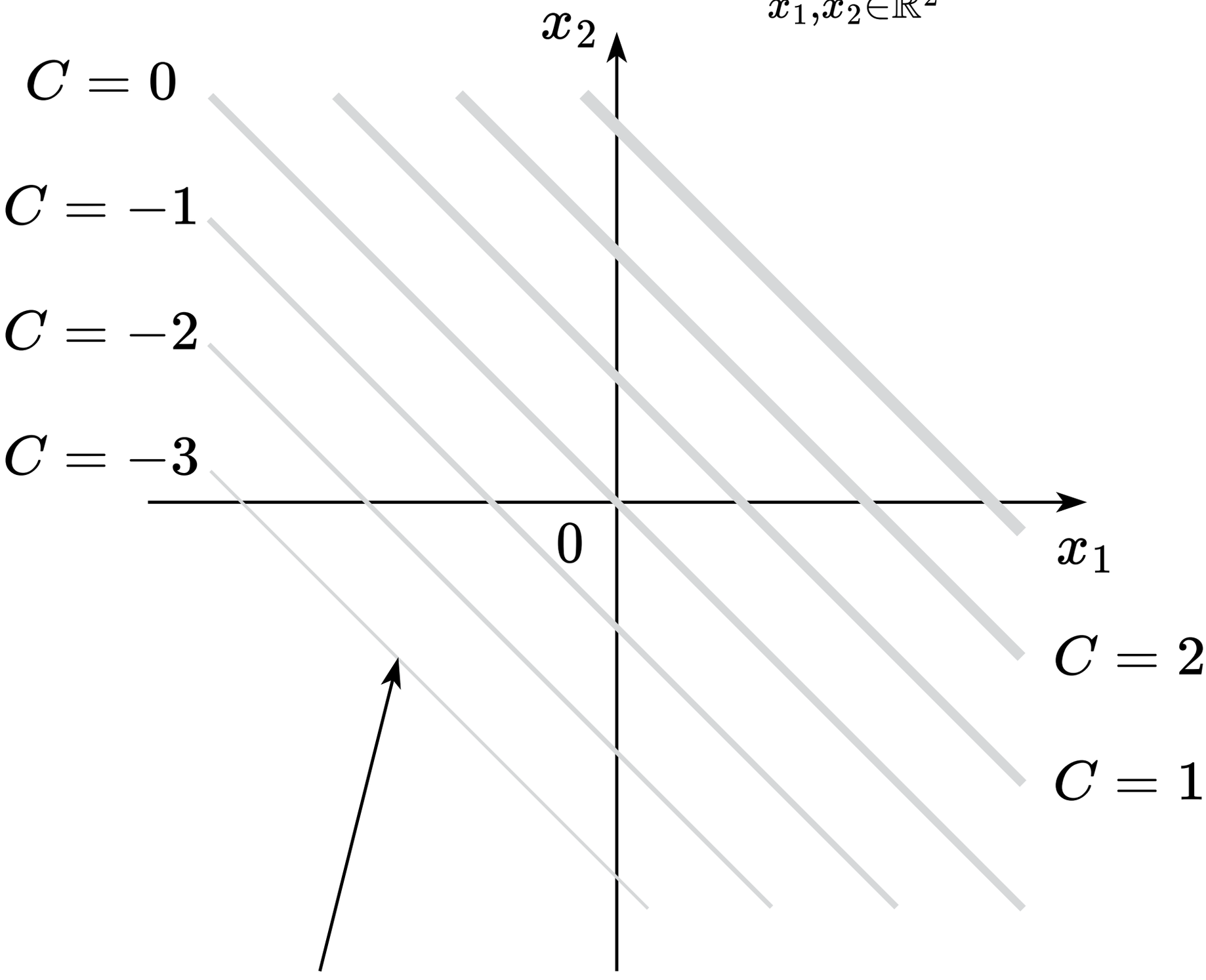
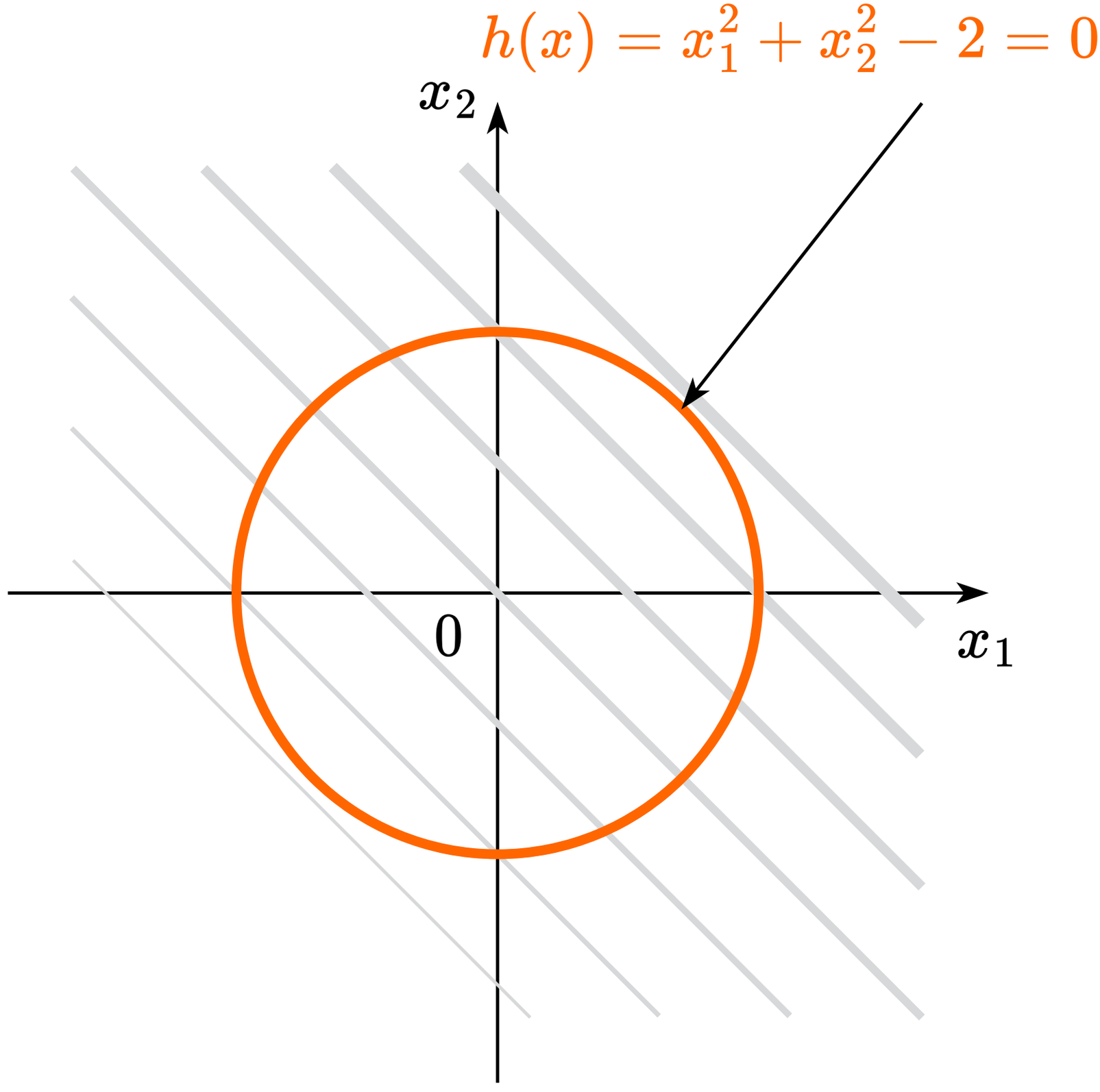
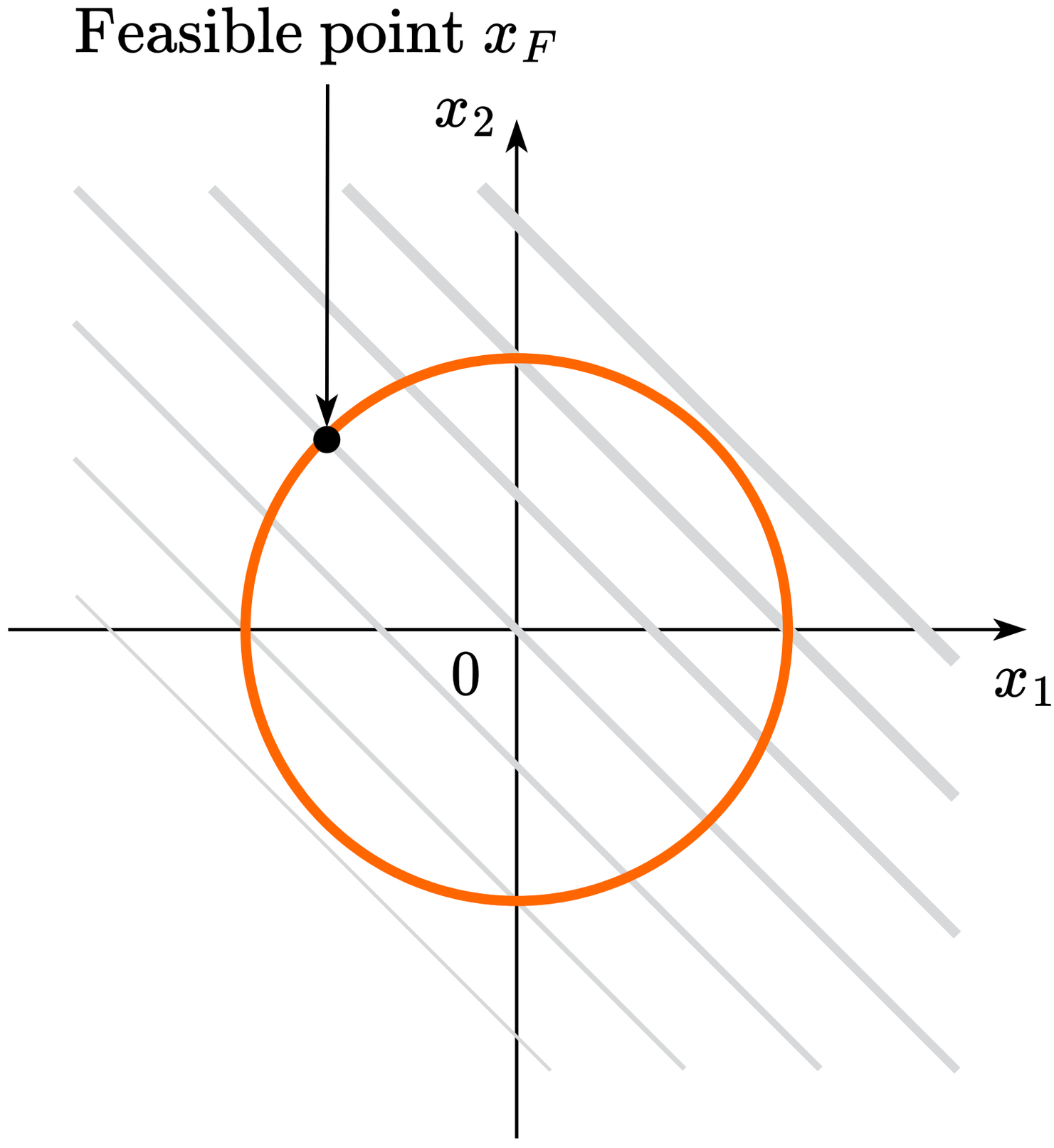
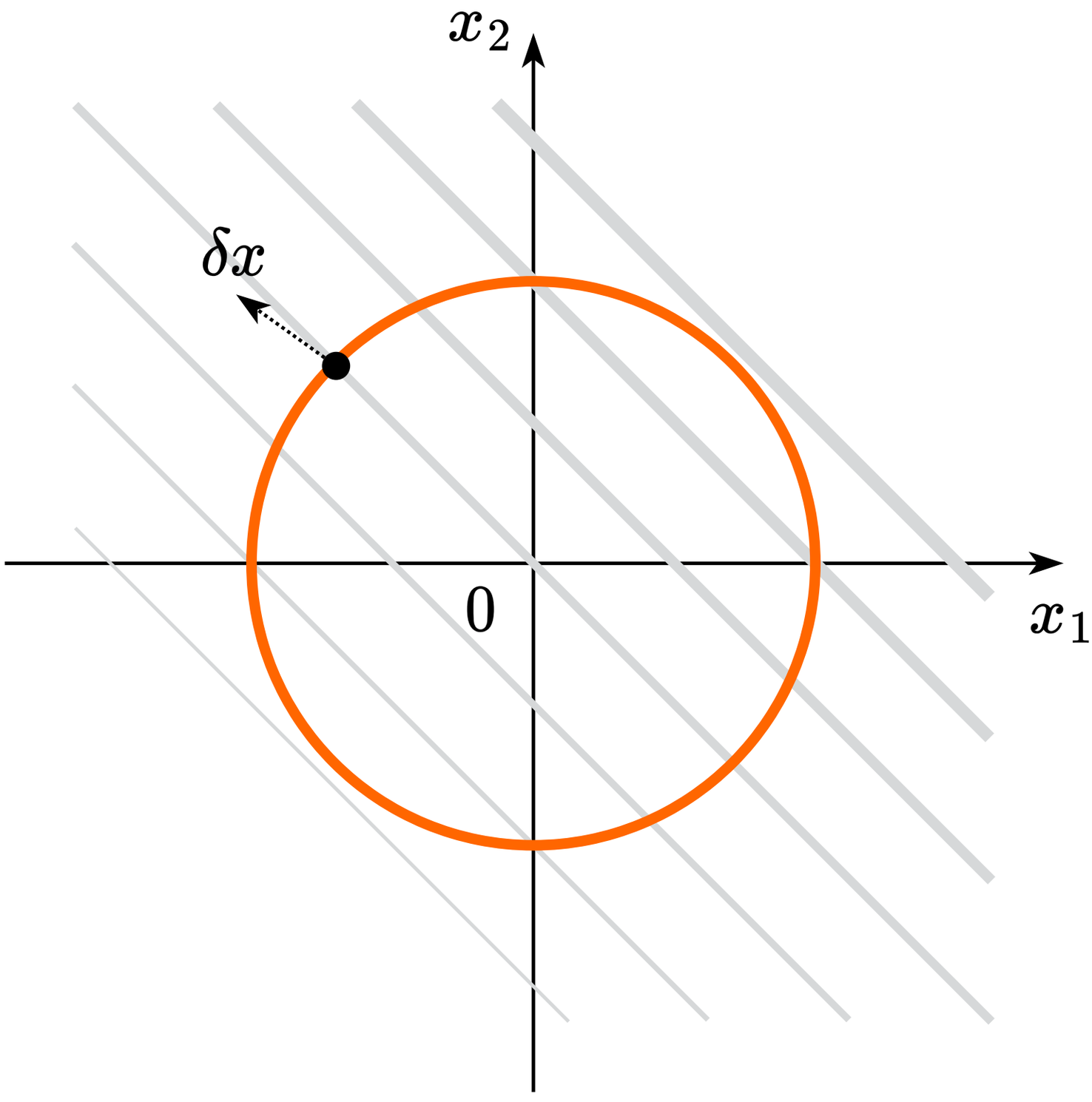
$$f(x) = x_1 + x_2 \to \min_{x_1, x_2 \in \mathbb{R}^2}$$

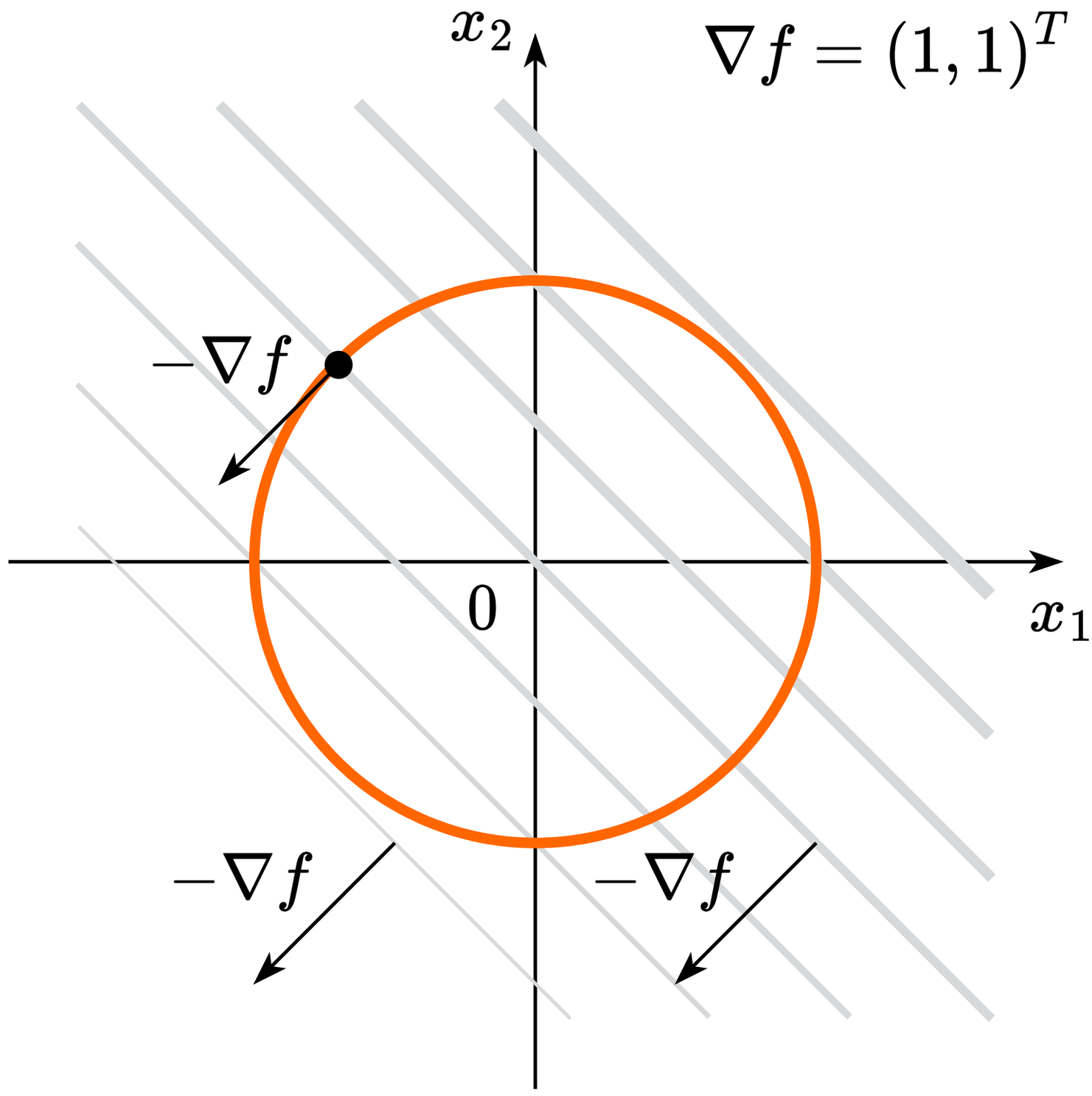


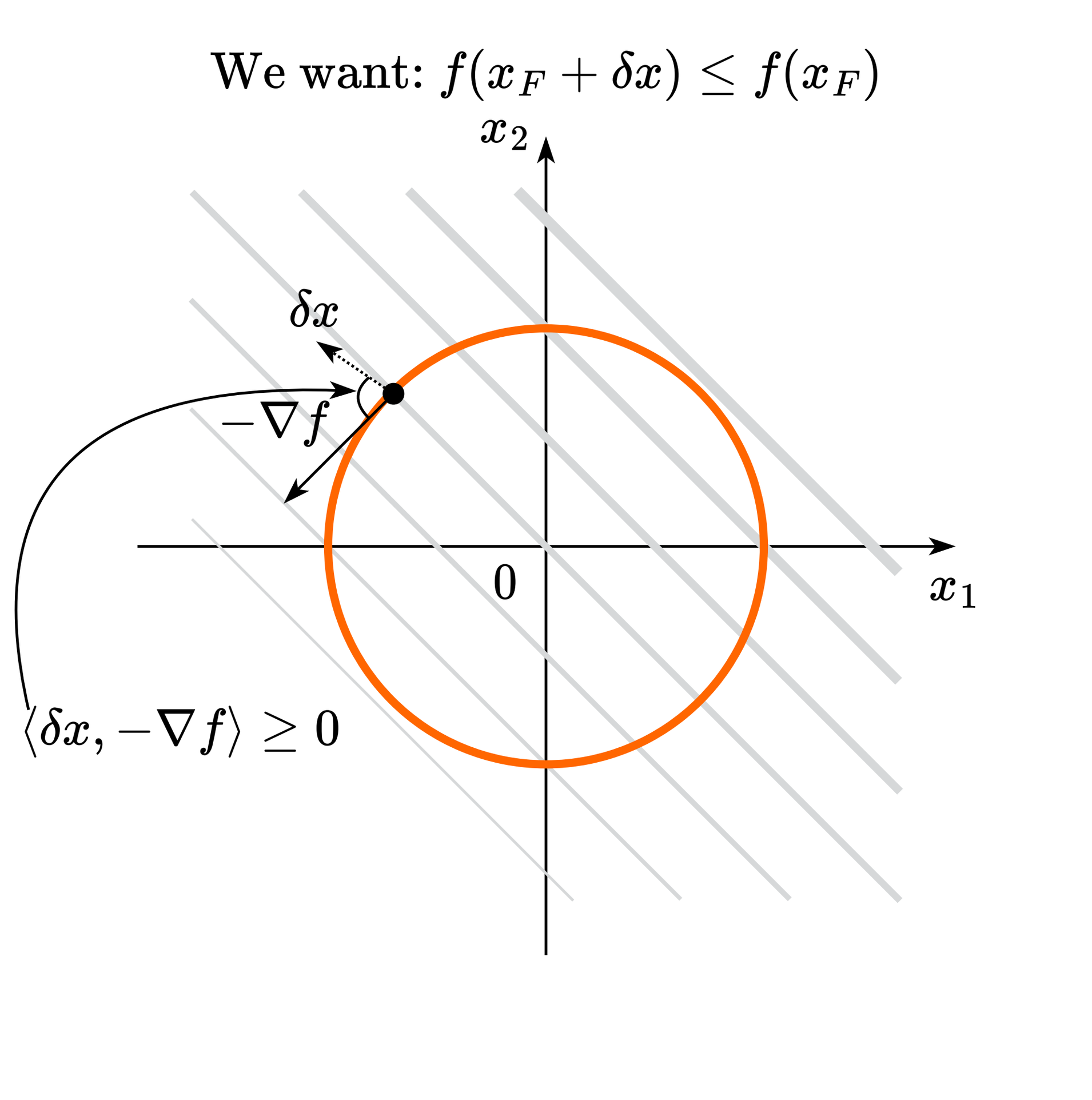
Contour lines of $f(x) = x_1 + x_2 = C$


$h(x) = x_1^2 + x_2^2 - 2 = 0$
x_2
x_1
0

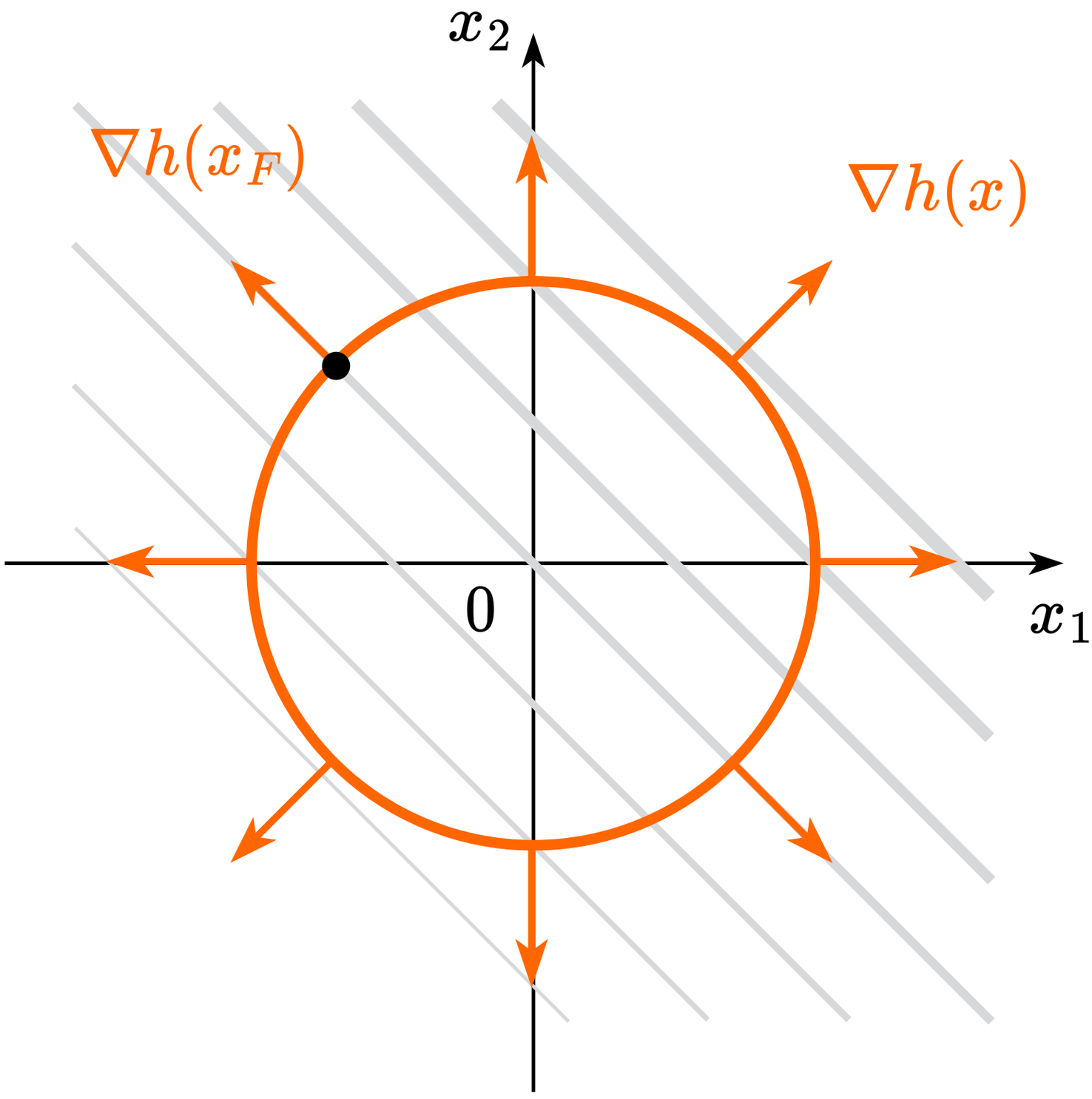

Feasible point x_F
x_2
x_1
0

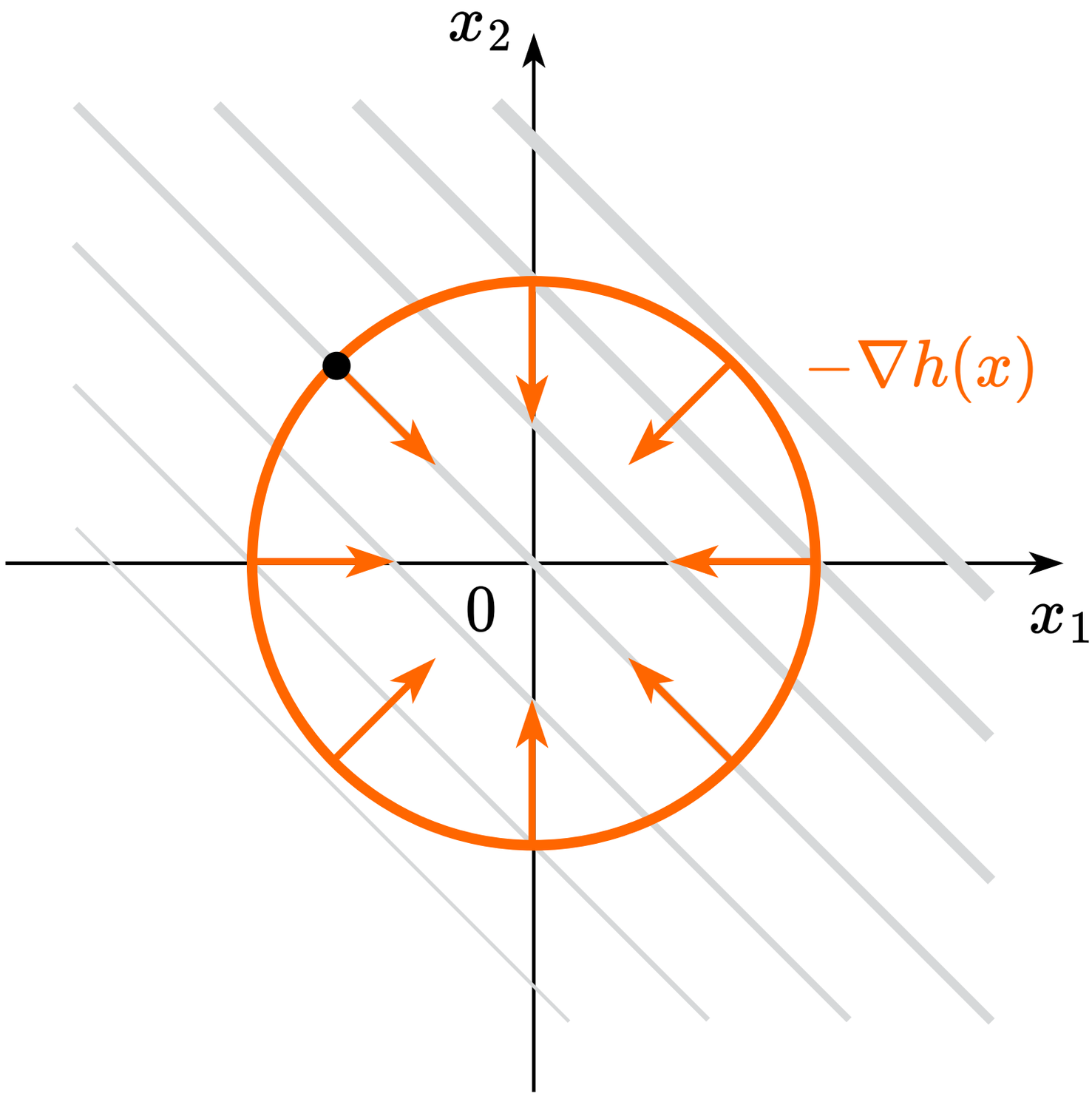

x_2
δx
0
x_1

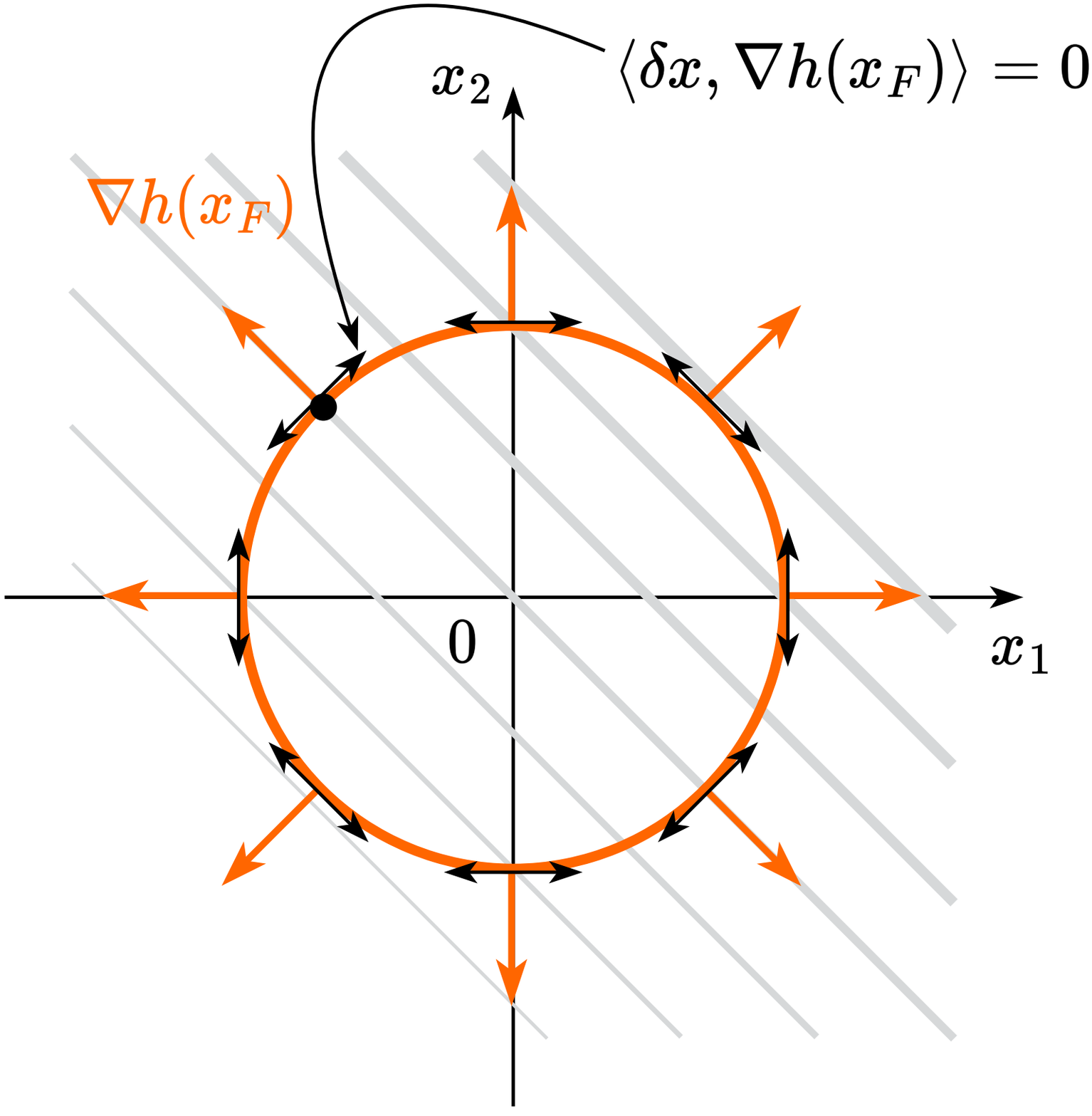

x_2
$\nabla f = (1,1)^T$
$-\nabla f$
0
x_1
$-\nabla f$
$-\nabla f$

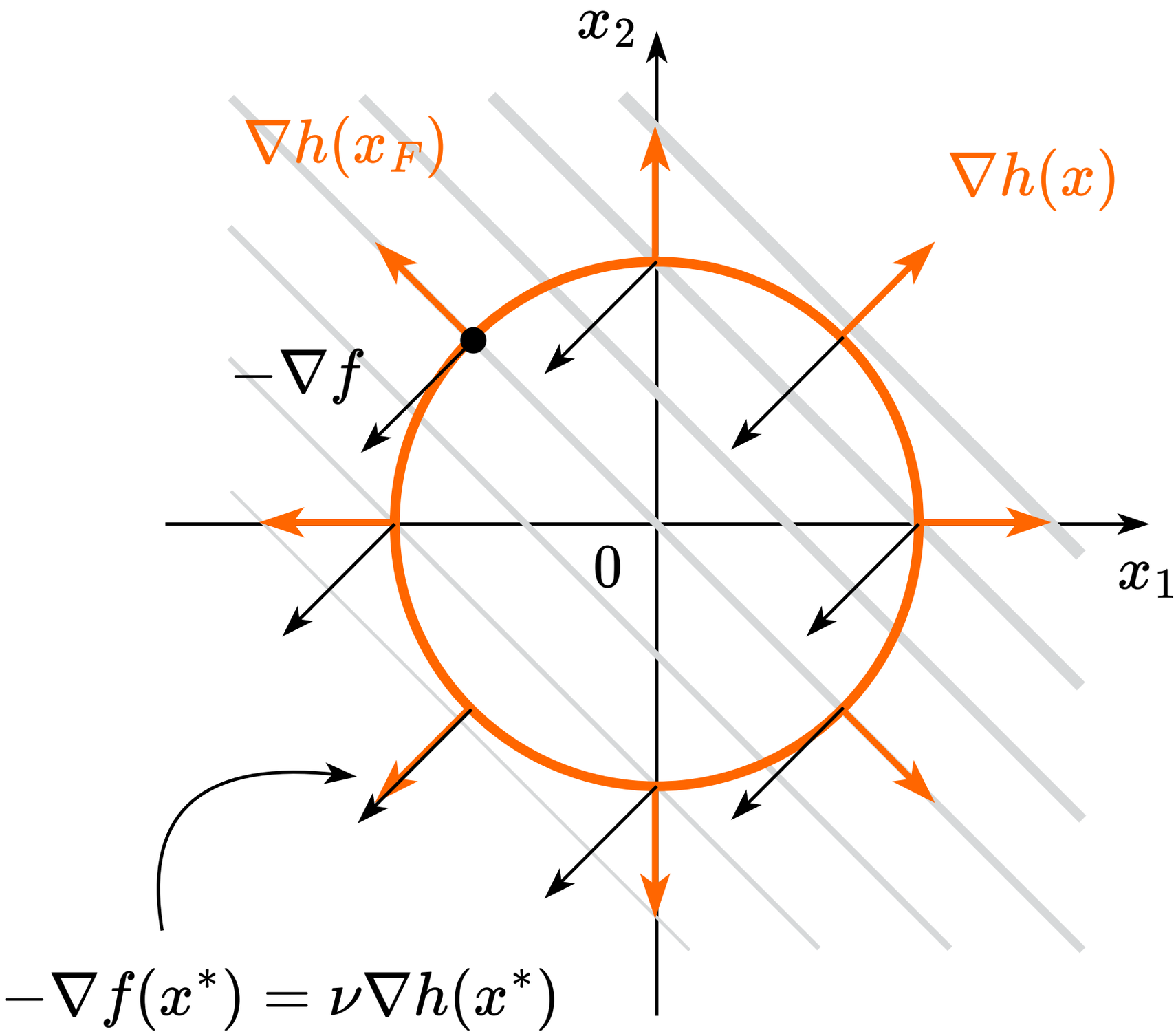

We want: $f(x_F + \delta x) \leq f(x_F)$
x_2
δx
$-\nabla f$
0
x_1
$\langle \delta x, -\nabla f \rangle \geq 0$

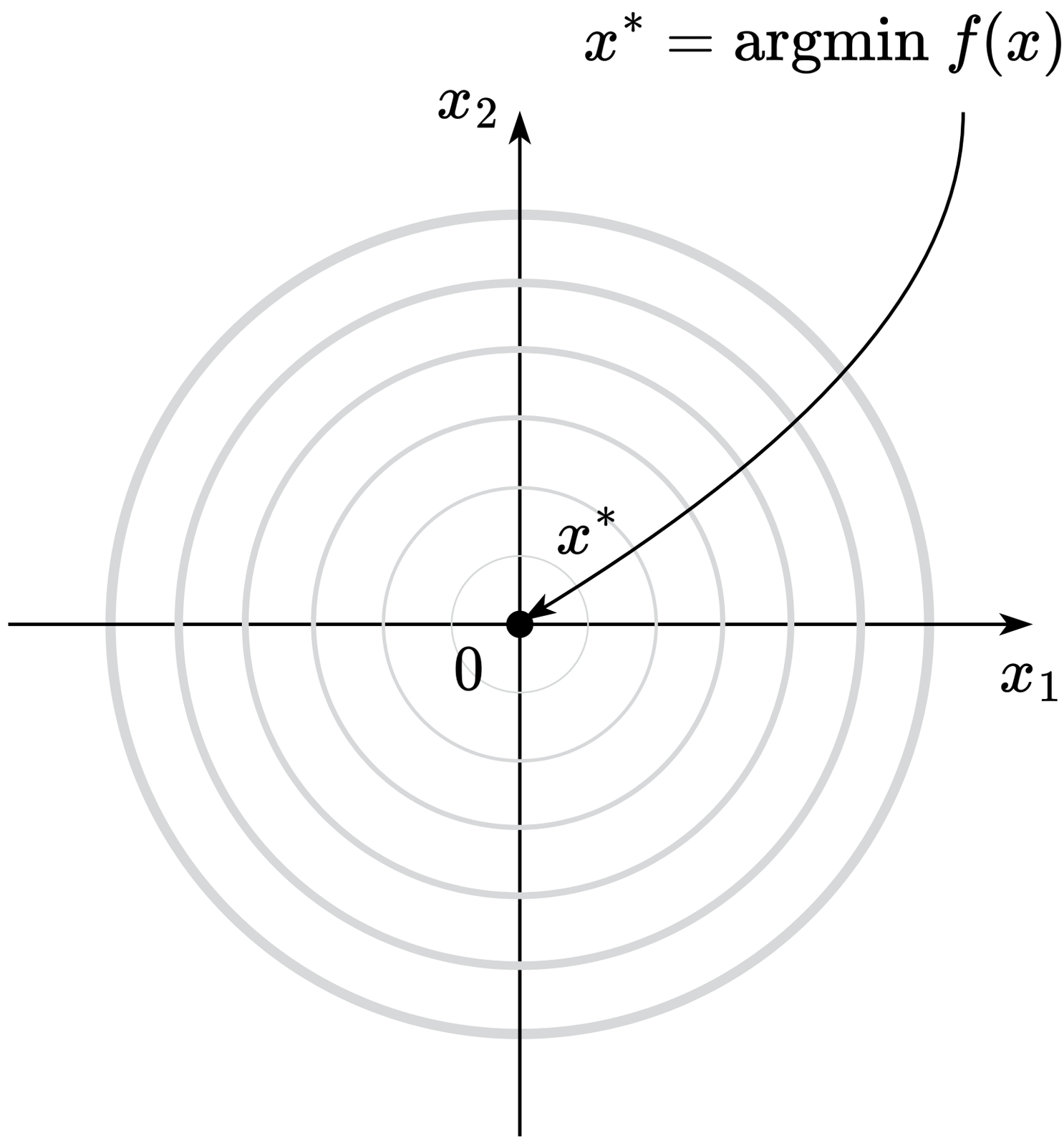
$$\nabla h = (2x_1, 2x_2)^T$$




x_2
$-\nabla h(x)$
0
x_1


$\langle \delta x, \nabla h(x_F) \rangle = 0$
x_2
$\nabla h(x_F)$
0
x_1


x_2
$\nabla h(x_F)$
$\nabla h(x)$
$-\nabla f$
0
x_1
$-\nabla f(x^*) = \nu \nabla h(x^*)$



Contour lines of $f(x) = x_1^2 + x_2^2 = C$