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To solve the Laplace's equation using separation of variables, we make the following assumptions:

$$u(r,\theta) = R(r)\Theta(\theta)$$

Substituting this into Laplace's equation, we get:

$$\frac{1}{R(r)}\frac{d^2R(r)}{dr^2} + \frac{1}{r}\frac{dR(r)}{dr} + \frac{1}{R(r)r^2}\frac{d^2\Theta(\theta)}{d\theta^2} = 0$$

Dividing both sides by $R(r)\Theta(\theta)$, we have:

$$\frac{1}{R(r)}\frac{d^2R(r)}{dr^2} + \frac{1}{r}\frac{dR(r)}{dr} = -\frac{1}{r^2}\frac{d^2\Theta(\theta)}{d\theta^2}$$

We can separate the equation into two ordinary differential equations by equating the coefficients of $\frac{d^2R(r)}{dr^2}$ and $\frac{d^2\Theta(\theta)}{d\theta^2}$:

$$\frac{1}{R(r)}\frac{d^2R(r)}{dr^2} + \frac{1}{r}\frac{dR(r)}{dr} = n^2$$

$$\frac{1}{\Theta(\theta)}\frac{d^2\Theta(\theta)}{d\theta^2} = -n^2$$

The general solution to the first equation is:

$$R(r) = C_1r^n + C_2r^{-n}$$

The general solution to the second equation is:

$$\Theta(\theta) = A_1\cos(n\theta) + A_2\sin(n\theta)$$

Since the solution is required to be an odd function of θ, we take only the $\sin(n\theta)$ terms:

$$\Theta(\theta) = B_n\sin(n\theta)$$

So the general solution for $u(r,\theta)$ is:

$$u(r,\theta) = \sum_{n=1}^{\infty} B_n\left(C_1r^n + C_2r^{-n}\right)\sin(n\theta)$$

Since $u(r,\theta)$ is finite in the region $r \geq 2$, we can set $C_1 = 0$:

$$u(r,\theta) = \sum_{n=1}^{\infty} B_nr^{-n}\sin(n\theta)$$

Regenerate response