Fixed Income Securities

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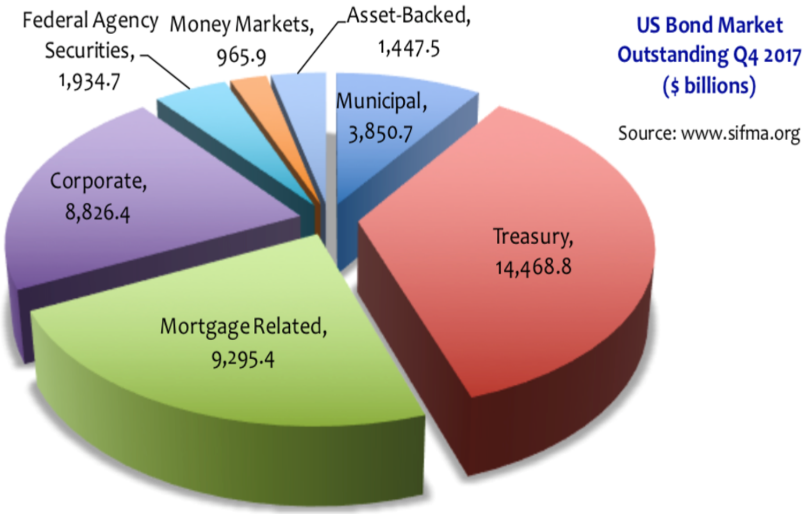
Outline

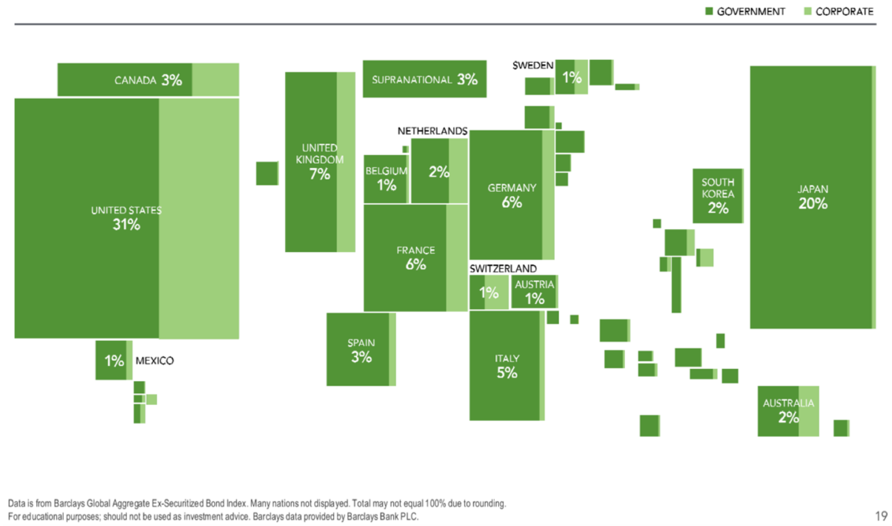
- What are bonds? What kind of bonds are traded?
- Bond Pricing.
- Bond Yields.
- Zero-coupon bonds.
- The Term Structure of Interest Rates & Forward Rates.
- Interest Rate Risk and Duration.
- Default Risk and Bond Ratings.

What is a bond?

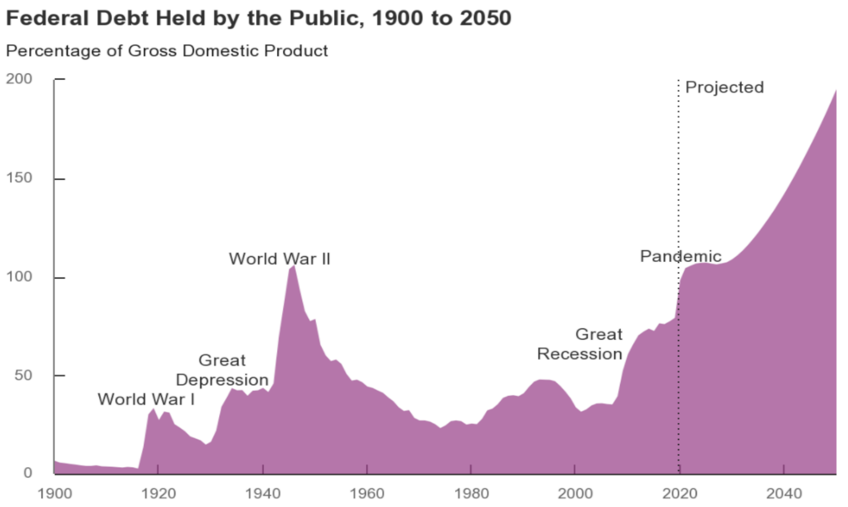
A form of borrowing by governments, corporates and others



Global Bond Markets



The 2020 Long-Term Budget Outlook



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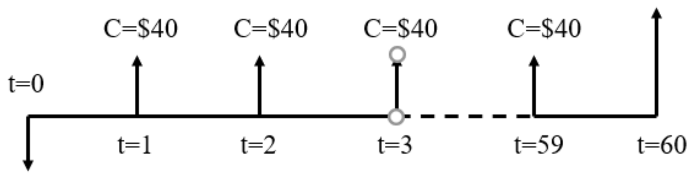
What is a bond?

- A **bond** is a financial security that promises the holder a set of pre-specified payments (fixed amounts or based on a formula) at known future dates
- Example: a 3-yr bond with notional 1,000 and 2% annual coupon
  - **Maturity** or **Redemption Date**: date of final cash flow, here 3yrs from now
  - **Notional, Face Value or Principal**: cash flow at maturity, here 1,000
  - **Coupon** (stated as APR): periodic cash flows expressed as a percentage of notional, here due at the end of each of the next 3 years
  - Other features: embedded options, covenants etc

Promised Cashflows	t = 0	t = 1	t = 2	t = 3
Coupon		20	20	20
Principal				1000
Total		20	20	1020

Example: Coupon Bonds

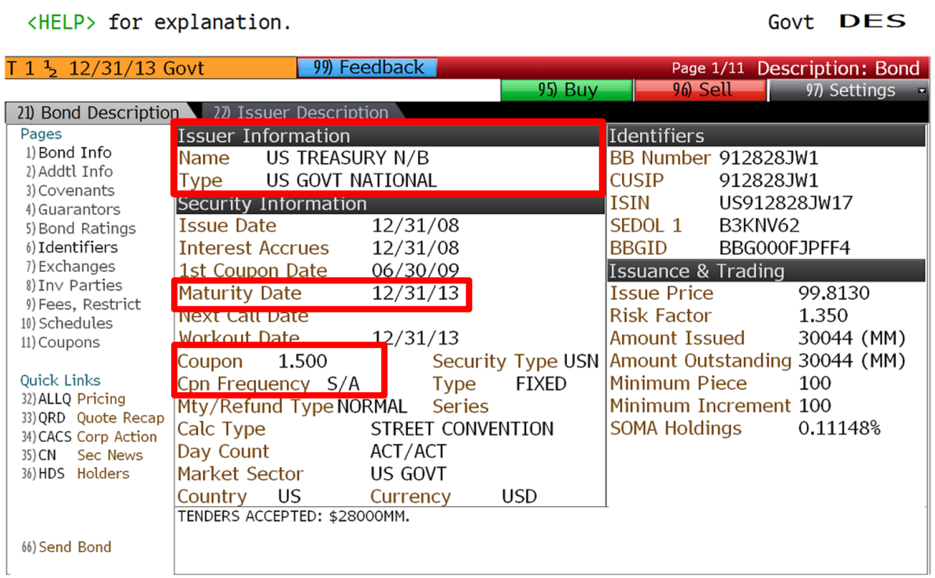
- **Example**: bond with par value of \$1000, coupon rate of 8%, and maturity of 30 years.
- The bondholder is entitled to semiannual payments of \$40 $(= \frac{0.08}{2} x1000)$ for the life of the bond, plus \$ 1000 par value at maturity.



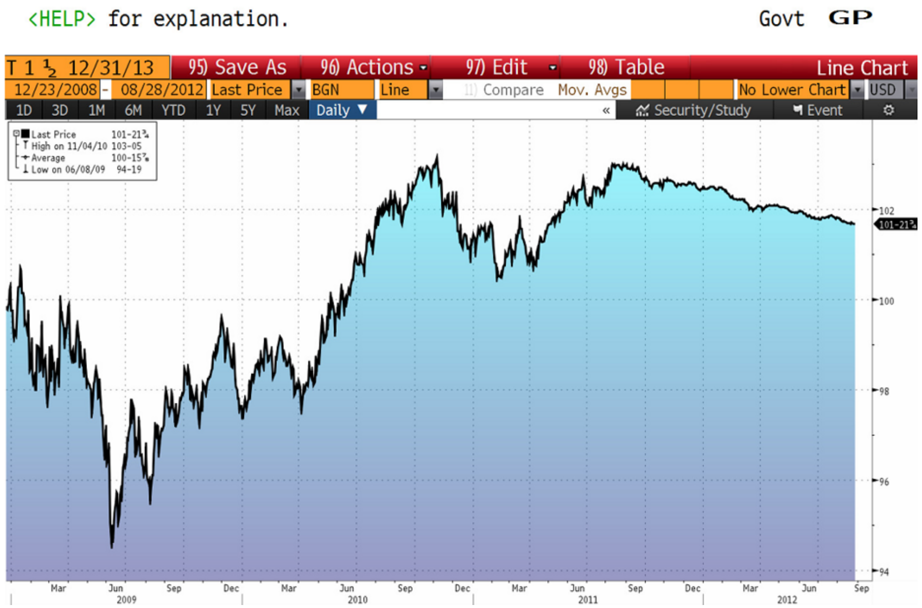
Example: Coupon Bonds

- 20-year bond issued by US state of S.C. January 1867.
- Face value of \$1000
- Interest rate of 6% paid semiannually - each of the coupons could be turned in for \$30 every 6 months.

Treasury Note

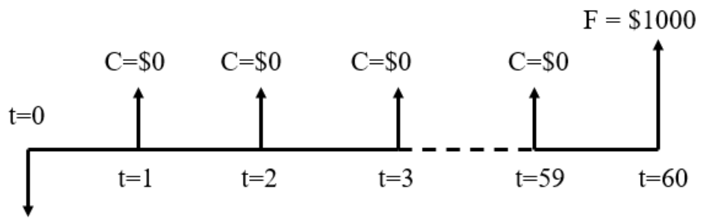


Treasury Note



Example: Zero-Coupon Bonds

- Zero-coupon bonds (or discount bonds) make no coupon payments (zero - coupon rate). Investors receive par value at maturity but no interest until then.
- Why would you buy such a bond?

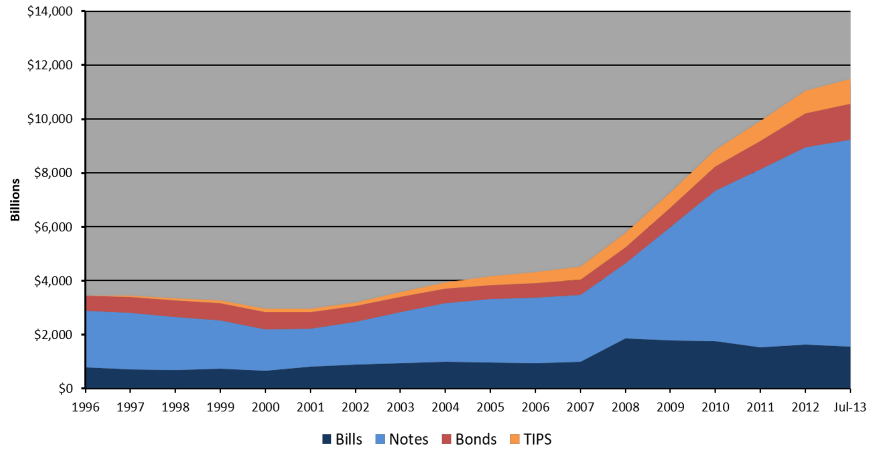


US Government Securities ("Treasuries")

- Mostly fixed-rate nominal bonds:
  - **T-Bills**: original maturity of 1 year or less (typically 1m, 3m, 6m and 1yr) discount (ie zero coupon) securities
  - **T-Notes**: original maturity 1 to 10 years (typically 2, 3, 5, 7 and 10 yrs) coupon-paying (semi-annual)
  - **T-Bonds**: original maturity of more than 10 years (typically 30 yrs) coupon-paying (semi-annual)
- Inflation-protected (**TIPS**)
  - Original maturity of 5, 10 and 30 years
  - Coupon-paying (semi annual), coupon and principal adjusted for inflation
- Other (Floating Rate Notes, STRIPS etc)

Outstanding U.S. Treasury Debt

July 2013: $ 11,464 Billion held by public



Bond Pricing

Bond Pricing

How do we value bonds with **guaranteed** cash flows?

- Assuming cashflows are default-free then we should price them using interest rates/discount factors for riskless borrowing/lending at various maturities
- In theory, all bonds must be priced by the **same** set of riskfree interest rates/discount factors otherwise there is an **arbitrage opportunity**

Bond Pricing

Assuming cashflows are guaranteed (default-free) then value bonds using riskfree interest rates/discount factors

- E.g. Value of 3yr, 1,000 notional, 2% annual coupon bond is given by:

$$P = \frac{20}{(1+r_1)} + \frac{20}{(1+r_2)^2} + \frac{1020}{(1+r_3)^3} = 20 \cdot d_1 + 20 \cdot d_2 + 1020 \cdot d_3$$

- $\mathbf{r_t}$ denotes the **spot interest rate** for maturity t: ie the rate for borrowing and/or lending between today and t (here, compounded once annually)
- For now, assume the spot rates r_t are known. I will show you how to compute them later
- $\mathbf{d_t}$ denotes the **discount factor** for maturity t, where $d_t = \frac{1}{(1+r_t)^t}$

IMPORTANT: for each time period t, we have a different spot rate $\mathbf{r_t}$

Bond Pricing

- To value a security we discount its cash-flows by the appropriate discount rate:

Bond value = Present value of coupons + Present value of par value

- Using discount factor notation

$$\text{Price } = \sum_{t=1}^{T} d_t C_t$$

d_t is the discount factor, C_t is the cash flow at time t

Example: Bond Pricing

Consider a 30-year bond with a 8% coupon rate (annual payments) and a \$1000 face value. The annual interest rate (annual compounding) is 5%

- What is the initial price of this bond?

  Answer: $P = \$1,461.2$

- If the annual interest rate is unchanged, what will the price be immediately before and after the first coupon is paid?

  Answer:

$$P(\text{just before first coupon}) = \$1,534.2$$

$$P(\text{just after first coupon}) = \$1,454.2$$

Example 2: Bond Pricing

- 8% coupon, 30-year maturity bond, par value of \$1,000,annual coupon payments. Suppose the annual interest rate with annual compounding (APR) is 8%.
- The value of the bond:

$$\text{Price} = \sum_{t=1}^{30} \frac{1}{(1+0.08)^t} \times 80 + \frac{1}{(1+0.08)^{30}} \times 1,000 = 1,000$$

- In this example the coupon rate equals the discount rate, and the bond price equals par value.
- What if the discount/interest rate is higher?

Bond Pricing 2

- P $>$ Face Value : bond sells at a **premium**
- P $<$ Face Value : bond sells at a **discount**
- P $=$ Face Value : bond sells at **par**

Intuition (discount): Since the coupon is below the discount rate, investors will demand more than their initial investment at maturity (i.e., they'll pay less than face because the coupons are low)

Bond Price at different interest rates

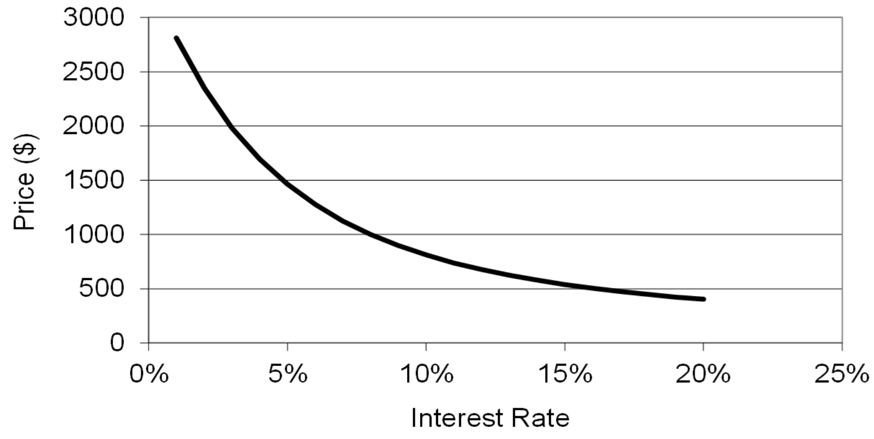
Discount Rate	Bond Price
4 %	$ 1,691.7
5 %	$ 1,461.2
6 %	$ 1275.3
8 %	$ 1,000.00
10 %	$ 811.46
12 %	$ 677.79

Price ($= \mathrm{PV}($ payments $)$) falls as interest rates rise.

- Crucial general rule in bond valuation.

Bond Pricing

Price of the 30-year, 8% coupon bond for a range of interest rates:



Graph – Bond Pricing: Price vs. r

- Negative slope means price falls as r rises.
  - If you buy a bond at par for 8% and market interest rates rise, then you suffer a loss.
  - Intuition is "you tied up your money earning 8% when alternative investments now earn higher returns."
- The convex shape means that an increase in interest rates results in a price decline that is smaller than the price increase resulting from an interest rate fall of equal magnitude.
  - From prior example, compare rate changes of +/- 2%
  - This property is called "convexity" (more on this later)

Bond Pricing

- For Treasuries, coupons and compounding are, by convention, semi-annual
- Discount factor for cash flows in t years:

$$d_t = \frac{1}{\left(1 + \frac{r_t}{2}\right)^{2t}}$$

- Where the spot rate r_t is the **annual** interest rate with semi-annual compounding (APR) for borrowing between now and t years
- We have thus far assumed constant discount rates, but in reality we often have different discount rates, and hence spot rates, for each time period. That is r_t is different for each time period t.

Bond Pricing

- Write down cash-flows (C_t) from coupon payments (C/2) and par value (F)
- Get spot rates and compute discount factors
- Multiply and add up via:

$$P_T = \sum_{t=1}^{2T} d_{\frac{t}{2}} \frac{C}{2} + d_T F = \sum_{t=1}^{2T} d_{\frac{t}{2}} C_{\frac{t}{2}} = \sum_{t=1}^{2T} \frac{1}{\left(1+\frac{r_t}{2}\right)^t} C_{\frac{t}{2}}$$

[t = indexed cash flow, t = 1, 2, 3 ... 2T]

Example: 1-year, $ 100 par, 5% Coupon Bond

1. Get cash flows:

Time	6-M (Coupon)	1-year (Coupon)	1-year (Par)
Cashflow	$2.50	$2.50	$100

2. Assume $r_{.5} = 0.04$ and $r_1 = 0.03$: Discount rates are

$$d_{.5} = \left(1+\frac{r_{.5}}{2}\right)^{-1} = (1+0.02)^{-1} = 0.980$$

$$d_1 = \left(1+\frac{r_1}{2}\right)^{-2} = (1+0.015)^{-2} = 0.971$$

3. Multiply and add up:

$$P_1 = d_{.5}C_{.5} + d_1 C_1 = 0.98 \times 2.5 + 0.971 \times 102.5 = \$101.94$$

Bond Yields

Bond Yields: Comparing bonds with different characteristics

- Consider three bonds:
  - Bond 1: T=2, C=2%, P=90.92
  - Bond 2: T=2, C=8%, P=102.26
  - Bond 3: T=2, C=0%, P=87.14
- A natural metric is to compare the expected returns:

  a.k.a. computing the "yield"

Yield-To-Maturity (YTM)

- The **yield-to-maturity** of a bond is the single discount rate, y, that sets the PV of a bond's *promised* cashflows equal to the observed price
  - E.g. consider the 3 year, 1,000 notional, 2% annual coupon bond. If the bond trades at 985.72 then YTM is the number y such that:

$$985.72 = \frac{20}{(1+y)} + \frac{20}{(1+y)^2} + \frac{1020}{(1+y)^3} \Rightarrow y = 2.5\%$$

- As before we have that:
  - Bond price and yield are inversely related
  - Bond price is convex in the yield
  - Bond trades above/at/below par if coupon rate is above/at/below the yield

Yield-To-Maturity (YTM)

- For a semi-annual coupon paying bond (market convention):

$$P_T = \sum_{t=1}^{2T} \frac{\frac{C}{2}}{\left(1+\frac{y}{2}\right)^t} + \frac{F}{\left(1+\frac{y}{2}\right)^{2T}}$$

- y is the internal rate of return (IRR) of the bond
- The IRR is defined as the interest rate that sets the net present value of the cash flows equal to zero. That is, y, equates the PV of the cashflows with the market price of the bond.

Back to the example

- Consider the (semi-annual coupon paying) Bond 1: T=2, C=2%, P=90.92
- Yield solves:

$$90.92 = \frac{1}{\left(1+\frac{y}{2}\right)^1} + \frac{1}{\left(1+\frac{y}{2}\right)^2} + \frac{1}{\left(1+\frac{y}{2}\right)^3} + \frac{101}{\left(1+\frac{y}{2}\right)^4}$$

- Which gives yield to maturity 6.941% (annualized)

Calculator: PV = 90.92, Coup. PMT = 1, F =100, T = 4,

y = ? = 3.47 gives YTM = 2 x 3.47 = 6.94%

By hand: Trial and error

See canvas for pyhton and R codes

Yields of the 3 Bonds

- The yields of the three bonds are:

$$\text{Bond 1}: \mathrm{C} = 2\%, \mathrm{P} = 90.92, \quad \mathrm{y} = 6.941\%$$
$$\text{Bond 2}: C = 8\%, P = 102.26, \quad y = 6.773\%$$
$$\text{Bond 3}: C = 0\%, P = 87.14, \quad y = 7.003\%$$

(Exercise: compute these y's at home)

How should we interpret the yield?

Interpreting the yield

- Interpreting the yields of individual bonds:
  - Yields and the "cost of borrowing"
  - Yields and interest rates
  - Yields and returns
- Later: understanding yield curves (plots of yield by maturity)

Interpreting the yield

- Yield is the expected rate of return on a bond **if**
  - Coupons are paid
  - You re-invest coupons at y
  - You hold the bond to maturity
- Is this realistic?
- Should we always invest in the bond with the highest yield?
- Either way, yields provide a useful benchmark for comparison

Interpreting the yield

- For coupon paying bonds held to maturity, returns depend on the rate at which you reinvest coupons
- The YTM on a coupon paying bond is equal to the compound return if held to maturity and the coupons can be reinvested at the original yield

E.g. a 3yr 2% annual coupon bond with face value 1,000 trading at 985.72

- Yield: $985.72 = \frac{20}{(1+y)} + \frac{20}{(1+y)^2} + \frac{1020}{(1+y)^3} \implies y = 2.5\%$
- Rearrange (multiply both sides by $(1+y)^3$):

$$\underbrace{985.72(1+y)^3}_{\text{Value of investing 985.72 at the yield, compounding for 3 years}} = \underbrace{20(1+y)^2}_{\text{Reinvest t=1 coupon for 2 yrs at y}} + \underbrace{20(1+y)}_{\text{Reinvest t=2 coupon for 1 yr at y}} + \underbrace{1020}_{\text{Cashflows in year3}}$$

Interpreting the yield

If the coupons are reinvested at lower/higher rates than the YTM, then the realised return will end up lower/higher than the YTM:

	Value at t=3 if coupons reinvested to maturity at		
	1%	**2.5%**	**5%**
t=1 coupon reinvested for 2 years	$20 \times (1.01)^2 =$ 20.402	$20 \times (1.025)^2 =$ 21.0125	$20 \times (1.05)^2 =$ 22.05
t=2 coupon reinvested for 1 year	$20 \times 1.01 =$ 20.2	$20 \times 1.025 =$ 20.5	$20 \times 1.05 =$ 21
t=3 coupon plus principal	1020	1020	1020
Total at t=3	1060.602	1061.5125	1063.05
Compound Return, R	2.47% (lower)	2.50% = YTM	2.55% (higher)

$$\text{Compound Return} = \frac{\text{Total at } t=3}{985.72} = (1+R)^3$$

Bonds Returns and Investment Horizon

- So far we have looked at compound returns if a bond is held to maturity
- If the investment horizon differs from the bond maturity then:
  - If the investment horizon is longer than the original bond maturity, then all bonds (coupon paying or not) will face reinvestment risk on the principal
  - If the investment horizon is shorter than the bond maturity, then all bonds (coupon paying or not) will face "price risk" in that the value of the bonds at the end of the horizon will not be known ex-ante (as they depend on interest rates at that future date)

Bonds Returns

- The total return earned on the bond over a period, sometimes called the holding period return (HPR), consists of:
  - capital gain/loss
  - return due to cash income (coupon)
- The simplest HPR to calculate is that between coupon dates in which case:

$$HPR_t = \frac{P_t - P_{t-1} + C}{P_{t-1}}$$

- Where:
  - P_t is the ending price
  - P_{t-1} is the initial price
  - C is the coupon income received at the end of the period

Example: Bonds Returns

- Bond: A 3 year, 2% annual coupon bond with face value of 1,000
  - Initial price of 985.72 i.e, P_0 =985.72
  - Price one year later (after first coupon of 20 has been paid) is P_1 =1019.70, and C =20
  - Holding Period Return/Total Return:

$$HPR_1 = \frac{P_1 - P_0 + C}{P_0} = \frac{1019.70 - 985.72 + 20}{985.72} = 5.48\%$$

Note: HPR of 5.48% exceeds initial yield of 2.5% (from previous slides)

Example: Bonds Return and Yields

Bond: a 3 year, 2% annual coupon bond with face value of 1,000

- Initial price of 985.72, implies an original yield of 2.5%:

$$P_0 = 985.72 = \frac{20}{1+y_0} + \frac{20}{(1+y_0)^2} + \frac{1020}{(1+y_0)^3} \Rightarrow y_0 = 2.5\%$$

- Price one year later (after first coupon) of 1019.70, implies a new yield of 1%:

$$P_1 = 1019.70 = \frac{20}{1+y_1} + \frac{1020}{(1+y_1)^2} \Rightarrow y_1 = 1\%$$

- Note that: New yield of 1% lower than initial yield of 2.5%

Bonds Return and Yields

- In general, relationship between HPR and change in yields:

  HPR $=>$ initial yield $\implies$ new yield is less than initial yield

  HPR $=$ initial yield $\implies$ new yield equals initial yield

  HPR $=<$ initial yield $\implies$ new yield is greater than old yield

- Mathematically true, intuition?

Government Bond Yields

BONDS: BENCHMARK GOVERNMENT

	Red Date	Coupon	Bid Price	Bid Yield	Day chg yield	Wk chg yield	Month chg yld	Year chg yld
Australia	11/20	1.75	99.21	2.11	0.01	0.03	0.09	0.12
	11/28	2.75	100.42	2.70	0.04	0.01	0.06	-0.09
Austria		-	-	-	-	-	-	-
	10/19	0.25	100.84	-0.44	0.02	0.03	0.10	0.08
Belgium	12/20	1.80	105.15	-0.40	-0.02	0.02	0.07	-0.01
	03/28	5.50	145.13	0.66	0.06	0.10	0.11	-0.05
Canada	02/20	1.25	98.87	2.03	-0.02	0.02	0.14	-
	06/28	2.00	96.73	2.37	0.03	0.11	0.20	0.33
Denmark	11/20	0.25	101.62	-0.45	0.02	0.06	0.10	-0.03
	11/27	0.50	100.67	0.43	-0.02	0.08	0.10	-0.14
Finland	09/20	0.38	101.81	-0.47	0.02	0.05	0.10	-0.02
	09/27	0.50	99.25	0.59	0.05	0.09	0.10	-
France	11/20	0.25	101.60	-0.44	0.03	0.05	0.09	-0.04
	05/23	1.75	108.63	-0.05	-0.01	0.07	0.13	-0.07
	05/28	0.75	100.27	0.72	-0.01	0.08	0.12	-
Germany	10/20	0.25	101.78	-0.56	-0.02	0.04	0.11	0.01
	08/23	2.00	110.85	-0.15	0.04	0.09	0.13	-0.04
	08/27	0.50	101.19	0.37	0.05	0.10	0.12	-0.12
	08/48	1.25	103.50	1.11	-0.02	0.05	0.10	-
United Kingdom	01/21	1.50	101.62	0.84	-0.01	0.04	0.12	0.45
	07/23	0.75	98.27	1.11	-0.01	0.07	0.11	0.34
	12/27	4.25	124.97	1.39	0.06	0.12	0.11	0.18
	07/47	1.50	92.91	1.82	0.05	0.11	0.08	-0.02
United States	11/19	1.75	98.93	2.57	0.01	0.01	0.09	-
	10/22	2.00	96.69	2.83	-0.03	-0.01	0.09	-
	11/27	2.25	94.05	2.99	-0.02	0.01	0.12	-
	11/47	2.75	92.84	3.13	0.00	0.03	0.14	-

Interactive Data Pricing and Reference Data LLC, an ICE Data Services company.

Source: Financial Times, 4 Aug 2018

Government Bond Yields and the coronavirus

Bond rally sends UK short-term yields below zero

Bank of England under pressure to cut interest rates to offset coronavirus impact

Tommy Stubbington in London MARCH 9 2020

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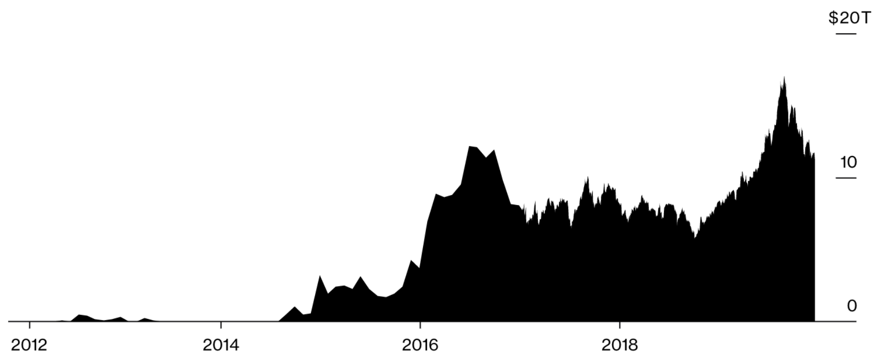
The UK joined the club of nations with negative-yielding debt for the first time on Monday as investors bet on further interest rate cuts to ease the effects of the coronavirus outbreak.

Link to the Financial times

Negative-yield drama

Market Value of Negative-Yielding Bonds in the Bloomberg Barclays Global-Aggregate Index



How can a bond have a negative yield? If you buy these bonds and hold them to maturity, you will get less money than what you paid for them, even including interest.

Making sense of negative bond yields

1. Income vs capital gain
   - Usually, if you buy a 30 year bond, you do not intend to hold it to maturity. Hence, if you think yields are likely to fall further you can buy negative yielding bonds in the expectation that you will benefit from capital gains. In fact, bonds beat stocks over the last 20 years. **From previous slides: When yields go down, bond prices go up.**
2. Liability matching
   - Liability-relative investors, such as insurance companies and pension funds use bonds to match the payments of their liabilities. **More on this in the next class.**
3. For portfolio diversification reasons
   - Strong negative correlation between the bond returns and stock returns. That is, when bonds tend to perform well when equities to badly and vice-versa. **More on this later in the course**

Making sense of negative bond yields

- What is the alternative, if you are not happy about paying to lend money?
  1. Retail deposits do not offer a real shelter
     - In the UK, deposit insurance schemes only protect bank deposits up to £85,000. Hence, not suitable for large investors
  2. You can store the money in cash
     - Storing larger amounts in cash is not option
  3. You may shift further along the risk spectrum and go into corporate bonds?

Zero-Coupon Bonds

Zero-Coupon Bonds (ZCB)

- Recall that ZCB make just one payment at maturity.
- The price of zero coupon bond is given by discounting the face value at the yield or equivalently at the interest rate for the bond maturity
- Therefore **zero coupon yields are equivalent to interest rates**
  - E.g. a 3 year zero coupon bond with face value of 1,000, trading at 921.84

$$921.84 = \frac{1000}{(1+y)^3} = \frac{1000}{(1+r_3)^3} \Rightarrow y = r_3 = 2.75\%$$

- Moreover, a zero coupon bond earns the yield as a compound return if held to maturity
  - E.g. return if invest 921.84 in the 3 yr zero coupon bond to get 1000 in 3 years' time

$$1,000 = 921.84 \cdot (1+R)^3 \Rightarrow R = 2.75\%$$

Price of a Zero is the discount factor

- Simplest bonds:
  - Cash flows are just the face value F (i.e., no coupons)
  - PV formula:

$$Z_T = d_T C_T = \frac{1}{(1+\frac{r_T}{2})^{2T}} \times F$$

  - From Zeros we can get discount factors:

$$d_T = \frac{Z_T}{F}$$

The YTM of a Zero is the spot rate

- What is the difference between yields on zeros (F=par) and spot rates? No difference
  - Yield on T-year zero bond:

$$Z_T = \frac{1}{(1+\frac{y_T}{2})^{2T}} F$$

  - Zero expiring in T-years

$$Z_T = \frac{1}{(1+\frac{r_T}{2})^{2T}} F$$

- Yields on zeros (y_T) are spot rates (r_T)!

Zero-Coupon Bonds (ZCB)

- The following table gives prices and YTM (annual compounding) of a ZCB with a par value of $1,000:

Time to Maturity	Price	YTM
1	925.93	8.000%
2	841.76	8.995%
3	758.32	9.660%
4	683.19	9.993%

- Recall that: 841.76=1,000/(1+YTM$_2$)^2 or YTM$_2$ = 8.995%. This is the (annual) spot rate that prevails today for a period of two years.
- Question: What is the PV today of receiveing $25M in two years?

Zero-Coupon Bonds (ZCB)

Question: What is the PV today of receiving $25M in two years? There are two alternatives:

- Divide the price of a two-year ZCB by its principal value to obtain the discount factor = 841.75/1000 = 0.84175. Multiply the cash-flow by the discount factor to obtain the PV:

$$PV = 0.84175 \times \$25M = \$21.044M$$

- Discount the cash-flow using the appropriate discount rate:

$$PV = \$25M/(1.08995)^2 = \$21.044M$$

Zero-Coupon Bonds (ZCB)

- The prices (and YTM) of zero coupon bonds is also important because it allows us to price (value) coupon bonds.
- What is the price of a 7% coupon (annual coupon payments), 4 years to maturity, \$100 face value bond? Use the information in the previous table.

Zero-Coupon Bonds (ZCB)

What is the price of a 7% coupon (annual coupon payments), 4 years to maturity, \$100 face value bond?

Maturity	Zeros	Coupon	Principal	PV
1y	0.92593	7	0	6.482
2y	0.84175	7	0	5.892
3y	0.75833	7	0	5.308
4y	0.68318	7	100	73.100
				90.782

Pricing Zeroes vs. Coupon Bonds

- Consider 1-year, \$100 par, 5% coupon bond

Time	6-Month	1-Year (Coupon + par)
Cashflow	\$2.50	\$102.50

- Same cash flows as portfolio with a 6-Month \$2.50 par zero and a 1-Year \$102.50 par zero
- **Implication**: the price of the coupon bond is equal to the price of a portfolio of zeroes with the right maturities and par values

Example of zero/coupon arbitrage

- Consider 1-year, 5% coupon bond
  - $r_{.5}$=0.04 and r_1=0.03 ($Z_{.5}$=0.980 and Z_1=0.971)
  - The price of the 1-year bond should be \$101.9438

Time	6-Month	1-year (Coupon + par)
Cash flow	\$2.50	\$102.50

- Suppose that the "market" price of 1-year bond is 100 and you can trade in the zeroes at the above prices

Arbitrage!

- What should you do?
  - Buy the "cheap" 1-year coupon bond and replicate the cash flows by trading in the zeroes
  - Sell 2.5 zeroes maturing in 0.5 year and 102.5 zeroes maturing in 1-year

Action	cost today:	Cash flow at $t = 1/2$	Cash flow at $t = 1$
Buy 1 Coupon bond	-100	2.5	102.5
Sell 2.5 zeroes	2.45	-2.50	
Sell 102.5 zeroes	99.49		-102.50
Net	1.94	0	0

- Arbitrage: 1.94 with no risk!

Arbitrage in real life: 3 Coupon Bonds

- Bond Triples
  - On February 15, 2001 there were three bonds maturing on the same day: August 15, 2003.

Bond Triplet	Coupon	Original Maturity	Price (15-Feb-01)
Bond 1	5.25%	3 years	100.84375
Bond 2	5.75%	10 years	102.020
Bond 3	11.125%	20 years	114.375

- Arbitrage opportunity?

Arbitrage in real life: 3 Coupon Bonds

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Bond 3	11.125%	20 years	114.375

Idea: Replicate Cashflows of Bond 2 via a portfolio composed of Bond 1 and Bond 3

- Let c_1, c_2 and c_3 be the coupon rates of the three bonds.
- Let F_1 and F_3 be the face amount of bond 1 and bond 3 needed to replicate a unit face amount of the second bond:
- In Math: To replicate cashflows of Bond 2 we need:
  - To match principal payment at maturity: $F_1 + F_3 = 1$
  - and match coupon payments: $c_1 F_1 + c_3 F_3 = c_2$

Arbitrage in real life: 3 Coupon Bonds

Bond Triplet	Coupon	Original Maturity	Price (15-Feb-01)
Bond 1	5.25%	3 years	100.84375
Bond 2	5.75%	10 years	102.020
Bond 3	11.125%	20 years	114.375

Idea: Replicate Cashflows of Bond 2 via a portfolio of Bond 1 and 3

- Solution: $F_1 = \frac{c_2 - c_3}{c_1 - c_3} = 91.49\%$ & $F_3 = \frac{c_1 - c_2}{c_1 - c_3} = 8.51\%$ $(C_1 = 5.25\% C_2 = 5.75\%, C_3 = 11.125\%)$
- In words: A portfolio with 91.49% of its face value in the 5.25s and 8.51% of its value in the 11.125s will replicate one unit face value of the 5.75s.
- What's the price of the portfolio? $P_1 \times F_1 + P_3 \times F_3 = 101.995$
- What's the price difference? 0.025 cheaper. How would you exploit this difference?

The Term Structure of Interest Rates and Forward Rates

The Term Structure of Interest Rates

- The term structure of interest rates is the set of yields to maturity, at a given time, on zero-coupon bonds of different maturities.
- The (pure) yield curve is a plot of the term structure.
- "Spot" curves across countries?

Issues with comparing different countries

- A couple of practical issues when looking at different countries
  - Are they riskless? (What is the credit rating of Japan?)
- When do countries pay coupons?
  - UK, US, Japan, Australia, Switzerland, Canada: semi-annual coupons
  - Germany, France, and Euroland: annual coupons

Yield curve across different countries (Date: 22 August 2019)

Figure 1: The aftermath of recent drop in bond yields
German and Swiss yield curves wholly negative

Schroders

	2 Year	3 Year	5 Year	7 Year	10 Year	20 Year	30 Year
US	1.6	1.5	1.5	1.6	1.6	1.9	2.1
Canada	1.5	1.4	1.3	1.3	1.3	1.5	1.5
Australia	0.8	0.7	0.7	0.8	0.9	1.4	1.6
UK	0.5	0.4	0.4	0.4	0.5	0.9	1.1
Japan	-0.3	-0.3	-0.3	-0.4	-0.2	0.1	0.2
Sweden	-0.6	-0.7	-0.7	-0.5	-0.3	0.2	-
Germany	-0.9	-0.9	-0.9	-0.8	-0.6	-0.3	-0.1
Switzerland	-1.1	-1.1	-1.0	-1.0	-1.0	-0.6	-0.5

Constructing yield curves

- Thus far, we have assumed spot rates are available
  - e.g., via zeroes
- In practice, only 3 and 6-month zeroes are available
  - 1-year zeroes used to be available
  - No zeroes after 1 year
- How do we construct zeros/spot rates beyond six months?

Could we use Treasury STRIPS?

- **S**eparate **T**rading of **R**egistered **I**nterest and **P**rincipal of **S**ecurities (since Feb' 85)
  - STRIPS let investors trade coupons and principal separately
  - STRIPS can give you the zero yield beyond the first year
- However: STRIPS are not as liquid as the regular bonds

U.S. Treasury Strips

Monday, September 09, 2013

U.S. zero-coupon STRIPS allow investors to hold the interest and principal components of eligible Treasury notes and bonds as separate securities. STRIPS offer no interest payment; investors receive payment only at maturity. Quotes are as of 3 p.m. Eastern time based on transactions of $1 million or more. Yields calculated on the ask quote.

Maturity	Bid	Asked	Chg	Asked yield
Treasury Bond, Stripped Principal				
2015 Feb 15	99.563	99.577	0.051	0.30
2015 Aug 15	99.143	99.162	0.038	0.44
2015 Nov 15	98.866	98.887	0.057	0.51
2016 Feb 15	98.443	98.466	0.092	0.64
2016 May 15	98.068	98.094	0.109	0.72
2016 Aug 15	97.476	97.504	0.148	0.87
2016 Nov 15	96.872	96.902	0.185	0.99
2017 May 15	95.551	95.586	0.210	1.23
2017 May 15	95.569	95.604	0.144	1.23
2017 Aug 15	94.950	94.987	0.219	1.31
2018 May 15	92.736	92.779	0.329	1.61
2018 Nov 15	91.063	91.110	0.309	1.81
2019 Feb 15	90.251	90.300	0.303	1.89
2019 Aug 15	88.473	88.525	0.295	2.07
2020 Feb 15	86.719	86.774	0.339	2.22
2020 May 15	85.842	85.898	0.340	2.29

What to do?

- In practice, spot rates are obtained from the coupon bond prices through a procedure known as bootstrapping
  - We always have the first spot rate, $r_{.5}$, from the 6-month T-bill.
  - Suppose we have a 1-year coupon bond (e.g., a two year bond that is one year old). Can we solve for the next spot rate from a coupon bond expiring in 1 year?

$$P(1) = \frac{\frac{C}{2}}{(1+\frac{r_{.5}}{2})} + \frac{F+\frac{C}{2}}{(1+\frac{r_1}{2})^2}$$

Bootstrapping the Yield Curve

- Suppose $r_{.5}$=.08 and there is a \$100 par 8.5% bond maturing in 1 year whose price is \$100.19
- The cash flows for this bond are
  - 4.25 at time 1 (6 months)
  - 104.25 at time 2 (1 year)
- What is r_1?

$$100.19 = \frac{4.25}{(1+\frac{0.08}{2})^1} + \frac{104.25}{(1+\frac{r_1}{2})^2}$$

Bootstrapped Spot Rates

- Solve for r_1:

$$100.19 = \frac{4.25}{(1+\frac{0.08}{2})^1} + \frac{104.25}{(1+\frac{0.083}{2})^2}$$

- Solve for r_1: 0.083
- Can continue with this procedure all the way out the yield curve.
  - Get a 1.5 year bond and solve for $r_{1.5}$.

Example: Pricing bonds from term structure

- Price a T-note with exactly 2 years to maturity and 5.5% coupon from the current spot curve.
- $r_{.5} = 1.90\%, r_1 = 2.1\%, r_{1.5} = 2.25\%$ and $r_2 = 2.4\%$

Time	6 Months	1 Year	1 1/2 Years	2 Years
Cashflow	\$2.75	\$2.75	\$2.75	\$102.75

- Price each coupon as a zero, then add them up

Where are we?

$$d_t = \frac{1}{(1+\frac{r_t}{2})^{2t}} \qquad P_t = \sum_{t=1}^{2T} d_{\frac{t}{2}} \frac{C}{2} + d_t F$$

Time	Cash Flow	Spot rate	Discount factor	Present Value
0.5	2.75	1.90%	0.9906	2.7241
1	2.75	2.10%	0.9793	2.6931
1.5	2.75	2.25%	0.9670	2.6592
2	102.75	2.40%	0.9534	97.9625
			Price	106.0390

Where are we?

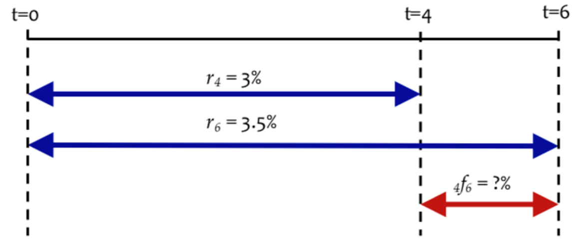
- We now know the basics of pricing Treasury securities
- Can we lock a rate of an investment which starts in 4 years from now and lasts for 6?
- The answer is "forward rates".

Forward rates

Forward Rates

An interest rate forward contract is a contract today that fixes the interest rate for a loan in the future.

- Let r_t be the spot interest rate between today and a future date t
- Then the forward rate ${}_xf_y$ is the interest rate per annum between future dates t=x and t=y that can be locked in today:
  - E.g. spot rates for t=4 and t=6 years are 3% and 3.5% pa, respectively. What is the forward rate that can be locked in for borrowing/lending between t=4 and t=6?



Example: Forward Rates

- Spot rates for t=4 and t=6 years are 3% and 3.5% p.a., respectively. What is the forward rate that can be locked in between t=4 and t=6?
  - Consider zero coupon bonds maturing at t=4 and t=6, priced at:

$$P_4 = \frac{1000}{(1.03)^4} = 888.5 \quad P_6 = \frac{1000}{(1.035)^6} = 813.5$$

  - Cashflows on zero coupon bonds:

	t = 0	t = 4	t = 6
Buy 4yr ZCB	-888.5	1000	0
Buy 6yr ZCB	-813.5	0	1000

Forward Rates

- Consider the following strategy that locks in the forward rate:

	t = 0	t = 4	t = 6
Short one 4 − yr ZCB	888.5	-1000	0
Buy $P_4/P_6 = 1.09219$ of 6 − yr ZCB	-888.5	0	1092.19
Net	0	-1000	1092.19

- Net: invest 1000 at t=4, receive 1092 at t=6, implying a fwd. rate of:

$$1000\,(1 + {}_4f_6)^2 = 1092.19 \quad {}_4f_6 = 4.51\%$$

- And, substituting back that P_4/P_6=1.09219 we get the relationship between spot and forward rates

$$(1 + r_4)^4 \cdot (1 + {}_4f_6)^2 = (1 + r_6)^6$$

- Investing at 4yr spot rate for four years and then at 2yr forward rate starting year 4 is equivalent to investing at 6yr spot rate for 6 years

Forward Rates

- Note that the relationship between spot and forward rates has to hold by no arbitrage arguments
- Relationship between spot and forward rates and shape of the zero coupon yield curve:
  - If zero yield curve is flat, forward rates equal the spot rate
  - If zero yield curve is upward (downward) sloping then forward rates exceed (are below) spot rates
  - E.g. previous example:

    $r_4 = 3.0\%$

    $r_6 = 3.5\%$

$\Longrightarrow \; {}_4f_6 = 4.51\%$

${}_4f_6 > r_6 > r_4$

Example: Forward Rates vs future spot rates

Example: You want to invest for two years in risk-free bonds. The current one-year, zero-coupon yield is 5%. The one-year forward rate is 6%. What is the best strategies: (1) buy a two-year bond, (2) buy a one-year bond and enter into an interest rate forward contract to guarantee the rate in the second year, or (3) buy a one-year bond and forgo the forward contract, reinvesting at whatever rate prevails next year. Under what scenarios would you be better off following the risky strategy?

Answer: Both strategies (1) and (2) lead to the same risk-free return of

$$(1+r_2)^2 = (1+r_1)^1 (1+{}_1f_2)^1 = (1.05)(1.06)$$

Strategy (3) is given by: $(1+0.05)(1+\tilde{r}_1)$ where $\tilde{r}_1$ is the one-year interest rate next year.

Example: Forward Rates

- You are given the following spot rates, annual compounding:

Year	Spot Rate
1	5.0%
2	5.4%
3	5.7%
4	5.9%
5	6.0%
6	4.8%

- How could you make a with this term structure?

Forward Rates

- Let us compute the forward rates

Year	Spot Rate	ZCB Price	Forward Rate
1	5.0%	$1/(1.05^1)=0.952381$	
2	5.4%	$1/(1.054^2)=0.900158$	0.952381/0.900158-1=5.8%
3	5.7%	$1/(1.057^3)=0.846789$	0.900158/0.846789-1=6.3%
4	5.9%	$1/(1.059^4)=0.79509$	0.846789/0.79509-1=6.5%
5	6.0%	$1/(1.06^5)=0.747258$	0.79509/0.747258-1=6.4%
6	4.8%	$1/(1.048^6)=0.754801$	0.747258/0.754801-1=-0.999%

- The last forward rate is negative. If you are able to borrow and lend at these rates, this is an arbitrage opportunity:
  - Borrow \$1Billion at 4.8% for 6 years.
  - Invest \$1Billion at 6% for 5 years and keep the cash in the last year.

Forward Rates

- Debt to be repaid $= \$1\text{B}\times(1.048)^6 = \$1.32485\text{B}$
- Revenues from investment $= \$1B \times (1.06)^5 = \$1.33823B$
- The total profits is 0.01338B or $13 M.
- This is one of the most typical trades of investment banks.

Generalizing Forward Rate Notation

- The rate on a loan beginning in t periods and lasting for x periods
  - "in t periods for x periods forward rate"
- Use ${}_tf_{t+x}$ to denote the forward rate on a loan
  - Starts t years from now
  - Lasts for x years (until t+ x)
- Examples:
  - ${}_1f_{1.5}$: rate on a 6-month loan, 1 year from now
  - ${}_5f_{5.5}$: rate on a 6-month loan, 5 years from now
  - ${}_{.5}f_3$: rate on a 2.5-year loan, 6 months from now
  - ${}_7f_{10}$: rate on a 3-year loan, 7 years from now

The generalized case

- We can obtain any forward rate ${}_tf_{t+x}$ by solving a simple equation:

$$\left(1+\frac{r_{t+x}}{m}\right)^{m(t+x)} = \left(1+\frac{r_t}{m}\right)^{mt} \times \left(1+\frac{{}_tf_{t+x}}{m}\right)^{mx}$$

where m is the number of compounding periods per year

Example: T-bills

- On 09/09/13:
  - 6 month: $r_{.5}$=0.04%
  - 1 year: r_1=0.12%
- What is the "in 6M for 6M" forward rate?
  - Solve the following equation:

$$\left(1+\frac{0.0012}{2}\right)^2 = \left(1+\frac{0.0004}{2}\right)\left(1+\frac{{}_{0.5}f_1}{2}\right)$$

- Simplifying, we have

$${}_{0.5}f_1 = 2\left[\frac{\left(1+\frac{0.0012}{2}\right)^2}{\left(1+\frac{0.0004}{2}\right)} - 1\right] = 0.002 = 0.2\%$$

Summary of what we have done.

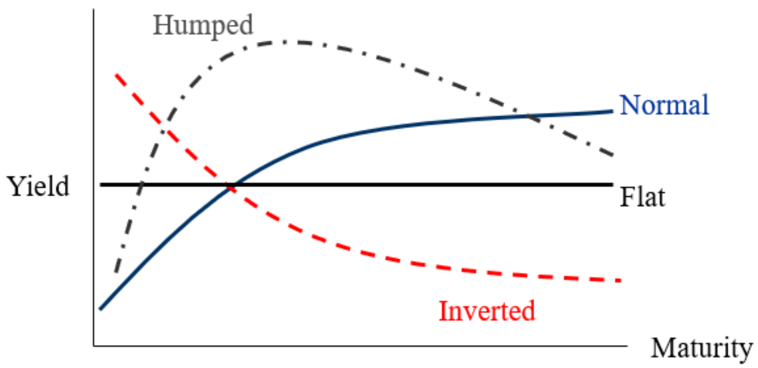
- Bonds are priced by computing the present value of all their cash flows at appropriate spot rates
- These discount rates can be obtained through zeros
- Plotting the zeros, or spot rates, against time to maturity gives the term structure of interest rates
- Forward interest rates are arbitrage-free rates that allow locking in future rates

Facts about the Term Structure.

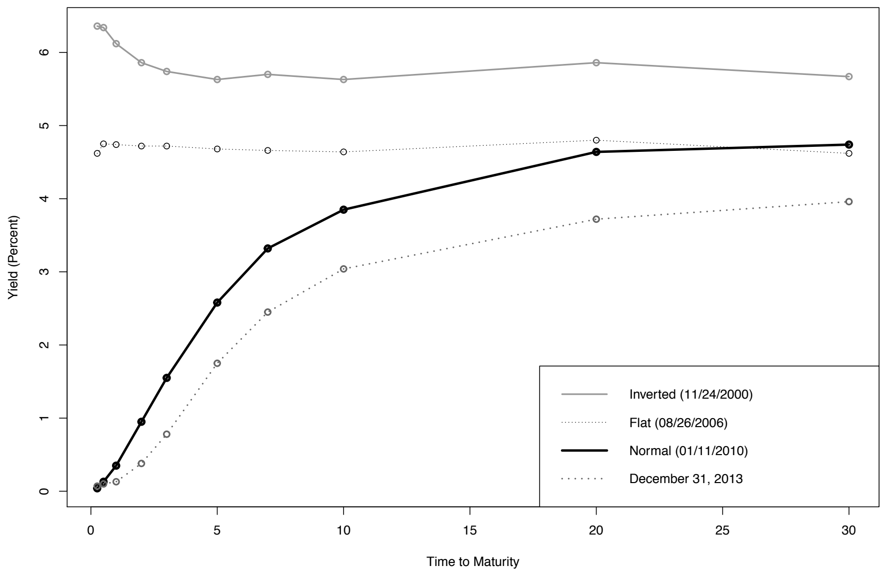
- The term structure can have many shapes
  - Inverted yield curves thought to be a predictor of recessions (Harvey 1986).
- Spot rates, both for short and long maturities, move substantially over time

What about the shape?

What might explain different shapes?



What about the shape?



Shape of the Yield Curve

- Short end determined by monetary policy (e.g. target Fed Funds rate in US)
- Remainder depends on several factors, primarily
  - Market's expectations of future rates
  - Risk premium
  - Supply and demand
- "Theories" of the term structure propose explanations based (mostly) on one of these factors
- Empirical evidence indicates all three play a role

Role of Expectations

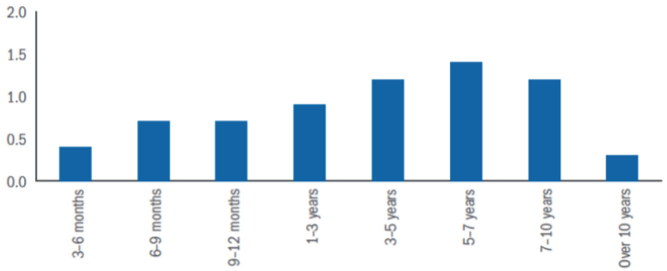
- If future spot rates were known with certainty then, by no arbitrage the forward ratefor a future period is the future spot rate in that period
- In the expectations hypothesis the forward rate $({}_1f_2)$ is the expected future spot rate $(\tilde{r}_1)$: $(1+r_2)=(1+r_1)(1+{}_1f_2)=(1+r_1)(1+\tilde{r}_1)$
  - therefore if spot curve is upward sloping (implying forward rates are higher than spot rates), you expect future spot rates to be higher than today
  - inconsistent with yield curves that are mostly upward sloping
- Evidence: forward rates tend to overstate future spot rates
- Expectations do matter: all else equal, yield curve steepens (flattens) if expected future spot rates go up (down)

Role of Risk Premium

- Investors in longer in longer maturity bonds need to earn a risk premium to compensate them for greater interest rate risk
- All else equal, this should result in an upward sloping yield curve
- Historical excess returns by maturity:

Exhibit 3: In the past, longer term bonds have outperformed bills in the U.S...

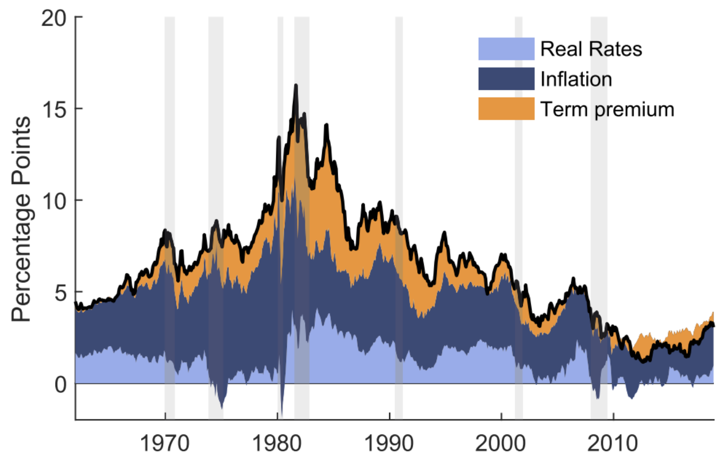
Treasury excess returns, 1952–2009 (%)



Source: Antti Ilmanen. *Expected Returns*. John Wiley & Sons, 2011

Decomposition of a 5 year bond yield (Based on my research)

(joint work with Amir Yaron)



bond yield = expected real short rates + average inflation + term premium

Role of Supply/Demand

Preferred habitat or market segmentation hypothesis:

- Curve is shaped by supply and demand at various maturities
- This can lead to any shape of curve – e.g. if most investors prefer short-run investing then this could also explain the traditional upward sloping shape of the yield curve
- Commonly accepted that high demand from pension funds and insurance companies for long dated bonds depresses yields at the long end

Shape & Dynamics of Yield Curve

In summary, all components matter:

- Market's expectations of future rates
- Bond risk premia
- Supply and demand

Exact drivers of (changes in) the shape of the yield curve hard to pin down

- E.g. curve steepening can reflect either an increase in market's expectations of future rates or an increase in risk premia, or both. . .

Slope of the Yield Curve

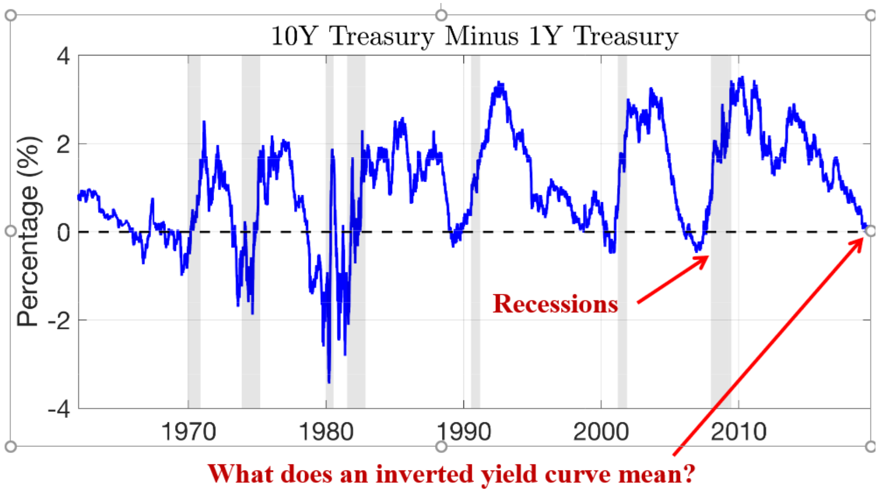
What we observe:

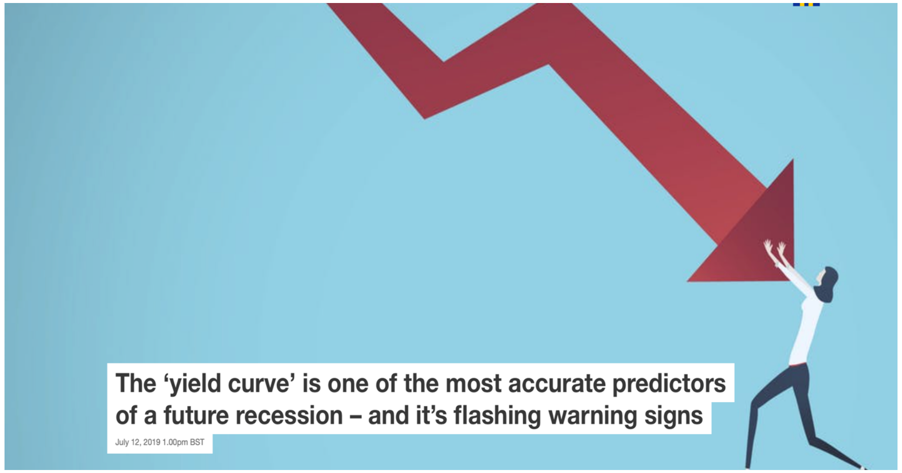
- Usually upward sloping
- Yields on short maturity bonds are more volatile than yields on long-maturity bonds
- Negative slope tends to precede recessions

Focus on the slope of the yield curve

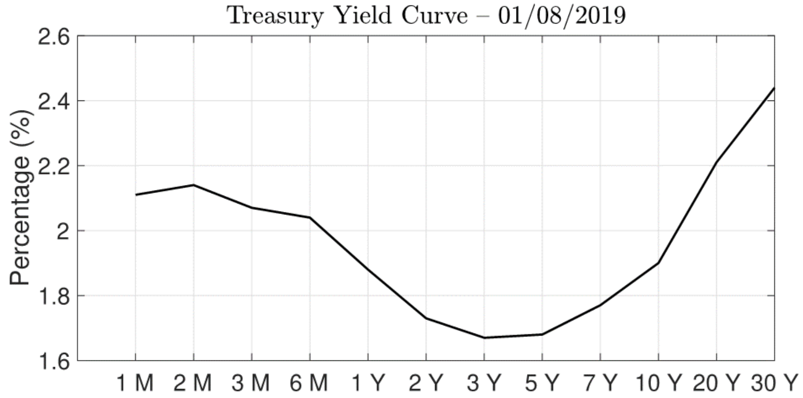
- Slope=long yield-short yield

Time Series of the Slope (10Y-1Y)

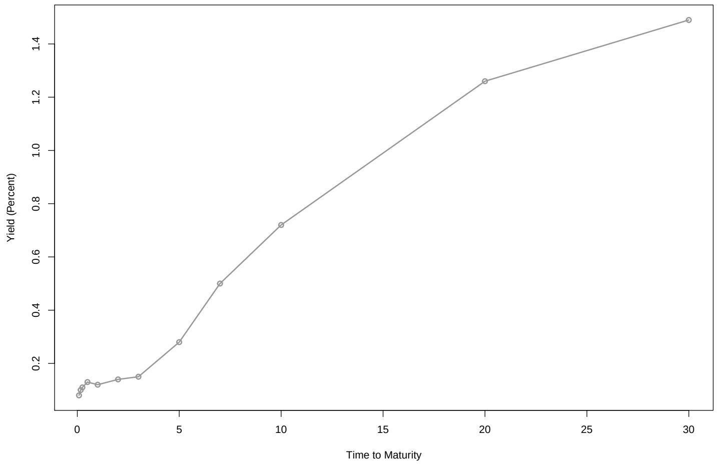




Yield Curve last year

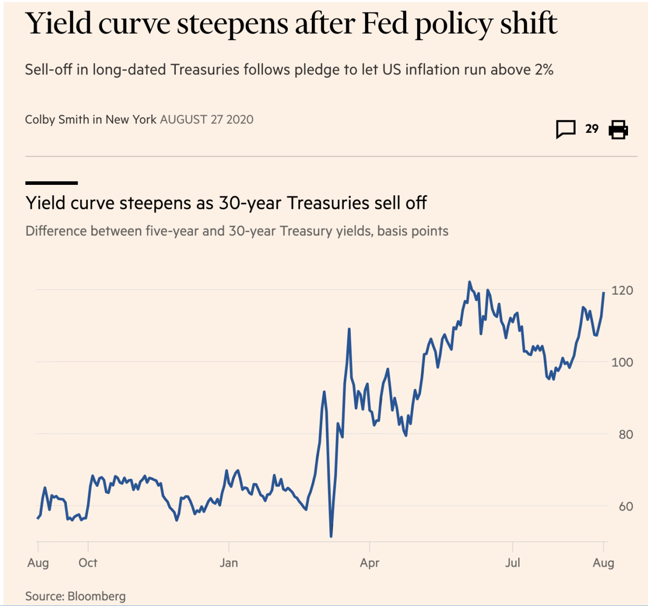


Yield Curve – 31/08/2020



30-year Treasury yields – 27/08/2020

Link to the Financial times



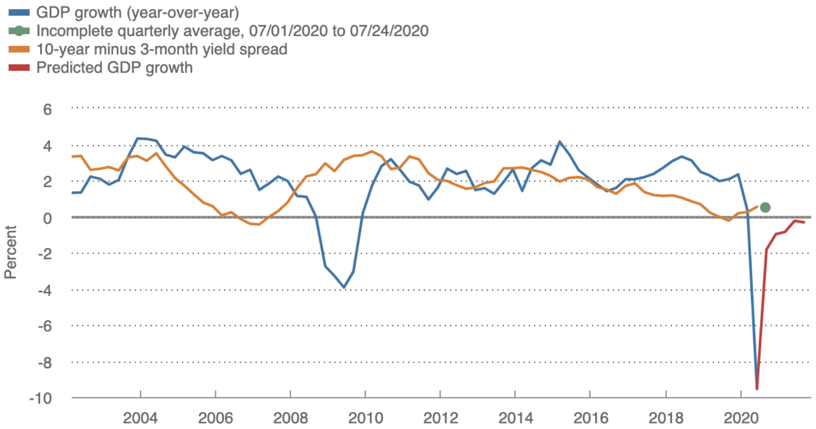
Yield curve steepens after Fed policy shift

Sell-off in long-dated Treasuries follows pledge to let US inflation run above 2%

Colby Smith in New York AUGUST 27 2020

29

Yield Curve predicted GDP Growth



Group project idea?

Does the yield curve predict market returns?

- Idea: recessions are times of significant market downturns. We could avoid those downturns by being out of the market during these times?
- Strategy?
  - Stay out of the market when the yield curve predicts we are in a recession?

Do you see any problem with this strategy?

Interest Rate Risk and Duration

Interest Rate Risk and Duration

We saw that the bond prices depend on interest rates.

- The returns of all fixed income securities are highly correlated, because they all depend on the interest rate.

How do interest rate changes affect my portfolio?

- Portfolio managers and firms need to know how the value of their bond portfolio responds to changes in interest rates and then manage this sensitivity.

Interest Rate Risk: Maturity

How does bond price sensitivity to interest rates depend on bond maturity?

- Suppose the yield curve is flat at 8%. Consider two 8% annual coupon bonds. Bond #1 has 15 years to maturity, bond #2 has 30 years. Both sell for $1,000.
- Now let's see what happens if the yield curve moves up by 1% (or 100 basis points):
  - Bond #1 will now sell for $919.39 (a 8.1% drop in value)
  - Bond #2 will now sell for $897.26 (a 10.3% drop in value).

Other things equal, the longer the time to maturity, the more sensitive is the price of a bond to changes in the interest rate.

- A rare exception to this general rule: deep discount bonds.

Interest Rate Risk: coupon rates

How does bond price sensitivity to interest rates depend on coupon rates?

- Suppose the yield curve is flat at 8%. Consider two 15 year bonds. Bond #1 has an 8% annual coupon rate and sells for $1,000, bond #2 has a 3% annual coupon rate and sells for $572.03.
- Now let's see what happens if the yield curve moves up by 1% (or 100 basis points):
  - Bond #1 will now sell for $919.39 (a 8.1% drop in value)
  - Bond #2 will now sell for $516.36 (a 9.7% drop in value).

Other things equal, the lower the coupon rate, the more sensitive is the price of a bond to changes in the interest rate.

- Long maturity pure discount bonds are most sensitive to changes in the interest rate.

Duration: Overview

Duration: measuring and managing interest rate risk

- Duration of an individual bond
- Duration of a portfolio
- Duration-neutral strategies (hedging and speculation)

Duration: Introduction

- Price of a bond is the future cash flows discounted at the relevant interest rates:

$$P = \frac{C}{(1+r_1)} + \frac{C}{(1+r_2)^2} + \frac{C}{(1+r_3)^3} + \dots \frac{C+100}{(1+r_T)^T}$$

- How do we measure sensitivity of bond price to interest rate changes?
- **Problem**: bond prices change due to changes in many different interest rates.
- Simplify: the bond price is also the future cash flows discounted at the bond yield:

$$P = \frac{C}{(1+y)} + \frac{C}{(1+y)^2} + \frac{C}{(1+y)^3} + \dots \frac{C+100}{(1+y)^T}$$

- For a single bond define duration as the sensitivity of the bond price with respect to the bond's own yield

Duration Measures

Common duration measures:

- Dollar Duration, \$D – dollar change in the bond price for a unit change in yield
- Modified Duration D_{MOD} (=\$D/P) – percentage change in the bond price for a unit change in yield
- Macaulay Duration (=$D_{MOD} \times (1+y)$) – weighted average time to cash flows (weights equal to fraction of total bond price contributed by PV of the given cash flow)

They are:

- valid for small changes in yields
- defined to be positive for standard bonds (whose prices are inversely related to yields)

Duration Measures– Coupon Paying Bonds

Mathematical definitions of duration (wrt yield, y) for an individual coupon paying bond (assuming annual cashflows CF_t):

- Bond Price:

$$P = \sum_{t=1}^{T} \frac{CF_t}{(1+y)^t} = \sum_{t=1}^{T} PV^*(CF_t)$$

- \$Duration:

$$\$D = -\frac{dP}{dy} = \frac{1}{1+y} \sum_{t=1}^{T} t \times PV^*(CF_t)$$

- Modified Duration:

$$D_{MOD} = -\frac{1}{P}\frac{dP}{dy} = \frac{1}{1+y} \sum_{t=1}^{T} t \times \frac{PV^*(CF_t)}{P}$$

where PV^* denotes PV when discounting at the yield (NOT interest rates)

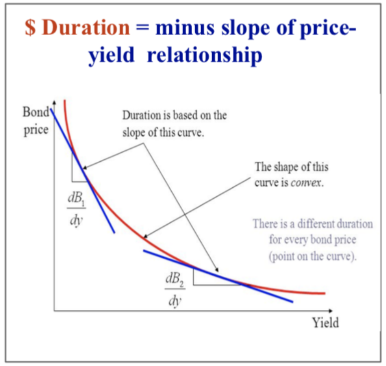
Duration Measures– Coupon Paying Bonds

In addition, we have the notion of Macaulay Duration, which is closely related to Modified Duration:

$$D_{MAC} = (1+y) \times D_{MOD} = -\frac{1+y}{P}\frac{dP}{dy} = \sum_{t=1}^{T} t \times \frac{PV^*(CF_t)}{P}$$

- Not really a sensitivity (but close)
- The weighted average time to cash flows (weights equal to fraction of total bond price contributed by PV of the given cash flow and therefore sum to 1)
- Useful for understanding properties of duration rather than for measuring interest rate sensitivities, e.g.:
  - Macaulay Duration of a zero coupon bond is equal to its maturity
  - Macaulay Duration of a coupon paying bond is less than its maturity

Duration Relationships: Review



Modified Duration
= \$ Duration/Price

Macaulay Duration
= Modified Duration x (1+y)
weighted average time to cashflows

Modified Duration
= Macaulay Duration/(1+y)

\$ Duration
= Modified Duration x Price

Coupon Paying Bond Example

- **Example**: 6yr, 10% annual coupon bond - notional 1,000, at 8% yield

$$D_{MAC} = \sum_{t=1}^{T} t \times \frac{PV^*(CF_t)}{P}$$

- Approach: Start by calculating Macaulay Duration:

t	CF	PV*(CF) at 8%	"weight" = PV*(CF)/Price	weight times t
1	100	92.59	0.08	0.08
2	100	85.73	0.08	0.16
3	100	79.38	0.07	0.22
4	100	73.50	0.07	0.27
5	100	68.06	0.06	0.31
6	1100.00	693.19	0.63	3.81
Sum		1092.46	1.00	4.85
Price	1092.46			
$ Duration	4903.36	=Mod Dur × Price		
Mod Duration	4.49	=Mac Dur/1.08		
Mac Duration	4.85			

Properties of Modified Duration

Modified Duration is a function of coupon, yield and maturity:

	coupon			
yield	0%	5%	10%	20%
4%	5.77	5.14	4.77	4.34
8%	5.56	4.88	4.49	4.06
12%	5.36	4.63	4.23	3.79

- Maturity and yield held constant, lower coupon $\Longrightarrow$ higher duration
- Maturity and coupon held constant, lower yield $\Longrightarrow$ higher duration
- Holding coupon and yield constant, a bond's duration generally increases with maturity (always the case for bonds selling at par or at a premium to par)

D_{MAC} for semi-annual bonds

- Formula:

$$D_{\mathrm{mac}}(T) = \sum_{t=1}^{2T} \frac{t}{2} \times w_t = \sum_{t=1}^{2T} \frac{t}{2} \frac{\left(\frac{\frac{c}{2}}{\left(1+\frac{y_T}{2}\right)^t}\right)}{P_T} + \frac{2T}{2} \frac{\left(\frac{F}{\left(1+\frac{y_T}{2}\right)^{2T}}\right)}{P_T}$$

$$D_{mac} = \left(1 + \frac{y}{2}\right) D_{MOD}$$

- Notice we have time to cash flows (t/2 or T) weighted by present value of the cash flow received.
- In general modified duration equals Macaulay Duration divided by

$$\left(1 + \frac{y}{n}\right)$$

Where n is the number of coupon periods per year

D_{MAC} Example

Compute the duration of an 8% coupon (semiannual coupon payments) and a zero-coupon bond, each with two years to maturity. Assume that the yield to maturity on each bond is 10%.

(1) Time (yrs)	(2) CF	(3) Discounted CF	(4) Weight	(1)*(4)
0.5	40	38.095	0.0395	0.0197
1	40	36.281	0.0376	0.0376
1.5	40	34.554	0.0358	0.0537
2	1040	855.611	0.8871	1.7741
Sum		964.540	1.000	**1.8852**$=D_{mac}$

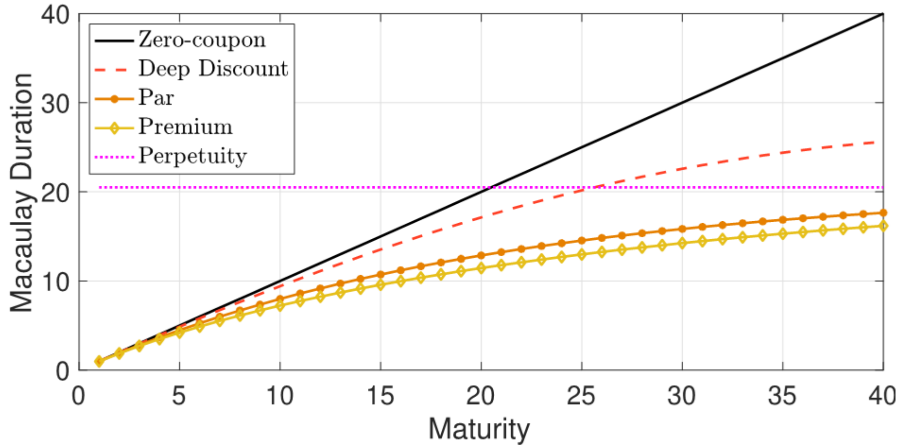
Macaulay Duration for Zeros

- Zero-coupon Bond

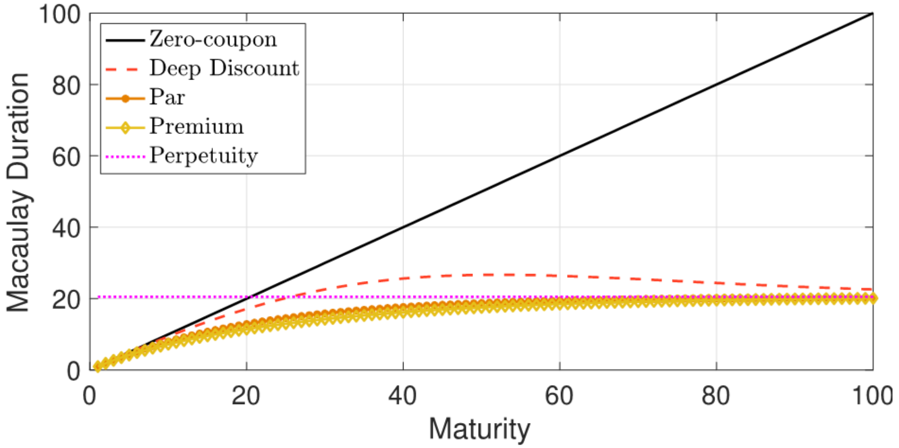
(1) Time (yrs)	(2) CF	(3) Discounted CF	(4) Weight	(1)*(4)
0.5	0	0.000	0.000	0.000
1	0	0.000	0.000	0.000
1.5	0	0.000	0.000	0.000
2	1000	822.702	1.000	2.000
Sum		822.702	1.000	**2.000**$=D_{mac}$

- The Macaulay duration of a zero-coupon bond is exactly exactly equal to its time to maturity.

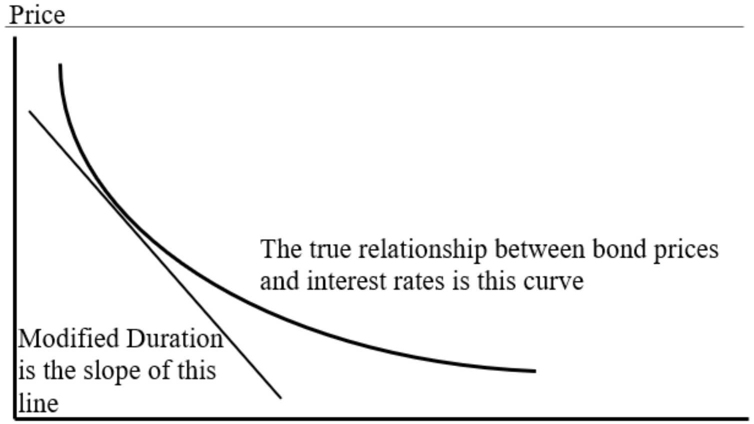
Coupon and Macaulay Duration



Coupon and Macaulay Duration



Interest Rate Risk and Duration



Duration of a Portfolio

- Consider a portfolio of n_A units of bond A and n_B units of bond B with value:

$$\Pi = n_A \cdot P_A + n_B \cdot P_B$$

- Define/derive durations of the portfolio:

$$\begin{aligned} \$\text{Duration} &= n_A \cdot \$D^A + n_B \cdot \$D^B \\ \text{Modified Duration} &= \tfrac{n_A P_A}{n_A P_A + n_B P_B} \cdot D^A_{MOD} + \tfrac{n_B P_B}{n_A P_A + n_B P_B} \cdot D^B_{MOD} \\ &= \omega_A \cdot D^A_{MOD} + \omega_B \cdot D^B_{MOD} \end{aligned}$$

where ω_i is the portfolio weight (percentage of total \$) of bond i

- Portfolio durations have the standard interpretations assuming all bond yields move up and down together (parallel shifts of the yield curve) by the same (small) amount.

Duration-Neutral Strategies

General idea of duration-neutral strategies:

- Set up a position so that it is duration neutral ie has zero duration
- Therefore, you will be hedged against small parallel changes in yields
- If yields move in a different manner, then the hedge will be less effective (could either make or lose money)

Duration neutral positions can be used in different contexts:

- **Hedging**: primary objective is to hedge interest rate risk of position
- **Speculation**: set up a position so that it is unaffected by small parallel changes in yields with a primary objective of taking a view on a different type of yield shift (e.g. steepening or flattening of the yield curve)

Three types of hedging

1. Construct a bond portfolio with zero duration
2. Immunisation
3. Zero-cost hedging

1- Construct a bond portfolio with zero duration

- Suppose we want to hedge a portfolio with YTM y and price $P(y)$.
- The idea is to consider an asset with a different YTM, y_1, with price denoted by $Q(y_1)$, and to build a *joint* portfolio with value equal to H.

$$H = P(y) + \delta Q(y_1)$$

where δ denotes the amount invested in the hedging instrument

- The goal is to make the *joint* portfolio insensitive to small interest-rate variations

$$(P'(y) + \delta Q'(y_1))\, dy = 0 \implies \$D^P + \delta \$D^Q = 0$$

- Hence, we can solve for δ to get

$$\delta = -\frac{P'(y)}{Q'(y_1)} = -\frac{\$D^P}{\$D^Q} = -\frac{P(y) D^P_{MOD}}{Q(y_1) D^Q_{MOD}} \tag{1}$$

See Question 3(c) of Problem Set 2.

- How to invest assets to fund fixed income(-like) liabilities while hedging or minimising interest rate risk
- Of great importance to banks (ALM), defined benefit pension funds (LDI), insurance companies
- Cleanest solution is to invest in zero coupon bonds that exactly match the liabilities ("cash-matching"):

$$PV(assets) = PV(liabilities)$$

where the cash flow of the assets (here zero coupon bonds) will be used to match our expenses.

- In practice "cash-matching" may be hard to implement given that zero coupon bonds are only available upto 1 year and STRIPS are relatively illiquid

- Alterntaively, invest in a portfolio of bonds today such that

$$\text{Rest. 1:} \qquad PV(assets) = PV(liabilities)$$

- However, choosing our bonds such that they satisfy Restriction 1 does not guarantee that we are also matching the duration of our liabilities.
- Therefore, we need an extra restriction that matches the duration of our assets (portfolio of bonds) with the duration of our liabilities:

$$\text{Rest. 2:} \quad \$ \text{ Duration of assets} = \$ \text{ Duration of liabilities} \quad \textbf{or}$$

$$\text{Rest. 2':} \quad \text{Mod. Duration of assets} = \text{Mod. Duration of liabilities}$$

This will "immunise" the liabilities against small parallel moves in yields.

Matching Dollar Duration

- In order to match the dollar duration we need to solve:

$$PV(assets) = PV(liabilities) \implies \eta_A P_A + \eta_B P_B = P_l$$

$$\$\text{Dur. of assets} = \$\text{Dur. of liab.} \implies \eta_A \$D^A + \eta_B \$D^B = \$D^l$$

- where η_i denotes the number of bonds that we need to buy of bond i, while $\$D^i$ denotes the dollar duration of bond i
- By construction, the *total cost* of this hedge ($\eta_A P_A + \eta_B P_B$) is equal to the value of the liabilities (P_l). That is, $P_l = \eta_A P_A + \eta_B P_B$

Matching Modified Duration

- To match modified duration we need to transform the previous equations
- Let's start from $PV(assets) = PV(liabilities)$:

$$\eta_A P_A + \eta_B P_B = P_l$$

$$\text{divided by } P_l \implies \frac{\eta_A P_A}{P_l} + \frac{\eta_B P_B}{P_l} = 1$$

$$\implies \omega_A + \omega_B = 1$$

- where ω_i denotes the fraction of your wealth (liabilities) in bond i (i.e., portfolio weights).

Matching Modified Duration

- Let's move to the second restriction: Again, start from

  \$ Duration of assets = \$ Duration of liabilities:

$$\eta_A \$D^A + \eta_B \$D^B = \$D^l$$

$$\text{div. by } P_l: \implies \frac{\eta_A \$D^A}{P_l} + \frac{\eta_B \$D^B}{P_l} = \frac{\$D^l}{P_l}$$

$$\text{div. and mult. } P_i: \implies \frac{\eta_A P_A \frac{\$D^A}{P_A}}{P_l} + \frac{\eta_B P_B \frac{\$D^B}{P_B}}{P_l} = \frac{\$D^l}{P_l}$$

$$\text{by def. of Modified duration: } \implies \frac{\eta_A P_A}{P_l} D^A_{Mod} + \frac{\eta_B P_B}{P_l} D^B_{Mod} = D^l_{Mod}$$

$$\text{by def. of Portafolio weights: } \implies \omega_A D^A_{Mod} + \omega_B D^B_{Mod} = D^l_{Mod}$$

Matching Modified Duration

- Hence, putting everyhting together we arrive to the following equations

$$\text{Rest. } 1 \implies \omega_A + \omega_B = 1$$

$$\text{Rest. } 2' \implies \omega_A D^A_{Mod} + \omega_B D^B_{Mod} = D^l_{Mod}$$

- Because it is the same system of equations as *Matching Dollar Duration*, the solution will be the same.
- $\omega_i \times 100\%$ will tell you how much of bond i you need to buy as percentage of your liabilities. To go from ω_i to the numbers of bonds i we just need to solve:

$$\omega_i = \frac{\eta_i P_i}{P_l} \implies \eta_i = \frac{\omega_i P_l}{P_i}$$

- To sum up: invest in a portfolio of bonds today such that:
  - PV(assets) = PV(liabilities)
  - (\$ or Modified) Duration of assets = (\$ or Modified) Duration of liabilities
- This will "immunise" the liabilities against small parallel moves in yields
- For big moves, the hedge will not be as effective and will need to be "reset"
- Note that there are other issues with this approach e.g. reinvestment risk etc

- The XYZ Pension Fund has an obligation to pay \$1m in 10 years
- Assume that all interest rates/yields are 5% pa
- What positions in the two following bonds should XYZ invest in so that:

PV(assets)=PV(liabilities)

Duration(assets)=Duration(liabilities)

	Mat.	Coupon (Annual)	Price	\$Duration	$D_{MOD.}$	D_{MAC}
Bond A	8	4%	935.37	6,201.94	6.6305	6.9620
Bond B	20	3%	750.76	10,348.87	13.7846	14.4738
Liability	10	–	613,913	5,846,792.89	9.5238	10

2- Hedging – Immunisation: Example

- To be immunised, solve either for number of bonds such that:

$$\begin{aligned} PV\text{ (assets)} = PV\text{ (liabilities)} \quad & n_A 935.37 + \quad n_B 750.76 \quad = 613,913 \\ \$D\text{(Assets)} = \$D\text{(liabilities)} \quad & n_A 6,201.94 + \quad n_B 10,348.89 \quad = 5,846,729.89 \end{aligned}$$

Solution: $n_A = 390.89$ and $n_B = 330.71$

- Or for portfolio weights such that:

$$\omega_A + \omega_B = 1$$
$$\omega_A 6.6305 + \omega_B 13.7846 = 9.5238$$

Solution: $\omega_A = 59.6\%$ and $\omega_B = 40.4\%$

2- Hedging – Immunisation: Example

How does the duration hedge perform?

Yields	**0.04**	**0.05**	**0.06**
PV(liab)	675,564.17	613,913.25	558,394.78
Bond A			
· Price	1,000.00	935.37	875.80
- Number	390.89	390.89	390.89
- Total Value	390,893.79	365,629.49	342,346.39
Bond B			
- Price	864.10	750.76	655.90
- Number	330.71	330.71	330.71
- Total Value	285,766.95	248,283.77	216,914.62
PV(assets)	676,660.73	613,913.25	559,261.01
PV(assets) - PV (liab)	**1,096.56**	**0**	**866.23**

3- Zero-cost hedging

- In this hedging we do not need any additional money to set up the hedge and hence the name “zero-cost hedging”.
- In contrast, for **Hedging – Immunisation** we needed to buy our assets such that $PV(assets) = PV(liabilities)$.
- Therefore we will necessarily need to take a short possition somewhere
- Let’s work with the modified duration case

3- Zero-cost hedging

- First we need that the duration of the portfolio is equal to zero:

$$\omega_I D^I_{Mod} + \omega_A D^A_{Mod} + \omega_B D^B_{Mod} = 0.$$

Note that we are adding new bonds to the original portfolio I

- In addition, because we want a zero cost of this hedge, the sum of the weights should be equal to 1:

$$\omega_I + \omega_A + \omega_B = 1.$$

- Finally, the initial portfolio I must be kept intact, hence we have the following restriction

$$\omega_I = 1.$$

- This implies that: $\omega_A + \omega_B = 0$ (or in \$ $\eta_A P_A + \eta_B P_B = 0$)

Hence, it does not cost us!

3- Zero-cost hedging

- Putting the pieces together, we have that the system of equations that the zero-cost hedging solves is

$$D^I_{Mod} + \omega_A D^A_{Mod} + \omega_B D^B_{Mod} = 0$$
$$\omega_A + \omega_B = 0$$

- As long as there is a difference in duration between the two bonds A and B, there will be corresponding weights in the two bonds that hedges the duration of the portfolio.

See Question 2 and Question 6 of Problem set 2.

3- Zero-cost hedging: Example - Final exam type of question

Example: Assume the following prices for \$100 face value zero-coupon bonds:

- six month zero = \$96
- eighteen month zero = \$90

Assume the following semi-annual coupon bond with twelve months to maturity:

- coupon rate = 6%
- face value = \$1,000
- yield to maturity = 9%

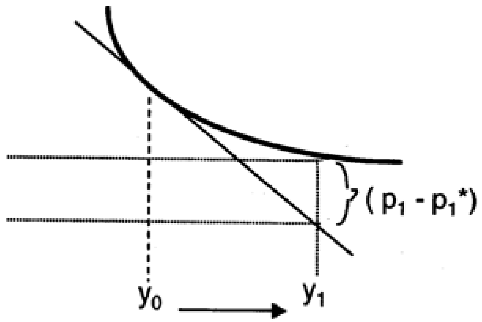
Use the zero-coupon bonds to create a zero-cost hedge portfolio (i.e., portfolio with present value = 0) that has the same interest rate risk as the coupon bond.

Problems with Duration

- Duration estimate assumes a parallel shift in the entire yield curve.
  - Short end is more volatile than the long end of curve
  - Parallel shift explains 66% of T-Bill yield variance
- **Duration is a straight line estimate to a curved (convex) price-yield function**

Problems with Duration

- Duration overestimates the change in bond price following an increase in yield from y_0 to y_1
- Error gets bigger as Δy increases
- What if the yield decreases?



Summary

- Bonds are priced by computing the present value of all their cash flows at appropriate spot rates
- These discount rates can be obtained through zeros
- Plotting the zeros, or spot rates, against time to maturity gives the term structure of interest rates
- Forward interest rates are arbitrage-free rates that allow locking in future rates
- Interest rate risk is measured by a bond's duration

Default Risk and Bond Ratings

Bonds and Credit Risk – Overview

- Introduction to credit risk and default
- Measures of credit risk
- Understanding yields and credit spreads of risky bonds
- Credit risk and duration

Fixed Income? Credit Risk & Default

- Bonds promise the holder pre-specified cashflows, but in practice the issuer (sovereign or corporate) may default
- A default typically sets in motion either:
  - Liquidation (corporates only)
  - Restructuring/reorganisation
- In default, the investor tipically gets some positive recovery value (less than face value)

Types of Credit Risk

- There are two main types of credit risk:
  - credit default risk is the risk of financial loss due to an issuer default
  - credit spread risk is the risk of financial loss due to changes in the level of an issuer's credit spreads/creditworthiness
- The key determinants of both types of credit risk are:
  - the probability of default
  - the severity of default – loss or recovery in default

Measures of Credit Risk

Traditional measures of credit risk in capital markets:

- **Credit Ratings**
- Credit Spreads
- Default Rates
- Recovery Rates

Credit Ratings

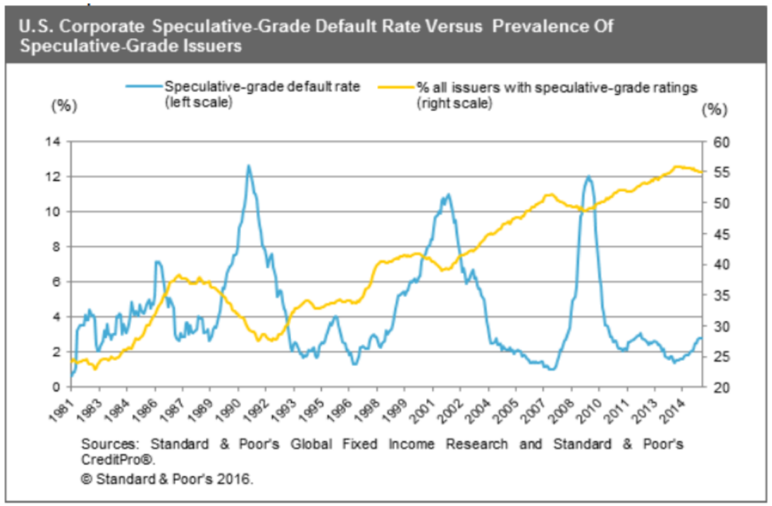
Credit Risk	Moody's*	Standard & Poor's	Fitch†
Investment Grade			
Highest quality	Aaa	AAA	AAA
High quality (very strong)	Aa	AA	AA
Upper medium grade (strong)	A	A	A
Medium grade	Baa	BBB	BBB
Below Investment Grade			
Lower medium grade (somewhat speculative)	Ba	BB	BB
Low grade (speculative)	B	B	B
Poor quality (may default)	Caa	CCC	CCC
Most speculative	Ca	CC	CC
No interest being paid or bankruptcy petition filed	C	C	C
In default	C	D	D

Source: Moody's, Standard & Poor's, Fitch

*The ratings from Aa to Ca by Moodys may be modified by the addition of a 1, 2 or 3 to show relative standing within the category, e.g., Baa2.

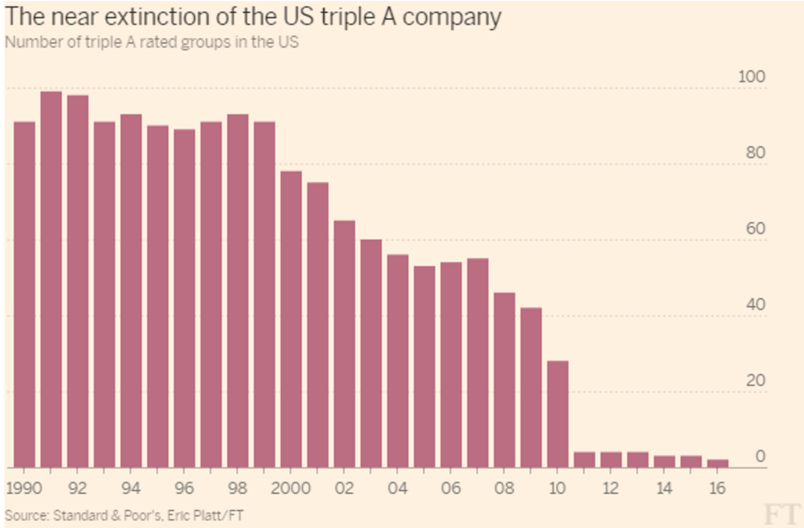
†The ratings from AA to CC by Standard & Poor's and Fitch may be modified by the addition of plus (+) or minus (-) sign to show relative standing within the category, e.g., A-.

Credit Ratings



Credit Ratings

Link to the Financial times



Determinants of Bond Ratings

Determinants of bond safety:

- Coverage ratios: ratios of earnings to fixed costs.
- Leverage ratio: debt-to-equity ratio.
- Liquidity ratios: current assets/current liabilities and current assets excluding inventories/current liabilities.
- Profitability ratios: return on assets.
- Cash flow-to-debt ratio

Bond Ratings and Financial Ratios

Median ratio data by bond rating: there is significant variation across industry and over time

Ratio	AAA	AA	A	BBB	BB	B	CCC
EBIT interest cover *	21.4	10.1	6.1	3.7	2.1	0.8	0.1
return on capital %	34.9	21.7	19.4	13.6	11.6	6.6	1
Gross profit margin %	27	22.1	18.6	15.4	15.9	11.9	11.9
Total debt/capital %	22.9	37.7	42.5	48.2	62.6	74.8	87.7
* Earnings before interst and tax divided by interest							

Measures of Credit Risk

Traditional measures of credit risk in capital markets:

- Credit Ratings
- **Credit Spreads**
- Default Rates
- Recovery Rates

Standard credit spread measures for corporate bonds:

1. **Difference in Yield-to-Maturity**: of the corporate bond and equivalent maturity government bond

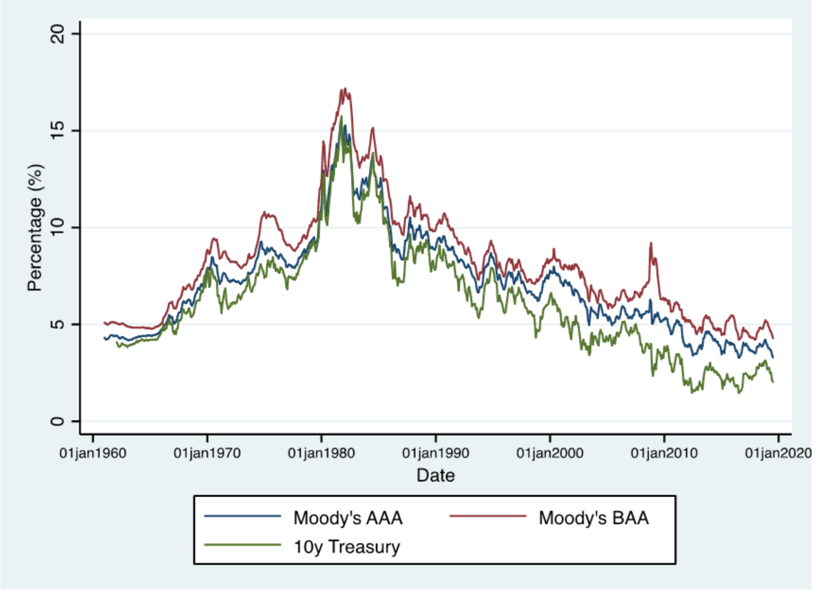
   3yr Government Bond with 2.5% annual coupon, trading at 100.925

$$100.925 = \frac{2.5}{(1+y)} + \frac{2.5}{(1+y)^2} + \frac{102.5}{(1+y)^3} \implies y = 2.1785\%$$

   3yr Corporate Bond with 4.5% annual coupon, trading at 99.325

$$99.325 = \frac{4.5}{(1+y)} + \frac{4.5}{(1+y)^2} + \frac{104.5}{(1+y)^3} \implies y = 4.7467\%$$

$$\text{Credit Spread} = 2.5682\%$$



2. **Z-spread**: the flat spread one needs to add to the riskfree zero coupon yield curve, so that the PV of the 'corporate' bond cash flows (using the adjusted yield curve) equals the market price of the bond

   3yr Government Bond with 2.5% annual coupon, trading at 100.925

   Spot interest rates for 1, 2 and 3 years: 1%, 1.5% and 2.2% pa

$$100.925 = \frac{2.5}{(1.01)} + \frac{2.5}{(1.015)^2} + \frac{102.5}{(1.022)^3}$$

   3yr Corporate Bond with 4.5% annual coupon, trading at 99.325

$$99.325 = \frac{4.5}{(1.01+z)} + \frac{4.5}{(1.015+z)^2} + \frac{104.5}{(1.022+z)^3} \implies z = 2.5853\%$$

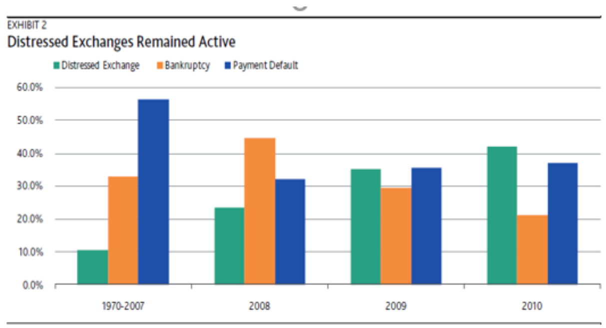
3. **Option Adjusted Spread (OAS)**: the flat spread one needs to add to the riskfree zero coupon yield curve, **in a pricing model that accounts for the embedded options** so that the PV of the bond cash flows equals the market price of the bond

Measures of Credit Risk

Traditional measures of credit risk in capital markets:

- Credit Ratings
- Credit Spreads
- **Default Rates**
- **Recovery Rates**

Defaults – by Default Type

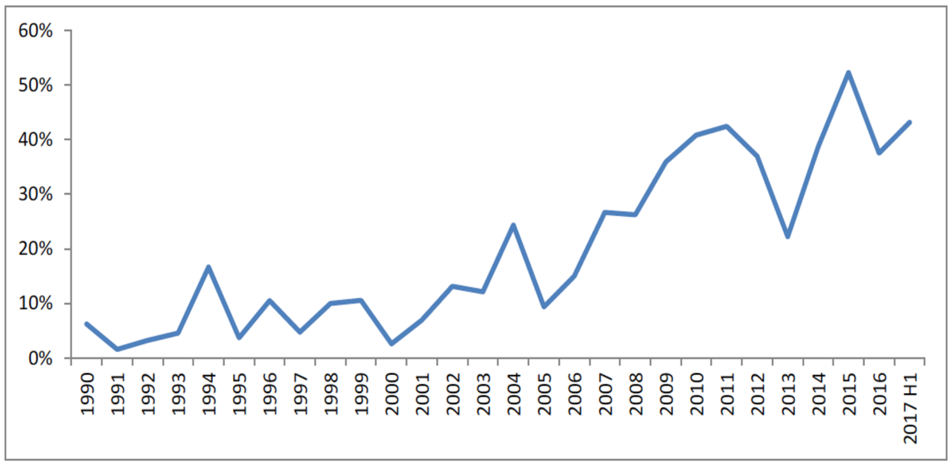


Moody's definition of a distressed exchange:

- The issuer offers bondholders a new package of securities that amount to a diminished financial obligation (such as debt of preferred stock, or debt with a lower coupon or par amount)
- The exchange had the apparent purpose of helping the borrower avoid default

Distressed Exchanges

Share of distressed exchanges on the rise



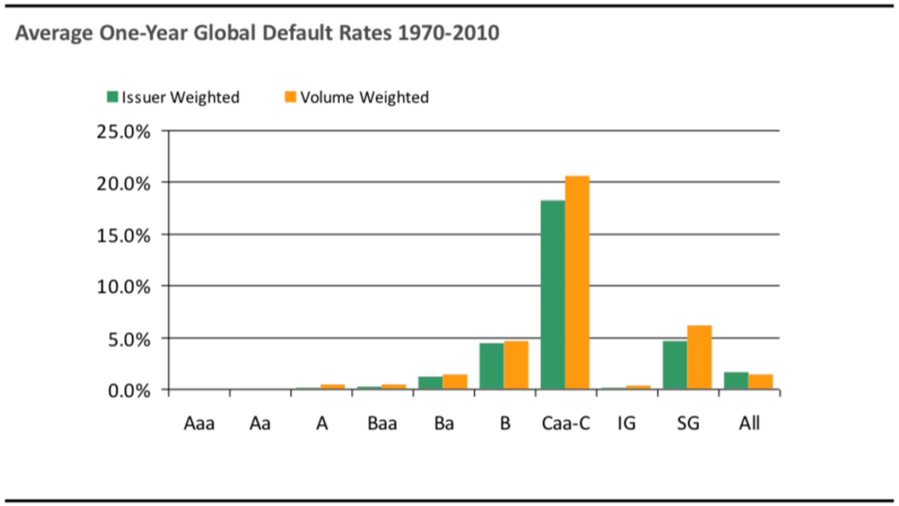
Source: Moody's Investors Service

Default Rates

Historical default rates are typically measured as follows:

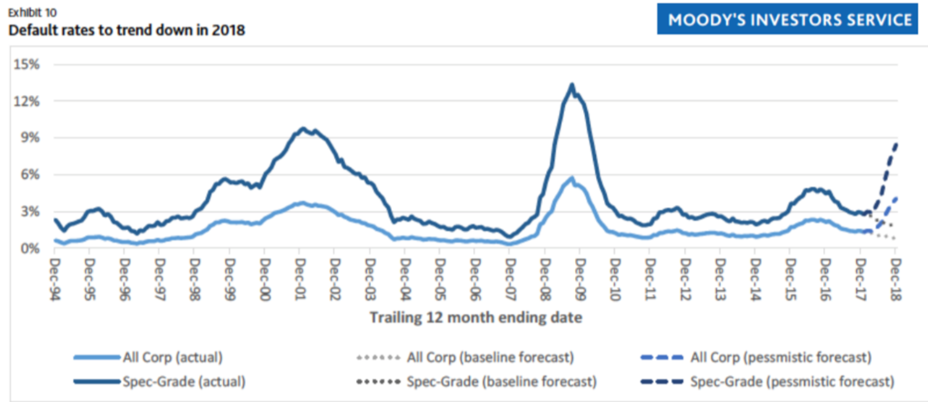
- group issuers by rating category at a point in time
- determine percentage of issuers (issuer-weighted default rate) or face value of debt (volume-weighted default rate) that defaulted
- within 1 yr (annual default rate) or within n years (n>1: cumulative default rate)

Annual Default Rates



Source: Moody's

Default Rates



Recovery Rates

Recovery rates are typically measured by seniority and security:

Exhibit 7

Average corporate debt recovery rates measured by trading prices

Panel A - Recoveries	Issuer-weighted recoveries			Volume-weighted recoveries		
Priority Position	2017	2016	1983-2017	2017	2016	1983-2017
1st Lien Bank Loan	69.04%	75.05%	67.07%	74.73%	77.95%	63.74%
2nd Lien Bank Loan	17.87%	22.50%	30.38%	30.29%	22.50%	27.73%
Sr. Unsecured Bank Loan	9.00%	n.a.	45.87%	9.00%	n.a.	40.21%
1st Lien Bond	62.43%	48.72%	53.62%	66.21%	40.89%	53.80%
2nd Lien Bond	52.75%	34.07%	45.18%	36.61%	35.82%	43.63%
Sr. Unsecured Bond	53.85%	31.45%	37.74%	39.79%	27.10%	33.48%
Sr. Subordinated Bond	38.00%	36.72%	31.10%	50.62%	56.10%	26.34%
Subordinated Bond	74.38%	24.50%	32.05%	76.37%	24.50%	27.55%
Jr. Subordinated Bond	17.50%	0.63%	22.79%	4.84%	0.63%	13.97%

Source: Moody's

Bonds and Credit Risk – Overview

- Introduction to credit risk and default
- Measures of credit risk
- **Understanding yields and credit spreads of risky bonds**
- Credit risk and duration

Understanding Yields of Risky Bonds

For a bond with credit risk is the bond's yield:

- A cost of borrowing? Yes, in the usual sense
- An interest rate? No
- A (compound) rate of return if you buy and hold the bond to maturity?

Understanding Yields of Risky Bonds

Recall the simple YTM-based spread calculations:

- 3yr Government Bond with 2.5% annual coupon, trading at 100.925

$$100.925 = \frac{2.5}{(1+y)} + \frac{2.5}{(1+y)^2} + \frac{102.5}{(1+y)^3} \Rightarrow y = 2.1785\%$$

- 3yr Corporate Bond with 4.5% annual coupon, trading at 99.325

$$99.325 = \frac{4.5}{(1+y)} + \frac{4.5}{(1+y)^2} + \frac{104.5}{(1+y)^3} \Rightarrow y = 4.7467\%, cs = 2.5682\%$$

Is the YTM on a government or corporate bond the return you earn on the bond by holding it to maturity?

Understanding Yields of Risky Bonds

- For a riskfree bond, the yield is the return you earn by buying and holding the bond to maturity if and only if you can reinvest the coupons at the original yield
- For a risky bond, the yield is calculated from the promised cashflows, therefore it is the return you earn by buying and holding the bond to maturity if and only if:
  - you can reinvest the coupons at the original yield
  - and the bond does not default
- Therefore the yield on the risky bond is a best case (no default) scenario return, not even the expected return!
- Problem: the yield on the risky bond is calculated from the promised cash flows only, completely ignoring the probability and severity of default. . . .

Example: Understanding Yields of Risky Bonds

Example: Suppose that the one-year, zero-coupon Treasury bill has a yield to maturity of 4%. What are the price and yield of a one-year, $1000, zero-coupon bond issued by Firm A? First, suppose that all investors agree that there is **no possibility** that Firm A will **default** within the next year.

Answer: Because this bond is risk free, the Law of One Price guarantees that it must have the same yield as the one-year, zero-coupon Treasury bill. The price of the bond will therefore be

$$P_0 = \frac{FV}{1+y} = \frac{1000}{1+0.04} = \$961.54$$

Example: Understanding Yields of Risky Bonds

Example: Now suppose that investors believe that Firm will **default with certainty** at the end of one year and will be able to pay only 90% of its outstanding obligations.

Answer: Even though the bond promises \$1000 at year-end, bondholders know they will receive only \$900. Investors can predict this shortfall perfectly, so the \$900 payment is risk free, and the bond is still a one-year risk-free investment. Therefore, we compute the price of the bond by discounting this cash flow using the risk-free interest rate as the cost of capital:

$$\hat{P}_0 = \frac{FV}{1+y} = \frac{900}{1+0.04} = \$865.38$$

The prospect of default lowers the cash flow investors expect to receive and hence the price they are willing to pay.

Example: Understanding Yields of Risky Bonds

Example: Compute the yield to maturity and the bond return.

Answer: Given $\hat{P}_0$, we can compute the bond's yield to maturity by using the **promised** rather than the **actual cash flows**. Thus,

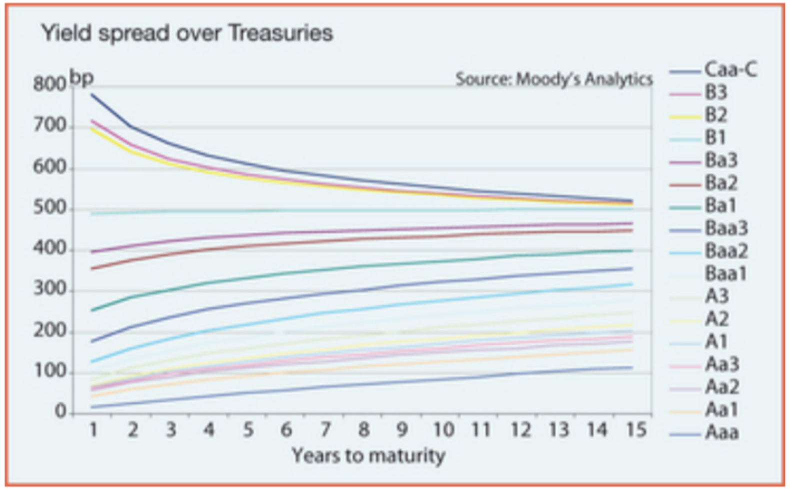
$$\hat{y} = \frac{FV}{\hat{P}_0} - 1 = \frac{1000}{865.38} = 15.56\%$$

Note that $\hat{y} = 15.56\% > 4\% = y$. But this does not mean that investors will earn a 15.56% return. Because Firma A will default, the expected return of the bond equals its 4% cost of capital:

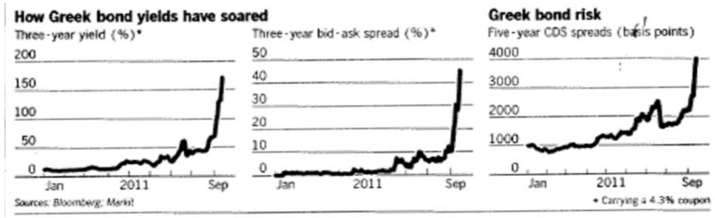
$$\text{bond return} = \frac{\text{actual cash flows}}{\hat{P}_0} - 1 = \frac{900}{865.38} = 4\%$$

Hence the yield to maturity of a defaultable bond exceeds the expected return of investing in the bond.

Credit spreads by maturity and rating as of mid-January 2011



What happened to Greece?



Greek yields off the scale as 3-year bond hits 172%

By David Oakley in London

Greek bond markets have gone off the scale. As investor concerns over a potential debt default by Athens mount, the country's debt has entered territory previously uncharted by a European sovereign.

Yields have risen by 150 percentage points on some bonds in the space of three months and volumes have slumped, almost to nothing on some days.

A trader at one big bank said: "Yield levels are unprecedented.

One three-year bond, which was trading at 20 per cent in June, is trading at a yield of 172 per cent, with a bid-offer spread – the difference between what a bank is offering to buy and sell the bonds at – of 47 percentage points.

As a comparison Belize, which investors warn is close to default, has 18-year bonds trading at 15 per cent, while Venezuela, shunned by mainstream investors, has nine-year bonds trading at 14 per cent.

Bankers say volumes have

For example, three-year bonds that mature in March next year are quoted at a price of 53 per cent of par, a heavy discount that suggests Greece may default soon.

In the credit default swaps market, which offers investors a kind of protection against default, prices for Greece have jumped to levels that many traders would have considered fanciful only a few months ago.

Greek CDSs have risen above 4,000 basis points – more than double levels at the start of

Understanding Credit Spreads

- Does this mean that credit spreads (calculated as difference of YTMs) are not particularly useful as measures of credit risk for bonds that are quite risky?
- Example: Consider two bonds from the same issuer, same seniority, both with a 5% coupon, one with 1 year to maturity, the other with 10 years to maturity:
  - YTM on 1yr bond = 162.5% pa
  - YTM on 10yr bond = 18.68% pa

Assume the government yield curve is essentially flat.

Is the 1yr bond much riskier than the 10yr bond?

Yields of Distressed Bonds

- As bonds of an issuer become increasingly distressed, their price will be determined mainly by what bondholders receive in default ie recovery rate
- Bonds representing the same claim (e.g. that rank "pari passu" for a corporate) should start trading at similar levels
- For the same (low) price today, the YTM – calculated from promised cashflows only – on a short maturity bond has to be higher than that on a long maturity bond
- Therefore, as YTM not very informative, distressed bonds "trade on price"

Bonds and Credit Risk – Overview

- Introduction to credit risk and default
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- **Credit risk and duration**

Credit Risk and Duration

Recall the simple YTM-based spread calculations:

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- 3yr Corporate Bond with 4.5% annual coupon, trading at 99.325

$$99.325 = \frac{4.5}{(1+y)} + \frac{4.5}{(1+y)^2} + \frac{104.5}{(1+y)^3} \Rightarrow y = 4.7467\%, cs = 2.5682\%$$

Modified duration (calculated the usual way wrt bond's own yield) = 2.746

Percentage change in bond price for x bps (small) change in yield is -2.746 x%

Credit Risk and Duration

- Modified duration of 2.746 is the sensitivity with respect to the bond's yield, y
- But the corporate bond's yield is made up of two components:

$$y = g + s$$

here: $4.75\% = 2.18\% + 2.57\%$

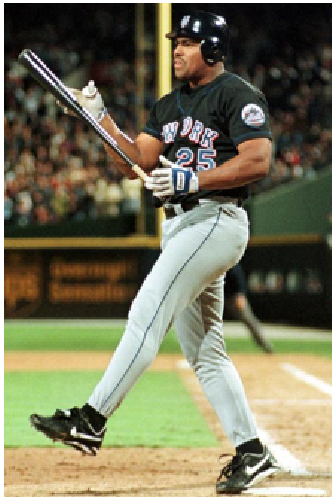
  - the yield on the government bond of the same maturity, g
  - the credit spread on the corporate bond, s
- Therefore the standard modified duration of 2.746 is both the:
  - Interest rate duration: the sensitivity of the corporate bond to changes in government bond yields (all else equal)
  - Spread duration: the sensitivity of the corporate bond to changes in credit spreads (all else equal)

Bonds with Credit Risk – Summary

- Measures of credit risk: ratings, spreads, default and recovery rates
- Understanding yields and credit spreads of risky bonds:
  - Yields are calculated from promised cash flows
  - Yields overstate expected returns
  - If corporate bonds are trading at distressed levels, credit spreads are a poor measure of (relative) credit risk
- Two (modified) durations – interest rate and spread duration:
  - Sensitivity to riskfree yield and credit spread
  - Same expression/value and equal to normal (modified) duration wrt bond's yield

Appendix

Appendix: Bobby Bonilla contract



Bonilla agreed to the Mets deferring the \$5.9m owed him for the summer 2000 in exchange for 25 annual payments of \$1,193,248.20 beginning July, 2011.

They used an 8% discount rate. Prime rate was 8.5%, long-term government bonds yielded ~6.5%.

"This is like a gift," Bonilla said. "I'm ecstatic now. Hey, I'm doing the macarena." Chicago Tribune, 5/21/2000.

Contracts have same PV @ 8%.

$$\$5.9m = \frac{1{,}193{,}248.20}{1.08^{10}}\left(\frac{1}{.08} - \frac{1}{.08(1.08^{25})}\right)$$

Appendix: Risk of the new contract

- The new contract has a lot more interest rate risk.
  - Imagine interest rates go down by 3% (to 5%).

$$\$10.32m = \frac{1,193,248.20}{1.05^{10}}\left(\frac{1}{.05} - \frac{1}{.05\,(1.05^{25})}\right)$$

  - The value of the contract almost doubles.
- Imagine you are the CFO of the Mets.
  - Are you worried about this risk?
  - How can you hedge this risk?

Appendix: Cash-flow matching

- We can hedge this exactly by purchasing zeros with face value of \$1.19m maturing each year from 2011 to 2025. The risk-free rate was 6.5%, so the cost is (note Treasuries so semi-annual compounding):

$$\sum_{t=1}^{25} \frac{1,193,248.20}{\left(1+\frac{0.065}{2}\right)^{20+2t}} = \$7.6m$$

- This portfolio replicates the payouts to Bonilla exactly, and requires no rebalancing.
- Question: Where do we get a 35-year risk-free zero coupon bond?

Appendix: Immunization

- The exact hedge illustrates the intuition, but has downsides.
  - Must purchase 25 bonds.
  - You cannot buy risk-free bonds with 31, 32, ... year maturities.
- Alternatives
  - "Bullet" hedge: Buy 1 bond with the same duration as the liability
  - Portfolio hedge: Buy any two bonds, say 5y and 10y zeros.
- Steps.
  - Find duration of the liability to hedge
  - Find duration of hedging instruments
  - Solve system of equations for weights in hedging instruments.

Appendix: Duration of the liability

We need D_{mod} or DV01 to hedge. The first step to getting these is calculating the D_{mac} for the liability:

- Recall, we weight the time (t) to each cash flow by the present value of the cash flow as a% of the total value.

$$\sum_{t=1}^{25}(10+t)\times w = D_{mac}$$
$$\sum_{t=1}^{25}(10+t)\left(\tfrac{1,193,248.20}{1.0325^{2\times(10+t)}}/7,602,942.22\right) = 19.81$$

- This implies

$$D_{\text{mod}} = \frac{19.81}{1.0325} = 19.18$$
$$\text{DV01} = \frac{19.81}{1.0325}\times\frac{\$7.6\text{m}}{10{,}000} = \$14,585.60$$

The liability is \$7,602,942.22 at 6.5%. What about at +/- 0.1%?

$$6.4\%\text{ semi } = 1.032\hat{\ }2 - 1 = 6.502\%\text{ compound rate:}$$

$$\frac{1{,}193{,}248.2}{1.06502^{10}} \times \left(\frac{1}{0.06502} - \frac{1}{0.06502(1.06502)^{25}}\right) = \$7,750,409.60$$

$$6.6\%\text{ semi } = 1.033\hat{\ }2 - 1 = 6.709\%\text{ compound rate:}$$

$$\frac{1{,}193{,}248.2}{1.06709^{10}} \times \left(\frac{1}{0.06709} - \frac{1}{0.06709(1.06709)^{25}}\right) = \$7,458,671.555$$

$$\frac{\%\Delta P}{\Delta y}\Delta y \approx D_{mod} \times 0.1\% = 19.18 \times 0.1\% = 0.01918$$

$$\frac{\Delta P}{\Delta y}\Delta y \approx DV01 \times \Delta y \text{ in bps} = \$14{,}585.60 \times 10 \text{ bps}=\$145{,}856$$

Using D_{mod} we should be +/- 1.918%

$$\$7{,}602{,}942.22 \times (1 \pm 0.01918) = \$7{,}457{,}086.25/\$7{,}748{,}798.19$$

Using DV01 we should be +/- \$145,855.97

$$\$7{,}602{,}942.22 \pm \$145{,}855.97 = \$7{,}457{,}086.25/\$7{,}748{,}798.19$$

Due to convexity, we overestimate the price decline for +0.1% and underestimate the price increase for -0.1%

- Simple: Buy a 20-year zero, you're done . . . if available.
- More complex: with a 5y and 10y zero, with y_5= 6.5%, y_{10}=6.58%.

$$D_{\text{mod }5} = \frac{y_5}{1+\frac{y_5}{2}} = 4.84$$

$$DV01_5 = D_{\text{mod }5} \times \frac{PV}{10{,}000} = 4.84 \times \left(\frac{\$100}{1.032510}\right)/10,000 = 0.035$$

$$D_{\text{mod }10} = \frac{10}{1+\frac{y_{10}}{2}} = 9.68$$

$$DV01_{10} = D_{\text{mod }10} \times \frac{PV}{10{,}000} = 9.68 \times \left(\frac{\$100}{1.0329^{20}}\right)/10,000 = 0.051$$

- We could use either D_{mod} or DV01. Here we'll use D_{mod}.
- We want gain/loss on hedge to offset the liability:

$$\left(\%\frac{\Delta P_{5y}}{\Delta y}\right)V_{5y} + \left(\%\frac{\Delta P_{10y}}{\Delta y}\right)V_{10y} = \left(\%\frac{\Delta P_{\text{Bonilla}}}{\Delta y}\right)V_{\text{Bonilla}}$$

- We also want the value of the hedge portfolio to be the same as for the liability:

$$V_{5y} + V_{10y} = V_{Bonilla}$$

- Plugging in D_{mod} and the liability value, we have two equations and two unknowns: V_{5y} and V_{10y}.

Appendix: Solving the system

$$4.8426 \times V_{5y} + 9.6815 \times V_{10y} = 19.1841 \times \$7{,}602{,}942.22$$
$$V_{5y} + V_{10y} = \$7{,}602{,}942.22$$

The solution is

- Short $14,930,607.94 of the 5y (i.e., negative position)

  That is equivalent to $\frac{-\$14{,}930{,}607.94(1.0325)^{10}}{\$100} = -205{,}578.69$ bonds

- Long $22,533,550.16 of the 10y.

  That is equivalent to $\frac{\$22{,}533{,}550.16(1.0329)^{20}}{\$100} = 430{,}521.82$bonds

Appendix: Check Solution

- What is the value of the hedge portfolio if there is parallel downward shift in yields of 0.1%?

$$-\frac{\$20{,}557{,}869.02}{\left(1+\frac{6.5\%-0.1\%}{2}\right)^{10}} + \frac{\$43{,}052{,}182.10}{\left(1+\frac{6.58\%-0.1\%}{2}\right)^{20}}$$
$$= \$7{,}749{,}716.98$$

Pretty close to actual value of

$$\frac{1{,}193{,}248.2}{1.06502^{10}} \times \left(\frac{1}{0.06502} - \frac{1}{0.06502(1.06502)^{25}}\right)$$
$$= \$7{,}750{,}409.60$$

Appendix: "The Carry Trade"

- Suppose you knew that the shape of the U.S. yield curve is going to stay the same
- Consider the following trade:
  - Borrow at the short end: Federal Funds @2%
  - Buy long-dated Treasuries, Mortgages or AAA's @ 4-6%
- Profit? If nothing moves, you get about 2-4% profit per dollar.
- What happens when rates change?