HARSH CHAN

First, let's perform the Fourier transform with respect to 't' and find the expression for $\tilde{p}_1(r,\omega)$:

$$\tilde{p}_1(r,\omega) = \mathcal{F}p_1(r,t)$$

$$= \int_{-\infty}^{\infty} p_1(r,t)e^{-i\omega t}dt$$

$$= \frac{2A_1}{\pi w_1^2} \exp\left(-\frac{2r^2}{w_1^2}\right) \sum_{n=-\infty}^{\infty} e^{i(\omega_0-\omega)nT_s} e^{-i(\omega-\omega_0)t_0}$$

$$= \frac{2A_1}{\pi w_1^2} \exp\left(-\frac{2r^2}{w_1^2}\right) \sum_{n=-\infty}^{\infty} \delta_{\omega,\omega_0+2\pi n/T_s} e^{-i(\omega-\omega_0)t_0}$$

where $\mathcal{F}p_1(r,t)$ is the Fourier transform of $p_1(r,t)$, and $\delta_{\omega,\omega_0+2\pi n/T_s}$ is the Kronecker delta function.

Now, let's perform the Hankel transform with respect to 'r' and find the expression for $\tilde{p}_1(k,\omega)$:

$$\tilde{p}_1(k,\omega) = \mathcal{H}\tilde{p}_1(r,\omega)$$

$$= \int_0^{\infty} \tilde{p}_1(r,\omega)J_0(kr)rdr$$

$$= \frac{2A_1}{\pi w_1^2} \sum_{n=-\infty}^{\infty} \delta_{\omega,\omega_0+2\pi n/T_s} e^{-i(\omega-\omega_0)t_0} \int_0^{\infty} \exp\left(-\frac{2r^2}{w_1^2}\right) J_0(kr)rdr$$

$$= \frac{2A_1}{\pi w_1^2} \sum_{n=-\infty}^{\infty} \delta_{\omega,\omega_0+2\pi n/T_s} e^{-i(\omega-\omega_0)t_0} \frac{\sqrt{\pi}w_1}{2} J_1\left(\frac{kw_1}{\sqrt{2}}\right)$$

where $\mathcal{H}\tilde{p}_1(r,\omega)$ is the Hankel transform of $\tilde{p}_1(r,\omega)$