

Pyraformer

Low-Complexity Pyramidal Attention
for Long-Range Time Series
Modeling and Forecasting

23.5.25

Presented by Yyyq

- 时序预测任务的难点：长时依赖问题
- 现有方法的缺陷：复杂度较高

Method	Complexity per layer
CNN (Munir et al., 2018)	$\mathcal{O}(L)$
RNN (Salinas et al., 2020)	$\mathcal{O}(L)$
Full-Attention (Vaswani et al., 2017)	$\mathcal{O}(L^2)$
ETC (Ainslie et al., 2020)	$\mathcal{O}(GL)$
Longformer (Beltagy et al., 2020)	$\mathcal{O}(L)$
LogTrans (Li et al., 2019)	$\mathcal{O}(L \log L)$
Pyraformer	$\mathcal{O}(L)$



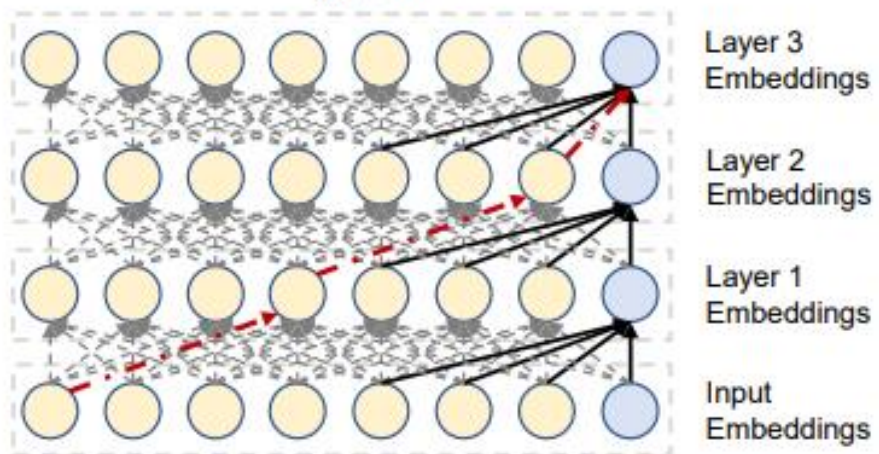
Connection example



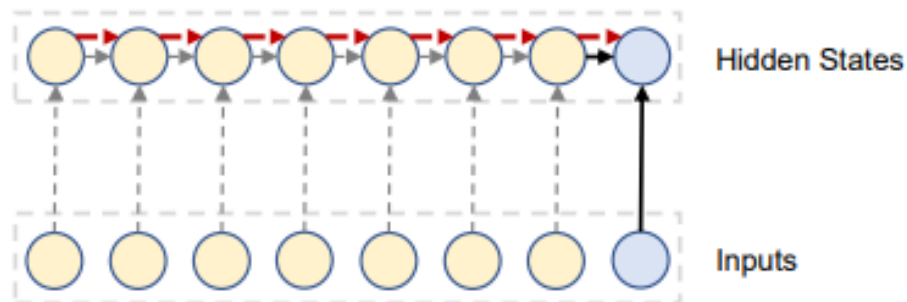
Maximum signal traversing path

最大信号传递路径

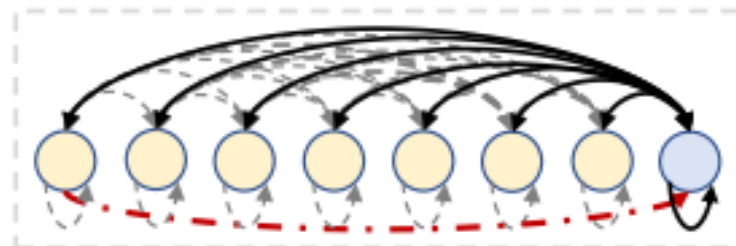
(b) CNN

 $O(L)$

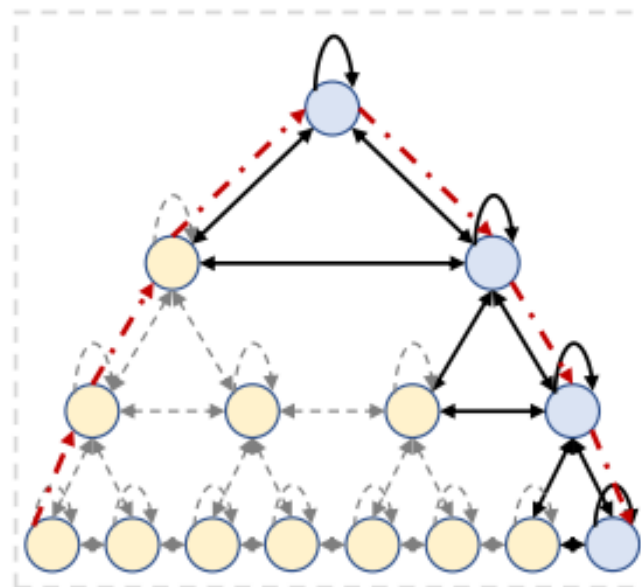
(c) RNN

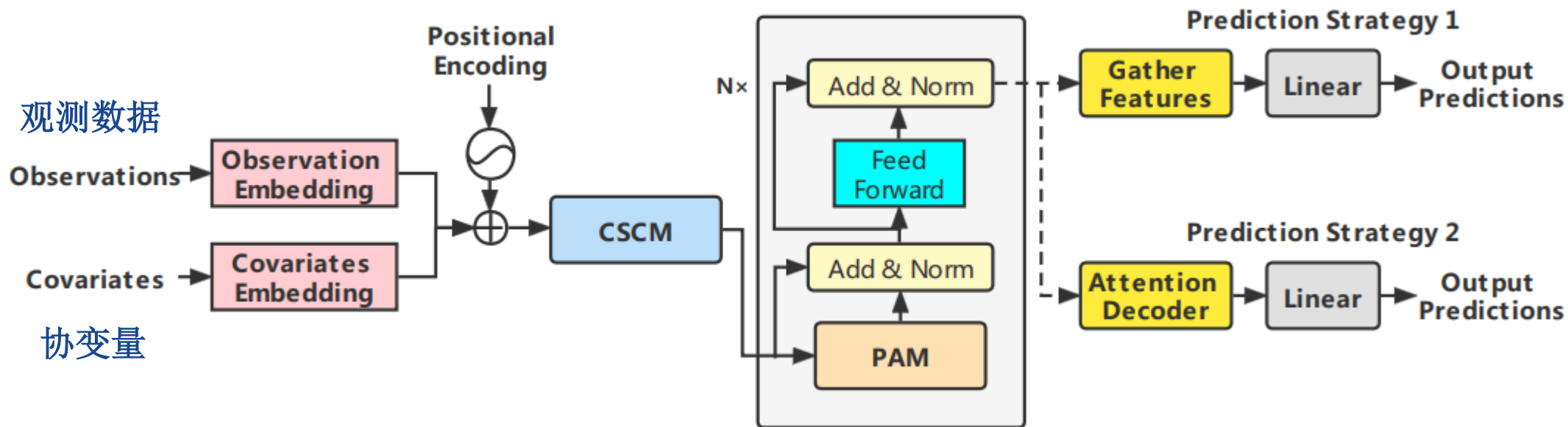
 $O(L)$

(a) Full Attention

 $O(1)$

(d) Pyraformer

 $O(1)$



03



算法描述: Pyramidal Attention Module (PAM)

尺度

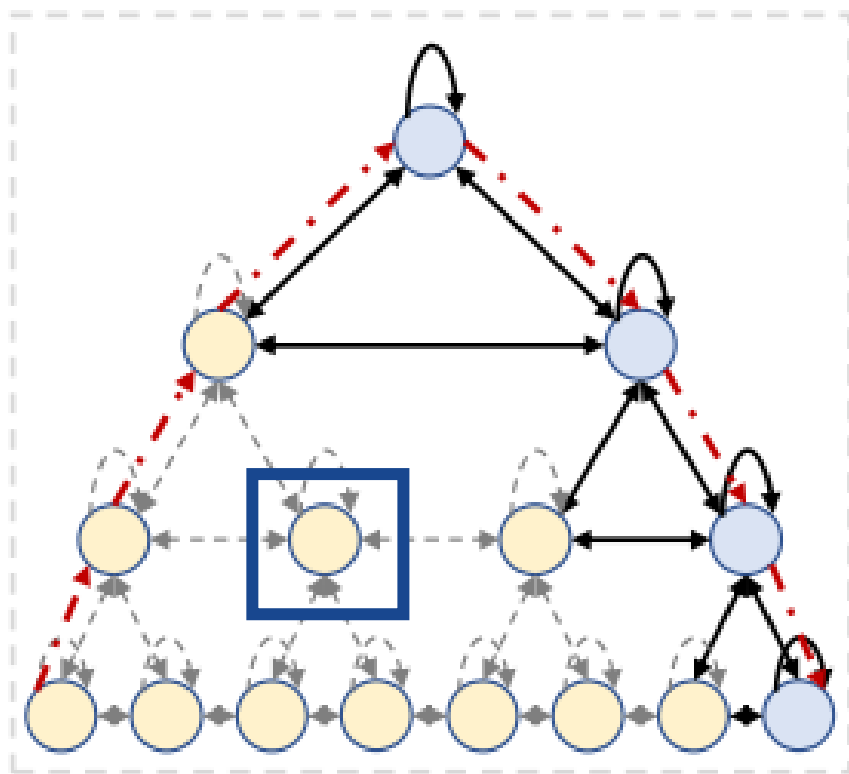
S

⋮

⋮

1

(d) Pyraformer



$$N_{\ell}^{(s)} \left\{ \begin{array}{l} A_{\ell}^{(s)} \text{ 表示同一级尺度上的相邻节点} \\ \quad \text{(包括自己, } A=3/5) \\ C_{\ell}^{(s)} \text{ 表示 } C \text{ 个子节点} \\ P_{\ell}^{(s)} \text{ 表示父节点} \end{array} \right.$$

一个节点最多可以连接 $A + C + 1$ 个节点

03



算法描述: Pyramidal Attention Module (PAM)

尺度

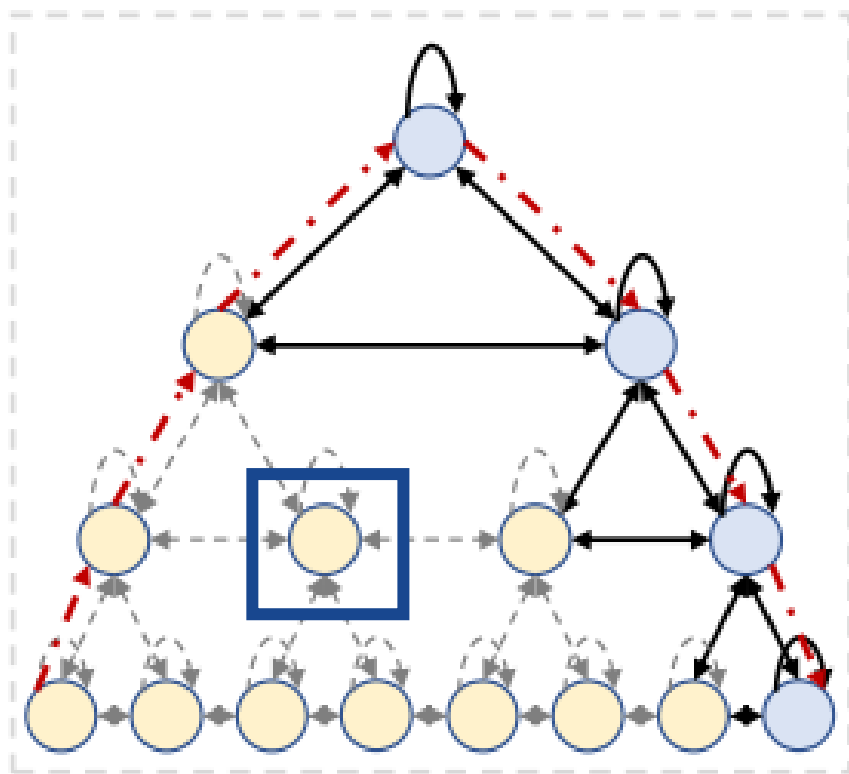
S

⋮

⋮

1

(d) Pyraformer



$$y_i = \sum_{\ell=1}^L \frac{\exp(\mathbf{q}_i \mathbf{k}_{\ell}^T / \sqrt{D_K}) \mathbf{v}_{\ell}}{\sum_{\ell=1}^L \exp(\mathbf{q}_i \mathbf{k}_{\ell}^T / \sqrt{D_K})},$$

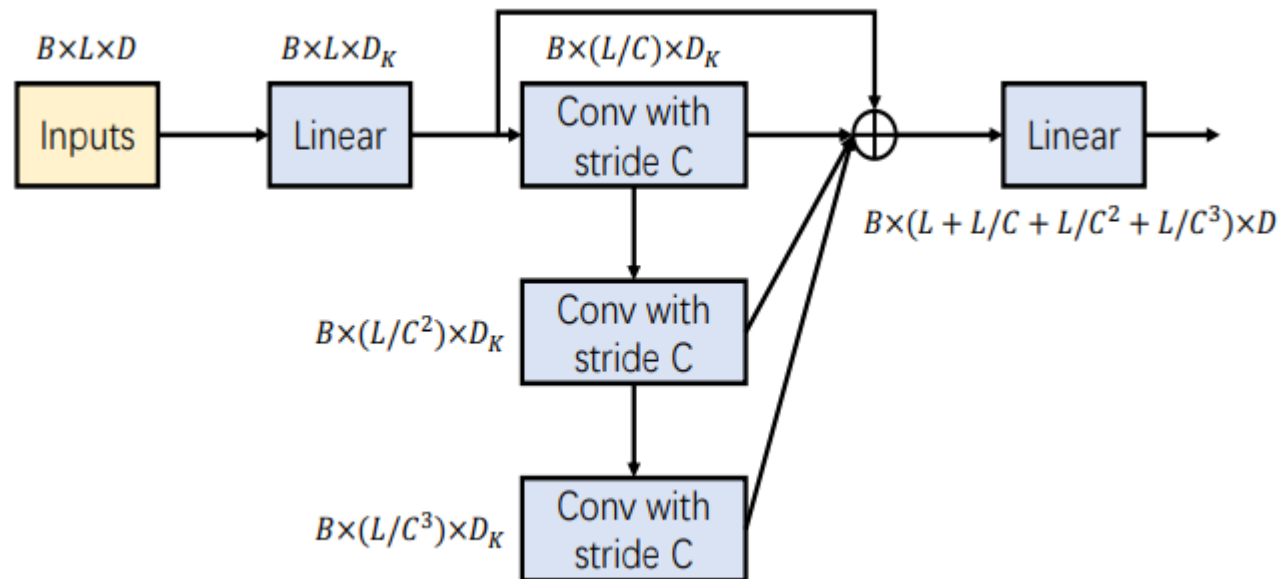


$$y_i = \sum_{\ell \in \mathbb{N}_{\ell}^{(s)}} \frac{\exp(\mathbf{q}_i \mathbf{k}_{\ell}^T / \sqrt{d_K}) \mathbf{v}_{\ell}}{\sum_{\ell \in \mathbb{N}_i^{(s)}} \exp(\mathbf{q}_i \mathbf{k}_{\ell}^T / \sqrt{d_K})},$$

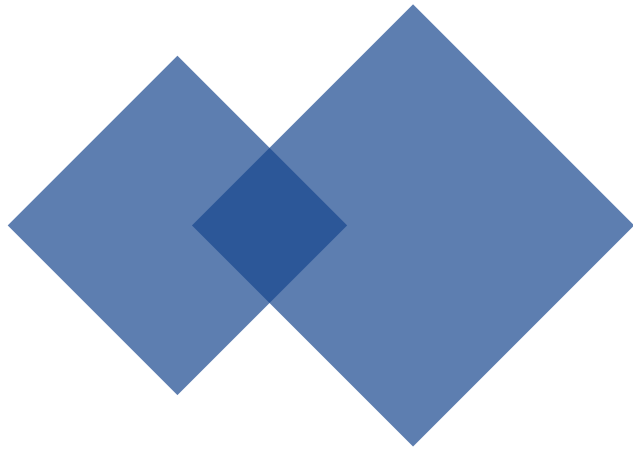
03



算法描述: Coarser-scale Construction Module (CSCM)

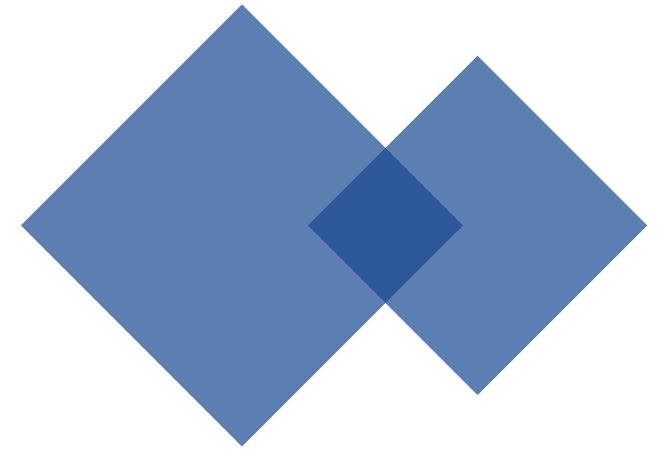


尺度为 S , 序列长度: $\frac{L}{C^S}$



ISTNet

Inception Spatial Temporal
Transformer for Traffic Prediction

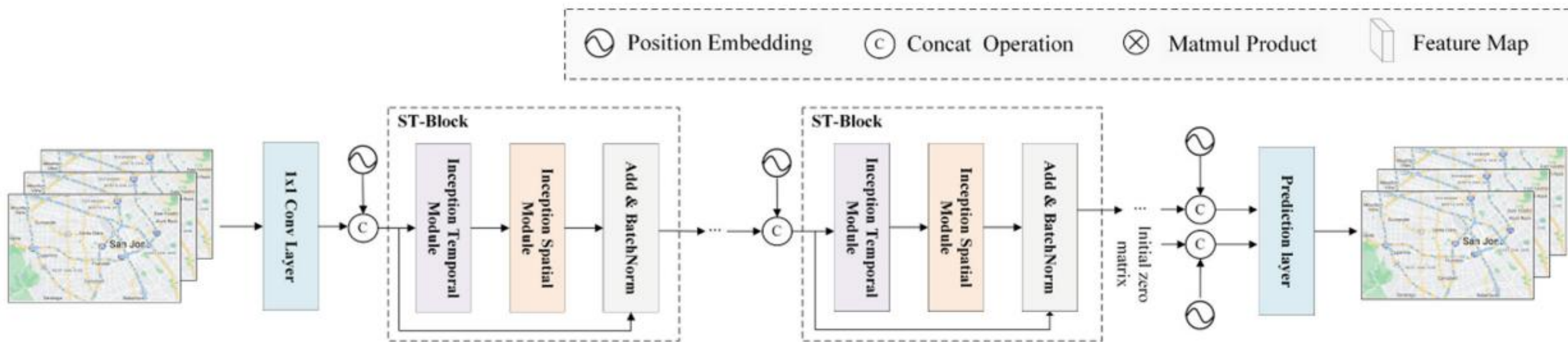


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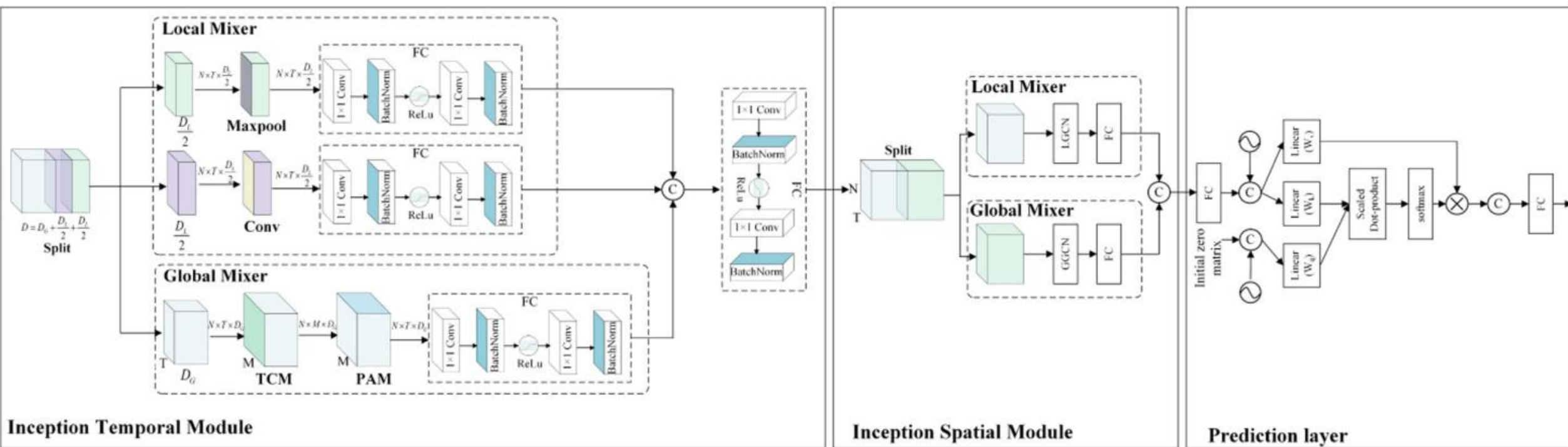
Presented by Yyyq



- 时间相关性
 - 局部相关性 CNN: 需要堆叠多层, 效率低
 - 全局依赖性 注意力机制: 偏好全局信息
- 空间相关性
 - 局部相关性
 - 潜在的全局相关性 (语义相关)



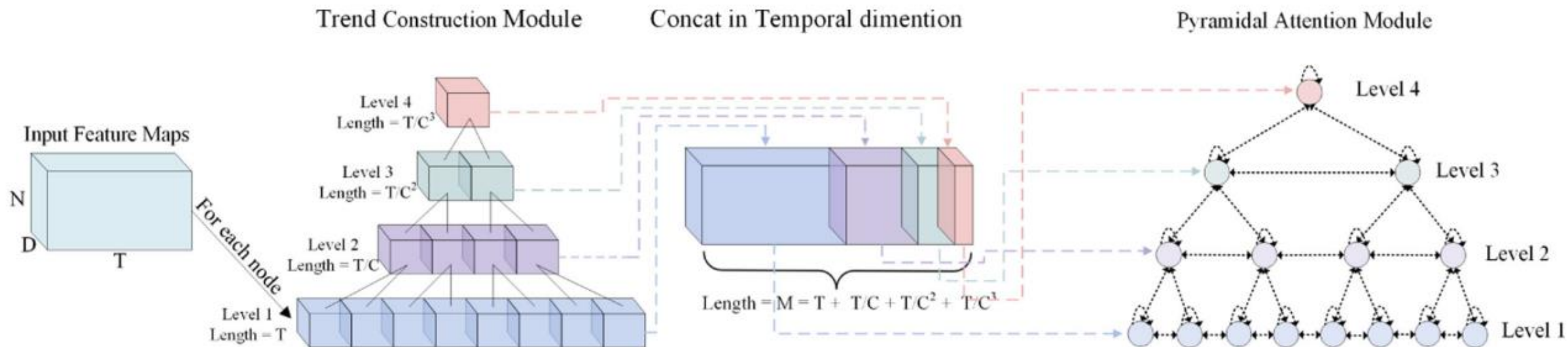
(a) Overall architecture of ISTNet



03



算法描述: Trend Construction Module (TCM) Pyramidal Attention Module (PAM)





➤ 局部混合器: 高斯核函数

$$\mathcal{A}_{i,j} = \exp\left(-\frac{\text{dist}(v_i, v_j)}{\mu^2}\right)$$
$$\text{dist}(v_i, v_j) \leq \varepsilon, \mathcal{A}_{i,j} = 0.$$

$$\mathcal{Y}_L^S = FC(\mathcal{A}\mathcal{X}^{local}W_1 + b_1)$$

➤ 全局混合器: 自适应邻接矩阵

$$\mathcal{M} = \text{softmax}(EE^T / \sqrt{D})$$

for $i = 1, 2, \dots, N$

$$nodeId = \text{argtopk}(\mathcal{M}[i, :])$$
$$\mathcal{M}[i, -nodeId] = 0$$

$$\mathcal{Y}_G^S = FC(\mathcal{M}\mathcal{X}^{global}W_2 + b_2)$$



➤ **ST-Block**堆叠的第一层到第 l 层，平衡局部和全局组件

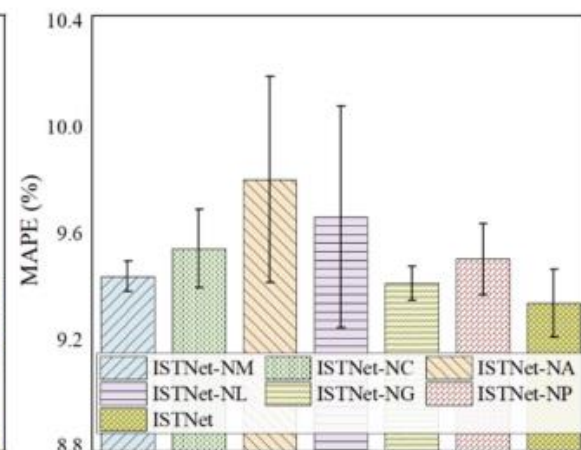
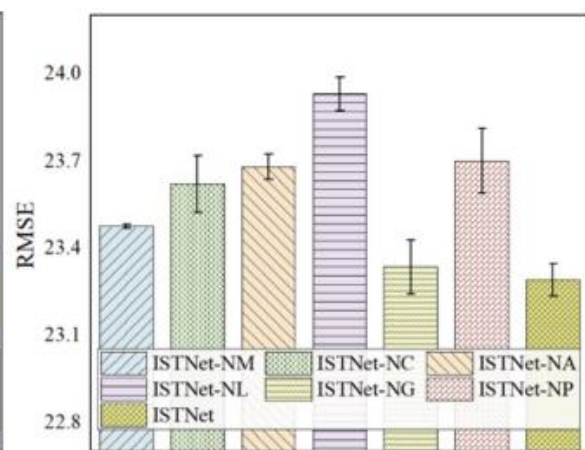
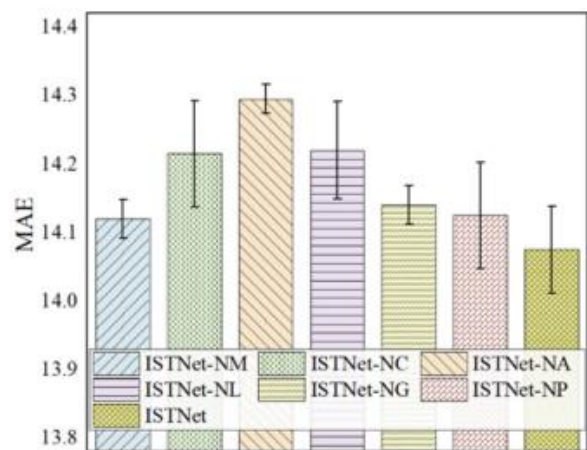
底层更喜欢局部信息，而顶层在捕获全局信息方面起着更重要的作用。

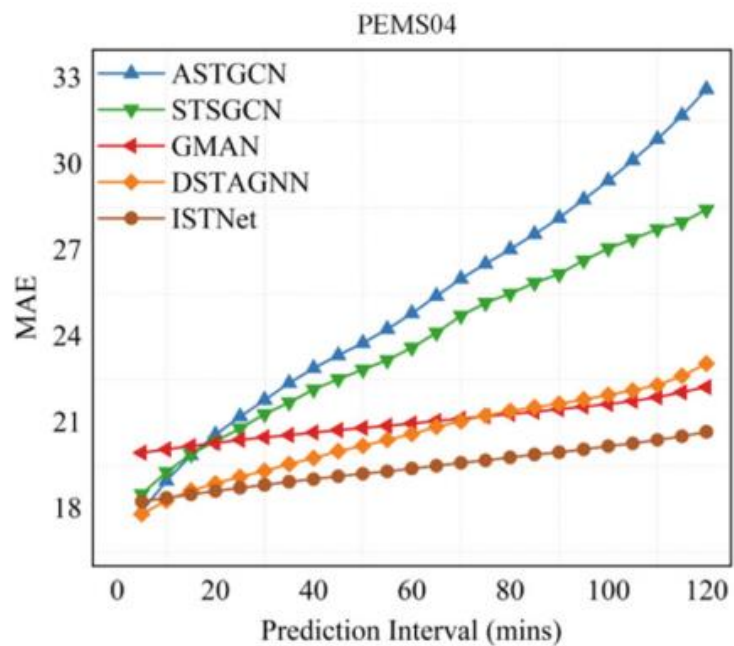
通道比率：局部 / 全局 = $[2, 1, 1/2]$



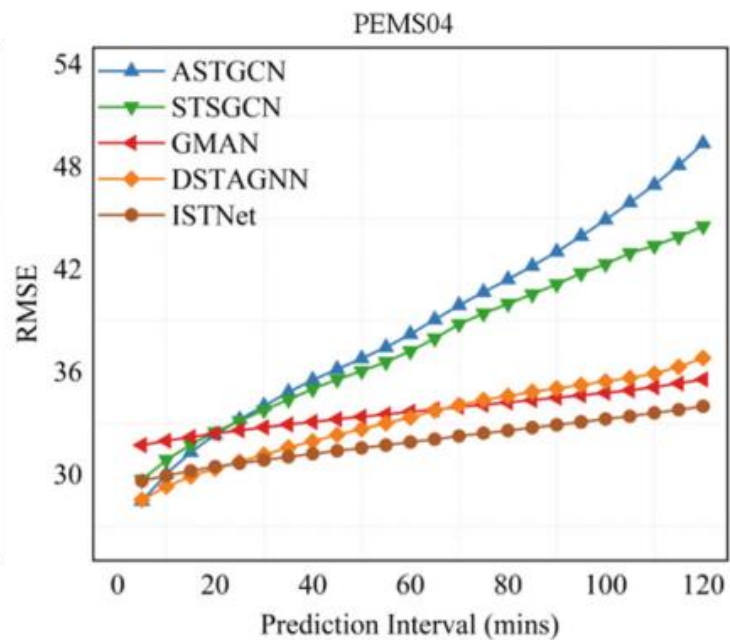
Datasets	Time Range	Time Steps	Time Interval	Nodes
PEMS03	09/01/2018 - 11/30/2018	26202	5-min	358
PEMS04	01/01/2018 - 02/28/2018	16992	5-min	307
PEMS07	05/01/2017 - 08/31/2017	28224	5-min	883
PEMS08	07/01/2016 - 08/31/2016	17856	5-min	170

Dataset	Metrics	VAR	SVR	FC-LSTM	DCRNN	STGCN	ASTGCN	Graph WaveNet	STSGCN	GMAN	DSTAGNN	ISTNet
PEMS03	MAE	19.72	19.77	19.56	17.62	19.76	18.67	15.67	17.51	15.52	15.57	15.03±0.09
	RMSE	32.38	32.78	33.38	29.86	33.87	30.71	26.42	29.05	26.53	27.21	24.89±0.25
	MAPE(%)	20.50	23.04	19.56	16.83	17.33	19.85	15.72	16.92	15.19	14.68	15.24±0.19
PEMS04	MAE	24.44	26.18	23.60	24.42	23.90	22.90	19.91	21.52	19.25	19.30	18.51±0.03
	RMSE	37.76	38.91	37.11	37.48	36.43	33.59	31.06	34.14	30.85	31.46	30.36±0.10
	MAPE(%)	17.27	22.84	16.17	16.86	13.67	16.75	13.62	14.50	13.00	12.70	12.36±0.16
PEMS07	MAE	27.96	28.45	34.05	24.45	26.22	28.13	20.83	23.99	20.68	21.42	19.67±0.13
	RMSE	41.31	42.67	55.70	37.61	39.18	43.67	33.62	39.32	33.56	34.51	32.96±0.04
	MAPE(%)	12.11	14.00	15.31	10.67	10.74	13.31	9.10	10.10	9.31	9.01	8.57±0.20
PEMS08	MAE	19.83	20.92	21.18	18.49	18.79	18.72	15.57	17.88	14.87	15.67	14.08±0.05
	RMSE	29.24	31.23	31.88	27.30	28.2	28.99	24.32	27.36	24.06	24.77	23.27±0.12
	MAPE(%)	13.08	14.24	13.72	11.69	10.55	12.53	10.32	11.71	9.77	9.94	9.34±0.09

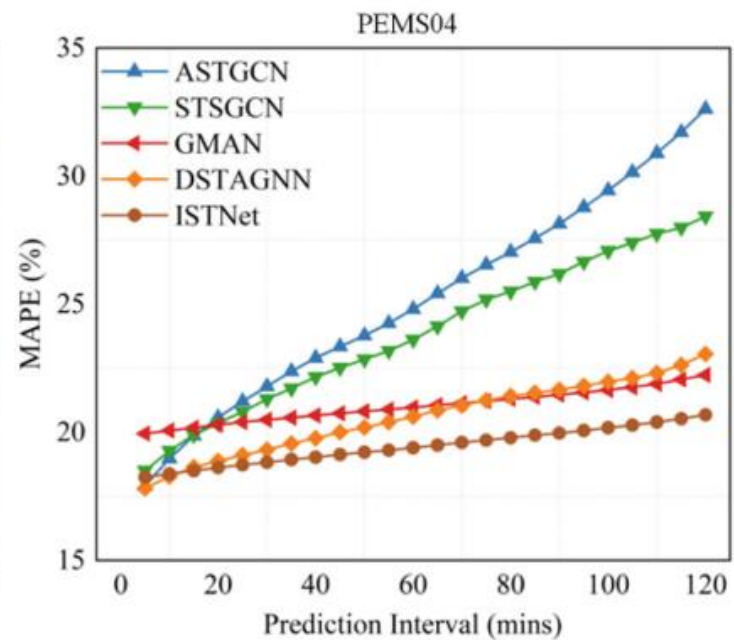




(a) MAE per prediction step



(b) RMSE per prediction step



(b) MAPE per prediction step



谢谢观看

MANY THANKS !

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