# Solutions to Assignment in Chapter 6

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**6.1** Given a random sample of size $n$ from a normal population with the known variance $\sigma^2$, show that the null hypothesis $H_0 : \mu = \mu_0$ can be tested against the alternative hypothesis $H_1 : \mu \neq \mu_0$ with the use of a one-tailed criterion based on the chi-square distribution.

**Solution**. From the Proposition 2.1.2 we have that the test statistic

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0, 1),$$

and a critical region of size $\alpha$ for a two-tailed test is given by

$$|z| = \Big|\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}\Big| \geq z_{\alpha/2}.$$

On the other hand, from Proposition 2.3.2, we have that

$$Z^2 = \Big(\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}\Big)^2 = \frac{n(\bar{X} - \mu_0)^2}{\sigma^2} \sim \chi^2(1).$$

This means that

$$\alpha = \mathbb{P}\Big(\frac{n(\bar{X} - \mu_0)^2}{\sigma^2} \geq \chi^2_{\alpha,1}\Big),$$

and hence, an one-tailed criterion based on the chi-square distribution is given by

$$\frac{n(\bar{x} - \mu_0)^2}{\sigma^2} \geq \chi^2_{\alpha,1}. \quad \square$$

**6.2** Suppose that a random sample from a normal population with the known variance $\sigma^2$ is to be used to test the null hypothesis $H_0 : \mu = \mu_0$ against the alternative hypothesis $H_1 : \mu = \mu_1$, where $\mu_1 > \mu_0$, and that the probabilities of type I and type II errors are to have the preassigned values $\alpha$ and $\beta$. Show that the required size of the sample is given by

$$n = \frac{\sigma^2(z_\alpha + z_\beta)^2}{(\mu_1 - \mu_0)^2}.$$

**Solution**. Using the Proposition 3.2.1 (Neyman-Pearson lemma), similarly to Example 12. 4 in Section 3.2, we have that the most powerful critical region $C$ is given by

$$\bar{x} \geq K, \text{ inside } C; \quad \bar{x} < K, \text{ outside } C.$$

Since the probabilities of type I and type II errors are to have the preassigned values $\alpha$ and $\beta$, we

$$K = \mu_0 + z_\alpha \frac{\sigma}{\sqrt{n}} \quad \text{and} \quad K = \mu_1 - z_\beta \frac{\sigma}{\sqrt{n}}.$$

This implies

$$\mu_0 + z_\alpha \frac{\sigma}{\sqrt{n}} = \mu_1 - z_\beta \frac{\sigma}{\sqrt{n}},$$

or

$$\mu_1 - \mu_0 = (z_\alpha + z_\beta)\frac{\sigma}{\sqrt{n}}.$$

Thus, we get

$$\sqrt{n} = \frac{\sigma(z_\alpha + z_\beta)}{\mu_1 - \mu_0},$$

or

$$n = \frac{\sigma^2(z_\alpha + z_\beta)^2}{(\mu_1 - \mu_0)^2}. \quad \square$$

**6.3** Based on certain data, a null hypothesis is rejected at the 0.05 level of significance. Would it also be rejected at the

(1) 0.01 level of significance;

(2) 0.10 level of significance?

**Solution**. (1) It is not necessarily rejected at the 0.01 level of significance since $0.05 > 0.01$. (2) Yes, it will be rejected at the 0.10 level of significance since $0.05 < 0.10$. $\square$

**6.4** In the test of a certain hypothesis, the $P$-value corresponding to the test statistic is 0.0316. Can the null hypothesis be rejected at the

(1) 0.01 level of significance;

(2) 0.05 level of significance;

(3) 0.10 level of significance?

**Solution**. (1) No, since $0.0316 > 0.01$. (2) Yes, since $0.0316 < 0.05$. (3) Yes, since $0.0316 < 0.10$. $\square$

**6.5** According to the historical records of a reading comprehension test, eighth graders should average 84.3 with a standard deviation of 8.6. If 45 randomly selected eighth graders from a certain school district averaged 87.8, test the null hypothesis $H_0 : \mu = 84.3$ against the alternative hypothesis $H_1 : \mu > 84.3$ at the 0.01 level of significance.

**Solution I**.

(1) $H_0 : \mu = 84.3$ against $H_1 : \mu > 84.3$; $\alpha = 0.01$.

(2) Reject $H_0$ if $z \geq z_{0.01} = 2.33$, where

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}.$$

(3) Substituting the given data: $n = 45$, $\bar{x} = 87.8$, $\mu_0 = 84.3$, and $\sigma = 8.6$, we get

$$z = 2.73.$$

(4) Since $z = 2.73 > 2.33$, the null hypothesis $H_0$ must be rejected. $\square$

**Solution II**. (Using the $P$-value method).

(a) $H_0 : \mu = 84.3$ against $H_1 : \mu > 84.3$; $\alpha = 0.01$.

(b) Use the test statistic $Z$ given by

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}.$$

(c) Substituting the given data: $n = 45$, $\bar{x} = 87.8$, $\mu_0 = 84.3$, and $\sigma = 8.6$, we get an observed value $z$ of the test statistic $Z$:

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = 2.73.$$

The corresponding P-value is:

$$\mathbb{P}\big(Z \geq 2.73\big) = 0.5 - \mathbb{P}\big(0 \leq Z \leq 2.73\big) = 0.5 - 0.4968 = 0.0032.$$

(d) Since the $P$-value $0.0032 \leq 0.01 = \alpha$, the null hypothesis $H_0$ must be rejected. □

**6.6** The security department of a factory wants to know whether or not the true average time required by the night guard to walk his round is 30 minutes. If, in a random sample of 32 rounds, the night guard averaged 30.8 minutes with a standard deviation of 1.5 minutes, test the null hypothesis $H_0 : \mu = 30$ minutes against the alternative hypothesis $H_1 : \mu :\neq 30$ minutes at the 0.01 level of significance.

**Solution I**. Since the size of sample $n = 32 \geq 30$ we use the large-sample test.

(1) $H_0 : \mu = 30$ against $H_1 : \mu \neq 30$; $\alpha = 0.01$.

(2) Reject $H_0$ if $|z| \geq z_{0.005} = 2.575$, where

$$z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}.$$

(3) Substituting the given data: $n = 32$, $\bar{x} = 30.8$, $\mu_0 = 30$, and $s = 1.5$, we get

$$z = 3.02.$$

(4) Since $z = 3.02 > 2.575$, the null hypothesis $H_0$ must be rejected. □

**Solution II**. (Using the $P$-value method).

(a) $H_0 : \mu = 30$ against $H_1 : \mu \neq 30$; $\alpha = 0.01$.

(b) Use the test statistic $Z$ given by

$$Z = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}.$$

(c) Substituting the given data: $n = 45$, $\bar{x} = 30.8$, $\mu_0 = 30$, and $s = 1.5$, we get an observed value $z$ of the test statistic $Z$:

$$z = \frac{\bar{x} - \mu_0}{x/\sqrt{n}} = 3.02.$$

The corresponding P-value is:

$$\mathbb{P}\big(|Z| \geq 3.02\big) = 1 - 2\mathbb{P}\big(0 \leq Z \leq 3.02\big) = 0.0026.$$

(d) Since the $P$-value $0.0026 \leq 0.01 = \alpha$, the null hypothesis $H_0$ must be rejected. □

**6.7** In 12 test runs over a marked course, a newly designed motorboat averaged 33.6 seconds with a standard deviation of 2.3 seconds. Assume that it is reasonable to treat the data as a random sample from a normal population. Test the null hypothesis $H_0 : \mu = 35$ seconds against the alternative hypothesis $H_1 : \mu < 35$ seconds at the 0.05 level of significance.

**Solution**.

(1) $H_0 : \mu = 35$ against $H_1 : \mu < 35$; $\alpha = 0.05$.

(2) Reject $H_0$ if $t \le -t_{\alpha,n-1} = -t_{0.05,11} = -1.796$, where

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}.$$

(3) Substituting the given data: $n = 12$, $\bar{x} = 33.6$, and $s = 2.3$, we get

$$t = -2.11.$$

(4) Since $t = -2.11 < -t_{0.05,11} = -1.796$, the null hypothesis $H_0$ must be rejected. □

**6.8** Five measurements of the tar content of a certain kind of cigarette yielded 14.5, 14.2, 14.4, 14.3, and 14.6 mg/cigarette. Assume that it is reasonable to treat the data as a random sample from a normal population. Test the null hypothesis $H_0; \mu = 14.0$ mg/cigarette against the alternative hypothesis $H_1 : \mu \neq 14.0$ mg/cigarette at the 0.05 level of significance.

**Solution**. From the given sample data, we have that $\bar{x} = 14.4$ and $s = 0.1581$.

(1) $H_0 : \mu = 14.0$ against $H_1 : \mu \neq 14.0$; $\alpha = 0.05$.

(2) Reject $H_0$ if $|t| \ge t_{\alpha,n-1} = -t_{0.05,4} = 2.776$, where

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}.$$

(3) Substituting the given data: $n = 5$, $\bar{x} = 14.4$, and $s = 0.1581$, we get

$$t = 5.66.$$

(4) Since $|t| = 5.66 > t_{0.05,4} = 2.776$, the null hypothesis $H_0$ must be rejected. □

**6.9** Sample surveys conducted in a large county in a certain year and again 20 years later showed that originally the average height of 400 ten-year-old boys was 53.8 inches with a standard deviation of 2.4 inches whereas 20 years later the average height of 500 ten-year-old boys was 54.5 inches with a standard deviation of 2.5 inches. Test the null hypothesis $H_0 : \mu_1 - \mu_2 = -0.5$ cm against the alternative hypothesis $H_1 : \mu_1 - \mu_2 < -0.5$ cm at the 0.05 level of significance.

**Solution I**.

(1) $H_0 : \mu_1 - \mu_2 = -0.5$ against $H_1 : \mu_1 - \mu_2 < -0.5$; $\alpha = 0.05$.

(2) Since $n_1 = 400$ and $n_2 = 500$ are both greater than 30, we use the large-sample test, i.e., reject $H_0$ if $z \le -z_\alpha = -z_{0.05} = -1.645$, where

$$z = \frac{\bar{x}_1 - \bar{x}_2 + 0.5}{\sqrt{(s_1^2/n_1) + (s_2^2/n_2)}}.$$

(3) Substituting the given data: $\bar{x}_1 = 53.8$ and $s_1 = 2.4$, $\bar{x}_2 = 54.5$ and $s_2 = 2.5$, we get

$$z = -1.21942 \approx -1.22.$$

(4) Since $z = -1.22 > -1.645$, $H_0$ cannot be rejected. $\square$

**Solution II.**

(a) $H_0 : \mu_1 - \mu_2 = -0.5$ against $H_1 : \mu_1 - \mu_2 < -0.5$; $\alpha = 0.05$.

(b) Since $n_1 = 400$ and $n_2 = 500$ are both greater than 30, we use the large-sample test $Z$

$$Z = \frac{\bar{X}_1 - \bar{X}_2 + 0.5}{\sqrt{(S_1^2/n_1) + (S_2^2/n_2)}}.$$

(c) Substituting the given data, we get

$$z = \frac{\bar{x}_1 - \bar{x}_2 + 0.5}{\sqrt{(s_1^2/n_1) + (s_2^2/n_2)}} = -1.21942 \approx -1.22.$$

Then, the corresponding $P$-value is given by

$$P = \mathbb{P}(Z \leq -1.22) = \mathbb{P}(Z \geq 1.22)\big) = 0.1112.$$

(d) Since the $P$-valued $P = 0.1112 \geq 0.05 = \alpha$, $H_0$ cannot be rejected. $\square$

**6.10** To compare two kinds of bumper guards, six of each kind were mounted on a car. Then each car was run into a concrete wall at 5 km per hour, and the following are the costs of the repairs (in dollars)

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Bumper guard 1 | 127 | 168 | 143 | 165 | 122 | 139 |
| Bumper guard 2 | 154 | 135 | 132 | 171 | 153 | 149 |

Assume that it is reasonable to treat the data as two independent random samples from two normal populations.

(1) Test at the 0.02 level of significance whether it is reasonable to assume that the two populations sampled have equal variances, i.e., test the null hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$ against the alternative hypothesis $H_1 : \sigma_1^2 \neq \sigma_2^2$ at the 0.02 level of significance.

(2) Test at the 0.01 level of significance whether the difference between the means of these two samples is significant, i.e., test the null hypothesis $H_0 : \mu_1 = \mu_2$ against the alternative hypothesis $H_1 : \mu_1 \neq \mu_2$ at the 0.01 level of significance.

**Solution**. Using Excel, we can get that $\bar{x}_1 = 144$ with $s_1 = 19.06$, and $\bar{x}_2 = 149$ with $s_2 = 14.21$

(1) Test the null hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$ against the alternative hypothesis $H_1 : \sigma_1^2 \neq \sigma_2^2$ at the 0.02 level of significance.

(a) $H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 : \sigma_1^2 \neq \sigma_2^2$; $\alpha = 0.02$.

(b) Since $H_1$ is two-sided and $s_1^2 > s_2^2$, reject $H_0$ if

$$f = \frac{s_1^2}{s_2^2} \geq f_{\alpha/2, n_1-1, n_2-1} = f_{0.01,5,5} = 11.0.$$

(c) Substituting the given data, we obtain that

$$f = \frac{s_1^2}{s_2^2} = 1.80.$$

(d) Since $f = 1.80 < 11.0 = f_{\alpha/2, n_1-1, n_2-1}$, $H_0$ can not be be rejected.

(2) Test the null hypothesis $H_0 : \mu_1 = \mu_2$ against the alternative hypothesis $H_1 : \mu_1 \neq \mu_2$ at the 0.01 level of significance.

(a) $H_0 : \mu_1 - \mu_2 = 0$ against $H_1 : \mu_1 - \mu_2 \neq 0$; $\alpha = 0.01$.

(b) According to the result of (1) we can assume that $\sigma_1^2 = \sigma_2^2$. Reject $H_0$ if $|t| \geq t_{\alpha/2, n_1+n_2-2} = t_{0.005,10} = 3.169$, where

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{(1/n_1) + (1/n_2)}}$$

with

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}.$$

(c) Substituting the given data: $n_1 = n_2 = 6$, $\bar{x}_1 = 144$ and $s_1 = 19.06$, and $\bar{x}_2 = 149$ and $s_2 = 14.21$, we first get

$$s_p = 16.802,$$

and then, we obtain

$$t = -0.52.$$

(d) Since $|t| = 0.52 < 3.169 = t_{\alpha/2, n_1+n_2-2}$, the null hypothesis $H_0$ can not be rejected. □

**6.11** In a study of the effectiveness of certain exercises in weight reduction, a group of 16 persons engaged in these exercises for one month and showed the following results (in pounds):

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Weight before | 211 | 180 | 171 | 214 | 182 | 194 | 160 | 182 |
| Weight after | 198 | 173 | 172 | 209 | 179 | 192 | 161 | 182 |
| | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| Weight before | 172 | 155 | 185 | 167 | 203 | 181 | 245 | 246 |
| Weight after | 166 | 154 | 181 | 164 | 201 | 175 | 233 | 242 |

Assume that it is reasonable to treat the data as two independent random samples from two normal populations.

(1) Test at the 0.02 level of significance whether it is reasonable to assume that the two populations sampled have equal variances, i.e., test the null hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$ against the alternative hypothesis $H_1 : \sigma_1^2 \neq \sigma_2^2$ at the 0.02 level of significance.

(2) Use the 0.05 level of significance to test the null hypothesis $H_0 : \mu_1 = \mu_2$ against the alternative hypothesis $H_1 : \mu_1 > \mu_2$, and thus judge whether the exercises are effective in weight reduction.

**Solution**. Using Excel, we can get that $\bar{x}_1 = 190.50$ with $s_1 = 27.12$, and $\bar{x}_2 = 186.38$ with $s_2 = 25.01$

(1) Test the null hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$ against the alternative hypothesis $H_1 : \sigma_1^2 \neq \sigma_2^2$ at the 0.02 level of significance.

(a) $H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 : \sigma_1^2 \neq \sigma_2^2$; $\alpha = 0.02$.

(b) Since $H_1$ is two-sided and $s_1^2 > s_2^2$, reject $H_0$ if

$$f = \frac{s_1^2}{s_2^2} \geq f_{\alpha/2,n_1-1,n_2-1} = f_{0.01,15,15} = 3.52.$$

(c) Substituting the given data, we obtain that

$$f = \frac{s_1^2}{s_2^2} = 1.18.$$

(d) Since $f = 1.18 < 3.52 = f_{\alpha/2,n_1-1,n_2-1}$, $H_0$ can not be be rejected.

(2) Test the null hypothesis $H_0 : \mu_1 = \mu_2$ against the alternative hypothesis $H_1 : \mu_1 \neq \mu_2$ at the 0.05 level of significance.

(a) $H_0 : \mu_1 - \mu_2 = 0$ against $H_1 : \mu_1 - \mu_2 > 0$; $\alpha = 0.05$.

(b) According to the result of (1) we can assume that $\sigma_1^2 = \sigma_2^2$. Reject $H_0$ if $t \geq t_{\alpha,n_1+n_2-2} = t_{0.05,30} = 1.645$, where

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{(1/n_1) + (1/n_2)}}$$

with

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}.$$

(c) Substituting the given data: $n_1 = n_2 = 16$, $\bar{x}_1 = 190.50$ with $s_1 = 27.12$, and $\bar{x}_2 = 186.38$ with $s_2 = 25.01$, we first get

$$s_p = 26.09,$$

and then, we obtain

$$t = 0.45.$$

(d) Since $t = 0.45 < 1.645 = t_{\alpha,n_1+n_2-2}$, the null hypothesis $H_0$ can not be rejected. Thus, the exercises are not effective in weight reduction. □

**6.12** Suppose that two independent random samples of size $n_1 = 21$ and $n_2 = 10$ from two normal populations having the distributions $N(\mu_1, \sigma_1^2)$ and $(\mu_2, \sigma_2^2)$, respectively, have the sample means $\bar{x}_1 = 465.13$ and $\bar{x}_2 = 422.16$, and the standard deviations $s_1 = 54.80$ and $s_2 = 49.20$, respectively.

(1) Test the null hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$ against the alternative hypothesis $H_1 : \sigma_1^2 \neq \sigma_2^2$ at the 0.02 level of significance.

(2) Test $H_0 : \mu_1 = \mu_2$ against the alternative hypothesis $H_1 : \mu_1 \neq \mu_2$ against at the 0.05 level of significance.

**Solution**. (1) Test the null hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$ against the alternative hypothesis $H_1 : \sigma_1^2 \neq \sigma_2^2$ at the 0.02 level of significance.

(a) $H_0 : \sigma_1^2 = \sigma_2^2$ against $H_1 : \sigma_1^2 \neq \sigma_2^2$; $\alpha = 0.02$.

(b) Since $s_1^2 > s_2^2$, reject $H_0$ if

$$f = \frac{s_1^2}{s_2^2} \geq f_{\alpha/2,n_1-1,n_2-1} = f_{0.01,20,9} = 4.81.$$

(c) Substituting the given data, we obtain that

$$f = \frac{s_1^2}{s_2^2} = 1.24.$$

(d) Since $f = 1.24 < 3.52 = f_{\alpha/2,n_1-1,n_2-1}$, $H_0$ can not be be rejected.

(2) Test the null hypothesis $H_0 : \mu_1 = \mu_2$ against the alternative hypothesis $H_1 : \mu_1 \neq \mu_2$ at the 0.05 level of significance.

(a) $H_0 : \mu_1 - \mu_2 = 0$ against $H_1 : \mu_1 - \mu_2 \neq 0$; $\alpha = 0.05$.

(b) According to the result of (1) we can assume that $\sigma_1^2 = \sigma_2^2$. Reject $H_0$ if $|t| \geq t_{\alpha/2,n_1+n_2-2} = t_{0.025,29} = 2.045$, where

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{(1/n_1) + (1/n_2)}}$$

with

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}.$$

(c) Substituting the given data: $n_1 = 21$, $\bar{x}_1 = 190.50$ with $s_1 = 27.12$, $n_2 = 10$, and $\bar{x}_2 = 180.13$ with $s_2 = 22.56$, we first get

$$s_p = 53.1253,$$

and then, we obtain

$$t = 2.1052.$$

(d) Since $t = 2.1052 > 2.045 = t_{\alpha/2,n_1+n_2-2}$, the null hypothesis $H_0$ must be rejected.. □

**6.13** In a random sample, the standard deviation $s = 2.53$ minutes for the amount of time that 30 women took to complete the written test for their drive's licenses. At the 0.05 level of significance, test the null hypothesis $H_0 : \sigma = 2.85$ minutes against the alternative hypothesis $H_1 : \sigma < 2.85$ minutes.

**Solution**.

(1) $H_0 : \sigma^2 = 2.85$ against $\sigma^2 < 2.85$; $\alpha = 0.05$.

(2) Reject $H_0$ if $\chi^2 \leq \chi^2_{1-\alpha,n-1} = \chi^2_{0.95,29} = 17.708$, where

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}.$$

(3) Substituting the given data: $n = 30$, $s^2 = 2.53$, and $\sigma_0^2 = 2.85$, we obtain

$$\chi^2 = 22.85.$$

(4) Since $\chi^2 = 22.85 > 17.708 = \chi^2_{1-\alpha,n-1}$, the hypothesis $H_0$ can not be rejected. □

**6.14** In a random sample of 600 cars making a right turn at a certain intersection, 157 pulled into the wrong lane. Use the 0.05 level of significance to test the null hypothesis that the actual proportion of drivers who make this mistake at the given intersection is $\theta = 0.30$ against the alternative hypothesis $\theta \neq 0.30$.

**Solution**.

(1) $H_0 : \theta = 0.30$ against $H_1 : \theta \neq 0.30$; $\alpha = 0.05$.

(2) Reject $H_0$ if $|z| \geq z_{\alpha/2} = z_{0.025} = 1.96$, where $z$ is given by

$$z = \frac{x - n\theta_0}{\sqrt{n\theta_0(1-\theta_0)}}.$$

(3) Substituting the data: $x = 157$, $n = 600$ and $\theta_0 = 0.30$, we get

$$z = -2.05.$$

(4) Since $|z| = 2.05 > 1.96 = z_{\alpha/2}$, the hypothesis $H_0$ must be rejected. □

**6.15** The manufacturer of a spot remover claims that his product removes 90% of all spots. In a random sample, only 174 of 200 spots were removed with his product. Use the 0.05 level of significance to test the null hypothesis $H_0 : \theta = 0.90$ against the alternative hypothesis $H_1 : \theta < 0.90$.

**Solution**.

(1) $H_0 : \theta = 0.90$ against $H_1 : \theta < 0.90$; $\alpha = 0.05$.

(2) Reject $H_0$ if $z \leq -z_\alpha = z_{0.05} = 1.645$, where $z$ is given by

$$z = \frac{x - n\theta_0}{\sqrt{n\theta_0(1-\theta_0)}}.$$

(3) Substituting the data: $x = 174$, $n = 200$ and $\theta_0 = 0.90$, we get

$$z = -1.41.$$

(4) Since $z = -1.41 > -1.645 = -z_\alpha$, the hypothesis $H_0$ can not be rejected. □

**6.16** In a random sample of 1000 families in the United States, it is found that 290 of the families contained at least one member with a college degree. Use the 0.05 level of significance to test the null hypothesis that the actual proportion of such United States families is $\theta = 0.35$ against the alternative hypothesis $\theta < 0.35$.

**Solution**.

(1) $H_0 : \theta = 0.35$ against $H_1 : \theta < 0.35$; $\alpha = 0.05$.

(2) Reject $H_0$ if $z \leq z_\alpha = z_{0.05} = 1.645$, where $z$ is given by

$$z = \frac{x - n\theta_0}{\sqrt{n\theta_0(1-\theta_0)}}.$$

(3) Substituting the data: $x = 290$, $n = 1000$ and $\theta_0 = 0.35$, we get

$$z = -3.978.$$

(4) Since $z = -3.978 < -1.645 = -z_\alpha$, the hypothesis $H_0$ must be rejected. □