MATH 4753 Laboratory 14: Simple Linear Regression

In this lab we will investigate more detail concerning the mathematical underpinning of ***Simple*** Linear Regression.
Why is it called “Simple”? The reason is because we use only one independent variable (one x). When more than one x is used we call this ***multiple*** regression.
As I will show you in class:

$$SS_{xx} = \sum_{i=1}^{n}(x - \bar{x})^2$$

$$SS_{xy} = \sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

$$\hat{\beta}_1 = \frac{SS_{xy}}{SS_{xx}}$$

$$\bar{y} = \hat{\beta}_0 + \hat{\beta}_1\bar{x}$$

Tasks

All output should be made through RMD. Please upload the following files:

- HTML
- RMD

All plots should be made through RMD and knitted into suitable formats.

You are expected to adjust the functions as needed to answer the questions within the tasks below.

- Task 1
    - Make a folder LAB14
    - Download the file “lab14.r”
    - Place this file with the others in LAB14.
    - Start Rstudio
    - Open “lab14.r” from within Rstudio.
    - Go to the “session” menu within Rstudio and “set working directory” to where the source files are located.
    - Issue the function `getwd()` and copy the output here.
    - Create your own R file and record the R code you used to complete the lab.

- Task 2
    - Make a function (`mylsq`) that will calculate estimates of the slope and intercept under least squares regression. The function will operate on two vectors of the same length, x and y, where x is the independent variable and y the dependent variable. It is partially made below. **Hint:** Use the formulae above.

```
mylsq=function(x,y){
```

```
ssxx=sum((x-mean(x))^2 )
ssxy=sum() ## fill in the missing portion
b1hat=ssxy/ssxx
b0hat=   ##  fill in the missing portion
return(list(b0hat=b0hat,b1hat= )) #fill in the missing portion
}
```

- Suppose `x=1:20` and `set.seed(29);y=4+6*x + rnorm(20,0,5)`
- Use `mylsq()` to calculate the least squares estimates of parameters $\beta_0, and\ \beta_1$.
- Plot the points and the least squares line, with a heading and appropriate x and y labels. Also make the line have lwd=2 and be blue in colour. Hint: You can use abline()
- Check your calculations using `slr=lm(y~x); summary(slr)`.

- Task 3
  - Now make a function that will predict the average y value from a given xnew. The function is mypred() will take three arguments, the x value, b0hat and b1hat.

```
mypred=function(x,b0,b1){
ym=b0+ ## fill in the gap
ym
}
```

- Use the same data in Task 2 and predict a new mean y value ($\hat{y}$) when xnew=15.5
- Plot this point (xnew,ym) with the previous data and least squares line. **Hint:** use `points(), cex=3,col="Green",pch=19`

- Use the functions you have created so far and answer **10.12 page 498 in MS 6th edition**

a. Find the least-squares line for the data.
b. Interpret $\hat{\beta}_0$ and $\hat{\beta}_1$ in the words of the problem.
c. Predict the sweetness index if the amount of pectin in the orange juice is 300 ppm.

OJUICE

Run	Sweetness Index	Pectin (ppm)
1	5.2	220
2	5.5	227
3	6.0	259

  - a)
  - b)
  - c)

- Use the functions you have created so far and answer **10.80 page 553 in MS 6th edition**

DRILLROCK

Depth at Which Drilling Begins x, feet	Time to Drill 5 Feet y, minutes
0	4.90
25	7.41
50	6.19

a. Construct a scattergram for the data.
b. Find the least-squares prediction equation.
c. Graph the least-squares line on the scattergram.
d. Interpret the values of $\hat{\beta}_0$ and $\hat{\beta}_1$.

    - a)
    - b)
    - c)
    - d)

- Task 4
  - On page 501 MS proves that the least squares estimator $\hat{\beta}_1$ is an unbiased estimator of β_1.
  - On page 503 MS shows that an unbiased estimator of σ^2 is

$$s^2 = \frac{SSR}{n-2}, where\ SSR = \sum(y_i - \hat{y}_i)^2$$

  - Complete the following function that calculates s^2

```
mysq=function(x,y){
n=length(x)
ssxx=sum((x-mean(x))^2 )
ssxy=sum() ## fill in the missing portion
b1hat=ssxy/ssxx
b0hat=   ##  fill
yhat=b0hat+  ##  fill
ssr=sum((y-##)^2) # fill
sq= ##  fill
return(list(ssr=ssr,sq=sq))
}
```

  - Using x and y from Task 2 estimate σ^2. How close did you get?

- Now answer MS page 506 10.25 below

a. Plot the data points on a scattergram.

b. Fit a simple linear model relating carbon content in a pilot test, y, to the carbon content in a lab furnace, x. Interpret the estimates of the model parameters.

c. Compute SSE and s^2.

d. Compute s and interpret its value.

CARBON

Carbon Content (%)		Carbon Content (%)	
Pilot Plant	Lab Furnace	Pilot Plant	Lab Furnace
1.7	1.6	3.4	4.3
3.1	2.4	3.2	3.6

  - a)
  - b)
  - c)
  - d)

################### LAB FINISHES HERE ################################