① let no. of signals amplified/sec be the states and x_n be the value of no. of signals amplified/sec after n sec.

Assuming poissons distribution and thus

$P_{\lambda x} = P(x|\lambda) = \frac{\lambda^x}{x!} \times e^{-\lambda}$

where λ is no of signals amplified in last sec. & x is no of signals amplified in next sec.

$P_{10} = \frac{1^0}{0!} \times e^{-1}$

$P_{11} = \frac{1^1}{1!} \times e^{-1}$ & similarly for others.

$P_{12} = \frac{1^2}{2!} e^{-1}$

⋮

$P_{1n} = \frac{1^n}{n!} \times e^{-1}$

$\sum_{\substack{i=0 \\ j\neq 0}}^{\infty} P_{ij} = 1$

$\therefore P_{0j} = 0$ since system stops at $x_n = 0$

Transition Matrix

$$A = \begin{array}{c|cccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & \cdots\ \infty \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 1 & e^{-1} & \frac{e^{-1}}{1!} & \frac{e^{-1}}{2!} & \frac{e^{-1}}{3!} & \frac{e^{-1}}{4!} & \frac{e^{-1}}{5!} & \frac{e^{-1}}{6!} & \cdots \\ 2 & \frac{2^0 \times e^{-2}}{0!} & \frac{2^1 \times e^{-2}}{1!} & \frac{2^2 \times e^{-2}}{2!} & \frac{2^3 \times e^{-2}}{3!} & \frac{2^4 \times e^{-2}}{4!} & \frac{2^5 \times e^{-2}}{5!} & \frac{2^6 \times e^{-2}}{6!} & \cdots \\ 3 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \\ 4 & & & & & & & & \\ 5 & & & & & & & & \\ 6 & & & & & & & & \\ \vdots & & & & & & & & \\ \infty & & & & & & & & \end{array}$$

So since the probability of ~~1st~~ current state only depends upon the prev. state, it is a markov model, with countably ∞ states for the problem.

2) let $P_{ij}(n)$ be the probability of going from state i to j in n seconds.

So by Chapman - Kolmogrov Theorem

$$P_{ij}(n) = \sum_{k} P_{ik}(r) \times P_{kj}(n-r)$$

g) $$P_{10}(2) = P_{10}(1) \times P_{00}(1) + P_{11}(1) \times P_{10}(1)$$

$$+ \sum_{i=2}^{\infty} P_{1i}(1) \times P_{i0}(1) = A^2_{10}$$

So $P_{ij}(n) = (A_{ij})^n$

A is the transition matrix.

For Bazoka 5000 to stop, $j=0$ & $i=1$ since it starts with state 1.

$$E(n) = \sum_{n=1}^{\infty} n \times P_{10}(n)$$

$$= \sum_{n=1}^{\infty} n A_{10}^{n}$$

$$E(n) = 1 \times (A)_{10} + 2 \times (A^2)_{10} + 3 (A^3)_{10} \cdots\cdots + n A_{10}^{n}$$

$$= e^{-1} + 2e^{-1}(e^{-1} + \delta(0)) + 3\left(\sum_{i=1}^{\infty} A^2_{1i} A_{i0}\right) \cdots\cdots$$

$$E(n) = e^{-1} + 2e^{-1}(e^{-1} + \delta(0)) + 3 \sum_{i=1}^{\infty} \frac{e^{-1}}{i!}(\delta(i) + e^{-1}) \times e^{-i}$$

$$+ \cdots n \sum_{i=1}^{\infty} A_{10}^{n-1} \times e^{-i}$$

⇒ Couldn't find a general term A_{10}^{n} calculations for A_{10}^{2}, A_{10}^{3} are shown.

$$A^2_{1\alpha} = \left(\sum_{i=2}^{\infty} A_{1i} \times A_{i\alpha}\right) + A_{11} A_{1\alpha} \cancel{+ A_{10} A_{0\alpha}}.$$

for $\alpha = 0$

$$A^2_{10} = \sum_{i=2}^{\infty} \frac{e^{-1}}{i!} \times \frac{i^{0} \times e^{-i}}{0!} + e^{-1} \times e^{-1} +$$

$$A^2_{10} = e^{-2} + e^{-1}\left(S(0)\right)$$

$$\boxed{S(x) = \sum_{k=2}^{\infty} \frac{k^{x} e^{-k}}{k!}}$$

for α

$$A^2_{1\alpha} = \sum_{i=2}^{\infty} \frac{e^{-1}}{i!} \times \frac{i^{\alpha} \times e^{-i}}{\alpha!} + e^{-1} \times \frac{e^{-1}}{\alpha!}$$

$$A^2_{1\alpha} = \frac{e^{-1}}{\alpha!} S(\alpha) + \frac{e^{-2}}{\alpha!}$$

$$A^2_{1\alpha} = \frac{e^{-1}}{\alpha!}\left(S(\alpha) + e^{-1}\right)$$

$$A^3_{10} = \sum_{i=1}^{\infty} A^2_{1i} A_{i0}$$

$$= \sum \frac{e^{-1}}{i!}\left(S(i) + e^{-1}\right) e^{-i}$$

4) Yes it also show martingale behaviour since the only change is it amplifies twice in one cycle.

$\therefore \quad E(X_{n+1} \mid X_n) = E(X_n)$

$\Rightarrow E(X_{n+m} \mid X_n) = E(X_n)$

$\Rightarrow E(\underbrace{X_{2n+2}}_{\text{next value after two amplifications}} \mid X_{2n}) = E(\underbrace{X_{2n}}_{\text{Value after n cycles.}})$