

# Chapter 10 – Ancient and Medieval India

## The Sutra and the Series

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### I. Cultural Invocation

- **Civilization:** Ancient and Medieval India
- **Time Period:** c. 800 BCE – 1600 CE
- **Figures:** Baudhāyana (Sulbasutra geometry), Aryabhata (sine tables and rotation), Brahmagupta (algebra and quadrilaterals), Bhaskara II (infinitesimals and proofs), Madhava (infinite series and trigonometry)

*In India, mathematics was not merely calculation — it was revelation. Numbers appeared in hymns, proofs were sung, and the infinite was summoned through rhythm.*

*Geometry emerged not in the abstract — but in ritual fire altars, bound by rope and rule. Zero was not a placeholder — it was a metaphysical insight. And infinity was not feared — it was named, shaped, and passed forward through verse.*

*Where the West saw mathematics as deduction, India saw it as **resonance** — a harmony of cosmos, computation, and consciousness.*

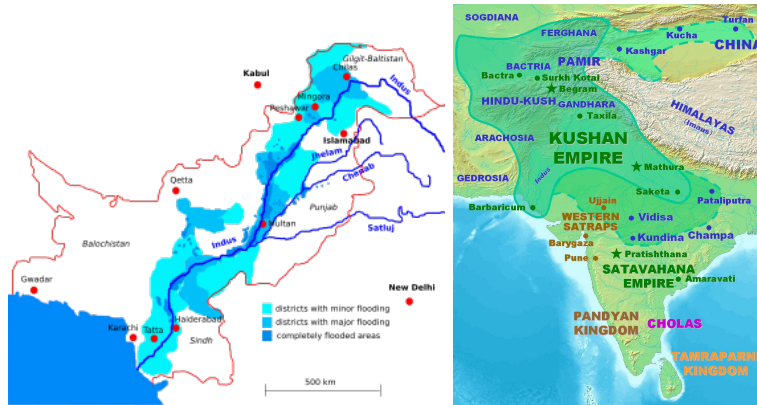


Figure 1: Indus River and Kushan Empire (c. 30 AD - c. 375 AD)

## II. Problems and Solutions

### Problem 1 — Can You Build with Ropes and Reason?

#### Concept Preview

When you build an altar for the gods, the angles must be right — or the fire will falter.

Before proofs, before theorems, India's Sulbasutras encoded geometry in ropes, knots, and ritual.

**Topic: Geometric Construction in the Sulbasutras**

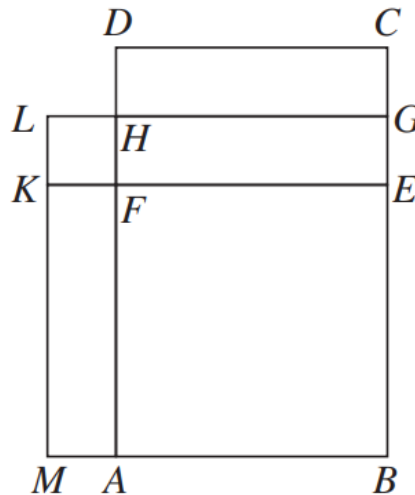


FIG. 10.1

Figure 2: Geometric construction of a square from a rectangle in Apastamba's Sulbasutra

- Rope-stretchers used ratios like 3-4-5 to construct right angles.
- Apastamba's method mirrors elements of Euclid II:
  - Extend shorter sides to match square dimensions.
  - Use midpoints and segment bisectors to construct the square.

#### Meta-Realization

This was not deduction — it was construction.

The altar was not symbolic of geometry.

*It was geometry.*

## Problem 2 — Can a Lattice Compute Faster Than You Can Carry?

### Concept Preview

Before calculators, before carrying digits — could a window of triangles make multiplication mechanical?

**Topic: Gelosia Multiplication (Lattice Method)**

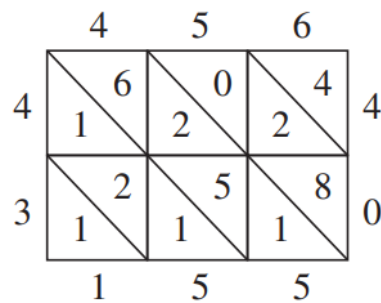


FIG. 10.2

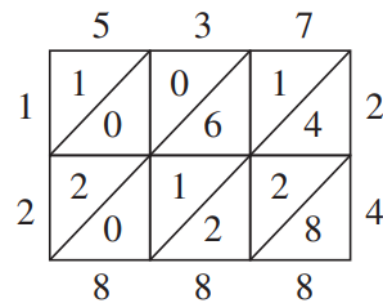


FIG. 10.3

Figure 3: Gelosia (lattice) multiplication grids from Indian arithmetic

- Diagonal grids segment partial products.
- Each cell handles multiplication of a digit pair.
- Carrying is replaced by visual summation across diagonals.

### Meta-Realization

The lattice is not just a method — it is a map of multiplication.

Digit meets digit not in chaos, but in a window of clarity.

*The grid remembers what your hands cannot.*

### Problem 3 — Can You Divide With a Boat?

#### Concept Preview

Long division feels tedious. But once, it was theatrical — shaped like a galley ship, carved into the page.

**Topic: The Galley Method of Division**

$$\begin{array}{r}
 117 \\
 382 \overline{)44977} \\
 \underline{382} \phantom{00} \\
 677 \phantom{00} \\
 \underline{382} \phantom{00} \\
 2957 \phantom{00} \\
 \underline{2674} \phantom{00} \\
 283
 \end{array}$$

FIG. 10.4

$$\begin{array}{cccccc}
 & & & & 2 & \\
 & & & & \cancel{2} & \cancel{3} \\
 & & & & \cancel{3} & \cancel{8} & 8 \\
 382 & \left| \begin{array}{ccccc} \cancel{4} & \cancel{4} & \cancel{9} & \cancel{7} & 3 \\ \cancel{4} & \cancel{4} & \cancel{8} & \cancel{7} & \cancel{7} \\ \cancel{3} & \cancel{8} & \cancel{2} & \cancel{2} & \cancel{4} \end{array} \right| & 117 & \\
 & & & & \cancel{3} & \cancel{8} & \cancel{7} \\
 & & & & \cancel{2} & \cancel{6} &
 \end{array}$$

FIG. 10.5

Figure 4: Figures 10.4 and 10.5: Modern vs. Galley division methods

- Subtractions appear above and below the central dividend.
- Galley shape mirrors the logic of recursive subtraction.
- Visual placement matters: differences above, subtrahends below.

#### Meta-Realization

Even division was once an art form — symmetrical, recursive, and spatial.

*The quotient sails from one reduction to the next — carried not by rules, but by rhythm.*

## Problem 4 — Can You Square What Cannot Be Seen?

### Concept Preview

Western mathematicians feared the irrational. Indian thinkers named it, shaped it, and used it — without apology.

### Topic: Brahmagupta's Algebra and Area Formulas

- Cyclic quadrilateral area:

$$K = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

- General quadrilateral:

$$K = \sqrt{(s-a)(s-b)(s-c)(s-d) - abcd \cos^2 \alpha}$$

- Embraced irrational roots, negative solutions, and even division involving zero (though inconsistently).

### Meta-Realization

Where others hesitated, Brahmagupta advanced.

He did not debate whether irrationals existed — he built with them.

*Not certainty — but courage — is the foundation of algebra.*

## Problem 5 — Can the Infinite Be Solved With a Name?

### Concept Preview

Europe speaks of Newton. Of Leibniz. Of calculus.

But centuries before, Indian mathematicians were already taming the infinite — not with limits, but with *verse*.

They called it by name. And they made it speak.

### Topic: Madhava and the Kerala School of Mathematics

*No temples. No diagrams. Just palm-leaf manuscripts and recursion.*

- **Timeframe:** 14th–16th centuries CE, Kerala (southwestern India)
- Madhava developed:

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

- Also expanded:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots, \quad \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

- **Sanskrit Verse Encoding:**

- Formulas embedded in poetic meter
- Mathematical recursion preserved via rhythmic phrasing

- Approximated  $\pi$  to 11+ decimal places:

$$\pi \approx 3.14159265359$$

- Used arc tangent convergence corrections and recursive finite differences to refine sine table entries.

#### Mathematical Note: How India Defined sin

Indian mathematicians defined  $\sin(\theta)$  as the length of the *half-chord* of a circle.

$$\text{jya}(\theta) = R \cdot \sin(\theta)$$

- This was rooted in circular, not triangular thinking.
  - Sine tables were computed using difference methods, not geometric construction.
  - Aryabhata compiled values for  $\theta$  from  $0^\circ$  to  $90^\circ$  in 24 equal steps.
  - This approach laid the groundwork for Madhava's infinite series centuries later.
- They did not define sine by sides — they summoned it from the circle itself.*

**Sidebar: Madhava vs. Archimedes on  $\pi$** 

- **Archimedes** (c. 250 BCE):  
Approximated  $\pi$  between  $\frac{223}{71}$  and  $\frac{22}{7}$ .  
Used a geometric method with 96-gon perimeter estimates.  
Accuracy: *2 decimal places*.
- **Madhava** (c. 1400 CE):  
Used infinite arctangent series with corrections.  
Approximated  $\pi$  to 11+ digits:  $\pi \approx 3.14159265359$ .  
Embedded in Sanskrit verse, not symbolic algebra.
- **Key Insight:**  
Archimedes *encircled*  $\pi$ . Madhava *summoned* it — digit by digit, from the infinite.

**Meta-Realization**

They did not define limits.

They did not name convergence.

They sang the infinite — and listened as it answered back in decimals.

*This was not approximation. This was invocation.*

*Infinity arrived — not by proof, but by verse.*

### III. Closing Dialectic

#### Summary

In India, mathematics was not abstracted — it was *evoked*. It emerged from fire altars, from lunar cycles, from poetry and planetary motion.

- The Sulbasutra rope-stretchers did not draw lines — they consecrated space.
- The lattice method of multiplication was not a trick — it was a ritual window.
- Brahmagupta did not fear irrationality — he gave it form, gave it rules, and let it live.
- The Kerala School did not define convergence — they sang it, digit by digit.

Indian mathematics threaded together:

- **Ritual and recursion**
- **Verse and verification**
- **The physical and the philosophical**

While the West sought certainty in proof, India sought continuity in pattern.

Not to dominate number — but to harmonize with it.

This was not just a body of results.

It was a lineage. A resonance. A memory handed down in palm leaves and planetary arcs.

#### Comparative Mathematical Cosmologies

**Greek:** Number as perfection. Geometry as proof. The irrational as disruption.

**Chinese:** Number as ritual. Computation as continuity. Memory encoded in form.

**Indian:** Number as vibration. Mathematics as reverberation, as resonance. The infinite invoked, not feared.

Each civilization did not merely compute — it expressed its metaphysics in method.

#### Exit Prompt

*You are Bhaskara. Or Madhava. Or the anonymous voice behind a Sulbasutra.*

No symbols. No algebra. Just recursion, ritual, and the desire to name the infinite.

*What will you encode — and how will it endure?*