Mean-Variance Analysis

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MFA - Investment Fundamentals

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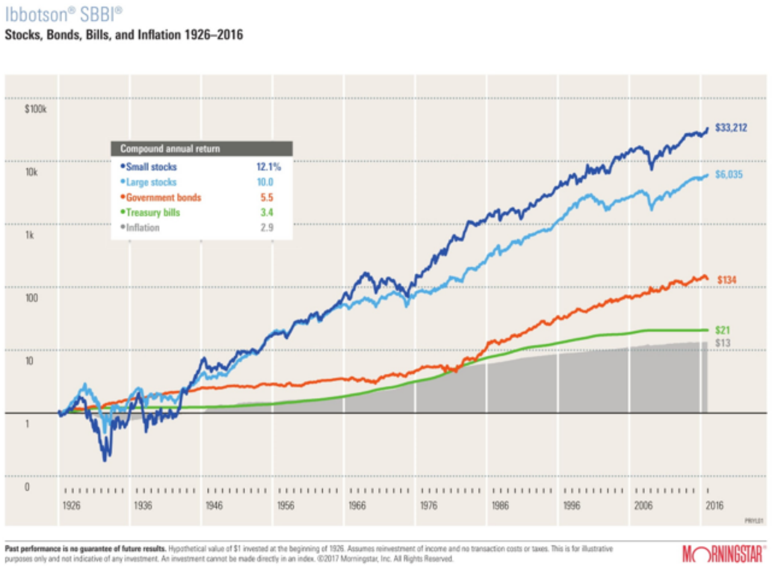
Outline

- Efficient frontier with multiple assets
- Two-fund separation
- Capital market line
- Sharpe Ratio
- Tangency portfolio
- Does it work in practice?

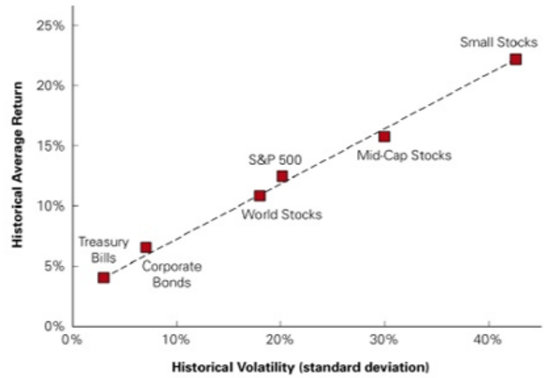
Historical Evidence on Risk and Return

Value of $1 invested in 1926 through 2016 (with reinvestment of income)



Relationship Between Risk and Return

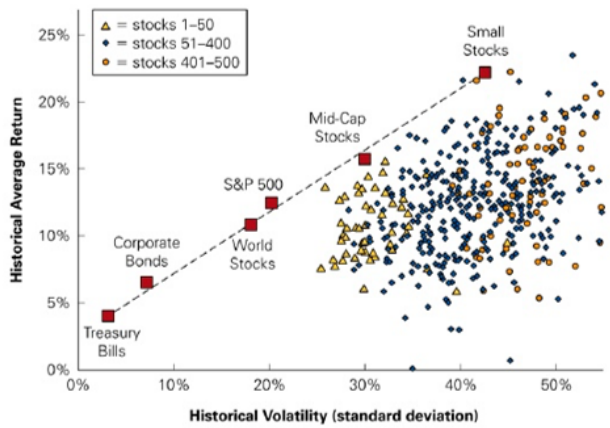
For "broad" portfolios there seems to be a general increasing relationship between historical volatility and average return:



Source: CRSP, Morgan Stanley Capital International

Relationship Between Risk and Return

However, no precise relationship between historical volatility and average return for individual stocks:



Context for Today's Lecture

- We reviewed the history of equity/bond/bill returns
  - Large variation in returns/variability among asset classes
    - Equities have historically vastly outperformed bonds and bills
  - Motivates need for a framework to analyse these investments
- We introduced tools for summarizing a portfolio's characteristics
  - Calculated portfolio means/variances
- Introduced the concept of risk
  - Portfolio variance as a measure of risk
  - Covariance terms are very important!
- Now we will study how to identify **the best portfolios** among those that are feasible

Context for Today's Lecture

- Equivalent problem
  - What is the optimal set of portfolio weights $(w_1, w_2, \ldots, w_j)$?
    - Gives you the best trade-off between risk and reward
- Optimal weights determined by preferences
  - Do you want to maximize your chance of having \$1m?
  - Do you prefer to minimize the chance you aren't poor?
- Solution requires a model for preferences

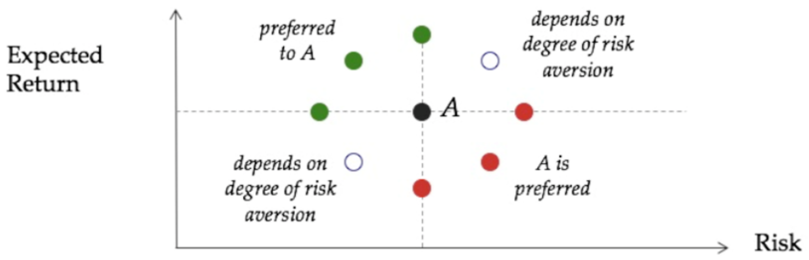
Modern Portfolio Theory (MPT)

Markowitz's Modern Portfolio Theory (published 1952, Nobel 1990)

- Start with individual securities whose investment properties are summarized by:
  - Expected return, eg estimated as the historical average return
  - Risk or volatility, eg estimated as the standard deviation of historical returns
- What are the risk and return of portfolios of securities?
- How should investors choose an optimal portfolio?

MPT - Investor Preferences

- Assume that investors are risk averse:
  - for the same expected return, prefer less risk
  - for the same risk, prefer more return
- Consider a risk averse investor at point A below. Which portfolios would (s)he prefer?



MPT - Investor Preferences

- What about other aspects of return distribution beyond the mean and standard deviation?
- MPT assumes that either:
  1. Investors do not care about higher moments (skewness, kurtosis etc)
  2. Security returns are normally distributed
     - All moments of the distribution are fully described by the mean and standard deviation

Validity of assumptions

- Expected Returns are good — uncontroversial
- Variance as a measure of risk?
  - Intuition: Investors prefer less dispersion in wealth outcomes
  - Also holds if returns are normally distributed
  - Potential concerns
    - Investors may care about other things
      Co-movement between the portfolio return and the rest of your wealth.
    - Security returns are not really normally distributed
      E.g., equity returns during 2008, options at any point in time, high-frequency returns, ...
- Frictionless markets
  - Securities may be traded at any price/quantity without transaction costs
  - Simplifies analysis, may or may not have an important effect

Validity of assumptions

- First question: What portfolio means and variances are possible?
- Go back to our equations for $\bar{R}_p$ and σ_p^2, and vary the weights w_j

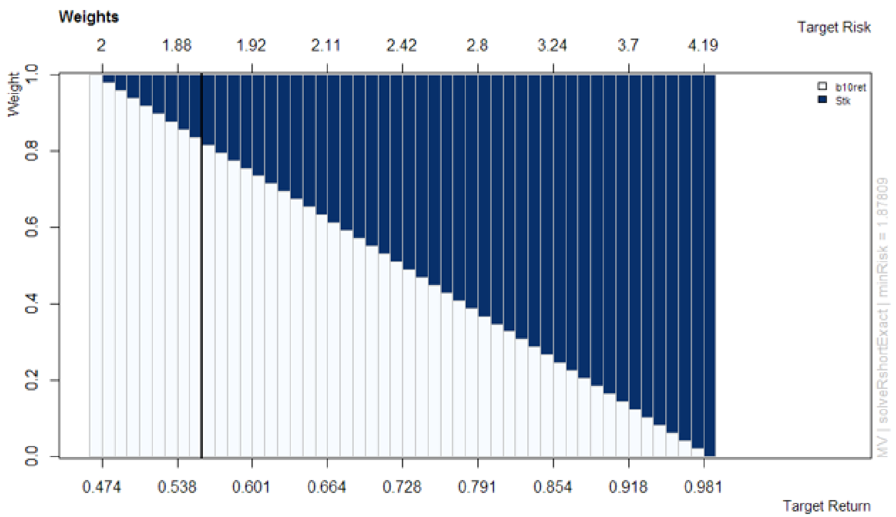
$$\bar{R}_p = \sum_{i=1}^{N} w_i \bar{r}_i$$

$$\sigma_p^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + 2 \sum_{j<i} w_i w_j \sigma_{ij}$$

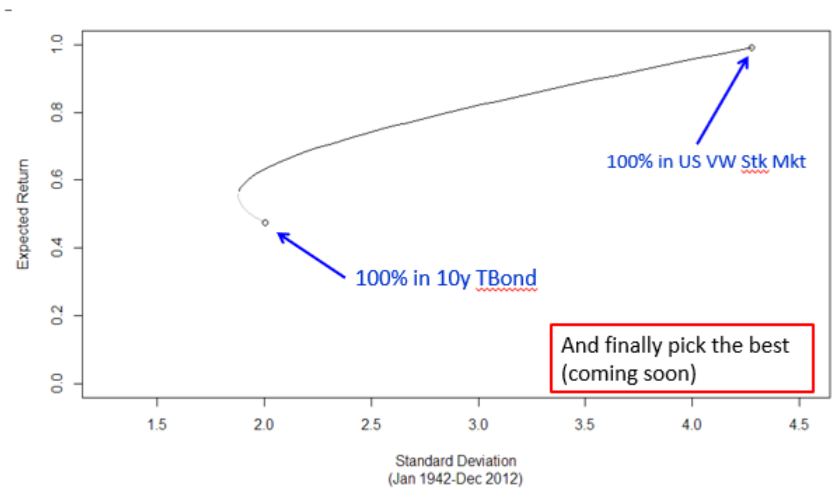
- Then figure out which is the best.

Consider a stock & bond portfolio

Vary weights from 100% bond (white) to 100% stock (blue)



Then plot mean vs. variance.

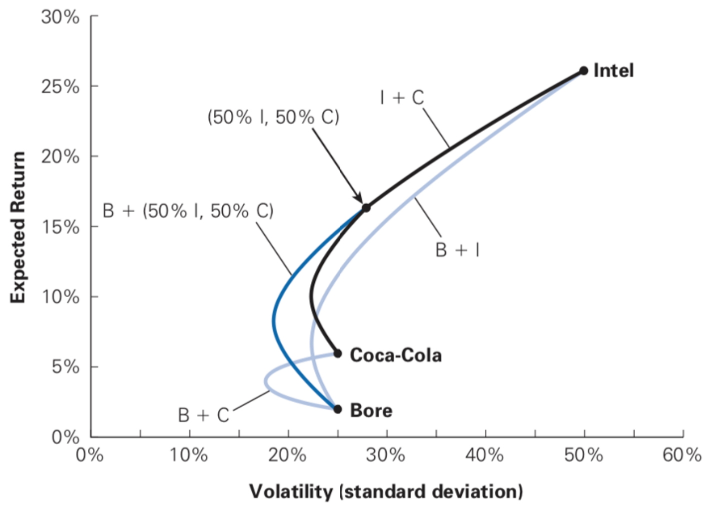


The efficient frontier

- Looks simple with two assets.
- With many assets:
  - Many more possibilities for means and variances.
  - Many portfolios will deliver the same expected return.
- Simplify problem by identifying Efficient frontier
  - Portfolios with the lowest variance for a given $E[\tilde{r}]$
  - Dominates all other portfolios with the same $E[\tilde{r}]$
  - Portfolios on the frontier are efficient portfolios

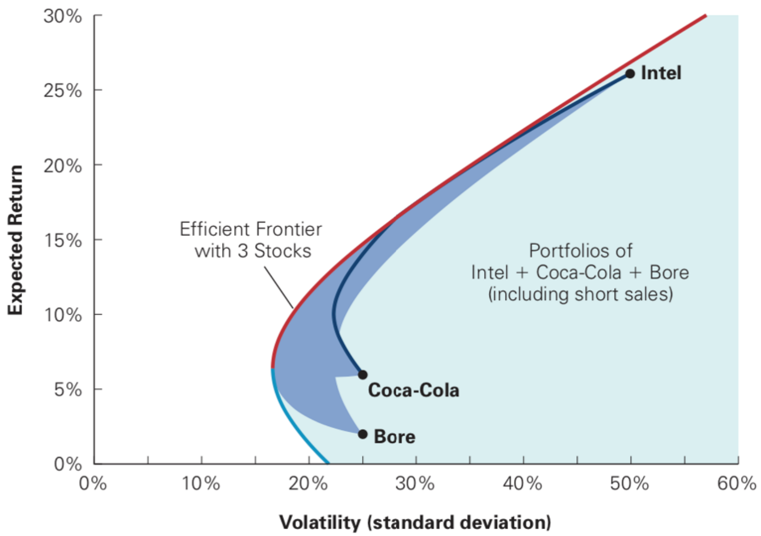
Portfolios of 3 Risky Securities

Starting from pairwise combinations of 3 stocks:



Portfolios of 3 Risky Securities

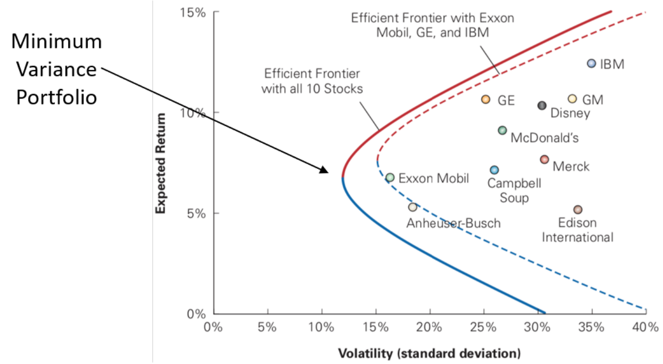
All possible combinations of 3 stocks and the efficient frontier



Large Portfolios Risky Securities

If you continue to add risky securities:

- efficient frontier improves but at a decreasing rate: decreasing marginal benefits of diversification
- efficient frontier dominates individual securities



Implications

- Adding securities **shifts the frontier to the left**
- As we add more and more securities, these **shifts become smaller and smaller**
- **Individual securities are in general dominated by the frontier!**

Limits to Diversification

- Total portfolio risk will typically not go down to zero even when you hold a very large number of securities:
  - Diversify away **firm-specific, non-systematic, idiosyncratic, unique** risk
  - Left with exposure to **non-diversiftable, systematic, "market" risk**
- Most of the diversification benefits come from adding the first 15-30 securities (chosen at random)

Limits to Diversification: Variance of an Equally Weighted Port.

- Recall for a portfolio of N stocks

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}\sum_{j \neq i} w_i w_j \sigma_{ij}$$

- If we take $w_i = w_j = 1/n$ we have *:

$$\text{Var}(R_P) = \frac{1}{n}(\text{Average Variance of the Individual Stocks})$$

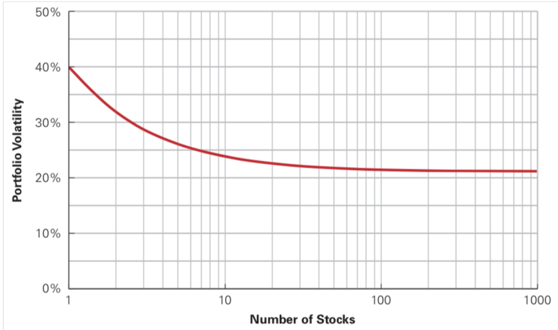
$$+\left(1 - \frac{1}{n}\right)(\text{Average Covariance between the Stocks})$$

*Note that there are n variances each with $1/n^2$ weight. There are $n^2 - n$ covariance terms each with weight $1/n^2$

Limits to Diversification

Risk reduction from adding more stocks in portfolio is marginal after approx. 20 stocks

Risk reduction from adding more stocks in portfolio is marginal after approx. 20 stocks



$$SD(R_P) = \sqrt{\frac{1}{n}(0.40^2) + \left(1 - \frac{1}{n}\right)(0.28 \times 0.40 \times 0.40)}$$

Limits to Diversification: When risks are Independent

Example: What is the volatility of an equally weighted average of n independent, identical risks?

- If risks are independent, the stock returns are uncorrelated and therfore their covariances are zero. Hence, the volatility of an equally weighted portfolio of risk is

$$\sigma_p = \sqrt{\text{Var}(R_P)} = \sqrt{\frac{1}{n}\text{Var}(\text{individual risk})} = \frac{\sigma}{\sqrt{n}} \to 0 \quad \text{as} \quad n \to \infty$$

- We can eliminate all risk because there is no common risk.

Diversification with general portfolio weights

- So far, we assumed equally weighted stocks. For a portfolio with arbitrary weights, we have

$$\sigma_p^2 = \sum_i \omega_i \text{Var}(R_i, R_P) = \sum_i \omega_i \sigma_i \sigma_p \text{Corr}(R_i, R_P)$$

- Which, simplifies to

$$\sigma_p = \sum_i \underbrace{\omega_i \sigma_i \text{Corr}(R_i, R_P)}_{\text{contr. of security i to the overall volatility of portfolio p}}$$

- Therefore, when combining stocks into a portfolio that puts positive weight on each stock, we reduce volatility

$$\sigma_p = \sum_i \omega_i \sigma_i \text{Corr}(R_i, R_P) < \sum_i \omega_i \sigma_i$$

Review of efficient frontier

- Gives the set of efficient portfolios
  - The portfolios with minimum variance given expected return.
- Does not tell us which efficient portfolio to choose
  - Depends on risk aversion
  - Very risk-averse investors will choose the minimum variance portfolio.
- How do we construct our ideal portfolio?
  - Perform an optimization to find unique portfolio weights
  - Or, use a short-cut property called two-fund separation

Two-fund Separation

- All frontier portfolios may be constructed from a weighted average of any two frontier portfolios.
- **Tobin (two-fund) separation theorem:** identifying the optimal portfolio for a risk averse investor can be broken down into two steps:
  - Find the two optimal portfolios of risky securities - this does not depend on risk preferences
  - Based on specific risk preferences of the investor, determine the appropriate amount to invest in the two optimal portfolios.

Risk-Free Saving and borrowing

We have considered the risk and return possibilities that result from combining only risky investments into portfolios.

- **Adding a risk-free asset** has a large effect on the frontier
  - Greatly expands the set of feasible portfolios
  - Frontier changes from a curve to a straight line
- The risk-free asset is on the frontier
  - It is the minimum variance portfolio when available.
  - Implies the frontier may be constructed from it and one risky portfolio on the frontier (due to two-fund separation)

Adding a Riskfree Asset

- Any portfolio Q (on the efficient frontier, or in general) can be combined with the **riskfree asset** that has:
  - Return r_f
  - zero variance and zero covariance with all risky assets (& with P)
- Expected return of portfolio Q:

$$\bar{R}_Q = w_{rf} r_f + w_p \bar{r}_P = (1 - w_p) r_f + w_p \bar{r}_P$$

- Variance of portfolio Q:

$$\sigma_Q = \sqrt{(1 - w_p)^2 \times var(r_f) + w_p^2 \times var(r_P) + 2(1 - w_p) w_p cov(r_f, r_P)}$$

$$\sigma_Q = \sqrt{w_p^2 \times var(r_P)} \quad \text{since} \quad var(r_f) = cov(r_f, r_P) = 0$$

Adding a Riskfree Asset

- The expected return and variance of the combined position are:

$$\bar{R}_Q = w_{rf} r_f + w_p \bar{r}_P \quad \text{and} \quad \sigma_Q = w_P \sigma_P$$

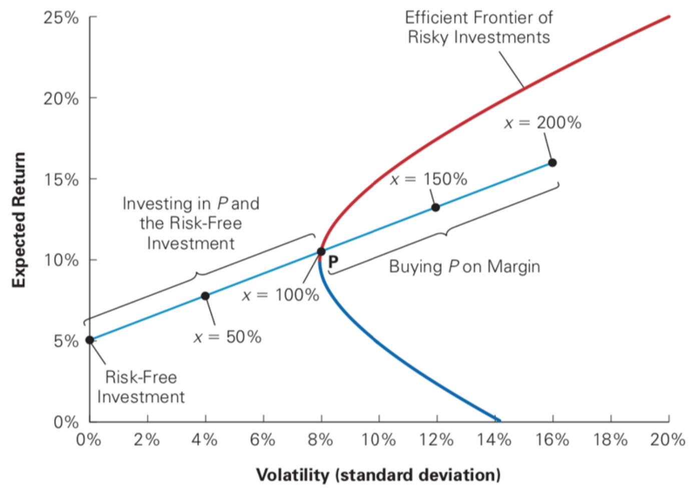
This implies: $w_P = \frac{\sigma_Q}{\sigma_P} \Rightarrow w_{rf} = 1 - \frac{\sigma_Q}{\sigma_P}$

- The possible risk-return combinations are given by a straight line between the risk free asset and the portfolio P, known as the Capital Allocation Line, with equation (combining the two equations above):

$$\bar{R}_Q = \left(1 - \frac{\sigma_Q}{\sigma_P}\right) r_f + \frac{\sigma_Q}{\sigma_P} \bar{r}_P = r_f + \frac{(\bar{r}_P - r_f)}{\sigma_p} \sigma_Q$$

Adding a Riskfree Asset

Investing in P and the riskfree asset (here $x = w_P$):



Example: Adding a Riskfree Asset

Example: Suppose you have \$10,000 in cash, and you decide to borrow another \$10,000 at a 5% interest rate in order to invest \$20,000 in portfolio P, which has a 10% expected return and a 8% volatility. What is the expected return and volatility of your investment?

Answer:

- You have doubled your investment in P using margin, so $\omega = 200\%$

$$\bar{R}_{P,\omega} = r_f + \omega \left(\bar{R}_P - r_f\right) = 5\% + 2 \times (10\% - 5\%) = 15\%$$

$$SD(\bar{R}_{P,\omega}) = \omega SD(\bar{R}_P) = 2 \times 8\% = 16\%$$

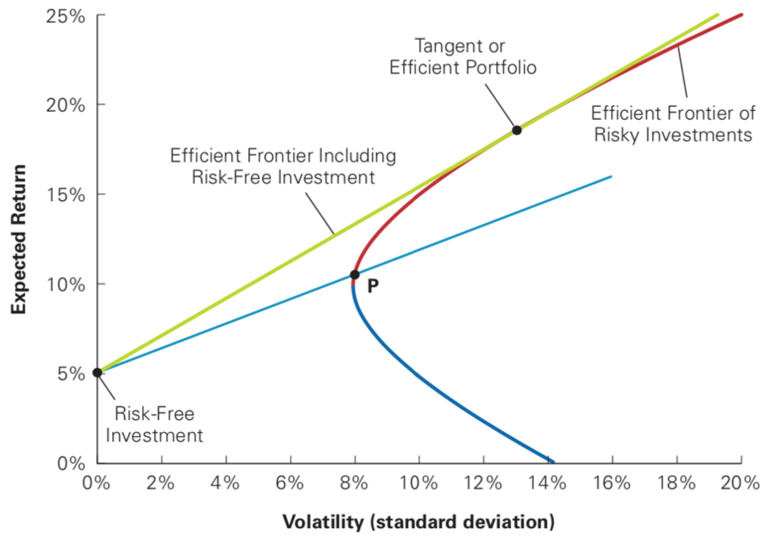
Adding a Riskfree Asset

- The Capital Allocation Line is: $\bar{R}_Q = r_f + \frac{(\bar{r}_P - r_f)}{\sigma_p} \sigma_Q$
- The slope of any Capital Allocation Line is given by:

$$\frac{\text{Portfolio Mean Excess return}}{\text{Portfolio Volatility}} = \frac{E(r_p) - r_f}{\sigma_p}$$

- This is known as the **Sharpe ratio** and gives the reward-to-risk ratio of portfolio P and all investments along the line
- Investors who prefer more return for the same level of risk, will choose the portfolio that combined with the risk free asset has the CAL with the highest slope or Sharpe Ratio
- This portfolio is the one for which the CAL is tangent to the efficient frontier - known as the **tangency or mean-variance efficient (MVE) portfolio**

Adding a Riskfree Asset



Example: Optimal portfolio choice

Example: Portfolio P has an expected return of 10.5% and a volatility of 8%. Suppose the risk- free rate is 5%, and the tangent portfolio has an expected return of 18.5% and a volatility of 13%. Which portfolio would you choose, if you want to maximize the expected return without increasing the volatility of portfolio P? Which portfolio would you choose, if you keep the expected return the same but minimize the risk?

- **Answer:** The best portfolios are combinations of the risk-free investment and the tangent portfolio. Investing ω in the tangent:

$$\bar{R}_{T,\omega} = r_f + \omega \times \left(\bar{R}_T - r_f\right) = 5\% + \omega \times (18.5\% - 5\%) = 15\%$$

$$SD(\bar{R}_{T,\omega}) = \omega \times SD(\bar{R}_T) = \omega \times 13\%$$

Example: Optimal portfolio choice

Maximize the expected return without increasing the volatility

- Hence, to maintain the volatility at 8%, we need $\omega = \frac{8\%}{13\%} = 61.5\%$
- The expected return will then be $5\% + 61.5\% \times (18.5\% - 5\%) = 13.3\%$

Keep the expected return the same but minimize the risk

- ω must satisfy: $5\% + \omega \times (18.5\% - 5\%) = 10.5\% \implies x = 40.7\%$
- The new volatility level will be $40.\%7 \times 13\% = 5.29\%$ the lowest possible given the expected return.

Key Result: Adding a Riskfree Asset

Tobin (two-fund) separation theorem: identifying the optimal portfolio for a risk averse investor can be broken down into two steps:

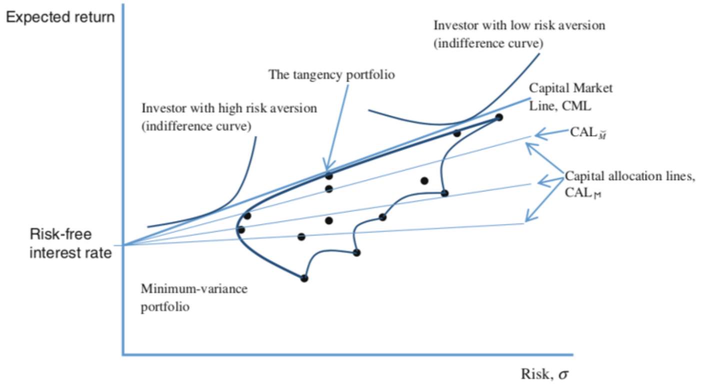
1. Find the optimal portfolio of risky securities - this does not depend on risk preferences
2. Based on specific risk preferences of the investor, determine the appropriate amount to invest in this portfolio versus the riskfree asset (cash)

Identifying the Tangency Portfolio

Tobin step 1: how do we identify the tangency portfolio?

- Solve for the portfolio that maximises the Sharpe ratio:

$$\max_{\omega} \frac{E(r_P)-r_f}{\sigma_P} \text{ s.t. } \quad E(r_P) = \sum_{i=1}^{N} \omega_i E(r_i)$$
$$\sigma_P^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} \omega_i\omega_j\sigma_{i,j} \quad \text{and} \quad \sum_{i=1}^{N}\omega_i = 1$$



Tobin step 1: 2 Asset case

- To compute the mean-variance efficient (MVE), or tangency portfolio, maximize the Sharpe ratio:

$$\max_{w} \frac{E(r_p)-r_f}{\sigma_p}$$
where $E(r_p) = wE(r_A) + (1-w)E(r_B)$
$$\sigma_p = \left[w^2\sigma_A^2 + (1-w)^2\sigma_B^2 + 2w(1-w)p_{AB}\sigma_A\sigma_B\right]^{1/2}$$

- Solution to this is ugly

Solution of the MVE problem

- The tangency portfolio weight for A:

$$w_T = \frac{E(r_A^e)\sigma_B^2 - E(r_B^e)\rho_{A,B}\sigma_A\sigma_B}{E(r_A^e)\sigma_B^2 + E(r_B^e)\sigma_A^2 - [E(r_A^e) + E(r_B^e)]\rho_{A,B}\sigma_A\sigma_B}$$

- Where r^e denotes the excess return

$$E(r_i^e) = E(r_i) - r_f \quad i = A, B$$

Numerical Example

Asset	**Expected Return**	**Volatility**	**Correlation**		
			Asset A	Asset B	Risk-free Asset
Asset A	10%	20%	1	0.5	0
Asset B	15%	30%	0.5	1	0
Risk-free Asset	3%	0%	0	0	1
Minimum variance	10.75%	19.64%	$\omega_A = 0.85$		
(only risky assets)	**we saw how to derive this port. last class**				

Numerical Example

- Tangency portfolio weights:

$$w_T = \frac{\underbrace{(0.07)}_{E(r_A^e)} \cdot \underbrace{0.30^2}_{\sigma_B^2} - \underbrace{(0.12)}_{E(r_B^e)} \cdot \underbrace{0.5 \cdot 0.2 \cdot 0.3}_{\rho_{A,B}\sigma_A\sigma_B}}{\underbrace{(0.07)}_{E(r_A^e)} \cdot \underbrace{0.30^2}_{\sigma_B^2} + \underbrace{(0.12)}_{E(r_B^e)} \cdot \underbrace{0.20^2}_{\sigma_A^2} - [\underbrace{(0.07)}_{E(r_A^e)} + \underbrace{(0.12)}_{E(r_B^e)}] \cdot \underbrace{0.5 \cdot 0.2 \cdot 0.3}_{\rho_{A,B}\sigma_A\sigma_B}} = .5$$

- How much you should put in B?

The tangency portfolio

Given the weights, we can compute the risk and return of the tangency portfolio:

$$E(r_T) = w_T E(r_A) + (1 - w_T) E(r_B)$$
$$= 0.5 \cdot 0.10 + 0.5 \cdot 0.15 = 0.125 = 12.5\%$$
$$\sigma_T = \left[w^2\sigma_A^2 + (1-w)^2\sigma_B^2 + 2w(1-w)\rho_{A,B}\sigma_A\sigma_B\right]^{1/2} = 21.79\%$$

Maximal Sharpe Ratio?

- This portfolio should have the maximal Sharpe ratio
- Does it? Compute Sharpe Ratios

The Sharpe Ratio

- Sharpe ratios of the stocks A and B

$$SR_A = \frac{E(r_A) - r_f}{\sigma_A} = \frac{0.10 - 0.03}{0.20} = 0.35$$
$$SR_B = \frac{E(r_B) - r_f}{\sigma_B} = \frac{0.15 - 0.03}{0.30} = 0.40$$

- Sharpe ratio of the tangency portfolio

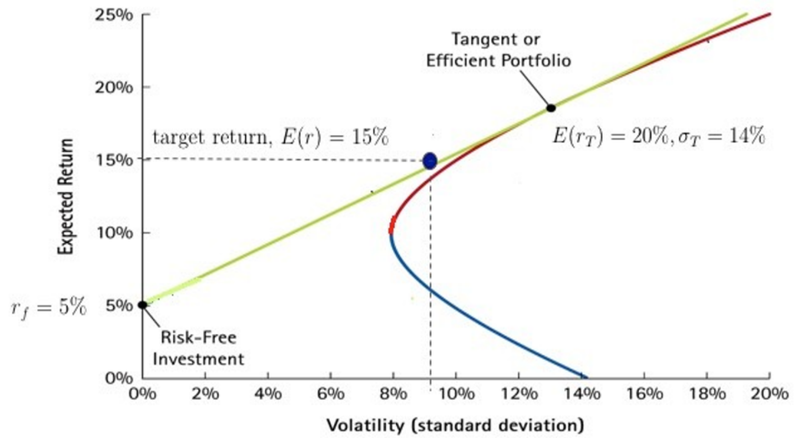
$$SR_T = \frac{E(r_T) - r_f}{\sigma_T} = \frac{0.125 - 0.03}{0.2179} = 0.4359$$

Tobin step 2: for a given investor, how do we identify how much to invest in the tangency portfolio versus riskfree?

Two approaches:

1. Specify a target portfolio expected return or risk
2. Based on risk preferences (utility functions) - higher risk aversion, invest more in the riskfree asset

Let's say you want a target expected return of 15%:



- For a portfolio with weight ω_T invested in the tangency portfolio and the remainder $(1-\omega_T)$ in the riskfree asset we know that:

$$E(r) = \omega_P E(r_P) + (1-\omega_P) r_f \quad \sigma^2 = \omega_P^2 \sigma_P^2$$

- Therefore, to get a target expected return of 15%, we solve for ω_T :

$$0.15 = \omega_T \cdot 0.20 + (1-\omega_T) \cdot 0.05 \Longrightarrow \omega_T = \frac{2}{3}$$

- To calculate the volatility of this position:

$$\sigma = \omega_T \cdot \sigma_P = \frac{2}{3} \cdot 0.14 = 9.33\%$$

- Utility theory quantifies the subjective desirability of something
  - Investors choose the portfolio that delivers the highest utility
- A model of mean-variance utility

$$u(r_t) = -e^{-Ar}$$

  - If returns are normally distributed

$$E[u(r_t)] = -e^{-A\left(\bar{r} - \frac{A}{2}\sigma^2\right)}$$

  - Maximizing is equivalent to maximizing $u(r_t) = \bar{r} - \frac{A}{2}\sigma^2$
  - A measures the investor's level of risk aversion:

    $A > 0$: risk averse

    $A = 0$: risk neutral

    $A < 0$: risk seeking

Optimal Portfolios

- CML gives possible risk-return trade-offs
- Find the portfolio weight that maximizes utility
- **Formally:**

$$U^* = max_w \left(E(r_p) - 0.5 * A * var(r_p) \right)$$

$$\implies U^* = max_w \left(r_f + \omega \left(E(r_T) - r_f \right) - 0.5 * A * var(r_T) \right)$$

Optimal weight?

- Problem is well defined because utility is a quadratic function of the portfolio weight
- To find the maximum, take derivative and set equal to zero to get the optimal weight in the tangency portfolio:

$$w^* = \frac{\bar{r}_T - r_f}{A\sigma_T^2}$$

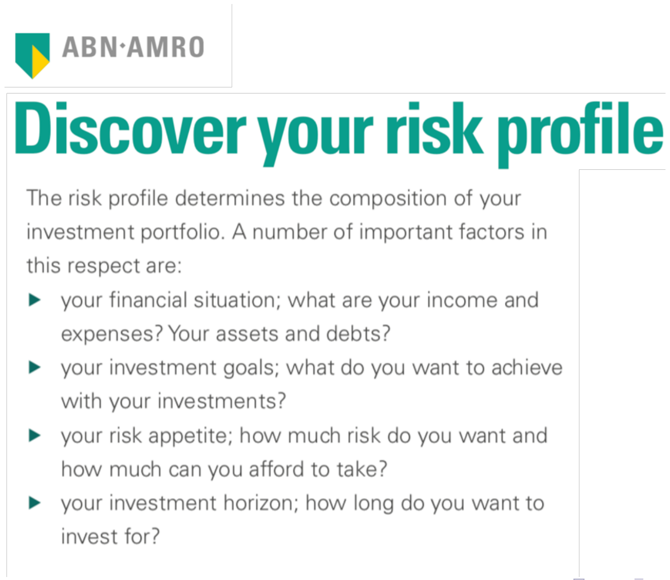
Example: VW US MKT (1927-2012)

- Assume means and variances are constant over time
  - $r_f = 4\%$,
  - $E(r) = 11\%$,
  - $\sigma(r) = 19\%$

A	w*	$E(r_p)$	σ
0.5	3.88	31.15	73.68
1.0	1.94	17.57	36.84
2.0	0.97	10.79	18.42
4.0	0.48	7.39	9.21
8.0	0.24	5.70	4.61

Two-fund separation in practice?

Private banking example:


ABN·AMRO

Discover your risk profile

The risk profile determines the composition of your investment portfolio. A number of important factors in this respect are:

- your financial situation; what are your income and expenses? Your assets and debts?
- your investment goals; what do you want to achieve with your investments?
- your risk appetite; how much risk do you want and how much can you afford to take?
- your investment horizon; how long do you want to invest for?

Two-fund separation in practice?

ABN-AMRO MeesPierson Private Banking asset allocation profiles:

Benchmark Investment Advice	Very defensive	Defensive	Moderately defensive	Moderately offensive	Offensive	Very offensive
Equities						
MSCI Europe tilt (50% MSCI-Europe/40% MSCI World ex Europe, 10% MSCI Emerging Markets)*	0%	15%	30%	50%	70%	85%
Fixed income						
BoA Merrill Lynch EMU Direct Government Bonds 1-10 years**	90%	70%	55%	35%	15%	0%
Alternative investments						
GPR-250 World	5%	10%	10%	10%	10%	10%
Liquidities						
1 Month Euribor	5%	5%	5%	5%	5%	5%

* MSCI Indices are Net Dividend
** Fixed Income benchmarks are Total Return

Example 1: Two-fund separation

Question: Consider the following four mutual funds (based on Vanguard's LifeStrategy offering). All four funds invest in the same two broadly diversified indices: one an equity index, the other a bond index. However, their target allocations to the two indices differs depending on the "risk profile" of the investor in the fund:

	Low to Moderate	Moderate	Moderate to High	High
Percentage in equities	20%	40%	60%	80%
Percentage in bonds	80%	60%	40%	20%

Answer: No, according to Tobin's two-fund separation, all investors regardless of risk preferences should hold the same tangency portfolio/mix of bonds and equities.

Example 2: Two-fund separation

Question: You are a financial advisor and have a very risk averse client and a relatively less risk averse client. You recommend the following asset allocations to these clients. Are your recommendations consistent with two-fund separation? Please explain briefly.

Asset	Very risk averse allocation	Less risk averse allocation
Risk-free asset (T-bills)	60%	20%
Bonds	30%	60%
Stocks	10%	20%

Answer: Yes they the same of bonds are. According to 2-fund separation, all investors should choose portfolio mix of risky securities. For both investors the ratio of bonds to stocks is 3:1.

Key Takeaways of MPT

- Benefits of diversification (free lunch):
  - By holding less than perfectly correlated securities together in a portfolio you can reduce risk without giving up expected return
  - Typically cannot diversify away all risk, left with systematic risk
- Optimal portfolio choice (Tobin two-fund separation):
  - Investors should hold a combination of the riskfree asset and the tangency portfolio that has the highest Sharpe ratio
  - First identify the tangency portfolio (does not depend on risk preferences)
  - Identify optimal combination of riskfree asset and the tangency portfolio based on target risk or return/risk preferences - more (less) risk averse investors should hold more (less) in the riskfree asset

Final type of question

Question: You are a portfolio manager who has access to two risky securities only. Security 1 has a mean return of 15% and a standard deviation of 28%, while security 2 has a mean return of 2% and a standard deviation of 17%. These securities have a correlation of-1.

1. What portfolio would you recommend for an investor who wants to minimize risk and doesn't care about return? What is the expected return and standard deviation of this portfolio?
2. What are the possible weights on security 1 that an investor would hold if he or she prefers more to less, is risk averse, and wants a portfolio with a standard deviation of less than that of Security 1? Give a range and show your answers on a mean-standard deviation diagram.

Advanced Portfolio Theory

Issues when implementing Mean-Variance Optimal Portfolios

How well does MVA work in practice?

- Perform an implementable test of optimization:
  - 10y-bond/Stocks 1942-2012
  - Every month, estimate based on past 10 years.
  - Form portfolio with same return as past equal-weighted average.
  - Hold for 1-month, after skipping a month between portfolio formation and holding period.
- Sharpe ratios for Mean-Variance vs. Equal-Weight
  - MV: 0.53
  - EW: 0.49 (In-sample SR = 0.55)
- Markowitz looks pretty good!

Add an asset far from frontier

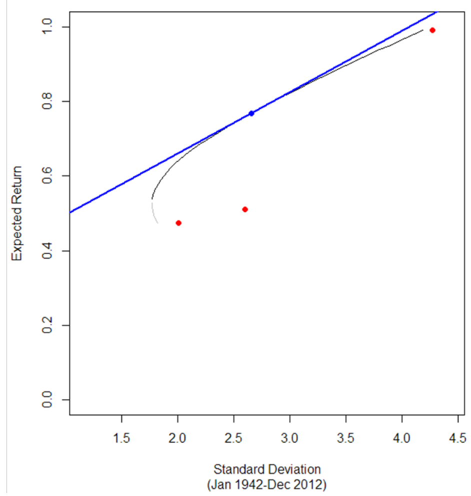
This makes optimization look better.

New 3 asset SR vs. Old 2 asset SR

MV: 0.54 vs. 0.53

EW: 0.45 vs. 0.49

(In Sample MV SR = 0.55)



Take-away from Bond + Stock example

- With only a few assets, optimization work well.
  - The Markowitz-style optimal portfolio had a higher (out of sample) Sharpe ratio than the naïve (1/N) portfolio.
- This example benefits from two things:
  - Long time series
  - Few covariances to estimate
- If the time series is shorter or we have more assets, problems arise . . .

Conventional view from practitioners:

"The logic of mean variance optimization is seductive, but the seduction unravels in the investment period."

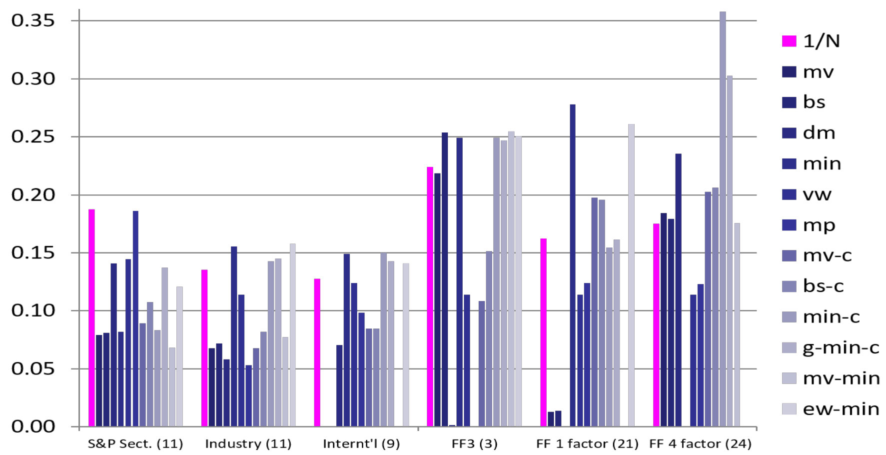
DeMiguel, Garlappi, Uppal (2009)

- What happens when you apply optimization in more complex settings?
  - More assets
  - Shorter time-series
- The authors compare 1/N to the classic mean-variance portfolio and more complicated variants.

Table 1
List of various asset-allocation models considered

#	Model	Abbreviation
Naive		
0.	$1/N$ with rebalancing *(benchmark strategy)*	ew or $1/N$
Classical approach that ignores estimation error		
1.	Sample-based mean-variance	mv
Bayesian approach to estimation error		
2.	Bayesian diffuse-prior	Not reported
3.	Bayes-Stein	bs
4.	Bayesian Data-and-Model	dm
Moment restrictions		
5.	Minimum-variance	min
6.	Value-weighted market portfolio	vw
7.	MacKinlay and Pastor's (2000) missing-factor model	mp
Portfolio constraints		
8.	Sample-based mean-variance with shortsale constraints	mv-c
9.	Bayes-Stein with shortsale constraints	bs-c
10.	Minimum-variance with shortsale constraints	min-c
11.	Minimum-variance with generalized constraints	g-min-c
Optimal combinations of portfolios		
12.	Kan and Zhou's (2007) "three-fund" model	mv-min
13.	Mixture of minimum-variance and $1/N$	ew-min
14.	Garlappi, Uppal, and Wang's (2007) multi-prior model	Not reported

Out-of-sample Sharpe Ratios



Take aways

- No method consistently better than 1/N
- How long a time-series for MV dominate 1/N?
  - Need 3000 months with 25 assets!
  - 6000 months with 50 assets!
- When will more complex methods dominate?
  - If estimation window is long
  - If efficient Sharpe ratio far from 1/N Sharpe ratio
  - When the number of assets is small

MPT in Practice

- Issues with inputs:
  - how to estimate the inputs for large N (answer: use factor models)
  - expected returns, variances, correlations all estimated with error
- Issues with outputs:
  - Portfolio weights very sensitive to small changes in inputs
  - Unconstrained optimisation often leads to portfolios with large short positions, levered positions etc
  - Typically deal with this by imposing constraints on portfolio weights
  - Other solutions (Google if interested): robust statistical estimators, resampling, economic models etc

Alternative Diversification Approaches

- Main idea is to diversify, so why not build a diversified portfolio in a different way, that relies less on having to estimate all the inputs?
- Some alternative diversification schemes:

Name	Description
• **Equally Weighted**	Portfolio weights of $1/N$
• **Risk Parity (1)**	Portfolio weights are proportional to inverse of volatility (or variance)
• **Risk Parity (2) Equal Risk Contribution**	Portfolio weights chosen so that each asset contributes equally to total portfolio variance
• **Minimum Variance**	Portfolio on the left most tip of the efficient frontier

Alternative Diversification Approaches

Inputs needed for alternative diversification schemes:

	Estimate means?	Estimate volatilities?	Estimate correlations?
Equally Weighted	✗	✗	✗
Risk Parity (1)	✗	✔	✗
RP (2) - Equal Risk Contribution	✗	✔	✔
Minimum Variance Portfolio	✗	✔	✔

Where are we?

- We now understand how to make capital allocation decisions given the required inputs:
  - Expected returns
  - Volatilities
  - Correlations
- A big part of the task is identifying the tangency portfolio. Difficult!
- Where do we get the inputs?
  - Historical information
  - **Models**
- After midterm: the CAPM

Key Terms

- Minimum variance frontier
- Diversification
- Idiosyncratic risk
- Systematic risk
- Tangency portfolio
- Capital Market Line
- Sharpe ratio

Housekeeping

- Today 7 of Oct at 9pm you will have access to the midterm.
- Next monday – 12 of Oct – due date of the midterm (25% of the grade)
- Next next monday – 19 of Oct – due date of the final group project (25% of the grade)
- And that Friday – 23 of Oct – Final exam (50% of the grade)

Housekeeping: Midterm

- Today 7 of Oct at 9pm you will have access to the midterm.
  - It's on the **Assignment** link
- Submit it before 11:59pm on Monday 12 of Oct.
- It covers the material we saw until now **(today including)**
- 7 questions in total:
  - 3 questions on Present Value Theory (20 points total)
  - 3 questions on Fixed Income (60 points total)
  - 1 questions on Portfolio Theory(20 points total). For this question, you need to look at the excel file link to the pdf
- If you know the material, it should take you around 3 hours.

To be fair to everyone, we won't take any questions until after the deadline. If somehting it's not clear, state your assumptions and work based on those assumptions.