

1 Syntax

$t =$

- c
- $t \text{ op } t$
- x
- $\lambda x. t$
- $t \ t$
- $\text{if } t \text{ then } t \text{ else } t$
- $\text{let rec } x = t \text{ in } t$

- $HCons \ t \ t$
- $HNil$
- $\text{lookup}[N]$

- new
- t
- $t := t$
- $!t$
- $t \Rightarrow t$

- $\text{pushHeap}[T]$
- $\text{popHeap}[T]$
- $>>=$
- return

- forkJoin

2 Types

$$T = \begin{array}{l} \tau \\ T \rightarrow T \\ T \times T \\ P \text{ \#may appear at most once in a type} \end{array}$$

$$\begin{array}{l} \textit{State } T \ T \ T \\ \textit{Ref } T \ \mathbb{N} \ D \ T \\ \textit{Heap } T \end{array}$$

$$\begin{array}{l} \textit{HNil} \\ \textit{HCons } T \ T \\ \textit{Lookup } T \ \mathbb{N} \ T \end{array}$$

$$P = \begin{array}{l} \textit{RW } T \\ \textit{RO } T \\ \textit{Locked } T \end{array}$$

$$D = \begin{array}{l} \textit{Deletable} \\ \textit{NotDeletable} \end{array}$$

3 Typing Rules

3.1 HList

$$\begin{array}{c} \overline{\Gamma \vdash \textit{HNil} : \textit{HNil}} \\ \frac{\Gamma \vdash \textit{hd} : \textit{hd} \quad \Gamma \vdash \textit{tl} : \textit{tl}}{\Gamma \vdash \textit{HCons } \textit{hd } \textit{tl} : \textit{HCons } \textit{hd } \textit{tl}} \\ \frac{\Gamma \vdash t : l \quad \textit{Length}(l) < n \quad \textit{At}(l, n) = a}{\Gamma \vdash \textit{lookup}[n] \ t : \textit{Label } l \ n \ a} \end{array}$$

$$\begin{aligned} \text{Length}(HNil) &= 0 \\ \text{Length}(HCons(hd, tl)) &= \text{Length}(tl) + 1 \end{aligned}$$

$$\begin{aligned} \text{At}(HCons(hd, tl), 0) &= hd \\ \text{At}(HCons(hd, tl), n) &= \text{At}(tl, n - 1) \end{aligned}$$

3.2 References

$$\begin{aligned} &\frac{\Gamma \vdash t : \text{Ref } n \text{ d } (Heap\ t) \quad \text{Writable}(s, n)}{\Gamma \vdash \text{new } t : \text{State } s \text{ s } (\text{Ref } n \text{ Deletable } t)} \\ &\frac{\Gamma \vdash t : \text{Ref } n \text{ Deletable } t \quad \text{Writable}(s, n)}{\Gamma \vdash \text{delete } t : \text{State } s \text{ s } unit} \\ &\frac{\Gamma \vdash t : \text{Ref } n \text{ d } t \quad \Gamma \vdash v : t \quad \text{Writable}(s, n)}{\Gamma \vdash t?v : \text{State } s \text{ s } unit} \\ &\frac{\Gamma \vdash t : \text{Ref } n \text{ d } t \quad \text{Readable}(s, n)}{\Gamma \vdash !t : \text{State } s \text{ s } t} \\ &\frac{\Gamma \vdash t : \text{Ref } n \text{ Deletable } a \quad l : \text{Label } a \text{ m } b}{\Gamma \vdash t \Rightarrow l : \text{Reference } n \text{ NonDeletable } b} \end{aligned}$$

$$\begin{aligned} \text{Writable}(HCons(hd, tl), 0) &= (RW\ t \in hd) \\ \text{Writable}(HCons(hd, tl), n) &= \text{Writable}(tl, n - 1) \end{aligned}$$

$$\begin{aligned} \text{Readable}(HCons(hd, tl), 0) &= (\text{Locked } t \notin hd) \\ \text{Readable}(HCons(hd, tl), n) &= \text{Readable}(tl, n - 1) \end{aligned}$$

3.3 State

$$\overline{\Gamma \vdash \text{pushHeap}[t] : \text{State } s \text{ } (HCons(t, s)) \text{ } (\text{Ref } (\text{Length}(s)) \text{ NotDeletable } (Heap\ T))}$$

$$\begin{aligned} &\overline{\Gamma \vdash \text{popHeap}[t] : \text{State } (HCons(t, s)) \text{ s } unit} \\ &\frac{\Gamma \vdash p : \text{State } p \text{ q } a \quad \Gamma \vdash k : a \rightarrow \text{State } q \text{ r } b}{\Gamma \vdash p \gg= k : \text{State } p \text{ r } b} \\ &\frac{\Gamma \vdash x : a}{\Gamma \vdash \text{return } x : \text{State } s \text{ s } a} \end{aligned}$$

3.4 Fork/Join

$$\frac{\begin{array}{c} \Gamma \vdash t_1 : unit \rightarrow State\ s_1\ s_1\ a \\ \Gamma \vdash t_2 : unit \rightarrow State\ s_2\ s_2\ b \\ s_1 \oplus s_2 = s \end{array}}{\Gamma \vdash forkJoin\ t_1\ t_2\ : State\ s\ s\ (a \times b)}$$

$$HNil \oplus HNil = HNil$$

$$hd_1 \odot hd_2 = hd \Rightarrow HCons(hd_1, tl_1) \oplus HCons(hd_2, tl_2) = HCons(hd, tl_1 \oplus tl_2 - 2)$$

h₁ and hd₂ are compatible (no write – write conflict) and hd is less restrictive than both \Rightarrow $hd_1 \odot hd_2 = hd$