = 0$.
  - Symmetric payoffs, no upfront payment
  - Unlimited gains / losses (except by zero asset / underlying price)
- Contingent claims (options) have
  - Positive value at issuance

$$\text{Option premium} = PV(\text{Expected payoff at expiry}) \tag{7.19}$$

  - Asymmetric payoffs

$$\begin{aligned} \text{Max. loss} =& \text{Option price for long put / call} \\ & \text{Long party has a capped loss} \\ \text{Max. gain} =& \text{Option price for short put / call} \\ & \text{Short party has a capped gain} \end{aligned}$$

- Arbitrage puts limits on the minimum and maximum values (premia) of options

Option	Minimum value	Maximum value
Call, c_t	$\text{Max}\big[0, S_t - X(1 + R_f)^{-(T-t)}\big]$	S_t
Put, p_t	$\text{Max}\big[0, X(1 + R_f)^{-(T-t)} - S_t\big]$	$X(1 + R_f)^{-(T-t)}$

Table 7.3: Option price (premia) minimum and maximum values. For the minimum value in both cases, we compare the current price to the present value of the future strike price.

- Consider a portfolio which has the following positions, given in table 7.4 The value of the portfolio at time t is given by

$$\begin{aligned} \text{Value}(t) &= c_t + PV(X) - S_t, \\ &= c_t + \frac{X}{(1 + R_f)^{-(T-t)}} - S_t \end{aligned} \tag{7.20}$$

  At expiry, depending on whether the asset price is above or below the strike price, the individual positions take on the following values:

Long	ATM call option ZCB, same maturity as option, par value = option strike
Short	Underlying stock

Table 7.4: Positions in a hypothetical portfolio involving a call option

	$S_T > X$	$S_T < X$
c_T	$S_T - X$	0
ZCB	X	X
Stock	S_T	S_T
PV	$S_T - X + X - S_T = 0$	$0 + X - S_T > 0$

Table 7.5: Value of the the various components of the call option portfolio at option expiry

- Using equation 7.20, and the results of table 7.5, we can establish a minimum value of the call option, c_t, which is that

$$c_t \geq S_t - \frac{X}{(1+R_f)^{-(T-t)}}. \tag{7.21}$$

- We can go through a very similar exercise for a portfolio that instead holds a put option. In this case, we find:

Long	ATM put option Underlying stock
Short	ZCB, same maturity as option, par value = option strike

Table 7.6: Positions in a hypothetical portfolio involving a put option

The value of the portfolio at time t is given by

$$\begin{aligned} \text{Value}(t) &= p_t + S_t - PV(X), \\ &= p_t + S_t - \frac{X}{(1+R_f)^{-(T-t)}} \end{aligned} \tag{7.22}$$

At expiry, depending on whether the asset price is above or below the strike price, the individual positions take on the following values:

	$S_T > X$	$S_T < X$
p_T	0	$X - S_T$
Stock	S_T	S_T
ZCB	X	X
PV	$0 + S_T - X > 0$	$X - S_T + S_T - X = 0$

Table 7.7: Value of the the various components of the put option portfolio at option expiry

- Using equation 7.22, and the results of table 7.7, we can establish a minimum value of the put option, p_t, which is that

$$p_t \geq \frac{X}{(1+R_f)^{-(T-t)}} - S_t. \tag{7.23}$$

7.13 Factors affecting option values

Factor	Impact on call	Impact on put	
High asset price	$c_t \uparrow$	$p_t \downarrow$	Intrinsic value
High exercise price	$c_t \downarrow$	$p_t \uparrow$	
High volatility	$c_t \uparrow$	$p_t \uparrow$	Time value
Long time to expiry	$c_t \uparrow$	$p_t \uparrow$	
High risk-free rate	$c_t \uparrow$	$p_t \downarrow$	High R_f lowers $PV(X)$. Think about intrinsic value
High benefit of holding	$c_t \downarrow$	$p_t \uparrow$	Opposite for costs

Table 7.8: Table contianing the impact of various factors on option prices. This holds in most instances, except for when $T \gg 1$, $X \gg S_t$, in which case an investor is better off investing at the risk-free rate.

7.14 Option replication using put-call parity

- We can use put-call parity of European options only, as there is no uncertainty in the time of exercise of the option, as opposed to an American option.
- A protective put position comprises

$$\text{Protective Put} = \text{Long stock} + \text{Long put} \tag{7.24}$$

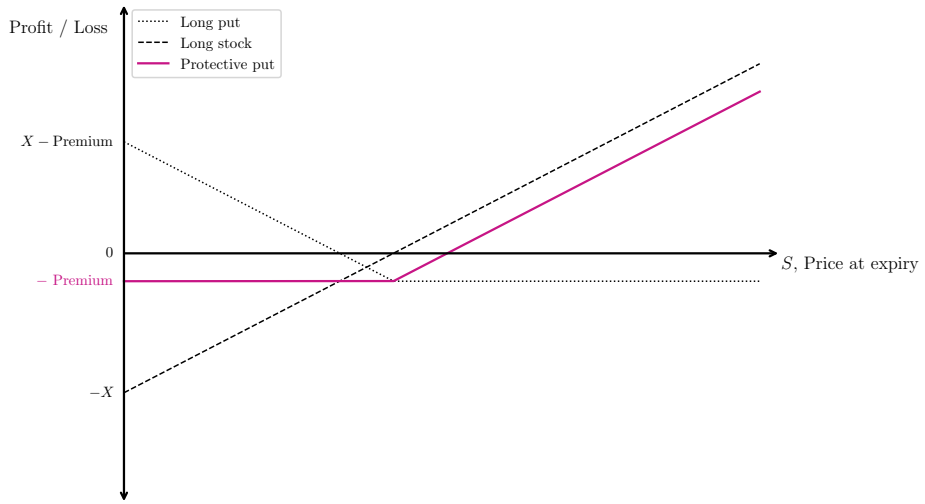


Figure 7.8: Protective put pay-off diagram

$$\text{If } S \leq X, \text{ payoff } = S + (X - S) = X \quad | \quad \text{If } S \geq X, \text{ payoff } = S + 0 = S$$

- A fiduciary call position comprises a long position in both a call option, and zero coupon bond with par value equal to strike price, and the same maturity of the option.

$$\text{If } S \leq X, \text{ payoff } = 0 + X = X \quad | \quad \text{If } S \geq X, \text{ payoff } = X + S - X = S$$

By construction, we can see that this has identical payoffs. As such, they must have the same total value of the portfolio, through the principal of no-arbitrage. Therefore,

$$c_t + \frac{X}{(1 + R_f)^T} = p_t + S. \tag{7.25}$$

This gives us an expression relating to the put-call parity of European options. This expression can be rearranged in order to create a synthetic put, call or stock position.

$$\text{Synthetic put} \qquad p_t = c_t - S + PV(X) \tag{7.26}$$
$$\text{Synthetic call} \qquad c_t = p_t + S - PV(X) \tag{7.27}$$
$$\text{Synthetic stock} \qquad s = c_t + PV(X) - p_t \tag{7.28}$$

EXAMPLE: Consider a stock trading at 52. The risk free rate is 5%. If a 3m put option with a strike price of 50 is valued at 1.50 today, what is the value of a 3m call today?

$$\begin{aligned} c_t &= p_t + S - PV(X) \\ &= 1.5 + 52 - \frac{50}{(1+5\%)^{0.25}} \\ &= 4.11 \end{aligned}$$

7.15 Put-call-forward parity

- An underlying asset may also be replicated through the use of a forward contract and a risk-free bond which pays the forward price at expiration. (Think back to the cash and carry model, §7.5). Here,

$$S_0 = \frac{F_0(T)}{(1+R_f)^T}. \tag{7.29}$$

Therefore, equation 7.25 can be rewritten as

$$\frac{F_0(T)}{(1+R_f)^T} + p_t = \frac{X}{(1+R_f)^T} + c_t. \tag{7.30}$$

- We can link these ideas back to the capital structure of a firm. Recalling that for any firm,

$$\text{Assets} = \text{Liability} + \text{Equity},$$

the "Assets" refers to the market value of firm assets, V_0, otherwise known as the firm value. The liabilities can be likened to a zero-coupon bond, which has a par value at redemption equal to the size of the outstanding debt.

	Solvency	**Insolvency**
	$V_T > D$	$V_T < D$
Equity value	$V_t - D$	0
Debt	D	V_T

Looking at this, we can draw some parallels with options.

  - The equity payoff is equivalent to a call option where D is the strike price (long call option)
  - The debt payoff is equivalent to going short a put option

- Risky debt, or debt of the company can be recreated through a combination of risk-free debt and short put option position:

$$\text{Risk debt} = \underbrace{\text{Risk-free debt}}_{\frac{X}{(1+R_f)^T}} - \text{Put option.} \tag{7.31}$$

An investor goes short a put option in order to receive the option premium. This is equivalent to the credit risk premium.

- Recalling pu-call parity, from equation 7.25, we find

$$c_0 + PV(D) = p_0 + V_0 \tag{7.32}$$

$$\text{Firm value, } V_0 = \underbrace{c_0}_{\text{Equity}} + \underbrace{PV(D) - p_0}_{\text{Debt}} \tag{7.33}$$

7.16 Derivative valuation using a one-period binomial model

- We recall

$$\text{Option value (Premium)} = PV(\text{Expected pay-off at expiry}).$$

Under a one-period binomial model, we allow the asset price to move exactly once, either up or down.

Today	Expiry	Pay-off
S_0	S_+ up move	$\text{Max}(0, S_+ - X) = c_+$
	S_- down move	$\text{Max}(0, S_- - X) = c_-$

Table 7.9: Pay-offs for a one-period binomial model used to price an option

For this, we need to know X, R_f, σ[5]

- To be precise, the value of $\S_\pm$ is given by

$$S_\pm = S_0 e^{\pm\sigma\sqrt{\delta t}},$$

however this is unlikely to be tested.

- A risk-free portfolio will hold a long position in an underlying and go short a call option. The fact that it is risk-free implies the value at the end of the period will be the same regardless of whether the price moves up or down. We must therefore determine the hedge ratio which results in $V_+ = V_-$,

$$V_T = V_\pm \tag{7.34}$$

$$V_T = \begin{cases} V_+ = hS_+ - c_+ \\ V_- = hS_- - c_- \end{cases} \tag{7.35}$$

EXAMPLE: Consider

$$S_0 = 50, \qquad S_+ = 60, \qquad S_- = 42, \qquad X = 55.$$

The value of the call option at expiry in each circumstance is

$$c_+ = \text{Max}(0, S_+ - X) = \text{MAX}(0, 60 - 55) = 5$$
$$c_- = \text{Max}(0, S_- - X) = \text{MAX}(0, 42 - 55) = 0$$

[5] σ drives the up / down move. It is assumed that the volatility is constant

Then, we can determine the hedge ratio by rearranging equation 7.35 to give

$$\begin{aligned} hS_+ - c_+ &= hS_- - c_-, \\ h(S_+ - S_-) &= c_+ - c_-, \\ h &= \frac{c_+ - c_-}{S_+ - S_-}, \\ &= \frac{5-0}{60-42}, \\ &= \frac{5}{18} = 0.278. \end{aligned}$$

Therefore, an investor needs 0.278 shares to offset each short call. We know that this portfolio should return the same value, regardless of whether the asset price moves up to S_+ or down to S_-. As such, it should return the risk-free rate. In other words,

$$\frac{V_T}{V_0} = 1 + R_f,$$

where V_T is given by

$$\begin{aligned} V_T &= \underbrace{hS_+}_{0.278 \cdot 60} - \underbrace{c_+}_{5} = \underbrace{hS_-}_{0.278 \cdot 42} - \underbrace{c_-}_{0} \\ &= 11.68 \end{aligned}$$

Given a risk-free rate of $R_f = 3\%$,

$$\begin{aligned} \frac{V_T}{V_0} &= \frac{11.68}{V_0} = 1 + 3\%, \\ V_0 &= \frac{11.68}{1.03} = 11.34. \end{aligned}$$

We can use the same expression but evaluated at time $t = 0$ to find the initial value of the portfolio in terms of the present value of the stock and the call option.

$$\begin{aligned} V_0 = hS_0 - c_0 = 0.278 \times 50 - c_0 = 11.34, \\ c_0 = 2.56, \end{aligned}$$

So the option premium is 2.56.

- We can use a very similar method to value a put option. In this case, the risk-free portfolio is constructed by going long in both the stock and a put option.

EXAMPLE: Consider

$$S_0 = 50, \qquad S_+ = 60, \qquad S_- = 42, \qquad X = 48.$$

The value of the call option at expiry in each circumstance is

$$\begin{aligned} p_+ &= \text{Max}(0, X - S_+) = \text{MAX}(0, 48 - 55) = 0 \\ p_- &= \text{Max}(0, X - S_-) = \text{MAX}(0, 48 - 42) = 6 \end{aligned}$$

Then, we can determine the hedge ratio by rearranging equation 7.35 to give

$$\begin{aligned} hS_+ + p_+ &= hS_- + p_-, \\ h(S_+ - S_-) &= -p_+ + p_-, \\ h &= \frac{-p_+ + p_-}{S_+ - S_-}, \\ &= \frac{-0+6}{60-42}, \\ &= \frac{6}{18} = 0.333. \end{aligned}$$

Therefore, an investor needs 0.333 shares to offset each long put position. Again, we know that this portfolio should return the same value, regardless of whether the asset price moves up to S_+ or down to S_-.

$$\begin{aligned} V_T &= \underbrace{hS_+}_{0.333\cdot 60} + \underbrace{c_+}_{0} = \underbrace{hS_-}_{0.333\cdot 42} - \underbrace{c_-}_{6} \\ &= 20 \end{aligned}$$

Given again, a risk-free rate of $R_f = 3\%$,

$$\begin{aligned} \frac{V_T}{V_0} &= \frac{20}{V_0} = 1 + 3\%, \\ V_0 &= \frac{20}{1.03} = 19.42. \end{aligned}$$

$$\begin{aligned} V_0 = hS_0 + p_0 = 0.333 \times 50 + p_0 = 19.42, \\ p_0 = 2.75, \end{aligned}$$

So the option premium is 2.75.

7.16.1 Risk-neutral pricing

- The value of the option is given by the present value of the expected pay-off of that option.

$$\pi_+ = \text{Risk-neutral probability of up-move} = \frac{1 + R_f - D}{U - D}, \tag{7.36}$$

$$\pi_- = \text{Risk-neutral probability of down-move} = 1 - \pi_u, \tag{7.37}$$

where U and D are the factors of an up and down move respectively.

Combining these in an expectation calculation, we find

$$\text{Option value} = \frac{\pi_+ c_+ + \pi_- c_-}{(1 + R_f)^T}, \tag{7.38}$$

Without loss of generality, we can assume a call option, but the same process is applicable to a put option.

EXAMPLE: Consider a stock priced at 30. The risk free rate is 7%, and the up / down move factors are 1.15 and 0.87 respectively. A call option has a strike price of 30, with an expiry in one year's time.

$$\begin{aligned} S_0 = \begin{cases} S_+ = 1.15 \times 30 = 34.50, \; c_+ = 4.50, \\ S_- = 0.87 \times 30 = 26.10, \; c_- = 0, \end{cases} \\ h = \frac{4.50 - 0}{34.50 - 26.10} = 0.536, \end{aligned}$$

So the portfolio must go long 0.536 units of stock to offset each short call contract. The risk free portfolio will be positioned long stock and short the call option.

$$\begin{aligned} V_T = V_\pm = 0.536 \times 26.10 = 13.99, \\ V_0 = \frac{13.99}{1.07} = 13.075, \\ = hS_0 - c_0, \\ c_0 = 16.08 - 13.075, \\ c_0 = 3.005. \end{aligned}$$

We can then use equation 7.38 to work out the probability of an up and down move.

$$\begin{aligned} c_0 &= \frac{\pi_+ c_+ + \pi_- c_-}{(1+R_f)^T}, \\ 3.005 &= \frac{\pi_+ \cdot 4.5 + (1-\pi_+)\cdot 0}{(1.07)}, \\ \pi_+ &= \frac{3.005 \times 1.07}{4.50}, \\ \pi_+ &= 0.715, \qquad \pi_- = 0.285. \end{aligned}$$

We could have used equations 7.36 and 7.37 instead, which recovers the same results.

$$\begin{aligned} \pi_+ = \frac{1+R_f-D}{U-D} = \frac{1+0.07-0.87}{1.15-0.87} = 0.715, \\ \pi_- = 1-\pi_u = 1-0.715 = 0.285. \end{aligned}$$

8 Alternative Investments

8.1 Feature, methods, and structures

- Alternative investments offer diversification to an investor, with respect to traditional investment. It expands the universe of potential investments, and typically carries low correlations with traditional assets
- Alternative investments are typically less liquid, and have longer time horizons that traditional investments. The minimum investment size is typically much greater, and often requires more specialised knowledge. As such, they ten to command higher fees (mangement and performance)
- Characteristics of alternative investments include
  - Information structures that facilitate direct investment by management
  - Information asymmetry between the fund mangers and investors in the fund
  - Difficulty in accurately measuring performance
- Types of alternative investments include
  - Private capital (Equity and debt)
  - Real estate
  - Natural resources (Commodities, farmland, timberland)
  - Infrastructure (Public-private partnerships)
  - Digital assets (Cryptocurrencies)
  - Hedge funds (Alpha-seeking strategies)
- Alterntative investment methods include
  - Fund investing – Investing in a pool of assets alongside other investors

Advantages	Disadvantages
Fund manager expertise	Large capital investment
Diversification	Long investment horizon
Lower investor involvement	Limited transparency and informational asymmetry
	Higher management / incentive fees

Table 8.1: Advantages and disadvantages of fund investing

  - A term sheet will detail
    - Investment policy
    - Fee structure
    - Requirements
  - Co-investing – Fund investing with the right to directly invest in assets alongside the fund manager
  - Direct investing – Investor purchases assets

Advantages to investors	Advantages to fund managers
Lower fees	Increase in availability of investment funds
More control	
Benefit from mager expertise	Expand scope and diversification of investments

Table 8.2: Advantages of co-investing to investors and fund managers

Advantages	Disadvantages
No fees	Requires expertise
Full control	High minimum investment
	Lack of diversification

Table 8.3: Advantages and disadvantages of direct investing

General partner	Fund manager
Limited partner	Accredited investors – Limited liability, no management responsibility
Limited partnership agreement	Sets out fund rules and guidance, alongside operational details
Side letters	Special terms negotiated by individual limited partners (LPs)
Master Limited Partnership	Specialises in natural resources and real estate

Table 8.4: The involved parties and agreements with regards to a typical compensation structure of a limited partnership

8.2 Compensation structures

- Typically, this refers to limited partnerships.:
- The compensation structure is typically split out into management and performance fees.
  - Management fees
    - Typically 1-2% of AUM (Fixed cost to the investor)
    - Independent of performance. For hedge funds, this refers to the AUM. For private capital funds, this refers to the committed capital (Not just the invested capital)
  - Performance fees
    - Paid to general partners / fund managers based on fund performance

8.2.1 Performance fees

- A fund may use either a soft hurdle or hard hurdle to determine performance fees
  - Soft hurdle (benefits GP)
    - % increase in the investment of value, contingent on beating a minimum performance
  - Hard hurdle (benefits LP)
    - Only paid on the excess above some threshold

EXAMPLE: Consider a fund that has returned 15% over the past year. The fund has a hurdle rate of 6%, and there is an 80/20 split between the LP and GP

- Under a soft hurdle:

$$15\% > 6\%$$
$$\text{GP} = 20\% \times 15\% = 3\%$$

- Under a hard hurdle:

$$GP = 20\% \times (15\% - 6\%)$$
$$= 1.8\%$$

- A catch-up clause may be implemented to benefit the LP. In this case, everything up to the hurdle rate goes to the LP. A "catch-up" clause is then implemented on the next portion of earnings to accelerate the GP's compensation up to the soft hurdle level. From then on, it behaves the same as a soft hurdle.
- Consider a fund which has a 10% hurdle rate with a catch-up clause. The performance fee is 20/80 in the LP's favour. In this case, the first 10% of gains goes to the LP. The next 2.5% go to the GP. Anything over 12.5% is then split 20:80 between the GP and LP.

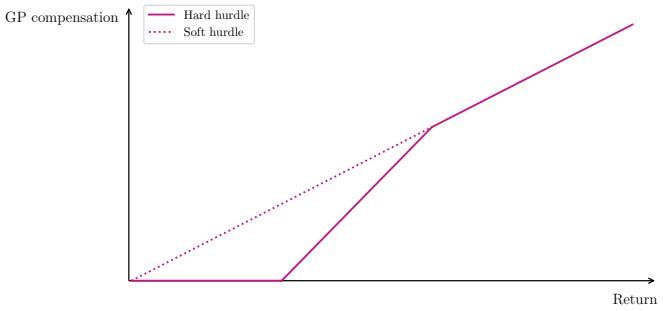


Figure 8.1: GP compensation as a function of performance.

- A high water mark is sometimes put in place to prevent double payment for the same gains. In this case, performance fees are only paid on gains over the previous high investment value.
- A clawback provision may also be implemented. In this instance, any losses can be recovered by the LPs by prior excess incentive payments
- The waterfall structure determines how cashflows are allocated to GPs and LPs in a limited partnership agreement.
  - Deal-by-deal "American Waterfall" (Better for GP)
    - Distributed as fund exits each investment
    - Shared between GP / LP
  - Whole-of-fund "European Waterfall" (Better for LP)
    - LP receives everything until the hudle rate is cleared
    - After the hurdle rate is cleared, the GP participates in further profits

8.3 Alternative investment performance and returns

- Risks of alternative investments include
  - Timing of cash flows over an investment's life cycle
  - Use of leverage by fund managers
  - Valuation of investments that do not necessarily have observable market prices
  - Complexity of fees, taxes, and accounting

- Timing of cash flows of a fund is broadly split into three phases. This is well represented by the "J-curve effect"

  1. Capital commitment phase
     - Identifying what to include in portfolio
       * Negative returns
       * Fees
       * No cash flows
  2. Capital deployment phase
     - Investing in assets

     Outflows > Inflows

  3. Capital distribution phase
     - Price appreciation of asset

     Inflows > Outflows

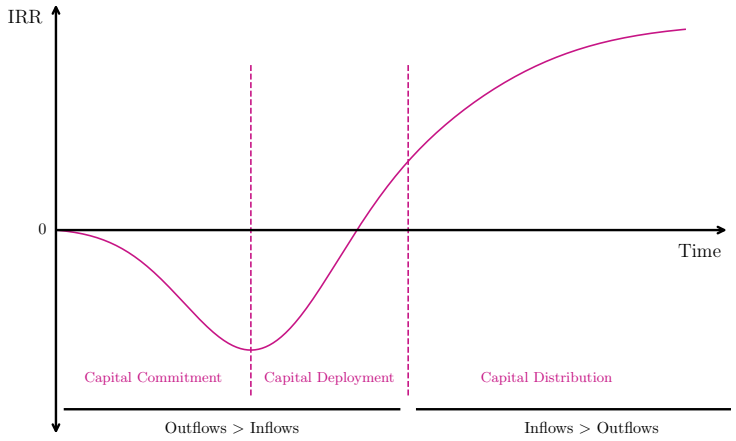


Figure 8.2: Typical J-curve for fund returns

- Performance appraisal
  - Private capital and real estate involve cash outflows and inflows over the life of an investment. We can therefore define

$$\text{Multiple of invested capital} = \frac{\text{Total capital returned} + \text{Value of remaining assets}}{\text{Total capital paid in}}. \tag{8.1}$$

    This does not however consider the timing of any cash flows. It does not accurately reflect the time value of money, or risk associated with the investment. To account for this, we can instead use the internal rate of return.

8.3.1 Use of leverage

- A private fund may use leverage to amplify returns.
- Given the unlevered portfolio return, r, the leveraged return is given by

$$r_L = \frac{r(V_O + V_B) - r_B V_B}{V_O}, \tag{8.2}$$

where the subscripts O and B refer to an investers own and borrowed funds respectively. The use of leverage amplifies any gains, but also any losses too.

EXAMPLE: Consider a fund which has 200mn capital. It adds leverage of 100mn at a cost of $r_B = 5\%$. Calculate the levered return if in the following year, the fund returns either 10% or -5%.

Using equation 8.2, in the case of 10% return,

$$\begin{aligned} r_L &= \frac{0.1 \times (200 + 100) - 0.05 \times 100}{200} \\ &= \frac{30 - 5}{200} \\ &= 0.125 = 12.5\% \end{aligned}$$

In the case of a -5% annual return,

$$\begin{aligned} r_L &= \frac{-0.05 \times (200 + 100) - 0.05 \times 100}{200} \\ &= \frac{-15 - 5}{200} \\ &= -0.10 = -10\% \end{aligned}$$

8.3.2 Valuation of investments

- The fair value hierarchy has three classification levels
  1. Assets trading in active markets. Quoted prices are readily available.
  2. Assets do not have readily-available quoted prices. Direct / indirect observable inputs may be used to value an asset
  3. Assets require estimates / unobservable inputs to value them

8.3.3 Redemptions

- Redemptions in alternative assets are not as simple as assets which are actively traded in markets. Funds may charge a redemption fee, and institute lock-up and notice periods.
  - Lock-up period – Time after initial investment over which LPs cannot request redemption without incurring significant fees
  - Notice period – Typically 30 – 90 days, this defines the time within which a fund must fulfil a redemption request.

8.3.4 Return calculations

- Hedge funds in particular are subject to both survivorship bias and backfill / selection bias. There is no requirement for them to report returns. Hedge funds that have failed are often not included either. Therefore, the hedge funds for which data is available tends to be those that have been successful over a long period of time. Indices that look at such funds therefore tend to overstate returns and understate risk of an average hedge fund.
- Before-fee returns are the same as those in traditional investments
- After-fee returns adjust for cash flows after management and performance fees have been levied.

$$\text{Total fee} = mV_1 + \text{Max}(0, p(V_1 - V_0) \tag{8.3}$$

$$\text{Rate of return after fees} = \frac{V_1 - V_0 - \text{Total fees}}{V_0} \tag{8.4}$$

EXAMPLE: Consider a fund which has has a standard 2/20 fee structure based on beginning assets. The performance fee is calculated net of the management fee. The fund employs a soft-hurdle approach to calculating the performance fee, and also has a high-water mark. The value of the fund at the beginning of the next three years is as follows.

$$V_0 = 110.0 \text{ mn} \qquad V_1 = 100.2 \text{ mn} \qquad V_2 = 119.0 \text{ mn}$$

- After year 1:
  - The management fee is
$$0.02 \times 110,000,000 = 2,200,000$$
  - The performance fee is calculated as follows.
$$\text{Value net of management fee} = 100,200,000 - 2,200,000 = 98,000,000,$$
$$\text{Return net of management fee} = \frac{98,000,000}{110,000,000} - 1 = -10.9\%. \qquad (< 5\%)$$
    As the return is less than 5%, there is no performance fee
- After year 2:
  - The mangement fee is
$$0.02 \times (100,200,000 - 2,200,000) = 0.02 \times 98,000,000 = 1,960,000$$
  - The performance fee is
$$\text{Value net of management fee} = 119,000,000 - 1,960,000 = 117,040,000,$$
$$\text{Return net of management fee} = \frac{117,040,000}{98,000,000} - 1 = 19.4\%, \qquad (> 5\%)$$
$$\text{Performance fee} = 0.2 \times (119,000,000 - 1,960,000 - \underbrace{110,000,000}_{\text{High-water mark}}) = 1,410,000,$$
$$\text{Total fee} = 3,470,000,$$
$$\text{After-fee return} = \frac{119,000,000 - 3,470,000}{98,000,000} - 1 = 18\%.$$

EXAMPLE: Consider an investor who has invested 60,000,000 in a fund of funds. The fee structure for this fund is 1/10 based on year-end values, and the performance fee is independent of management fees.

	T_0	T_1
α	40 mn	45 mn
β	20mn	28 mn

$$\text{Value before fees} = 45,000,000 + 28,000,000 = 73,000,000$$
$$\text{Gain in value} = 73,000,000 - 60,000,000 = 13,000,000$$

$$\text{Management fee} = 0.01 \times 73,000,000 = 730,000$$
$$\text{Performance fee} = 0.1 \times 13,000,000 = 1,300,000$$
$$\text{Total fee} = 730,000 + 1,300,000 = 2,030,000$$

$$\text{Value after fees} = 73,000,000 - 2,030,000 = 70,970,000$$
$$\text{Return after fees} = \frac{70,970,000 - 60,000,000}{60,000,000} = 18.28\%$$

The direct investment return would have been $\frac{73,000,000 - 60,000,000}{60,000,000} = 21.67\%$

EXAMPLE: Consider a private equity fund which invests 100 mn in VC in a firm, which it then sells for 130 mn. It also invesets 100 mn in an LBO which it then sell for 80 mn. The fund has an incentive fee of 20%, and no clawback provisions

- Under an American waterfall structure (deal-by-deal):

$$\begin{aligned} \text{VC firm:} &\quad 0.2 \times (130 - 100) = 6 \\ \text{LBO firm:} &\quad 80 < 100 \text{ so no performance fee} \end{aligned}$$

  The after-fee return is therefore

$$\text{After-fee return} = \frac{130 + 80 - 6}{200} - 1 = 2\% \text{ to the LP}$$

- Under a European waterfall structure (whole-of-fund):

$$0.2 \times (130 + 80 - 200) = 2$$

  The after-fee return is therefore

$$\text{After-fee return} = \frac{130 + 80 - 2}{200} - 1 = 4\%. \text{ to the LP}$$

  We can clearly see the American waterfall is better for the GP, and the European waterfall is better for the LP

Assuming the fund with an American waterfall structure exited the VC firm in Y1 and the LBO in Y2, what would the effect of a clawback provision be?

The total gain across two years is

$$\text{Total gain} = 130 + 80 - 200 = +10 \text{ mn.}$$

The performance fee is therefore $0.2 \times 10 = 2$ mn on the whole portfolio. We saw earlier that the performance fee on VC exit is 6 mn, so the LP can "claw-back" $6 - 2 = 4$ mn upon the exiting of the LBO position in Y2.

8.4 Investments in private capital (Equity & debt)

- Private capital finances "portfolio companies" <u>without</u> the issuance of publicly-traded securities. While it provides good diversification from traditional assets, it also requires management skill.
- Private equity involves investment in a private company or taking a public company private. Strategies for this include

- Leveraged buyout, LBO
  - Financed by debt
- Venture capital, VC
  - High risk / reward
- Private investment in private equity

8.4.1 LBO

- This is the most common private equity strategy, and is largely funded by debt. There are two types:
  - Management buyout, MBO: The current managers are involved with purchase and remain with the company
  - Management buyin, MBI: External investors replace the managers of the acquired company

VC	⟶	**Growth Capital**	⟶	**LBO**
Start-up		Established business		Mature business
		"Growth Equity"		MBO / MBI, high leverage
		Primary capital		Secondary capital

Table 8.5: The private equity spectrum of investments

8.4.2 Venture capital

1. Formative stage
    - Pre-seed capital / angel investing. Focus of company is geared toward business plans and market potential
    - Seed stage / seed capital. Focus of company is geared toward product development and market research
    - Early stage / start-up stage. Focus of company is geared toward beginning production and sales
2. Later stage
    - Company expansion and early growth
3. Mezzanine stage financing
    - Preparation for IPO. If a company makes it to IPO, it is considered to be a successful investment for the VC investor. The IPO is the most lucrative for the private investor, grants continued upside, and also generates good publicity for the P/E firm

8.4.3 Private equity exit strategies

- Trade sale
    - Sell portfolio company to a strategic buyer (i.e. a competitor)
- Public listing
    - IPO - direct listing, special purpose acquisition company (SPAC)
- Recapitalisation
    - Issue portfolio company "debt" to fund dividend payment to private equity owner
- Secondary sale
    - Sell portfolio company to another private equity investor
- Write-off / liquidation
    - Take a loss from an unsuccessful investment

8.4.4 Private debt

- Direct lending
    - Includes leveraged loans using money borrowed from other sources
- Venture debt

  - Lending to start-up companies. Often convertible or with warrants, therefore this carries an equity upside

- Mezzanine debt
  - Subordinated to existing debt
- Distressed debt
  - Buying a company in / near default. A distressed debt investor may be an active participant in the restructuring of a company
- Unitranche debt
  - Combines all classes of debt into a single loan with a representative interest rate

8.4.5 Real estate and infrastructure

- Real estate investments cover residential properties (75% of the market), which covers single-family homes, and commercial property, which covers office buildings, shopping centres, industrial / warehouse / distribution, and rental residential
  - With single-family homes, the property owner is responsible for maintenance, insurance, and mortgage principal and interest payments, with the home acting as collateral on the loan)

	Debt	**Equity**
Private	Mortgage debt Construction loans Mezzanine debt	Direct ownership Sole ownership Joint ventures Limited partnerships Indirect ownership Real estate funds Private REITs
Public	MBS / CMBS / CMOs Covered bonds Mortgage REITs Mortgage ETFs	Publicly traded shares Construction Operating Development Public REITs UCITS / Mutual funds / ETFs

- Direct real estate investment benefits include
  + Control over investments
  + Diversification from traditional assets
  + Favourable tax treatment of real estate investments

  and drawbacks include
  - Illiquidity of assets and opacity of pricing
  - Additional complexity of managing and maintaining property assets

  - Specialised knowledge required when choosing investments
  - High level of capital required upfront to invest
  - Concentration risk

- Indirect real estate investments may be made through REITs. REITs are:
  - Exempt from double taxation, (if ¿90% dividends paid out)
  - Exchange traded, (thus providing liquidity)
  - Managed by specialists in the asset class.

  Types of REITs include
  - Equity REITs – real estate
  - Mortgage REITs – lending
  - Hybrid REITs – Combination

- REIT stragegies involve
  - Core real estate strategies
    * High quality commercial and residential property to deliver stable returns
    * Open-ended structure
    * Indefinite lives
  - Riskier investment strategies
    * Core-plus real estate strategies [modest redevelopment]
    * Value-add real estate strategies [monetary development]
    * Opportunistic "Speculation" real estate strageies [large-scale development]

8.4.6 Infrastructure investments

- Infrastructure projects are long-lived assets providing essential economic or social public services. These include
  - Transport [economic]
  - Utilities [economic]
  - Communications [economic]
  - Hospitals [social]
  - Prisons [social]

- Cash flows from infrastructure investments include
  - Availability payments
  - Usage-based payments (tolls)
  - Take-or-pay (buyer pays a minimum purchase price for the asset)

- Direct investment in infrastructure requires large upfront investment size, low liquidity of the asset, and a requirement to operate / maintain the asset over its useful life

- Indirect investment may be made through ETFs, listed mutual funds, master limited partnerships (energy only), or publicly traded infrastructure securities

- Infastrucutre investments may be
  - Brownfield (built on existing sites) – High yielding, but lower growth potential

  - Greenfield (built on planned sites) – Lower yielding, but carry higher risk and potential reward

- Generally, infrastructure assets provide good diversification from traditional assets, but are only suitable for long-term investors, such as institutional investors
- Risks associated with infrastructure assets include regulatory risk which is intrinsic to the asset class, alongside risks stemming from financial leverage, cash flows, construction, and operation of the asset

8.4.7 Diversification benefits

- Private equity and private debt have a lower correlation to traditional investment returns.
- The "Vintage year" is the first year of a fund's investment
- From highest risk / return to lowest;

1. Private equity
2. Mezzanine debt
3. Unitranche debt
4. Senior direct lending
5. Senior real estate debt
6. Infrastructure debt

8.5 Natural resources

- Natural resource investments include investments in
  - Raw land – Price appreciation, lease, location, aternative use (Direct / partnership owned)
  - Commodities – Gain exposure through the use of derivatives
  - Farmland / timberland – Requires knowledge of the underlying resource
- Investments can be made through
  - Direct investments
  - ETFs
  - REITs
  - Limited partnerships
  - Limited liability corporations

8.5.1 Farmland / Timberland

- Commodities include
  - Metals
  - Agricultural products
  - Energy productss

  They do not provide any cash flows. Return comes from price changes in the underlying assets.
- Typically, farmland and timberland provides a higher average return with lower volatility than global stocks

8.5.2 Commodities

- Commodity exposure can be achieved through:
  - Derivatives: $\underbrace{\text{Futures}}_{\text{Exchange}}$, $\underbrace{\text{Forwards}}_{\text{OTC}}$, $\underbrace{\text{Options}}_{\text{OTC / Exchange}}$, $\underbrace{\text{Swaps}}_{\text{OTC}}$, where the benefit of exchange-traded derivatives is that there is no counterparty risk due to novation of contracts through a CCP.

- ETPs: Suitable for investors restricted to holding equity shares only
- Commodity valuation is given by

$$\text{Futures price} \approx \text{Spot price} \times (1 + \text{Risk free}) + \text{Storage cost} - \text{Convenience yield} \tag{8.5}$$

The convenience yield is the value of having a physical commodity available for use. [Non-monetary benefits]

$$\begin{array}{ccccc} \text{Low convenience yield} & \implies & \text{Contango} & \implies & \text{Future} > \text{Spot} \\ \text{High convenience yield} & \implies & \text{Backwardation} & \implies & \text{Future} < \text{Spot} \end{array}$$

- Risks of investing in commodities include
  - Lack of liquidity
  - High fixed cost of production of commodity
  - Physical assets subject to adverse weather and natural disasters
  - Supply / demand effects on underlying physical price
- Returns on commodities are typically higher than global stocks and bonds. They provide a good hedge against inflation, and have a characteristic low corrleation with global stocks and bonds
- Prices are more sensitive to geopolitical and weather-related factors

8.6 Hedge funds

- Hedge funds are privately held, and are limited to qualified and accredited investors (usually an income and net worth eligibility screen)
- Drivers of return are usually market inefficiencies and price volatility.
- They typically invest in traditional asset classes, sometimes enhancing returns through the use of leverage and / or derivatives
- They are usually evaluated on either a total or risk-adjusted return basis
- Hedge funds differ from ETFs and mutual funds in a number of ways:
  - Less regulation
  - Flexible mandates
  - High managment and performance fees
  - Low transparency and high information asymmetry between managers and investors
  - Low liauisity (lock-up periods, notice periods, liquidity gates)
- Hedge funds can be organised into:
  - Commingled funds
    - Master-feeder structure – Tax-efficient, economies of scale, allows for funding from global investors
  - Separately-managed accounts
    - Customside portfolio, appropriate for large / institutional investors

8.6.1 Hedge fund strategies

- Equities
  - Fundamental long / short – Capture α, net long exposure
  - Fundamental growth – Identify high growth companies, capital appreciation

  - Fundamental value – Identify undervalued companies (potential for increase in revenues and cash flows)
  - Market neutral – Equal values in long and short positions
  - Short bias – Net short exposure

- Event-driven
  - Merger-arbitrage – Buy shares of the target firm and short shares of the acquirer
  - Distressed / restructuring – Buy undervalued shares during restructuring if the restructure will increase the value
  - Activist shareholder – Gain board seats to influence and drive decsions and policy
  - Special situations – Spinoffs, asset sales, security issuance / repurchase

- Relative value
  - Convertible arbitrage fixed income – Convertible bonds vs underlying common stock
  - Specific fixed income – ABS, MBS, high yield
  - General fixed income – Various issuers and types (Look for inconsistencies in rates)
  - Multistrategy – Across asset classes and markets

- Opportunistic
  - Macros strategies – Trade securities, currencies, commodities based on global trends
  - Managed futures – "Commodity trading advisors" – Trade commodity futures, incorporated financial futures

8.6.2 Hedge fund structures

- Hedge funds may be organised as a
  - Limited partnership / limited liability structure
    * General partner (fund manager) receives compensation based on performance
    * Private placement memorandum – Contractual relationship
    * Indefinite life
  - Fund-of-funds
    * This is where a hedge fund invests in other hedge funds
    * While this strategy requires a lower minimum investment compared to the underlying hedge funds, and offers greater diversification, it also requires an additional layer of fees for the fund-of-funds hedge fund, on top of the underlying funds

8.6.3 Hedge fund returns

- This may come through any of
  1. Market beta – Return from broad index
  2. Strategy beta – Return from specific sectors
  3. Alpha – Manager-specific returns

- Performance measured by indices of hedge funds is often overstated. Hedge fund indices show biased returns from
  - Survivorship bias, (25% fail within 3 years)
  - Selection bias (Only strong returns are published, non-representative index)
  - Backfill bias, (Only strong returns disclosed)

  This creates an upward bias on returns and downward bias on risk

8.7 Digital Assets

- Digital assets are those which are electronically stored, created, and transferred
- Distributed ledger technology (DLT) is used to secure and validate the assets
- The distributed ledger is the register for all transactions:

Benefits

+ Accuracy
+ Transparency
+ Security
+ Rapid ownership transfer
+ Peer-to-peer interaction

Disadvantages

– Data protection concerns
– Privacy violation potential
– Requirement for computational power

8.7.1 DLT Networks

- DLT networks consist of a digital ledger, consensus netwrk, and a network of participants
- DLT networks use cryptography to encrypt and store data
- Smart contracts are self-executing computer programs based on pre-determined criteria
- Blockchain records information sequentially within blocks which are linked together. It is then secured through cryptography

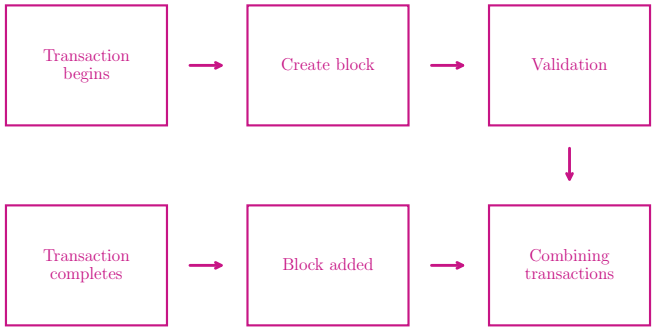


Figure 8.3: The steps involved with adding a transaction to a digital ledger

Consensus protocols

- This is a set of rules which determines how blocks may be chained together
  - Proof-of-work protocol
    - When a transaction is completed, miners use a computer to solve a cryptographic problem which verifies the transaction
  - Proof-of-state protocol
    - Network participants pledge collateral to guarantee the validity of a block

Forms of DLT networks

- Permissionless
  - Transactions are visible to all users

  - Any user can execute a transaction
  - Transactions are verified by consensus mechanisms, not the central authority

- Permissioned
  - Users are restricted from some activities
  - More cost-effective than open, decentralised, permissionless networks

8.7.2 Types of digital assets

- Digital assets include cryptocurrencies such as
  - Bitcoin, Ethereum, etc.
  - Altcoins (stablecoins / memecoins)
  - Central Bank Digital Currencies

  and tokens, such as
  - Non-fungible tokens (NFTs)
  - Security tokens (ICOs)
  - Utility tokens
  - Governance tokens

- Comaprison to other asset classes
  - Digital assets have inherent value differences – They yield no cash flows (interest / dividend payments) and thus have no fundamental value
  - Digital assets have transaction value differences – They are recorded on decentralised digital ledgers
  - Digital assets have different media of exchange – Their use may be restricted, and they primarily transact online
  - Digital assets have different regulations, and typically trade on unregulated exchanges
- Exchanges for trading in bitcoin and other cryptocurrencies include
  - Centralised exchanges
    * Privately-held, and offer trading platforms for price transparency and volume information
    * Most popular type of crypto exchange
    * Trade directly and electronically on private servers
  - Decentralised exchanges
    * Implement decentralised blockchain principles
    * No centralised authority – operates on distributed framework
- Direct investment in cryptocurrency occurs when a transaction is recorded on the blockchain (validated and permanently stored
  - Examples include purchasing tokens on a cryptocurrency exchange, trading an NFT, or investing in an ICO.
  - Fraud risk from this include scam ICOS, “pump-and-dump” schemes, market manipulation, and theft
- Indirect investment in crypto currency can be done through

- Crypto coin trusts
- Crypto futures contracts
- Crypto ETPs
- Crypto-related stocks
- Crypto-focused hedge funds

8.7.3 Digital investment in non-digital assets

- Digital investments in non-digital assets can be made through asset-backed tokens. These represent digital ownership of physical / financial assets
- Collateralised by the underyling asset, this may increase liquidity of expensifve assets
- Classified as securities, these asset-backed tokens allow for an immutable record of ownership

Returns

	BTC	S&P 500	MSCI World	BBG Agg
Average	8.84%	1.13%	0.66%	0.16%
Standard deviation	0.32	0.04	0.04	0.01
Coefficient of variation	3.66	3.43	6.09	8.16

Table 8.6: Table of long term characteristics of returns of Bitcoin compared to other commonly tracked indices

Correlations

	BTC	S&P 500	MSCI World	BBG Agg
BTC	1	—	—	—
S&P 500	0.21	1	—	—
MSCI World	0.22	0.97	1	—
BBG Agg	0.14	0.25	0.33	1

Table 8.7: Table of long term correlation of returns of Bitcoin compared to other commonly tracked indices. Note the lower correlations between BTC and others.

9 Portfolio management

9.1 Risk and return

- Historical risk and returns:

$$\text{Real return} = \text{Nominal return} - \text{Inflation}. \tag{9.1}$$

  This gives the return in real terms – how far will an investor's money go in terms of what is it's purchasing power "real" power in the world at the end of an investment's life

- While many models assume this, historically, returns typically do not follow a normal distribution. Insead, they are negatively skewed, by significant negative outliers, and exhibit positive excess kurtosis (fatter tails, kurtosis > 3)

- A portfolio manager should also consider the impact of liquidity on returns. This is evidenced by the bid-ask spread, as well as price impact when trading. Typically this is a bigger concern in EM investments, or assets which are infrequently traded

9.1.1 Risk aversion

- Assuming rational behaviour, an investor is assumed to prefer the least risky outcome that yields the same return. They would demand additional return for taking on greater risk. This behaviour is modelled using indifference curves

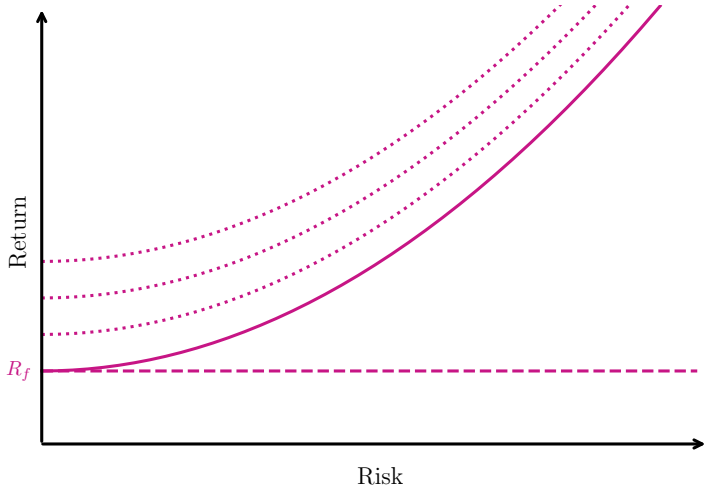


Figure 9.1: Indifference curves plotted in risk-return space. More generally known as utility curve, a more risk-averse investor will have a steeper curve - they will demand a higher return for every additional unit risk taken on.

- Given a choice of portfolios, an investor would choose the portfolio on the highest indifference curve. If returns are equal, and investor would be expected to choose the one with lowest risk. Given the same risk, an investor would be expected to choose the one offering the highest return.

9.2 Capital allocation line

- The capital allocation line is defined only for the combination of a risky asset with a risk-free asset. We recall

$$\text{Var}(R_P) = {w_A}^2{\sigma_A}^2 + {w_B}^2{\sigma_B}^2 + 2w_Aw_B\sigma_A\sigma_B\underbrace{\text{Cov}(A,B)}_{\sigma_A\sigma_B\cdot\rho_{A,B}}. \tag{9.2}$$

If $\sigma_B = 0$, in other words is risk free, then $E(R_B) = R_f$, and $\text{Cov}(A, B) = 0$. Equation 9.2 then reduces to

$$\text{Var}(R_p) = {w_A}^2 {\sigma_A}^2 \quad \equiv \quad \sigma_P = w_A \sigma_A \quad \Rightarrow \quad w_A = \frac{\sigma_P}{\sigma_A} \tag{9.3}$$

- Using

$$w_B = 1 - w_A \qquad \text{and} \qquad R_B = R_f,$$

we find

$$\begin{aligned} E(R_P) &= w_A R_A + w_B R_B \\ &= w_A R_A + (1 - w_A) R_f \\ &= R_f + w_A (R_A - R_f) \\ &= R_f + \frac{\sigma_P}{\sigma_A}(R_A - R_f) \\ &= R_f + \underbrace{\frac{R_A - R_f}{\sigma_A}}_{\text{Sharpe ratio}} \sigma_P \end{aligned} \tag{9.4}$$

where the Sharpe ratio is defined

$$\text{Sharpe ratio} = \frac{R_A - R_f}{\sigma_A}. \tag{9.5}$$

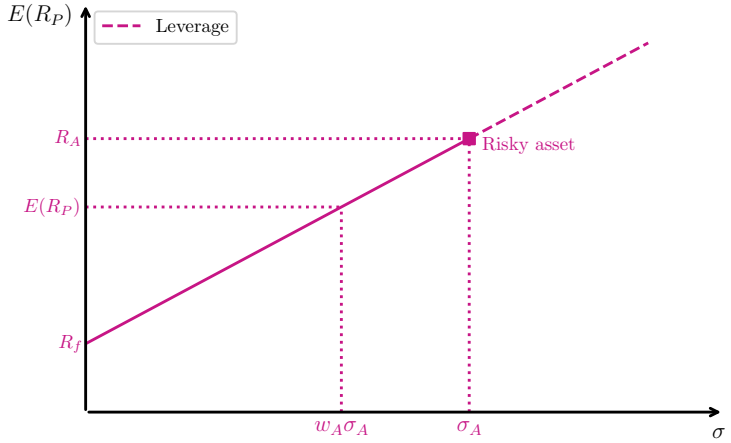


Figure 9.2: Capital allocation line for a portfolio consisting of a risky asset in combination with a risk-free asset. The dashed portion of the line is only achievable through the use of leverage.

- An investor's optimal portfolio will fall on the tangent point between a capital allocation line and the indifference / utility curve of that investor.
- Portfolio standard deviation measures can be calculated as follows

$$\text{Var}(X) = \frac{\sum (X_i - \overline{X})^2}{n - 1}, \tag{9.6}$$

$$\text{Cov}(X, Y) = \frac{\sum (X_i - \overline{X})(Y_i - \overline{Y})}{n - 1}, \tag{9.7}$$

$$\rho_{X,Y} = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}. \qquad \left[r_{X,Y} = \frac{\text{Cov}(X, Y)}{s_X s_Y} \right] \tag{9.8}$$

noting the use of sample standard deviation in the definitions here. This loses a degree of freedom.

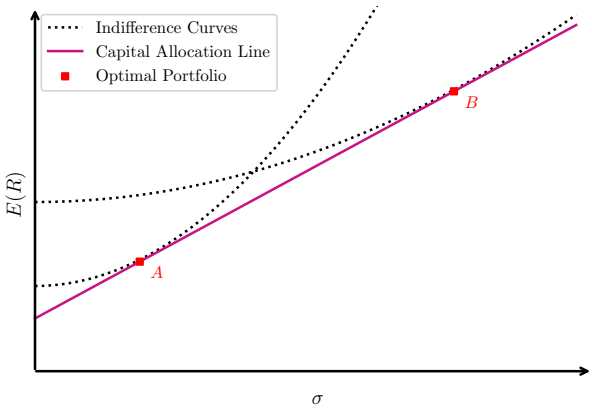


Figure 9.3: Optimal portfolio of a set risky asset in conjunction with a risk-free asset, for varying investor indifference curves. Investor A is more risk-averse (follows a steeper indifference curve), and so will select a less risky optimal portfolio

- We know from equation 9.2 that as correlations between assets fall, the overall portfolio risk also falls

$$\sigma_P{}^2 = \mathrm{Var}(R_P) = w_A{}^2\sigma_A{}^2 + w_B{}^2\sigma_B{}^2 + 2w_Aw_B\,\mathrm{Cov}(A,B),$$

  from which it is evident that a reduction in correlation of the assets results in a reduction of risk. These ideas lead on to the efficient frontier.

9.3 The efficient frontier

- Taking the square root of equation 9.2, we find

$$\sigma_P = \sqrt{w_A{}^2\sigma_A{}^2 + w_B{}^2\sigma_B{}^2 + 2w_Aw_B\sigma_A\sigma_B\rho_{A,B}}\,. \tag{9.9}$$

  If $\{w_i\}$ and $\{\sigma_i\}$ are all fixed, σ_P in equation 9.9 is minimised by a reduction in the correlation. Expected return however is not affected by correlation, merely the weighting of the assets and the expected return of the assets.

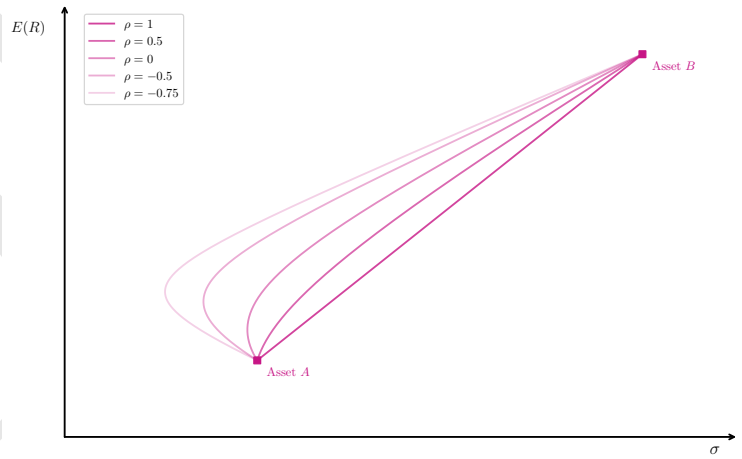


Figure 9.4: Each curve here shows values of R_P and σ_P for various weightings, w_A and w_B. We can see as $\rho \to 1$, the curve shows a greater bowing effect, and therefore a lower theoretical minimum risk achievable by a portfolio containing these two securities.

Extending this idea to whole portfolios, we can define the minimum variance frontier and efficient frontier.

- The efficient frontier, shown in figure 9.5, is the set of portfolios from the universe of all portfolios where return is maximised for a any given level of risk

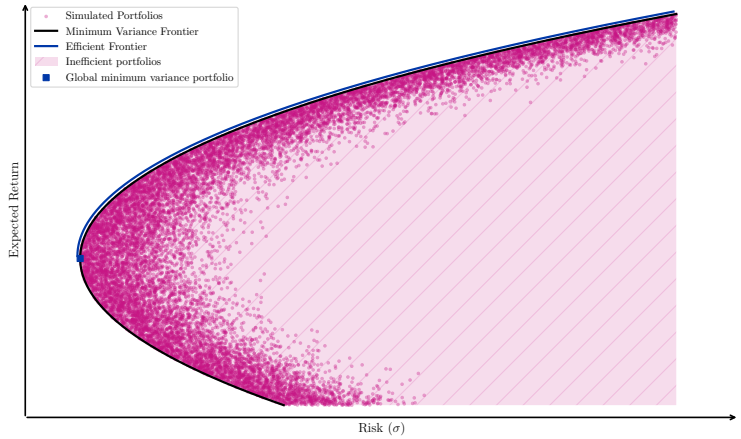


Figure 9.5: The minimum variance frontier and efficient frontier. The dots represent possible portfolios, and the shaded region represents the space of all inefficient portfolios.

9.4 Systematic risk and beta

- Combining risky assets does not necessarily result in a higher-risk portfolio. Recalling the capital allocation line,

$$E(R_P) = w_{\text{Risky}} \cdot R_{\text{Risky}} + w_{\text{f}} \cdot R_f, \tag{9.10}$$

$$\sigma_P = w_{\text{Risky}} \cdot \sigma_{\text{Risky}}. \tag{9.11}$$

  We also note that the gradient of the capital allocation line is the Sharpe ratio. The higher the Sharpe ratio, the better the risk-adjusted return.

- Assuming expectations are homogenous across all investors, all investors should have the same optimal risky portfolio. We define the capital market line as the capital allocation line for the optimal portfolio

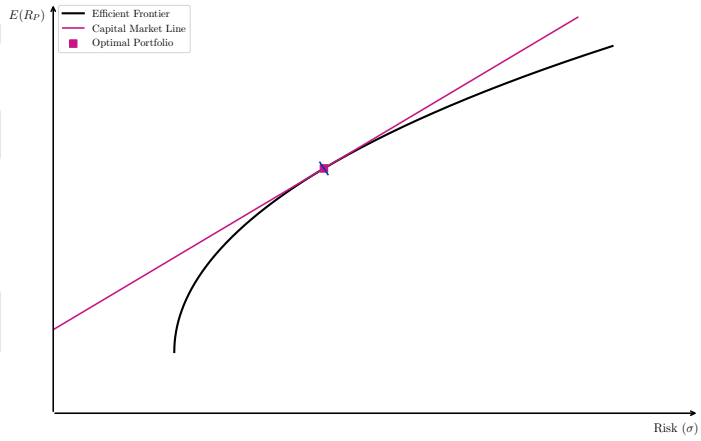


Figure 9.6: The capital market line is tangent to the efficient frontier. The intersection forms the optimal portfolio. For any position on the capital market line where $\sigma < \sigma_P$, the portfolio will lend money, receiving the risk-free rate. If $\sigma > \sigma_P$, then the portfolio will take on leverage, and so must borrow to finance this position.

- Recalling the Sharpe ration (equation 9.5), for the capital market line, we find

$$E(R_P) = R_f + \frac{R_M - R_f}{\sigma_M}\sigma_P, \tag{9.12}$$

$$E(R_P) = R_f + [E(R_M) - R_f] \cdot \frac{\sigma_P}{\sigma_M}. \tag{9.13}$$

  On a risk-adjusted basis, an investor cannot beat the market and cash. In practice this is possible, due to inefficiencies in the market.

- Systematic risk is cuased by macro factors (interest rtes, GDP growth, supply stocks, etc.). It is measured by the covariance of returns of a portoflio with market returns
- Unsystematic risk is stock-specific risk, and can be reduced by holding diversified portfolios.

$$\text{Total risk} = \underbrace{\text{Unsystematic risk}}_{\text{Can be removed}} + \text{Systematic risk} \tag{9.14}$$

- Assuming efficiency in market, this gives us the capital asset pricing model (CAPM),

CAPM: Only systematic risk is rewarded with higher returns

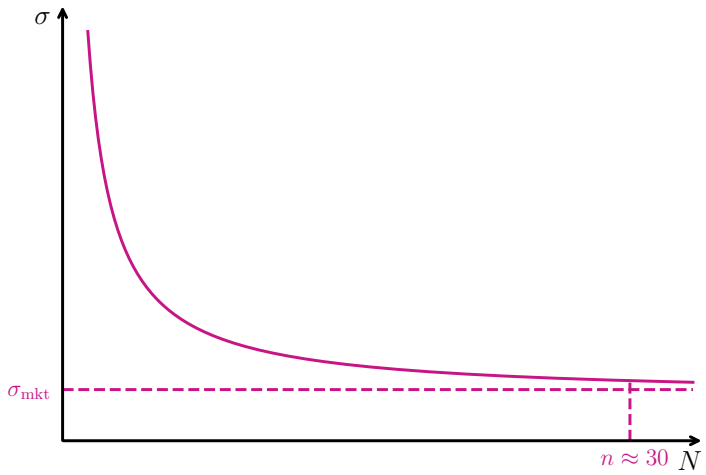


Figure 9.7: Additional securities added in a portfolio reduce the unsystematic risk of a portfolio

9.5 Returns-generating model

- The market model is defined by

$$R_i = \alpha_i + \beta_i R_m + \epsilon_i, \tag{9.15}$$

where the variables take on the following definitions

R_i	Return of a particular asset
α_i	The intercept (Unexplained)
β_i	The sensitivity of returns of asset i to returns of the market
R_m	Returns of the market
ϵ_i	The residual, defined such that $E(\epsilon_i) = 0$

What this gives us is a linear function linking security returns to market returns.. If however the market is not sufficient to explain all non-diversifiable risk, we can exted this to use a multi-factor model

$$E[R_i] - R_f = \beta_{i,1} \cdot E[F_1] + \beta_{i,2} \cdot E[F_2] + \cdots + \beta_{i,N} \cdot E[F_N], \tag{9.16}$$

where $\{F_n\}$ are the expected values of each risk factor, and $\{\beta_{i,n}\}$ is the sensitivity of factor i to each factor

- Broadly speaking, we can split factors into three groups
  - Macroeconomic factors
    - Unexpected GDP growth, inflation, consumer confidence, etc.
  - Fundamental factors
    - Earnings, earnings growth, firm size, dividend yield, etc.
  - Statistical factors
    - No basis in finance theory – uses principal component analysis
- The Fama and French model is a three factor model:
  1. Firm size
  2. Book-to-market ratio
  3. Excess return on the market portfolio

  Later, a fourth factor, momentum, was added
- In the market model, β is estimated as the slope of regression of asset returns on the market. These are "characteristic lines"

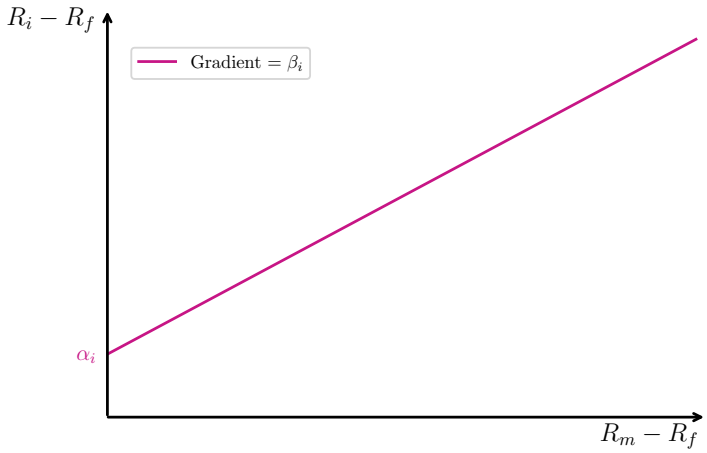


Figure 9.8: Returns of an asset against the market in excess return space.

The slope of the line is given by

$$\text{Slope} = \beta_i = \frac{\text{Cov}(i,m)}{{\sigma_m}^2} \qquad \beta_P = \sum w_i \beta_i \tag{9.17}$$

$$\beta_i = \rho_{i,m} \times \frac{\sigma_i}{\sigma_m} = \frac{\text{Cov}(i,m)}{\sigma_i \sigma_m} \frac{\sigma_i}{\sigma_m} = \frac{\text{Cov}(i,m)}{{\sigma_m}^2} \tag{9.18}$$

$$\beta_m = \underbrace{\rho_{m,m}}_{=1} \underbrace{\frac{\sigma_m}{\sigma_m}}_{=1} = 1 \tag{9.19}$$

9.6 The CAPM and SML

- Assumptions of capital market theory include
  - Investors use a mean-vriance framework
  - Unlimited lending / borrowing is possible at the risk-free rate
  - All investors have homogenous expectations ($\exists$ a market portfolio)

- One-period time horizon
- Assets can be bought in infinitessimally small increments
- Markets are completely frictionless
- No inflation, and stable interest rates
- Capital markets operate in equilibrium, and investors are price-takers

- The CAPM is defined

$$E(R_i) = R_f + \beta_i \underbrace{[E(R_m) - R_f]}_{\text{Market risk premium}}, \tag{9.20}$$

and defines the security market line. This is based on systematic risk only.

- β is defined as above, to be

$$\beta = \frac{\text{Cov}(i,m)}{{\sigma_m}^2} = \rho_{i,m} \times \frac{\sigma_i}{\sigma_m}. \tag{9.21}$$

This gives the expected return of an asset only taking into account the systematic risk. In equilibrium, we would expecte the required "fair" return to be equivalent to the expected return. Any instance when these are not equal is the result of mispriced securities.

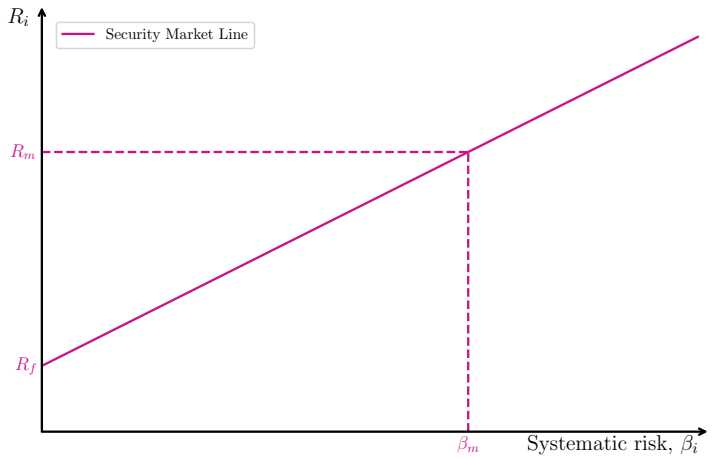


Figure 9.9: The security market line. According to CAPM, all securities should lie on this line, with the expected return determined by the security's beta to the market

CML	Efficiency of a portfolio. Refers to total risk
SML	Based on systematic risk only. It gives an appraisal of valus of securities against a fair value estimate.

Table 9.1: Difference between what the CML and SML represent

The CAPM is used for performance evaluation (risk / return of an active strategy), as well as attribution analysis (Sources of differences between portfolio returns and benchmark returns)

EXAMPLE: Consider the following three stocks. Determine whether they are underpriced, overpriced or fairly priced. $R_f = 7\%$ and $E(R_m) = 15\%$.

We can use the holding period return, $HPR = \frac{P_1 + D_1 - P_0}{P_0}$ to generate a forecast return, and compare this to the CAPM.

Stock	P_0	$E(P_1)$	$(E(D_1)$	β
A	\$25.00	\$27	\$1.00	1.0
B	\$40.00	\$45	\$2.00	0.8
C	\$15.00	\$17	\$0.50	1.2

Stock	Forecast return	CAPM required return	Jensen's Alpha	
A	12.0%	15.0%	−3.0%	Overpriced
B	17.5%	13.4%	+4.1%	Underpriced
C	16.6%	16.6%	0.0%	Fairly priced

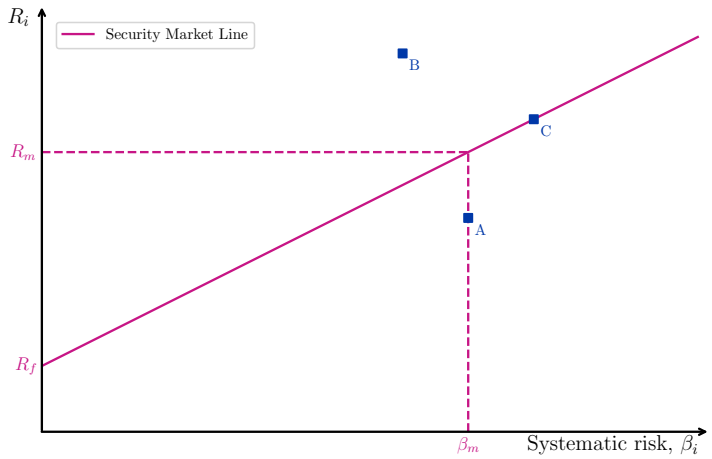


Figure 9.10: Assets A, B, and C, and their positions relative to the security market line. Assets below the line are undervalued, and assets are above the line when they are overvalued.

9.7 Risk adjusted measures of return

- The Sharpe ratio is a measure of the excess return a portfolio generates per unit of risk.

$$\text{Sharpe ratio} = \frac{E(R_p) - R_f}{\sigma_P} \tag{9.22}$$

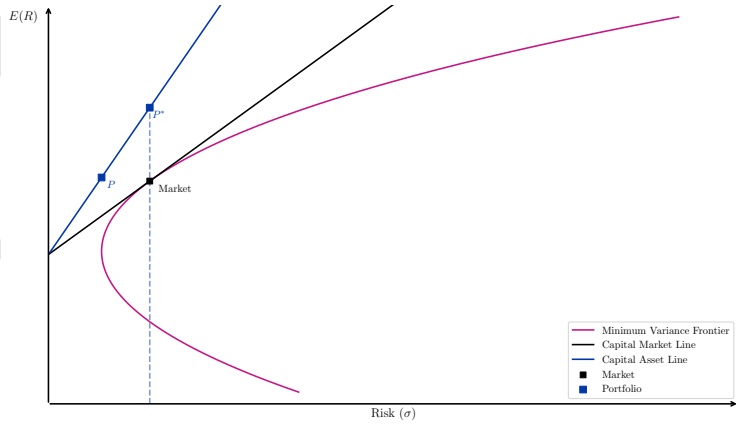


Figure 9.11: The gradient of the CAL / CML is the Sharpe ratio. If such a portfolio P exists, the Sharpe ratio exceeds that of the market. Thus, P will beat the market on a risk-adjusted return.

- Linked to the is the M^2 value. This is the portfolio return if the portfolio were to take on

the same risk as the market. This is shown by portfolio P^* in figure 9.11.

$$M^2 = R_f + \frac{\sigma_m}{\sigma_P}(R_P - R_f) \tag{9.23}$$

$M^2 = R_m$	Portfolio equivalent to market
$M^2 > R_m$	Portfolio better than market
$M^2 < R_m$	Portfolio worse than market

The M^2 alpha is the extra return a leveraged portfolio would make if it had the same risk as the market portfolio, and so is a measure of risk-adjusted performance

$$M^2 \text{ alpha} = M^2 - R_m \tag{9.24}$$

- Other risk adjusted measures include

$$\text{Sharpe ratio} = \frac{R_P - R_f}{\sigma_P} \tag{9.25}$$

$$\text{Treynor measure} = \frac{R_P - R_f}{\beta_P} \tag{9.26}$$

$$\text{Jensen's alpha} = R_P - [R_f + \beta_P(R_m - R_f)] \tag{9.27}$$

where the Sharpe ratio focuses on total risk, and the Treynor measure on systematic risk (β)

9.8 The porfolio management proces

- Portfolio management should always take a whole-portfolio approach, and any decisions should be made with that context in mind. Decisions should not be evaluated on a standalone asset without thinking about portfolio impacts

$$\text{Diversification ratio} = \frac{\text{Standard deviation of equally weighted portfolio}}{\text{Average standard deviation of assets}} \tag{9.28}$$

A lower diversification ratio indicates a greater marginal benefit from diversification.

- The steps of porfolio management are
  1. Planning
     - Understand investor objectives and constraints
     - Write an investor policy statement
  2. Execution
     - Asset allocation (top-down analysis)
     - Security selection (bottom-up analysis)
     - Portfolio construction (Target weightings (strategic / tactical), risk management, trading)
  3. Feedback
     - Monitor and update investment circumstances
     - Monitor and update market conditions
     - Rebalance portfolio
     - Measure and report performance to investors / clients

9.9 Types of investment clients

- Individual (retail) clients
  - Variety of needs, highly circumstance dependent
  - Investing for DC pension plans
- Institutional
  - Objectives constraints driven by institution mission
- Endowments / foundations [Institutional]
  - Provide ongoing support ot beneficiaries
    - Long time horizon
    - High risk tolerance
    - Low income needs
    - Low liquidity needs
- Insurance companies
  - Property and causalty (P&C), Life insurance

	P&C	**Life**
Time horizon	Short	Long
Risk tolerance	Low	
Income needs	Low	
Liquidity needs	High to meet claims	

Table 9.2: Typical investment constraints for insurance companies

- Banks
  - Loans are assets
  - Excess reserves are primarily invested in fixed-income and money-market securities

	Banks
Time horizon	Short
Risk tolerance	Low
Income needs	Must pay interest on deposits
Liquidity needs	High

Table 9.3: Typical investment constraints for banks

- Mutual funds (Regulated)
  - High liquidity needs
  - Time horizon, risk tolerance, and income needs are fund-sepcific, based on objectives and parameters of the investment strategy
- Sovereign wealth funds
  - Vairous investment goals
  - Invest for future generations

  - Manage foreign exchange reserves
  - Manage government assets (i.e. State pensions)

- Pensions (Defined Contribution)
  - Employee bears investment risk
  - No guarantee of future benefits (payments in retirement)

- Pensions (Defined Benefits)
  - Benefit is guaranteed to employee in perpetuity
  - Employer bears investment risk
  - Separate legal entity manages plan assets

9.10 Asset management industry

- Asset managers are buy-side firms, who manage investments on behalf of clients. They vary by size, and scope:
  - Full service
  - Specialists (Focused on style or asset class)
  - Multi-boutique (Holding company for specialists

  Firms may focus on traditional asset classes, as well as alternative investments

- Firms may follow
  - Active strategies, where manager skill is relied on to outperform some benchmark
  - Passive strategies, where managers aim to replicate performance of a pre-determined benchmark
  - Smart beta strategies, which focus on exposure to a specific market risk factor (size / value / growth / momentum)

- Trends in the industry:
  - Passive strategies now account for $\approx 20\%$ of total AUM
  - IT investments allow firms to make use of big data
  - Emergence of robo-advisors impact how products are marketed and disseminated to investors

9.11 Types of investment funds

- Pooled investments
  - Open-ended mutual funds
    * Investors purchase / redeem shares at NAV
    * Number of shares depends on subscription / redemption of shares in the market
    * Fee for ongoing management
    * Load funds – Up-front charges, redemption charges (Entry / exit fees to the fund)
    * No-load funds – No fees upon entry / exit of the fund
  - Closed-ended mutual funds
    * Fixed number of shares, issued at an "IPO"

        - Trade like shares in a company. Commission and spread, margin and shorting all apply
        - Fee for ongoing management
        - Market price may differ from NAV (trade at a premium / discount
        - Does not need to hold cash to handle redemptions. No primary issuance / recalling of shares after fund launches
    - Types of mutual fund by investment objective
        - Money market funds
        - Bond funds (HY, gobal, domestic, govt., corp, long / short-term, tax exempt)
        - Stock funds (Active / passive)
        - Balanced funds (multi-asset)

- ETFs
    - Typically index funds
    - Trade like shares on an exchange, and can be shorted or margined
    - Dividend typically paid out to investors
    - In-kind subscription and redemption keep market price close to NAV
        - Shares in an ETF are exchanged for baskets of securities in the underlying index by market-makers

- Separately managed accounts ("Wrap accounts", "Segregated mandates")
    - Owned by a single investor
    - High minimum investment amount
    - Does not benefit from economies of scale

- Private equity funds
    - Portfolio of privately held companies
    - May use high leverage
    - Restructure, improve cash flows, exit through IP / acquisition

- Venture capital funds
    - Start-up financing
    - Expect a degree of failure in investments, and some big successes
    - Active choices made in management of portfolio firms

- Hedge funds
    - Not registered / offered to public
    - Small number of accredited investors
    - High minimum investment,hight leverage, derivatives
    - Long - short, global-macro, event-driven strategies may all be employed

9.12 Portfolio planning and construction

- An Investment Policy Statement (IPS)
  - Identifies client objectives / constraints
  - Clearly states the accepted risk tolerance
  - Imposes investment discipline on client / manager
  - Indetifies risk arising from the investment strategy
  - Identifies a benchmark appropriate to the risk tolerance and other restrictions put in place
- Components of an IPS:
  - Describe client circumstances
  - Purpose of the IPS
  - Duty and responsibilities of all parties involved
  - Procdeures to update the IPS and resolve any problems
  - Investment objectives and constraints [Defines portfolio risk / return trade-off]
  - Investment guidelines
  - Evaluation of performance benchmark
  - Appendices:
    - Strategic / tactical asset allocations
    - Rebalancing procedures
- Factors affecting the risk tolerance
  - Psychological factors – Willingness to take risk
  - Personal factors – Ability to take risk

  Do not let a client take more risk than what they are able to take, regardless of their willingness to.
- Investment constraints
  - Liquidity – Potential need for cash
  - Legal and regulatory [More relevant for institutional investors and IRA accounts]
  - Time horizon – Time until proceeds required
  - Tax concerns – Taxable / tax-deferred / tax exempt investments
  - Unique needs and preferences of the client
- Strategic asset allocation
  - Based on risk, return and correlation of asset classes
  - Correlation within asset classes should be high
  - Correlation between asset classes should be low
- Combining the components of the IPS a portfolio manager should
  - Use risk / return / correlation of asset classes to construct an efficient frontier
  - Use objectives / constraints from IPS to select an optimal portfolio
  - Use tactical allocation where permitted

  - Budget risk within strategic allocaiton as appropriate

- ESG investing in portfolio planning
  - Negative screening – Exclusion of stocks
  - Positive screening – Only include highly-ranked stocks (thematic)
  - For active ownership, decide whether client / manager excercises voting rights

9.13 Behavioural biases of individuals

- Behavioural finance hypothesises two types of errors – cognitive errors and emotional biases.

Cognitive errors	Emotional biases
Faulty reasoning	Influenced by feelings / intuition
Memory errors	
Misunderstanding statistics	
Information processing errors	
Easier to mitigate	*Harder to mitigate*

Table 9.4: The two types of errors categorised by behavioural finance theory

9.13.1 Cognitive errors

- Belief perserverance is an irrational reluctance to change a current belief or question a prior decision. This can arise from avoiding cognitive dissonance – the discomfort experienced when newly-available information does not fit past patterns

- Processing errors are a flawed analysis of information

Belief perseverance	Processing errors
Conservatism bias	Anchoring / adjustment
Confirmation bias	Mental accounting
Representativeness bias	Framing
Illusion of control	Availability
Hindsight bias	

Table 9.5: The grouping of cognitive errors into belief perseverance errors processing errors

Belief perseverance

  - Conservatism bias – Ignoring new information when it arrives, after first forming a rational conclusion
    - Slow / reluctant to change opinions
  - Confirmation bias – Looking for evidence that agrees with a pre-existing view
    - Considers a position, and ignores any negative information
    - Mitigated by seeking out contrary view → May set up processes that support a preferred belief
  - Representativeness bias – Assign an investment to a category and assume it exhibits only characteristics of that category "Stereotyping"
    - Base rate neglect → mis-applying a label

    - Sample size neglect
  - Illusion-of-control bias – False belief that an investor has control over an outcome
    - Illusion of knowledge, i.e. an employee's impact on their employer's stock
  - Hindsight bias – Belief that past outcomes were more predictable than they actually were, based on selective memory
    - May distort earlier predictions
    - Trusting things that worked, regardless of merit

Processing errors

  - Anchoring and adjustment – Overweighting the importance of a prior value and comparing all new information to that prior value
    - May understimate the importance of new information
    - View security's value relative to it's current value / purchase price
  - Mental accounting – Treating money differently based on source / purpose
    - May result in holding investments that have offsetting risk / return
    - Conflict with total portfolio approach
  - Framing bias – Differing responses to information based on how information is presented
    - Risk tolerance based on potential gain vs potential loss
    - May overestimate significance of short-term volatility vs the long term
      Mitigate this by carefully considering the framing of questions which estimate risk tolerances
  - Availability – Overemphasising information that is easy to recall / readily accessible
    - Recency bias
    - Familiarity bias

9.13.2 Emotional biases

- Loss aversion – Feeling more pain from a loss than reward from an equally-sized gain
  - Investors tend to hold losing positions too long and sell gaining positions too quickly
  - Investors make frequent trades to realise small gains
- Self control – Overweighting short-term needs compared to long-term goals
  - Investors tend to prefer smaller short-term gains vs large future payoffs
- Status quo – resistance to change from existing situation, regardless of circumstance change
  - Ignoring new, relevant information in favour of past analysis and decision-making
- Endowment bias – Greater value placed on assets that are already owned
  - Investors may fail to sell assets which are no longer appropriate
- Regret aversion – Not acting due to fear of mistakes
  - Results in investors following the herd to minimise self-blame
- Overconfidence – Assuming that you have more knowledge than you do in reality
  - Investors place more value on their own analysis than is due

9.13.3 Bubbles and anomalies

- Bubbles may form in a mrket due to investors acting on behavioural biases
  - Overconfidence – underestimation of risk
  - Self-attribution – Claiming credit in a bull market
  - Confirmation —- Price rise gives validation to beliefs
  - Anchoring – Reliance on recent highs
  - Fear of regret – Slow to exit the market
- Market anomalies are phenomena which do not align with the efficient market hypothesis
  - Halo effect – Rapid growth ⇒ Good investment
  - Home bias – An investor overweighting stocks in their own country / companies whose products they use

9.14 Risk management

- The risk management process involves two steps;
  1. Defining a level of risk to be taken,
  2. Measuring the actual risk taken.

  It is about taking appropriate, deliberate risk, not just minimising risk
- The risk management framework involves the following steps:
  1. Establish risk governance policies and processes [At board level]
  2. Identify and measure risk [Risk drivers]
  3. Deploy risk infrastructure [People + systems]
  4. Define policies and processes [Day-to-day practices]
  5. Monitor, mitigate and manage risks
  6. Communicate ideas across organisation
  7. Perform strategic risk analysis – which risks are rewarded
- Risk governance
  - Directs risk management to act within risk tolerance
  - Enterprise-wide; Appointment of CRO
  - Establish a risk-management committee
- Risk tolerance
  - Specify acceptable / unacceptable risks, extent of risk exposure.
  - Factors include
    - Expertise in specific business lines
    - Ability to respond to external events
    - Financial strength
    - Regulatory environment
- Risk budgeting
  - Allocate risk tolerance to risk drivers, based on

1. Organisation risk tolerance
2. Risk characteristics of assets / investment

– Risk budget may be a single metric, such as VaR, portfolio β, scenario loss

- Financial risk
  - Credit risk
    - Counterparty being abil to fulfill obligations
  - Liquidity risk
    - Receiving less than fair value when selling an asset
  - Market risk
    - Asset prices / interest rates moving in adverse directions
- Non-financial risks
  - Operational risk
    - Human error / faulty processes / business interruptions / cyber
  - Solvency risk
    - Running out of cash
  - Regulatory / accounting / tax risk
    - Adverse changes in regulations
  - Settlement risk
    - Non-simultaneous exchange of obligations
  - Legal risk
    - Exposure to lawsuits / non-enforceable contracts
  - Model risk
    - Incorrect asset valuations
  - Tail risk
    - Underestimating probability of extreme outcomes

9.14.1 Measuring risk exposure

- Risk measures include
  - Standard deviation
  - Beta
  - Duration (Exposure to interest rates)
  - Value-at-risk (VaR)
    - Probability of a given loss – i.e. 5% VaR of \$5mn means an investor should expect a loss of > \$5 mn in 5% of time-periods
    - Does <u>not</u> give maximum loss
  - Conditional VaR
    - Average loss given that the loss exceeds a threshold
  - Stress testing
    - Impact of adverse conditions
  - Scenario analysis
- Risk measures for derivatives include
  - The price of a derivative is a function of the price of the underlying, the volatility of the underlying, and the risk-free rate,

$$P_{\text{Derivative}} = P(P_{\text{Underlying}}, \sigma_{\text{Underlying}}, R_f) \tag{9.29}$$

  - δ is defined as the sensitivity of derivative value to the price of the underlying

$$\delta = \frac{\mathrm{d}P_{\text{Derivative}}}{\mathrm{d}P_{\text{Underlying}}} \tag{9.30}$$

- γ is the sensitivity of δ to the price of the underlying

$$\gamma = \frac{\mathrm{d}\delta}{\mathrm{d}P_{\text{Underlying}}} = \frac{\mathrm{d}^2 P_{\text{Derivative}}}{\mathrm{d}P^2_{\text{Underlying}}} \tag{9.31}$$

- Vega is the sensitivity of the derivative value to the volatility of the underlying

$$\text{Vega} = \frac{\mathrm{d}P_{\text{Derivative}}}{\mathrm{d}\sigma_{\text{Underlying}}} \tag{9.32}$$

- ρ is the sensitivity of the derivative value to the risk-free rate

$$\rho = \frac{\mathrm{d}P_{\text{Derivative}}}{\mathrm{d}R_f} \tag{9.33}$$

- With any risk, an investor / company can choose to accept, avoid, or prevent a risk.
- An investor can transfer risk to another party through
  - Insurance
  - Surety bond (third-party obligations)
  - Fidelity bond (employeee dishonesty)
- An investor typically would shift risk through the use of derivative contracts

10 Ethics

10.1 Ethics

- Ethics can be defined as a set of shared beliefs which define acceptable and non-acceptable behaviour. In the investment profession, this covers how we treat clients and employers.
- The role of a code of ethics is to communicate to the public that a profession's members will use their skill to serve clients in an honest and thical manner
- A profession is an occupational group that require specialised knowledge, with a focus on ethical behaviour and service to community / society
- A profession may set or enforce standards for professional behaviour, continuing education, and / or putting clients first
- The need for high ethical standards is driven by a lack of trust in investment professionals, which increases the cost of capital. Providing false information can lead to slower growth of wider economy
- Suitability standard
  - Match investments to the risk / return preferences of the client
- Fiduciary standard
  - Act in the best interests of clients – investment professionals are placed in a position of trust
- Challenges to ethical behaviour
  - Individuals overestimate their ethics
  - External influences include social pressure, loyalty to employer / supervisor / coworker, and money / power / prestige. Client interests should always be put first
- Ethical standards and legal standards overlap, but are not always aligned
  - Some actions may be illegal but ethical, and some actions may be legal but unethical

  Ethical principles set a higher standard than laws.

$$\text{Legality} \not\Longleftrightarrow \text{Ethical}$$

- The framework for ethical decision makung involves
  1. Identify facts, ethical principles, and stakeholders / conflicts
  2. Consider alternatives and situational influences (seek guidance)
  3. Make a decision and act
  4. Evaluate the outcome

10.2 CFA guidance

- The CFA Institute Professional Conduct Program is a disciplinary review committee that enforces the code of standards. It handles inquiries as they relate to professional conduct
- An inquiry can be prompted by

  - Self-disclosure
  - Written complaints
  - Evidence of misconduct
    * Report by CFA exam proctor
    * Analysis of exam scores / materials

- Possible decisions include
  - No further action
  - Cautionary letter
  - Disciplinary action (sanctions)

10.3 Code of ethics

- Act in an ethical manner
- Integrity is paramount and clients should come first
- Use reasonable care, be independent
- Be a credit to the investment profession
- Uphold capital market rules / regulations
- Be competent

10.4 Standards of professional conduct

I Professionalism
  A Knowledge of the law
  B Independence and objectivity
  C Misrepresentation
  D Misconduct
  E Comptence

II Integrity of capital markets
  A MNPI
  B Market manipulation

III Duties to clients
  A Loyalty, prudence, care
  B Fair dealing
  C Suitability
  D Performance presentation (Fair / accurate / complete)
  E Preservation of confidentiality

IV Duties to employer
  A Loyalty
  B Additional compensation arrangements
  C Responsibilities of supervisors

V Investment analysis, recommendations, and actions
  A Diligence and reasonable basis
  B Communication with client / prospective clients [Disclosures]
  C Record retention

VI Conflicts of interest
  A Avoid / disclose conflicts in plain language
  B Priority of transactions
  C Referral fees

VII Responsibilities as a CFA member
  A Conduct as participants
  B Reference to CFA, designation, and program

10.5 I Professionalism

10.5.1 I-A Knowledge of the law

- Parameters
  - Understand / comply with all laws, rules, regulations, including the code / standards
  - Comply with the most strict applicable rules (CFA / local / foreign where relevant)
  - Do no knowingly assist in violations of the law. Otherwise, stop and dissasocitate from any such actions
- Guidance
  - Notify supervisor
  - May confront wrong-doer
  - Dissociate from those involved [Inaction = Participation]
  - Reporting to authorities is not always required
- Recommended procedures
  - Keep informed, review compliance procedures
  - Written procedures for reporting suspected violations
  - Member encouraged, but not required (unless by law), to report violations

10.5.2 I-B Independence and objectivity

- Parameters
  - Use reasonable care and judgement ot achieve and maintain independence in professional activities
  - Do not offer, solicit, accept any compensation that could compromise independence or objectivity
- Guidance
  - Toke gifts ok
  - Distinguish between gifts from clients and gifts from entities trying to influence a member's behaviour
  - May accept a gift from client – Must disclose to employer and obtain permission if the gift is contingent on future performance
  - Do not accept gifts that impair objectivity
  - Do not issue favourable research in return for anything
  - For issuer-baid research, a flat-fee structure s preferred, and must be disclosed
  - Credit rating firms should avoid influence by issuing firms
  - Pay for your own commercial travel
- Recommended procedures
  - Protect integrity of opinions
  - Create a restricted list
  - Restrict special cost arrangements
  - Limit gifts
  - Take care with IPO share allocations

10.5.3 I-C Misrepresentation

- Parameters
  - Do not make misreprentations of analysis, recommendations, actions, or other professional activities
- Guidance
  - Covers all forms of communication
  - Do not misrepresent qualifications, services, performance record, characteristics of an investment
  - Do not guarantee a certain return
  - Do not plagiarise others work or research
- Recommended procedures
  - Firms may provide a written list of services offered and qualifications held
  - Maintain records of materials used ot prepare research reports and quote soures, except for recognised financial / statistical reporting services
  - Models and anlaysis created by others at the same firm may be used without explicit attribution
  - Should encourage firm to establish procedures for verifying marketing claims of third parties which are then recommended to clients

10.5.4 I-D Misconduct

- Parameters
  - Do not engage in any professional conduct involving dishonesty, fraud, or deceit, or commit any act that reflects adversely on professional reputation, integrity, or competence
- Guidance
  - Conduct may not be illegal, but could impact ability to perform duty
- Recommended procedures
  - Adopt a code of ethics
  - Disseminate a list of vilations / sanctions
  - Conduct background check

10.5.5 I-E Competence

- Parameters
  - Act with and maintain the comptence necessary to fulfill professional responsibilities
- Guidance
  - Match abilities of an individual to the responsibilities they hold
  - Up to the individual o ensure they have the skills to carry out a role
- Recommended procedures
  - Participate in training

  - Make use of professional designations
  - Attend seminars / conferences
  - Participate in professional organisations
  - Engage in informal self-study

10.6 II Integrity of capital markets

10.6.1 II-A Material non-public information (MNPI)

- Parameters
  - Members in possession of MNPI that could cause an investment's value to change must not act on it, or cause someone else to act on it
  - "Material" refers to information on which a disclosure would affet a security's price, or if an investor would want to know about it before making investment decisions
  - If price effect is ambiguous, information may not be considered to be material
  - This extends to upcoming rating changes, or influential analysis that has yet to be released to the public
- Guidance
  - Information is non-public until it is made available to the marketplace
  - This includes swaps / options / mutual funds involving a given security
  - May use firm-provided information for specific use (i.e. due diligence)
  - Mosaic theory is permissible (Use of non-material, non-public information)
- Recommended procedures
  - Establishment of information barriers / firewalls within a company
  - Restricted lists
  - Review of employee trades
  - Restric proprietary trading when in possession of MNPI

10.6.2 II-B Market manipulation

- Parameters
  - Relates to price distortion artificial inflation of trading volumes with the intent to mislead market participants
- Guidance
  - Do not engage in transaction-based manipulation
    - False impression of activity / price movements
    - Gaining dominant position in an asset to manipulate the price of that asset or a related derivative
  - Do not distribute false, misleading information
- Recommended procedures
  - Establishing a code of conduct and rules concerning permissible behaviour

10.7 III Duties to clients

10.7.1 III-A Loyalty, prudence, and care

- Parameters
  - Duty of loyalty to clients – act with reasonable care, and exercise prudent judgement
  - Act for the benefit of clients and prioritise their interests above those of the employer or self
  - Determing and comply with the fiduciary duty
- Guidance
  - Take investment actions in the best interest of clients
  - Exercise prudence, care , skill and diligence in any actions and descision making
  - Follow applicable fiduciary duty
  - The "client" may be the investing public
  - Manage assets according to the IPS (governing documents of a fund)
  - Work in a total-portfolio view
  - Vote proxies responsibly, and disclose policies concerning proxy votes
  - Soft dollars (non-monetary rebates, i.e. research) must benefit the client
- Recommended procedures
  - Follow regulations
  - Establish client investment objectives
  - Diversify investments where possible within the investment constraints and guidelines
  - Deal fairly with all clients
  - Disclose all possible conflicts of interests
  - Vot proxies responsibly
  - Keep client information confidential from aall unless those who actively need to know their information
  - Seek best trading and execution practices

10.7.2 III-B Fair dealing

- Parameters
  - Deal fairly and objectively with all clients when
    - Providing investment analysis
    - Making investment recommendations
    - Taking investment action
    - Engaging in other professional activities
- Guidance
  - No discrimination when disseminating information
  - Fair dealing is not the same as equal treatment of clients. Different levels are ok as long as they are disclosed and does not disadvantage any other lcient
  - All clients must have a fair chance to act on every investment recommendation

  - If a client is unaware of recommendation changes, advise them of this before accepting orders
  - Treat all clients fairly
  - Disclose written allocation procedures
  - Do not disadvantage particular clients

- Recommended procedures
  - Limit the number of people aware of upcoming changes
  - Shorten the time frame from decsion making to dissemination of information
  - Have pre-dissemination guidelines for information
  - Ensure simultaneous dissemination of information
  - Maintain a list of clients and their holdings
  - Disclose trade allocation procedures clearly
  - Review accounts regularly to ensure fair client treatment
  - Disclose any service offerings in writing
  - Deviations from strict allocations pro-rata allowed if there is a minimum trade size required

10.7.3 III-C Suitability

- Parameters
  - Make reasonable inquiry about investment experience, risk / return objectives, financial constraints, before any investment recommendation / actions are made
  - Update information regularly
  - Ensure investments are suitable before any investment action
  - Look at suitability in a whole-portfolio context
  - Only make recommendations that are in line iwth portfolio objectives / restraints
- Guidance
  - Prepare an IPS, and update it annually
  - Determine whether the use of leverage and derivatives is suitable
  - If manging a fund to a mandate, ensure the mandate is followed
  - If a client requests an unsuitable trade, discuss the suitability with the client before executing
    * If not material to portfolio, follow a firm's policy for client approval
    * If material, discuss whether the IPS needs update
    * If client declines to update the IPS, reconsider advisory relationship
- Recommended procedures
  - IPS should include return objective and risk tolerance of a client
  - Constrainings include
    * Liquidity needs
    * Time horizon, tax considerations
    * Regulatory / legal constraints

10.7.4 III-D Performance presentation

- Parameters
  - When communicating investment performance information, ensure it is fair / accurate / complete
  - Brief presentations are acceptable if the limited scope is noted, and if more information is made available, upon request
- Guidance
  - Do not mis-state or mislead clients about performance
  - Do not misrepresent past performance
  - Provide fair and complete information
  - Do not guarantee ability to repeat past returns
- Recommended procedures
  - Consider audience sophistication
  - Use performance of similar, competitor portfolios to compare performance
  - Include terminated account in historical performance
  - Make all disclosures and maintain records

10.7.5 III-E Confidentiality

- Parameters
  - Keep client information confidential, unless
    - Illegal activity suspected
    - Disclosures required by law
    - Client fives permission to share information
- Guidance
  - In some cases, it may be required by law to report activities to the relevant authorities
  - Standard extends to former clients – May give information to CFA Institute for investigation
- Recommended procedures
  - Avoid discussing client information
  - Follow electronic data storage procedures

10.8 IV Duties to employer

10.8.1 IV-A Loyalty

- Parameters
  - Act for the benefit of employer, do not deprie employer of the advantage of skills, divulge confidential information, or otherwise cause harm to the employer
- Guidance
  - Client's interest come first, but consider the effet on firm integrity

  - Members are encouraged to give employer a copy of the Code of Standards
  - No incentive structure should be implementd which encourages unethical behaviour
- Guidance: Independent practice
  - Must disclose services, duration, and compensation of any independent work to employer
  - Must have employer consent
- Guidance: Leaving an employer
  - Employer records of any medium are property of the firm
  - No solicitation fo clients prior to leaving
  - No prohibition on use of knowledge / experience gained "human capital"
- Guidance: Whistleblowing
  - Permitted only if it protects clients or integrity of capital markets
- Recommended procedures
  - Policies for
    * Outside practices, non-compete
    * Leaving employer
    * Incident reporting
    * Employee classification

10.8.2 IV-B Additional compensation arrangements

- Parameters
  - Do not accept gifts / benefits / compensation / consideration that has a conflict of interes with employer unless written consent is obtained from all parties involved
- Guidance
  - Compensation and benefits covers direct compensation from clients, and other benfits from third parties
  - For written consent, email chains will suffice
- Recommended procedures
  - Written report of prosoed additional compensation
  - Includes details of incentives
  - Includes nature of compensation, amount, duration of agreement

10.8.3 IV-C Responsibilities of supervisors

- Parameters
  - Make reasonable efforts to ensure direct reports comply with applicable laws
- Guidance
  - Supervisors must actively prevent unethical behaviour item Supervirsors mus make reasonable efforts to detect violations
- Recommended procedures

  - Clear procedures
  - Designated compliance officer
  - Procedures to report violations
  - Checks and balances
  - Distribute these procedures clearly and prominently
  - Education of staff
  - Review of employee actions
  - Prompt initiation fo

10.9 V Investment analysis, recommendations, and actions

10.9.1 V-A Diligence and reasonable basis

- Parameters
  - Exercise diligence, independence, throughness in analysing investments, making recommendations, and taking investment action
  - Have a reasonable and adequate basis, supported by research, for any analysis, recommendation, or action
- Guidance
  - Make reasonable efforts to cover all relevant issues during analysis
  - Level of diligence required depends on product / service offered
  - On using second / third party research,
    * Determine soundness of the research (Assumptions / rigour / independence)
    * encourage firm policy to evaluate research
- Recommended procedures
  - Establish policy that research should have a reasonable and adequate basis
  - Review reports prior to circulation
  - Establish due diligence procedures
  - Devolop measurable criteria to assess quality of research
  - Consider tail risk events
  - Evaluate external advisors
  - Standard scenario testing, cash flow sensitivity to assumptions
  - Evaluate information providers
  - No need to dissociate from group research that an individual disagrees with

10.9.2 V-B Communication with client / prospective clients

- Parameters
  - Disclose nature and costs of services offered at initiation
  - Update if any changes
  - Disclose basic principles of investment process
  - Promptly disclose changes that materially affect processes
  - Disclose risk and limitations of the investment process

        * Use reasonable judgement in identifying relevant factors
        * Include in communication to clients
    - Distinguish facts and opinion
    - Clearly communicate potential gains / losses of an investment

- Guidance
    - Include basic characteristics of the security
    - Inform clients of any changes in the investment process
    - Consider portfolio context of assets
    - All forms of communication should be covered by these procedures

- Recommended procedures
    - Inclusion / exclusion of information depends on a case-by-case review
    - Maintenance of records

10.9.3 V-C Record retention

- Parameters
    - Develop and maintain appropriate records to support investment analyses, recommendations, actions, and otehr investment-related communications with clients

- Guidance
    - Maintain records to support research and rationale
    - Records are property of the firm
    - CFA institute suggests a 7-year retention policy

- Recommended procedures
    - Firm maintains records
    - Individuals must retain documents that support invesment-related communications
    - Cannot rely on materials from previous firms

10.10 Conflicts of interest

10.10.1 VI-A Avoid / disclose conflicts in plain language

- Parameters
    - Avoid, or make full, fair disclosure, of all matters that could reasonably be expected to impair independence / objectivity, or interfere with duties
    - Ensure disclosures are prominent, and that they are delivered in plain language

- Guidance
    - Disclose all matters that may impair objectivity
        * Firm – issuer member
        * Investment banking relations
        * Broker-dealer market-making activities
        * Significnt stock ownership
        * Board service

  - Disclose compensation arrangements that conflict with client to employer
  - Disclose to employers
    * Ownership of any stocks analysed
    * Board paricipation
    * Financial / other pressures
    * Conflicts that could damage employer's business

- Recommended procedures
  - Clearly stated and disclosed policies on conflicts of interest

10.10.2 VI-B Priority of transactions

- Parameters
  - Clients > Employers > Self in terms of transaction priority
  - Do not use knowledge of pending trades for personal gain ["Front-running"]
- Guidance
  - "Beneficial owner" has direct / indirect personal itnerest in securities
  - Client, employer transactions should take priority
  - Family accounts should be treated like regulat client accounts
- Recommended procedures
  - Firm plicyt ot
    * Limit participation in an IPO
    * Restrict purchase of securities through private placement
  - Establish blackout / restricted periods
  - Establish reporting procedures and prior clearance requirements
  - Disclose policies on personal investing to clients upon request

10.10.3 VI-C Referral fees

- Parameters
  - Disclose to employer / clients any compensation / consdieration received from / paid to others for recommendation of products / services
- Guidance
  - Disclosure allows clients and employers to evaluate full cost of service and any potential biases
  - Disclosures made before entering into any agreement
  - Disclose the nature of any consideration (cash or otherwise)
  - Firm should have a clear policy reagrding referrals
  - Clear approval process
  - Quarterly updates to employer on compensation disclosure
- Recommended procedures
  - Clear policies set out and made available to employees

10.11 VII Responsibilities as a CFA member

10.11.1 VII-A Conduct as participants

- Parameters
  - Do not engage in any conduct that compromises the reputation or integrity of the CFA institute / designation / programs
- Guidance
  - Honesty in exams
  - Repsect all examination conventions
  - Maintaining confidentiality of exam questions
  - No improper use of CFA desgination
  - No misrepresenting CA institute professional development program / conduct statement
  - Do not disclose any exam information (formulas / questions / topics tested)
- Recommended procedures
  - Familiarisation with professional standards and code of conduct

10.11.2 VII-B Reference to CFA, designation , and program

- Parameters
  - Do not misrepresent or exaggerate the meaning or implications of the CFA designation
- Guidance
  - Complete professional conduct statement annually
  - Pay membership dues (otherwise considered "inactive")
  - May reference participation, but no implication of achievement of partial completion
  - Factual statements permitted
- Recommended procedures
  - Make employer aware of candidacy / designation

10.12 Introduction to GIPS

- There are three components to the GIPS standards:
  - GIPS standards for firms
  - GIPS standards for asset owners
  - GIPS standards for verifiers
- GIPS were created to:
  - Make performance measurements directly comparable, using a standardised approach and methodology
  - Avoid misrepresentation of investment performance
    - Inclusion of all funds (Including underperforming / terminated accounts)
    - No manipulation of time periods
  - Convey useful information to clients

- Parties affected by GIPS
  - GIPS apply to investment management firms
  - Serve current / prospective clients [SMAs] of investment management firms
- Fundamental of compliance extends to a "distinct business entity" complying with all standards in order to claim compliance
- Input data and calculation methodology must be consistent and uniform across firms for fair, comparable presentations. This includes annual data, and showing of a performance benchmark. If > 6 portfolios, show the standard deviation and 3yr performance of the group
- Composite and pooled fund maintenance
  - Create meaningful asset-weighted composites
  - Include pooled funds in a composite where relevant
    - A composite is a is a group of portfolios with similar investment mandate and syle
  - Minimum asset level to warrant reporting

Composite time-weighted return Composite money-weighted return	Time-weighted adjusts for external cash flows
Pooled fund time-weighted return Pooled fund money-weighted return	If manager controls each flow, use money-weighted return

  All contain procedures for reporting fund performance (composites + pooled funds) as well as necessary disclosures
- GIPS advertising guidlines stiuplate requirements for any advertising that refers to a claim of GIPS compliance
- Composites are groupings of individual discretionary ("active") portfolios with the sam investment strategy, objective, or mandate
  - Must include all fee-apying discretionary portfolios (current + past) that the firm has maanged in this strategy
  - Groupings must be pre-identified
  - Client restrictions on an accounts mean it is non-discretionary as the fund manager does not have full discretion over investment decisions
- A firm is equivalent to a distinct business entity
- Independent verification
  - Volunatry process to verify compliance
  - Provides assurance that compliance is on a firm-wide basis
  - Must be performed by an indepenedent third party