Su2024MATH4991Lecture1Mk4

Origins of Mathematical Thought

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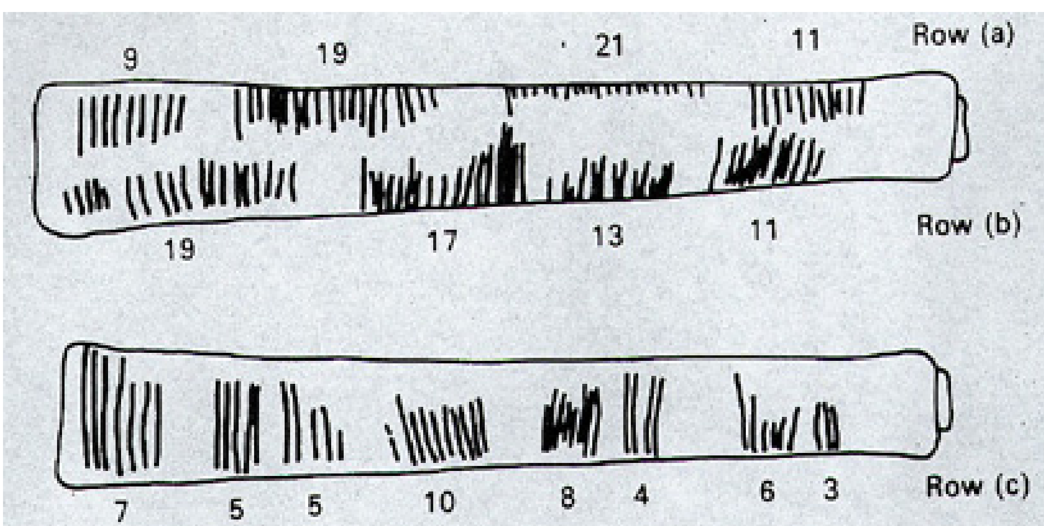
Chapter 1 – Traces

Approximate Range: *c. 300,000 BCE – c. 4000 BCE*

- Emergence of numerical intuition in hunter-gatherer societies.
- Earliest counting tools (e.g., Ishango Bone, ˜22,000 BCE).
- Development of pattern recognition and geometric reasoning in Neolithic art and craft.
- No writing systems yet — mathematics encoded through ritual, memory, and objects.



Lecture: How Early Humans Solved Problems Without Math

Guiding Premise

"Mathematics did not start with proofs. It started with problems."

This lecture presents Chapter 1: **Traces** not as a list of facts, but as a narrative of survival, intuition, and abstraction — solved problem by problem.

Problem 1: How many days have passed since the last full moon?

Situation: A hunter-gatherer group wants to track lunar phases for rituals or hunting.

Solution: They mark a bone or stick each day. Over time, they begin to group the notches — perhaps in fives or tens.

Artifact: The Ishango Bone shows notches arranged in what may be primes or lunar cycles.

Modern Insight:

This was the birth of arithmetic as *external memory*.
"They weren't doing math to be clever — they were trying not to starve."

Problem 2: How do I tell if one pile of berries is equal to another?

Situation:

Two foragers want to compare piles without counting.

Solution:

They use one-to-one matching: pair each berry from one pile with a berry from the other until one runs out.

Modern Insight:

This is proto-cardinality — the core of set theory.

"Before they knew what 'three' was, they knew what 'equal' meant."

Problem 3: How many people left camp?

Situation:

Ten warriors leave the village. How do we confirm they all return?

Solution:

Use ten pebbles or sticks — remove one for each departure, replace one for each return. Full pile = all returned.

Modern Insight:

This is the abstraction of identity — a symbolic representation system.

"Pebbles became numbers the moment they meant something."

Problem 4: How do I split this animal fairly?

Situation:

You kill a large boar and need to divide it among three families.

Solution:

You slice visually into equal parts, discovering symmetry and equivalence.

Modern Insight:

This becomes the foundation for division and fractional thinking.

Problem 5: How do I design a basket that doesn't collapse?

Situation:

Weavers learn that certain basket patterns are stronger.

Solution:

Use repeated geometric patterns (e.g., triangles, hexagons) to reinforce structure.

Modern Insight:

This is geometry through craft — using symmetry and congruence.

"They didn't draw triangles. They lived inside them."

Conclusion: The First Mathematicians Were Problem-Solvers

They didn't define number or shape. They used them, and abstraction followed.

Key Takeaways:

- Math is the afterthought of practical genius.
- Problems came first — solutions created patterns.
- Abstraction emerged through survival.

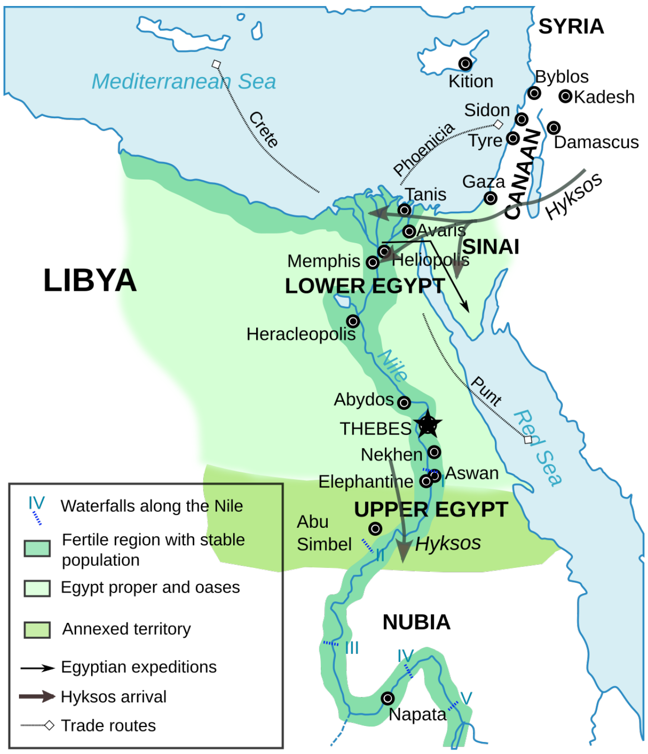
Exit Prompt:

"Can you invent a method to solve a survival problem using no words and no numbers — only objects and patterns?"

Chapter 2 – Ancient Egypt

Approximate Range: *c. 2700 BCE – c. 1600 BCE*

- **Old Kingdom:** *c. 2700–2200 BCE* — pyramid construction, land surveying.
- **Middle Kingdom:** *c. 2000–1650 BCE* — era of the Rhind and Moscow Papyri.
- Use of unit fractions, additive decimal system, false position method.
- Practical geometry developed for agriculture, taxation, and architecture.



Lecture: How the Nile Flooded Egypt into Inventing Math

Problem 1: The Nile erased all our farm boundaries. What now?

Situation: After the flood, a farmer claims 12 square rods of land. A scribe must verify this using geometric calculation.

Given: Base = 6 rods, Height = 4 rods

Egyptian Formula for Triangle Area:

$$\text{Area} = \frac{1}{2} \cdot \text{Base} \cdot \text{Height} = \frac{1}{2} \cdot 6 \cdot 4 = 12$$

Alternative (Doubling Table Method):

$$\begin{array}{r|r} 1 & 4 \\ 2 & 8 \\ 4 & 16 \end{array} \Rightarrow 6 = 2 + 4 \Rightarrow 8 + 16 = 24 \Rightarrow \frac{24}{2} = 12$$

Problem 2: How do I write and calculate with large numbers?

Situation: A scribe must compute 17×23 using Egyptian multiplication by doubling.

Doubling Table:

$$\begin{array}{r|r} 1 & 23 \\ 2 & 46 \\ 4 & 92 \\ 8 & 184 \\ 16 & 368 \end{array} \Rightarrow 17 = 16 + 1 \Rightarrow 368 + 23 = 391$$

Result:

$$17 \times 23 = 391$$

Problem 4: How do I find the slope of a pyramid face?

Situation: An architect needs the pyramid face to rise 7 cubits vertically for every 70 hands (10 cubits) of horizontal run.

Definition of Seqt:

$$\text{seqt} = \frac{\text{run in hands}}{\text{rise in cubits}} = \frac{70}{7} = 10$$

Interpretation:

A seqt of 10 means the pyramid face slopes inward 10 hands for every cubit of height.

Student Example:

If a pyramid is 50 cubits tall with a seqt of 7:

$$50 \times 7 = 350 \text{ hands} = 50 \text{ cubits} \Rightarrow \text{The face runs in as far as it rises — a } 45^\circ \text{ slope.}$$

Problem 5: A tax collector takes $\frac{1}{7}$ of a pile. What was the original heap?

Problem:

A heap and its seventh equals 19. Find the original heap using false position.

Step 1: Guess $x = 7$:

$$x + \frac{1}{7}x = 7 + 1 = 8$$

Step 2: Scale the guess:

$$\frac{19}{8} = 2.375 \Rightarrow x = 7 \cdot 2.375 = 16.625 = 16\frac{5}{8}$$

Check:

$$16.625 + \frac{1}{7}(16.625) = 16.625 + 2.375 = 19 \quad \checkmark$$

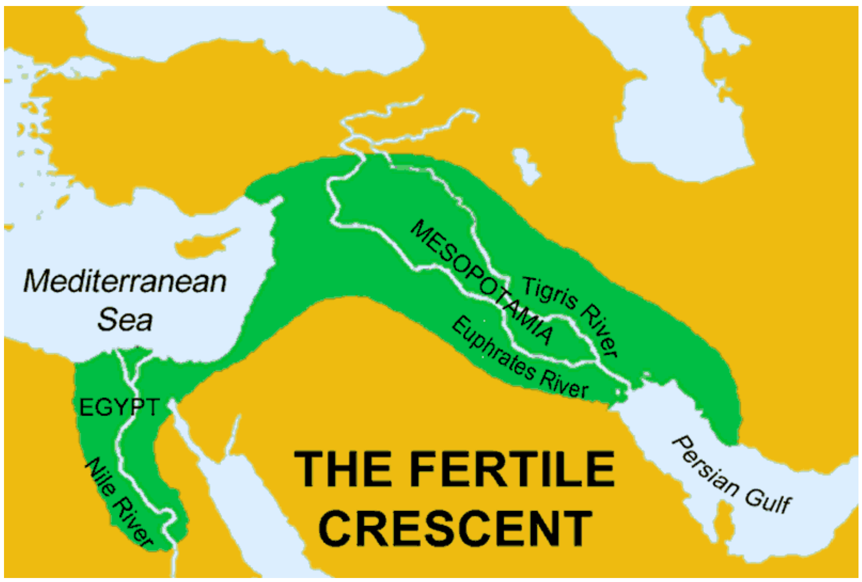
Result:

The original heap was $16\frac{5}{8}$.

Chapter 3 – Mesopotamia

Approximate Range: *c. 3000 BCE – c. 1600 BCE*

- **Sumerians:** *c. 3000–2300 BCE* — tokens, early number systems.
- **Akkadian Empire:** *c. 2300–2100 BCE* — administrative expansion.
- **Old Babylonian Period:** *c. 2000–1600 BCE* — peak mathematical activity.
- Base-60 positional notation, reciprocal tables, root approximations, quadratic equations, and geometric computations.



Lecture: Mud, Math, and Markets — How the Babylonians Computed the World

Guiding Premise

"Babylonian mathematics was born not from curiosity — but from contracts, clay, and cosmic patterns."

This lecture explores the development of math in Mesopotamia as a response to practical challenges in trade, measurement, surveying, and astronomy.

Problem 1: I need to split 34 minas of silver among 5 people.

Situation:

A merchant must divide 34 minas among 5 partners. There's no long division.

Solution:

Use reciprocal tables:

$$34 \div 5 = 34 \times \frac{1}{5} = 6.8 = 6;48 \text{ in sexagesimal}$$

Modern Insight: Babylonians preferred multiplication by reciprocals — a shift from division to algorithmic computation.

"They didn't divide. They reversed the problem."

Problem 2: How do I write 1 hour, 30 minutes?

Situation:

Astronomer-priests record time and angle fractions for planetary movement.

Solution:

Use base-60 positional numbers:

$$1;30 = 1 + \frac{30}{60} = 1.5$$

Modern Insight:

Their number system persists in clocks and circles.

"They chose 60 not for beauty — but because it plays nicely with division."

Problem 3: The king wants a right-triangle ramp. What's the diagonal?

Situation:

Engineers designing ziggurat ramps need consistent slope and right-angle reliability.

Solution:

Use Pythagorean triples (e.g., $3^2+4^2 = 5^2$), precomputed in tablets like Plimpton 322.

Modern Insight:

They used Pythagorean relationships without formal proofs — geometry by example.

"They didn't draw right triangles — they walked them."

Problem 4: A land grant must be divided under complex constraints.

Situation:

A legal condition: $(x + \frac{x}{7}) + \frac{1}{11}(x + \frac{x}{7}) = 1$ mina.

Solution:

$$y = x + \frac{x}{7} = \frac{8}{7}x, \quad \frac{12}{11}y = 1 \Rightarrow y = \frac{11}{12} \Rightarrow x = \frac{77}{96}$$

Sexagesimal:

$x = 0; 48, 7, 30$

Modern Insight:

This is algebra through verbal logic and scaling — pre-symbolic but structurally sound.

"Before they had x's and y's, they had portions and heaps — and solved just as well."

Problem 5: How do I find the diagonal of a square field?

Situation:

A land surveyor needs the diagonal to stretch rope across a square plot with side length 30 units.

Solution:

Use the Babylonian approximation:

$$\sqrt{2} \approx 1; 24, 51, 10 \approx 1.41421296$$
$$\text{Diagonal} = 30 \times \sqrt{2} \approx 42.42639 \Rightarrow \text{in sexagesimal: } 42; 25, 35, 49...$$

Modern Insight:

This is precise irrational approximation via algorithmic tables — supporting architecture and agriculture.

"They built right-angled space with the square root of 2 — no need to name it."

Conclusion: Babylonian Math Was Algorithmic Precision in Clay

Key Takeaways:

- Babylonian math was table-driven, abstract, and deeply practical.
- Base-60 enabled clean divisions and wide computational flexibility.
- They pioneered algorithmic thinking centuries before symbolic notation.

Exit Prompt:

"A rectangular warehouse must be built on a square plot. You know the side length. You have a table of square roots. What do you do next?"

Gives and Takes

Mathematics did not emerge fully formed. It grew over time — through abstraction, formalization, and cultural need. Each stage of mathematical history reflects a conscious or unconscious

sacrifice of the immediate, the concrete, or the ritual, in exchange for something more abstract, general, and powerful.

1. Prehistoric Humans

Gave up:

- Purely instinctual, context-bound perception of quantity.

Gained:

- The concept of number through contrast and sameness.
- Abstract thinking rooted in physicality (e.g., fingers, notches).
- Mathematics as a tool of memory, ritual, and pattern recognition.

Quote:

"They gave up immediacy, and gained the first glimmers of abstraction."

2. Egyptians

Gave up:

- Loose, oral, and purely concrete methods of counting and measuring.
- Dependence on myth and ritual to structure number relationships.

Gained:

- Formalized arithmetic and geometry, recorded in writing.
- Unit fractions, area and volume formulas, and method of false position.
- A mathematical culture rooted in administration and practicality.

Quote:

"They gave up the mythic, and gained the measurable."

3. Mesopotamians

Gave up:

- Rigid, symbol-heavy, additive systems.
- Sole focus on concrete or empirical problems.

Gained:

- A base-60 positional number system with powerful fractional capabilities.
- Algorithmic approaches to roots, equations, and interpolation.
- Generalized, table-driven, and abstract computational strategies.

Quote:

"They gave up repetition, and gained recursion. They gave up tallying, and gained theory."

4. In Sum

- Prehistoric peoples **named quantity**.
- Egyptians **measured the world**.
- Mesopotamians **calculated the abstract**.

Final Reflection:

Each civilization gave up something tangible to gain something powerful. As mathematics grew, it became less visible but more universal — evolving from physical objects to mental systems, from stone to symbol.