

homework 1, version 3

```
. md"_homework 1, version 3_"
```

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Homework I - convolutions

18.S191, fall 2020

This notebook contains *built-in, live answer checks*! In some exercises you will see a coloured box, which runs a test case on your code, and provides feedback based on the result. Simply edit the code, run it, and the check runs again.

For MIT students: there will also be some additional (secret) test cases that will be run as part of the grading process, and we will look at your notebook and write comments.

Feel free to ask questions!

```
student = (name = "Henry Luengas", kerberos_id = "HBot106")
```

```
. # edit the code below to set your name and kerberos ID (i.e. email without @mit.edu)
.
. student = (name = "Henry Luengas", kerberos_id = "HBot106")
.
. # press the ► button in the bottom right of this cell to run your edits
. # or use Shift+Enter
.
. # you might need to wait until all other cells in this notebook have completed running.
. # scroll down the page to see what's up
```

Let's create a package environment:

```
. begin
.   import Pkg
.   Pkg.activate(mktempdir())
. end
```

We set up Images.jl again:

```
begin
    Pkg.add(["Images", "ImageIO", "ImageMagick"])
    using Images
end
```

Exercise I - *Manipulating vectors (1D images)*

A Vector is a 1D array. We can think of that as a 1D image.

```
example_vector = Float64[0.5, 0.4, 0.3, 0.2, 0.1, 0.0, 0.7, 0.0, 0.7, 0.9]
```



```
. colored_line(example_vector)
```

Exercise I.I

👉 Make a random vector `random_vect` of length 10 using the `rand` function.

```
random_vect = Float64[0.565844, 0.810277, 0.774885, 0.65938, 0.175532, 0.672]
```

```
. random_vect = rand(10)
```



Got it!

Great!

Hint

Random Vector can be created using the `rand` function. See the documentation for more details.



👉 Make a function `mean` using a `for` loop, which computes the mean/average of a vector of numbers.

`mean` (generic function with 1 method)

```
. function mean(x)
.     sum = 0
.     for value in x
.         sum += value
.     end
.     return sum/length(x)
. end
```

2.0

```
. mean([1, 2, 3])
```

Got it!

Let's move on to the next section.

👉 Define `m` to be the mean of `random_vect`.

`m = 0.5325081771969846`

```
. m = mean(random_vect)
```

Got it!

Awesome!

👉 Write a function `demean`, which takes a vector `x` and subtracts the mean from each value in `x`.

`demean` (generic function with 1 method)

```
function demean(x)
    m = mean(x)
    output = [value - m for value in x]
    return output
end
```

Let's check that the mean of the `demean(random_vect)` is 0:

Due to floating-point round-off error it may not be exactly 0.

2.2204460492503132e-17

```
. mean(demean(copy_of_random_vect))
```

```
. copy_of_random_vect = copy(random_vect); # in case demean modifies `x`
```

Exercise 1.2

👉 Generate a vector of 100 zeros. Change the center 20 elements to 1.

`create_bar` (generic function with 1 method)

```
. function create_bar()
.     bar = [0 for i in 1:100]
.     bar[40:60] .= 1
.     return bar
. end
```

Got it!

Let's move on to the next section.

Exercise 1.3

👉 Write a function that turns a Vector of Vectors into a Matrix.

`vecvec_to_matrix` (generic function with 1 method)

```
. function vecvec_to_matrix(vecvec)
.     return hcat(vecvec...) #thuh fuck this took me forever and i dont understand it
. end
```

2×2 Array{Int64,2}:

```
1 3
2 4
```

```
. vecvec_to_matrix([[1,2], [3,4]])
```

Got it!

You got the right answer!

✎ Write a function that turns a Matrix into a Vector of Vectors .

matrix_to_vecvec (generic function with 1 method)

```
. function matrix_to_vecvec(matrix)
.
.     return [matrix[i,:] for i in 1:size(matrix,1)] #return a row for each row in the matrix
. end
```

Array{Int64,2} = Array{Int64,2}([6, 7], Int64[8, 9])

```
. matrix_to_vecvec([6 7; 8 9])
```

Got it!

Keep it up!

colored_line (generic function with 2 methods)

Exercise 2 - Manipulating images

In this exercise we will get familiar with matrices (2D arrays) in Julia, by manipulating images. Recall that in Julia images are matrices of RGB color objects.

Let's load a picture of Philip again.

```
philip_file = "/tmp/jl_Vjgw4o"
```

```
philip_file = download("https://i.imgur.com/VGPeJ6s.jpg")
```

```
philip =
```



```
. philip = let
.     original = Images.load(philip_file)
.     decimate(original, 8)
. end
```

Hi there Philip

Exercise 2.1

✎ Write a function `mean_colors` that accepts an object called `image`. It should calculate the mean (average) amounts of red, green and blue in the image and return a tuple `(r, g, b)` of those means.

```
. Enter cell code...
```

```
Normed{UInt8,8}
```

```
. typeof(philip[10,10].r)
```

```
mean_colors (generic function with 1 method)
```

```
function mean_colors(image)
```

```

.      sum = [0.0,0.0,0.0]
.      for pixel in image
.          sum[1] += pixel.r
.          sum[2] += pixel.g
.          sum[3] += pixel.b
.      end
.      avgerage = sum ./ prod(size(image))
.      return tuple(avgerage...)
.  end

```

```
(0.599363, 0.547754, 0.478205)
```

```
. mean_colors(philip)
```

Got it!

Great!

. Enter cell code...

Exercise 2.2

👉 Look up the documentation on the `floor` function. Use it to write a function `quantize(x::Number)` that takes in a value x (which you can assume is between 0 and 1) and "quantizes" it into bins of width 0.1. For example, check that 0.267 gets mapped to 0.2.

```
RGB{Normed{UInt8,8}}
```

```
. typeof(RGB(0,0,0))
```

quantize (generic function with 3 methods)

```

. begin
.     function quantize(x::Number)
.         return floor(x, digits=1)
.     end
.
.     function quantize(color::AbstractRGB)
.         return RGB(quantize(color.r), quantize(color.g), quantize(color.b))
.     end
.
.     function quantize(image::AbstractMatrix)
.
.         return broadcast(quantize, image)
.     end
. end

```

```
(0.2, 0.9)
```

```
quantize(0.267), quantize(0.91)
```

Got it!

You got the right answer!

Exercise 2.3

✎ Write the second **method** of the function `quantize`, i.e. a new *version* of the function with the *same* name. This method will accept a color object called `color`, of the type `AbstractRGB`.

Write the function in the same cell as `quantize(x::Number)` from the last exercise. 👉

Here, `::AbstractRGB` is a **type annotation**. This ensures that this version of the function will be chosen when passing in an object whose type is a **subtype** of the `AbstractRGB` abstract type. For example, both the `RGB` and `RGBX` types satisfy this.

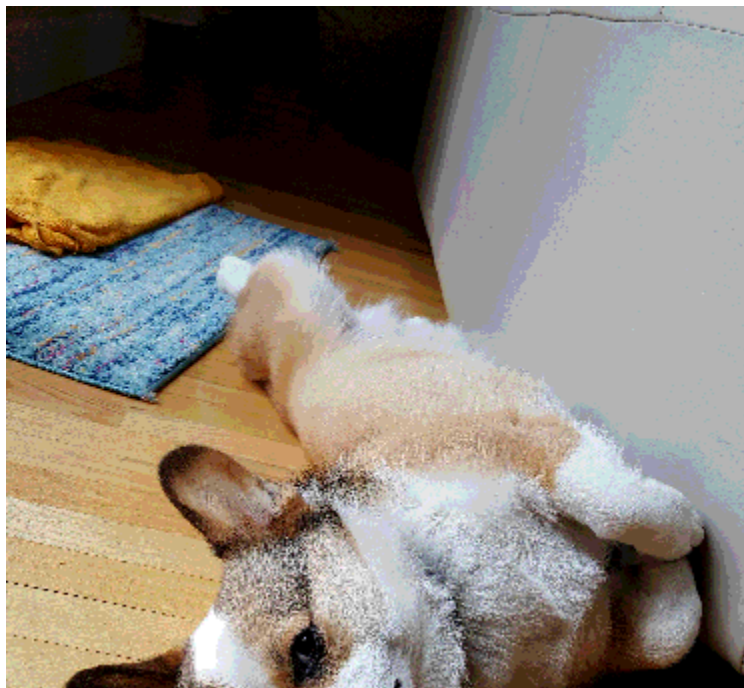
The method you write should return a new RGB object, in which each component (*r*, *g* and *b*) are quantized.

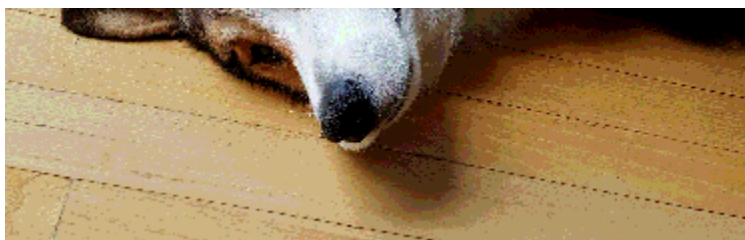
Exercise 2.4

✎ Write a method `quantize(image::AbstractMatrix)` that quantizes an image by quantizing each pixel in the image. (You may assume that the matrix is a matrix of color objects.)

Write the function in the same cell as `quantize(x::Number)` from the last exercise. 👉

Let's apply your method!





```
. quantize(philip)
```

Exercise 2.5

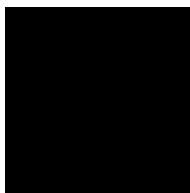
✎ Write a function `invert` that inverts a color, i.e. sends (r, g, b) to $(1 - r, 1 - g, 1 - b)$.

`invert` (generic function with 1 method)

```
. function invert(color::AbstractRGB)
.
.     return RGB(1-color.r, 1-color.g, 1-color.b)
. end
```

Let's invert some colors:

`black =`

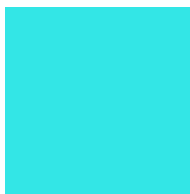


```
. invert(black)
```

`red =`



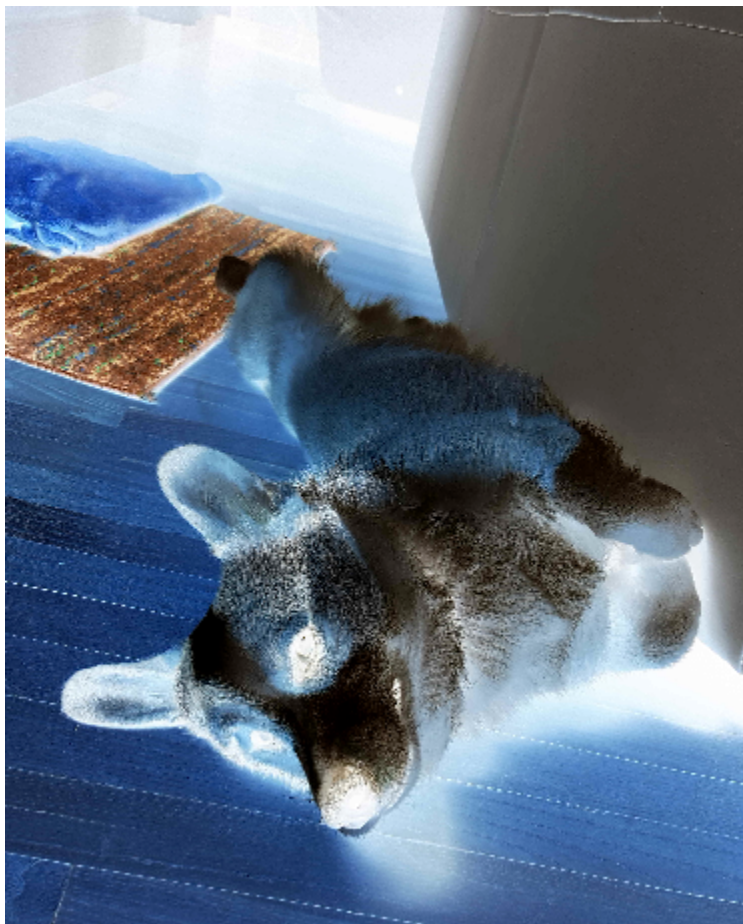
```
red = RGB(0.8, 0.1, 0.1)
```



```
. invert(red)
```

Can you invert the picture of Philip?

```
philip_inverted =
```



```
. philip_inverted = broadcast(invert, philip)
```

Exercise 2.6

✎ Write a function `noisify(x::Number, s)` to add randomness of intensity s to a value x , i.e. to add a random value between $-s$ and $+s$ to x . If the result falls outside the range $(0, 1)$ you should "clamp" it to that range. (Note that Julia has a `clamp` function, but you should write your own function `myclamp(x)`.)

```
0.5332243531835776
```

```
. rand()*2 - 1
```

noisify (generic function with 3 methods)

```
. begin
.
.   function myclamp(x)
.       if x >= 1.0
.           return 1.0
.       elseif x <= 0.0
.           return 0.0
.       else
.           return x
.       end
.   end
.
.   function noisify(x::Number, s)
.       noise = (rand()*2 - 1) * s
.       return myclamp(x + noise)
.   end
.
.   function noisify(color::AbstractRGB, s)
.       return RGB(noisify(color.r, s), noisify(color.g, s), noisify(color.b, s))
.   end
.
.   function noisify(image::AbstractMatrix, s)
.       return [noisify(pixel, s) for pixel in image]
.   end
. end
```

Hint

The second method generic function noisify(color::AbstractRGB, s) is defined as follows:

✎ Write the second method `noisify(c::AbstractRGB, s)` to add random noise of intensity s to each of the (r, g, b) values in a colour.

Write the function in the same cell as `noisify(x::Number)` from the last exercise. 🙌

 0.0



```
noisify(red, color_noise)
```

✎ Write the third method `noisify(image::AbstractMatrix, s)` to noisify each pixel of an image.

Write the function in the same cell as `noisify(x::Number)` from the last exercise. 🙌



```
. @bind philip_noise Slider(0:0.01:8, show_value=true)
```



```
. noisify(philip, philip_noise)
```

✎ For which noise intensity does it become unrecognisable?

You may need noise intensities larger than 1. Why?

`answer_about_noise_intensity =`

The image is unrecognisable with intensity 2.

```

. answer_about_noise_intensity = md"""
. The image is unrecognisable with intensity 2.
. """

```

```

. begin
.     Pkg.add("PlutoUI")
.     using PlutoUI
. end

```

decimate (generic function with 2 methods)

Exercise 3 - Convolutions

As we have seen in the videos, we can produce cool effects using the mathematical technique of **convolutions**. We input one image M and get a new image M' back.

Conceptually we think of M as a matrix. In practice, in Julia it will be a `Matrix` of color objects, and we may need to take that into account. Ideally, however, we should write a **generic** function that will work for any type of data contained in the matrix.

A convolution works on a small **window** of an image, i.e. a region centered around a given point (i, j) . We will suppose that the window is a square region with odd side length $2\ell + 1$, running from $-\ell, \dots, 0, \dots, \ell$.

The result of the convolution over a given window, centred at the point (i, j) is a *single number*; this number is the value that we will use for $M'_{i,j}$. (Note that neighbouring windows overlap.)

To get started let's restrict ourselves to convolutions in 1D. So a window is just a 1D region from $-\ell$ to ℓ .

Let's create a vector v of random numbers of length $n=100$.

```
n = 100
```

```
. n = 100
```

```
v = Float64[0.190681, 0.664396, 0.720537, 0.762361, 0.68362, 0.656818, 0.571
```

```
. v = rand(n)
```

Feel free to experiment with different values!

Exercise 3.1

You've seen some colored lines in this notebook to visualize arrays. Can you make another one?

👉 Try plotting our vector `v` using `colored_line(v)`.

```
. Enter cell code...
```

Try changing `n` and `v` around. Notice that you can run the cell `v = rand(n)` again to regenerate new random values.

Exercise 3.2

We need to decide how to handle the **boundary conditions**, i.e. what happens if we try to access a position in the vector `v` beyond `1:n`. The simplest solution is to assume that v_i is 0 outside the original vector; however, this may lead to strange boundary effects.

A better solution is to use the *closest* value that is inside the vector. Effectively we are extending the vector and copying the extreme values into the extended positions. (Indeed, this is one way we could implement this; these extra positions are called **ghost cells**.)

👉 Write a function `extend(v, i)` that checks whether the position i is inside `1:n`. If so, return the i th component of `v`; otherwise, return the nearest end value.

extend (generic function with 1 method)

```
. function extend(v, i)
.
.     return missing
. end
```

Some test cases:

missing

```
. extend(v, 1)
```

missing

```
extend(v, -8)
```


missing

```
. extend(v, n + 10)
```

Extended with 0:



Extended with your extend:

missing

Here we go!

Replace missing with your answer.

Exercise 3.3

✎ Write a function `blur_1D(v, l)` that blurs a vector `v` with a window of length `l` by averaging the elements within a window from $-\ell$ to ℓ . This is called a **box blur**.

blur_1D (generic function with 1 method)

```
. function blur_1D(v, l)
.
.     return missing
. end
```

Exercise 3.4

✎ Apply the box blur to your vector `v`. Show the original and the new vector by creating two cells that call `colored_line`. Make the parameter ℓ interactive, and call it `l_box` instead of just `l` to avoid a variable naming conflict.

```
. Enter cell code...
```

Hint

Use `colored_line` to show the original and blurred vectors. The `colored_line` function is defined in the `colored_line.jl` file in the `src` directory.

Exercise 3.5

The box blur is a simple example of a **convolution**, i.e. a linear function of a window around each point, given by

$$v'_i = \sum_n v_{i-n} k_n,$$

where k is a vector called a **kernel**.

Again, we need to take care about what happens if v_{i-n} falls off the end of the vector.

✎ Write a function `convolve_vector(v, k)` that performs this convolution. You need to think of the vector k as being *centred* on the position i . So n in the above formula runs between $-\ell$ and ℓ , where $2\ell + 1$ is the length of the vector k . You will need to do the necessary manipulation of indices.

`convolve_vector` (generic function with 1 method)

```
. function convolve_vector(v, k)
.
.     return missing
. end
```

Hint

`test_convolution = missing`

```
. test_convolution = let
.     v = [1, 10, 100, 1000, 10000]
.     k = [0, 1, 0]
.     convolve_vector(v, k)
. end
```

Edit the cell above, or create a new cell with your own test cases!

Here we go!

Replace missing with your answer.

Exercise 3.6

✎ Write a function `gaussian_kernel`.

The definition of a Gaussian in 1D is

$$G(x) = \frac{1}{2\pi\sigma^2} \exp\left(\frac{-x^2}{2\sigma^2}\right)$$

We need to **sample** (i.e. evaluate) this at each pixel in a region of size n^2 , and then **normalize** so that the sum of the resulting kernel is 1.

For simplicity you can take $\sigma = 1$.

gaussian_kernel (generic function with 1 method)

```
. function gaussian_kernel(n)
.
.     return missing
. end
```

Let's test your kernel function!

gaussian_kernel_size_1D = 3

```
. gaussian_kernel_size_1D = 3 # change this value, or turn me into a slider!
```

test_gauss_1D_a =

```
. test_gauss_1D_a = let
.     v = random_vect
.     k = gaussian_kernel(gaussian_kernel_size_1D)
.
.     if k != missing
.         convolve_vector(v, k)
.     end
. end
```

test_gauss_1D_b =

```
. test_gauss_1D_b = let
.     v = create_bar()
.     k = gaussian_kernel(gaussian_kernel_size_1D)
.
.     if k != missing
.         convolve_vector(v, k)
.     end
. end
```

Exercise 4 - Convolutions of images

Now let's move to 2D images. The convolution is then given by a **kernel** matrix K :

$$M'_{i,j} = \sum_{k,l} M_{i-k,j-l} K_{k,l},$$

where the sum is over the possible values of k and l in the window. Again we think of the window as being *centered* at (i, j) .

A common notation for this operation is $*$:

$$M' = M * K.$$

```

. md"""
.
.
. ## **Exercise 4** - _Convolutions of images_
.
. Now let's move to 2D images. The convolution is then given by a kernel matrix  $K$ :
.
.  $M'_{i,j} = \sum_{k,l} M_{i-k,j-l} K_{k,l}$ 
.
. where the sum is over the possible values of  $k$  and  $l$  in the window. Again we think of the
  window as being centered at  $(i, j)$ .
.
. A common notation for this operation is  $*$ :
.
.  $M' = M * K$ 
. """

```

Exercise 4.1

✎ Write a function `extend_mat` that takes a matrix `M` and indices `i` and `j`, and returns the closest element of the matrix.

`extend_mat` (generic function with 1 method)

```
function extend_mat(M::AbstractMatrix, i, j)
```

```
.    return missing
. end
```

Hint

(The hint is intentionally blurred in the original image)

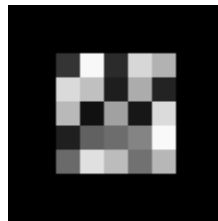
Let's test it!

```
small_image =
```



```
. small_image = Gray.(rand(5,5))
```

Extended with 0:



Extended with your extend:

```
9×9 Array{Missing,2}:
missing missing missing missing missing missing missing missing missing
missing missing missing missing missing missing missing missing missing
missing missing missing missing missing missing missing missing missing
missing missing missing missing missing missing missing missing missing
missing missing missing missing missing missing missing missing missing
missing missing missing missing missing missing missing missing missing
missing missing missing missing missing missing missing missing missing
missing missing missing missing missing missing missing missing missing
missing missing missing missing missing missing missing missing missing
```

Here we go!

Replace missing with your answer.

```
283×223 Array{Missing,2}:
missing missing missing missing missing ... missing missing missing missing
missing missing missing missing missing missing missing missing missing missing
missing missing missing missing missing missing missing missing missing missing
missing missing missing missing missing missing missing missing missing missing
missing missing missing missing missing missing missing missing missing missing
missing missing missing missing missing missing ... missing missing missing missing
missing missing missing missing missing missing missing missing missing missing
⋮
missing missing missing missing missing missing missing missing missing missing
missing missing missing missing missing missing missing missing missing missing
missing missing missing missing missing missing missing missing missing missing
missing missing missing missing missing missing ... missing missing missing missing
missing missing missing missing missing missing missing missing missing missing
missing missing missing missing missing missing missing missing missing missing
```

Exercise 4.2

👉 Implement a function `convolve_image(M, K)`.

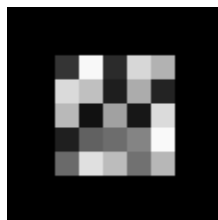
`convolve_image` (generic function with 1 method)

```
. function convolve_image(M::AbstractMatrix, K::AbstractMatrix)
.
.     return missing
. end
```

Hint

Let's test it out! 🎃

`test_image_with_border =`



```
K_test = 3×3 Array{Float64,2}:
 0.0  0.0  0.0
 0.5  0.0  0.5
 0.0  0.0  0.0
```

```
K_test = [
 0  0  0
 1/2 0  1/2
```

```
.      0  0  0
.  ]
```

missing

```
.  convolve_image(test_image_with_border, K_test)
```

Edit K_test to create your own test case!

missing

```
.  convolve_image(philip, K_test)
```

You can create all sorts of effects by choosing the kernel in a smart way. Today, we will implement two special kernels, to produce a **Gaussian blur** and a **Sobel edge detect** filter.

Make sure that you have watched [the lecture](#) about convolutions!

Exercise 4.3

👉 Apply a **Gaussian blur** to an image.

Here, the 2D Gaussian kernel will be defined as

$$G(x, y) = \frac{1}{2\pi\sigma^2} e^{\frac{-(x^2+y^2)}{2\sigma^2}}$$

```
.  md"""
.  #### Exercise 4.3
.  👉 Apply a Gaussian blur to an image.
.
.  Here, the 2D Gaussian kernel will be defined as
.
.  $$G(x,y)=\frac{1}{2\pi \sigma^2}e^{\frac{-(x^2+y^2)}{2\sigma^2}}$$
.  """
```

with_gaussian_blur (generic function with 1 method)

```
.  function with_gaussian_blur(image)
.
.      return missing
.  end
```

Let's make it interactive. 🌟



missing

```
. with_gaussian_blur(gauss_camera_image)
```

Exercise 4.4

👉 Create a **Sobel edge detection filter**.

Here, we will need to create two separate filters that separately detect edges in the horizontal and vertical directions:

$$G_x = \left(\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \otimes [1 \ 0 \ -1] \right) * A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} * A$$

$$G_y = \left(\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \otimes [1 \ 2 \ 1] \right) * A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} * A$$

Here A is the array corresponding to your image. We can think of these as derivatives in the x and y directions.

Then we combine them by finding the magnitude of the **gradient** (in the sense of multivariate calculus) by defining


$$G_{\text{total}} = \sqrt{G_x^2 + G_y^2}.$$

For simplicity you can choose one of the "channels" (colours) in the image to apply this to.

with_sobel_edge_detect (generic function with 1 method)

```
function with_sobel_edge_detect(image)

    return missing
end
```



Enable webcam

missing

```
. with_sobel_edge_detect(sobel_camera_image)
```

Exercise 5 - *Lecture transcript*

(MIT students only)

Please see the Canvas post for transcript document for week 1 [here](#).

We need each of you to correct about 100 lines (see instructions in the beginning of the document.)

👉 Please mention the name of the video and the line ranges you edited:

```
md """  
## **Exercise 5** - _Lecture transcript_  
_(MIT students only)_
```

```
  
Please see the Canvas post for transcript document for week 1 [here](https://canvas.mit.edu  
/courses/5637/discussion_topics/27880).
```

- We need each of you to correct about 100 lines (see instructions in the beginning of the document.)
-
- 🖱 Please mention the name of the video and the line ranges you edited:
- """

`lines_i_edited =`

Convolution, lines 100-0 (*for example*)

- `lines_i_edited = md"""`
- Convolution, lines 100-0 (*_for example_*)
- """

hint (generic function with 1 method)

almost (generic function with 1 method)

still_missing (generic function with 2 methods)

keep_working (generic function with 2 methods)

`yays =` MD[Great!, Yay ♥, Great! 🎉, Well done!, Keep it up!, Good job!, Awe:

correct (generic function with 2 methods)

not_defined (generic function with 1 method)

camera_input (generic function with 1 method)

process_raw_camera_data (generic function with 1 method)