# Chapter 9 – Ancient and Medieval China
The Jade Convergence

Mr. Harley Caham Combest

June 2025

## Cultural Invocation

- **Civilization:** Ancient and Medieval China
- **Time Period:** 14th century BCE to 14th century CE
- **Figures:** Zhou Gong (calendrical harmonics), Liu Hui (polygonal pi iteration), Zu Chongzhi (extreme precision in pi),
Qin Jiushao (congruence and indeterminate analysis), Yang Hui (numerical patterns and root extraction), Zhu Shijie (symbolic recursion and binomial logic)

*From bamboo rods to jade scrolls, Chinese mathematics was not written — it was enacted. Rooted in the Earth, attentive to the Heavens, its spirit endured dynastic collapse, imperial command, and the silence of ash. Yet it was also forged in necessity: the Yellow River, unpredictable and violent, demanded that measurement serve survival. What began as counting floods became calligraphy — and memory.*


Figure 1: Civilization between rivers (the Yellow and Yangtze): where geometry met governance and flood gave rise to form.

# II. Problems and Solutions

## Problem 1 — Can Geometry Approximate the Infinite?

### Concept Preview

We begin with a circle — not for perfection, but for convergence.

In an age without calculus, Liu Hui approached $\pi$ not through formula, but through form — refining a polygon into a ritual of iteration.

**Topic: Liu Hui's Polygon Approximation of $\pi$**

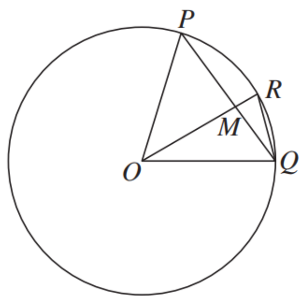



FIG. 9.1

Figure 2: Liu Hui's circle and polygon approximation diagram

- Begin with a hexagon inscribed in a circle of radius $r$.
- Construct triangle $\triangle PQM$, where $PQ$ is a side, $OM$ is the apothem, and $MR$ is the sagitta.
- Let:

$$\begin{aligned} \text{Apothem: } & OM = u \\ \text{Sagitta: } & MR = r - u \\ \text{Half-side: } & PQ/2 = s/2 \\ \text{New side (after doubling): } & w^2 = v^2 + \left(\frac{s}{2}\right)^2 \end{aligned}$$

- Simplified: $w^2 = 2rv$
- Liu Hui doubled sides iteratively to reach a 3072-gon: $\pi \approx 3.1416$

**Meta-Realization**
This was not brute calculation — it was recursive refinement.
Each triangle was a prayer, each iteration a deeper approximation of truth.
Liu Hui did not compute $\pi$ — he *approached it*, side by side, line by line.
*This was not analysis by function, but by faith in convergence.*

## Problem 2 — Can You Solve Equations Without Algebra?

### Concept Preview

If Liu Hui used geometry to approach the infinite, the Nine Chapters used it to master the finite.

They did not "solve for $x$" — they aligned columns, shifted numbers, and let logic emerge from symmetry.

This is algebra without letters. Mathematics, embodied.

**Topic: The Nine Chapters and Elimination by Columns**

The Chinese were especially fond of patterns; hence, it is not surprising that the first record (of ancient but unknown origin) of a magic square appeared there. The square

| | | |
|---|---|---|
| 4 | 9 | 2 |
| 3 | 5 | 7 |
| 8 | 1 | 6 |

was supposedly brought to man by a turtle from the River Luo in the days of the legendary Emperor Yii, who was reputed to be a hydraulic engineer. The concern for such patterns led the author of the *Nine Chapters* to solve the system of simultaneous linear equations

$$3x + 2y + z = 39$$
$$2x + 3y + z = 34$$
$$x + 2y + 3z = 26$$

by performing column operations on the matrix

$$\boxed{\begin{matrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 1 & 1 \\ 26 & 34 & 39 \end{matrix}} \quad \text{to reduce it to} \quad \boxed{\begin{matrix} 0 & 0 & 3 \\ 0 & 5 & 2 \\ 36 & 1 & 1 \\ 99 & 24 & 39 \end{matrix}}$$

The second form represented the equations $36z = 99$, $5y + z = 24$, and $3x + 2y + z = 39$, from which the values of $z$, $y$, and $x$ are successively found with ease.

Figure 3: Magic square and Gaussian-style elimination

- System of equations:

$$\begin{aligned} 3x + 2y + z &= 39 \\ 2x + 3y + z &= 34 \\ x + 2y + 3z &= 26 \end{aligned}$$

- Augmented matrix setup:

$$\begin{bmatrix} 3 & 2 & 1 & | & 39 \\ 2 & 3 & 1 & | & 34 \\ 1 & 2 & 3 & | & 26 \end{bmatrix}$$

- Column operations yield:

$$\begin{bmatrix} 0 & 0 & 3 & | & 99 \\ 0 & 5 & 2 & | & 24 \\ 3 & 2 & 1 & | & 39 \end{bmatrix}$$

**Meta-Realization**

This was not algebra — it was architecture.
Each rod was placed not by notation, but by alignment.
The solution emerged not from abstract variables, but from balance and elimination — a choreography of certainty.
*Algebra here was geometric, physical, and patterned — a ritual of clarity, not of calculation.*

## Problem 3 — Can You Compute by Touch?

### Concept Preview

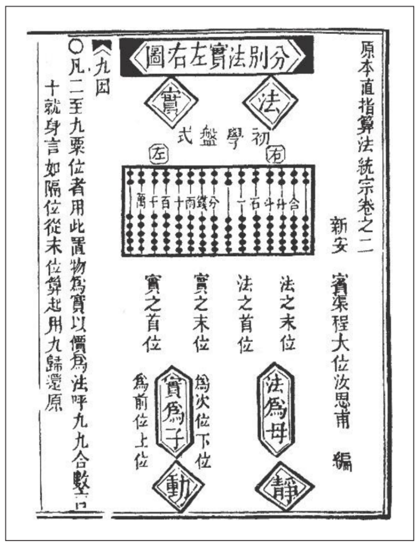
Before notation, there was placement.

Before abstraction, there was motion.

The Chinese did not merely calculate — they performed mathematics with rods, hands, and intuition.

This was not writing — it was *enactment*.

**Topic: The Rod Numerals and Abacus Tradition**




An early printed picture of an abacus, from the *Suan Fa Tongzong*, 1592 (Reproduced from J. Needham 1959, Vol. 3, p. 76.)

Figure 4: An early printed abacus from the Suan Fa Tongzong (1592)

- Numbers represented using rods in columns — each column denoting place value.
- Rod colors encoded polarity:
  - Red rods: positive numbers (yang)
  - Black rods: negative numbers (yin)
- Arithmetic performed through motion:
  - Rods were placed, removed, and shifted — choreography over notation.
  - Examples:

$$\sqrt{100a} = 10\sqrt{a}, \quad \sqrt[3]{1000a^3} = 10\sqrt[3]{a^3}$$

- Fractions used familial metaphors:
  - Numerator: “son”
  - Denominator: “mother”
- System evolved into the bead-based abacus.

**Meta-Realization**

This was not arithmetic on paper — it was arithmetic in motion.
Mathematics became choreography: hands moving with memory, rods aligned with place, polarity marked by color.
*The board remembered. The hands enacted. The math endured.*

## Problem 4 — Can You Summarize Infinity With a Pattern?

### Concept Preview

Some sequences grow too fast to count — yet the Chinese found a way to contain them.

Not with calculus. Not with variables. But with symmetry, structure, and a mind that saw numbers as shapes.

Zhu Shijie built no machine — he built a mirror.

**Topic: Series in the Jade Mirror**

A few of the many summations of series found in the *Jade Mirror* are the following:

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = n(n+1)\frac{(2n+1)}{3!}$$

$$1 + 8 + 30 + 80 + \cdots + n^2(n+1)\frac{(n+2)}{3!}$$
$$= n(n+1)(n+2)(n+3)\times\frac{(4n+1)}{5!}.$$

No proofs are given, however, nor does the topic seem to have been continued again in China until about the nineteenth century. Zhu Shijie handled his summations through the method of finite differences, some elements of which seem to date in China from the seventh century, but shortly after his work, the method disappeared for many centuries.

Figure 5: Summation formulas from Zhu Shijie's Jade Mirror

- Summation of squares:
$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$
- Higher-order sequence:
$$1 + 8 + 30 + 80 + \cdots = \frac{n(n+1)(n+2)(n+3)(4n+1)}{120}$$
- Example: $n = 3 \Rightarrow 1^2 + 2^2 + 3^2 = 14 = \frac{3(4)(7)}{6}$
- Patterns were visually encoded into triangle diagrams.

**Meta-Realization**

These were not formulas — they were ritual compressions.
Patterns were not decorative. They were the grammar of reality — the syntax by which number bent toward meaning.
*A triangle of rods could speak like calculus — but without needing its name.*

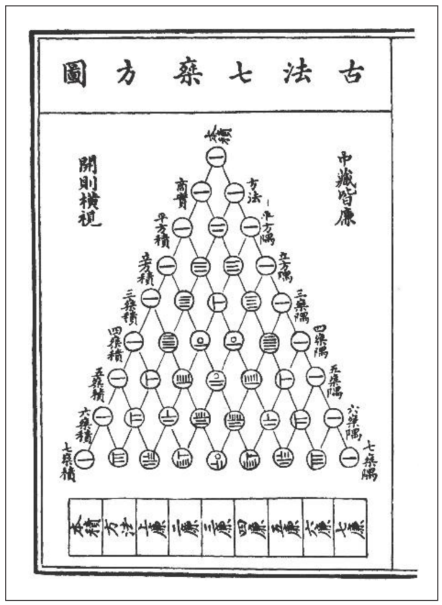
## Problem 5 — Can a Triangle Hold the Cosmos?

### Concept Preview

After Liu Hui's circles, Zhu Shijie's coefficients. But what if a shape could hold both structure and soul?

The Chinese arithmetic triangle wasn't merely a way to compute powers — it was a lattice of memory, a recursive key.

**Topic: The Arithmetic Triangle in Binomial Expansion**




The "Pascal" triangle, as depicted in 1303 at the front of Zhu Shijie's *Jade Mirror*. It is titled "The Old Method Chart of the Seven Multiplying Squares" and tabulates the binomial coefficients up to the eighth power. (Reproduced from J. Needham 1959, Vol. 3, p. 135.)

Figure 6: Zhu Shijie's arithmetic triangle with binomial coefficients

- Triangle encodes coefficients of $(a+b)^n$:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

- Constructed via recursive summation:
  - Each entry: sum of the two directly above
- Used for:
  - Root extraction
  - Solving equations up to 14th degree
  - Structuring series expansions
- More than computational — a visual cosmogram, a structure of recursive logic.

**Meta-Realization**

This was not Pascal's Triangle. It was China's triangle — recursive, recursive, recursive.
A visual invocation. A map of growth. A harmonic between algebra and cosmology.
*It was not combinatorics. It was continuity — carved into pattern.*

# III. Closing Dialectic

## Summary

These thinkers did not merely compute — they constructed *ritual engines* of number, form, and survival. Chinese mathematics was not confined to abstraction. It was **embodied** — enacted in rods, encoded in patterns, visualized in triangles.

- Liu Hui did not approximate $\pi$ through calculus — he refined it through recursion, triangle by triangle.
- The Nine Chapters solved systems not with symbols, but through rod alignments — a matrix of intention.
- Zhu Shijie's triangle was not notation — it was structure, cosmology, and recursion incarnate.

They built mathematics that could survive floods, serve emperors, predict eclipses, and preserve harmony — not in silence, but in form.

This was a tradition where:

- **The circle** mirrored heaven,
- **The square** embodied earth,
- And **the triangle** became the memory between them.

Chinese mathematics laid groundwork for:

- Korea and Japan's numerical pedagogy
- Islamic algebraic refinement
- European re-discovery of symbolic logic
- And modern recursion theory encoded in ancient geometry

But this chapter does not end in convergence — it ends in transmission.
The rod does not fall. The triangle does not close. The mirror does not crack.
It is passed — held in continuity, not conclusion.

## Exit Prompt

***You are Zhu Shijie.*** Not a theorist. A transmitter. No calculators. No variables. Just rods, memory, and the shape of recursion.

*What will you encode — and how will it endure?*