which is very much like recurrence (4.19). Indeed, this new recurrence has the same solution: $S(m) = O(m \lg m)$. Changing back from $S(m)$ to $T(n)$, we obtain

$$T(n) = T(2^m) = S(m) = O(m \lg m) = O(\lg n \lg \lg n) .$$

Exercises

4.3-1
Show that the solution of $T(n) = T(n-1) + n$ is $O(n^2)$.

4.3-2
Show that the solution of $T(n) = T(\lceil n/2 \rceil) + 1$ is $O(\lg n)$.

4.3-3
We saw that the solution of $T(n) = 2T(\lfloor n/2 \rfloor) + n$ is $O(n \lg n)$. Show that the solution of this recurrence is also $\Omega(n \lg n)$. Conclude that the solution is $\Theta(n \lg n)$.

4.3-4
Show that by making a different inductive hypothesis, we can overcome the difficulty with the boundary condition $T(1) = 1$ for recurrence (4.19) without adjusting the boundary conditions for the inductive proof.

4.3-5
Show that $\Theta(n \lg n)$ is the solution to the "exact" recurrence (4.3) for merge sort.

4.3-6
Show that the solution to $T(n) = 2T(\lfloor n/2 \rfloor + 17) + n$ is $O(n \lg n)$.

4.3-7
Using the master method in Section 4.5, you can show that the solution to the recurrence $T(n) = 4T(n/3) + n$ is $T(n) = \Theta(n^{\log_3 4})$. Show that a substitution proof with the assumption $T(n) \le cn^{\log_3 4}$ fails. Then show how to subtract off a lower-order term to make a substitution proof work.

4.3-8
Using the master method in Section 4.5, you can show that the solution to the recurrence $T(n) = 4T(n/2) + n^2$ is $T(n) = \Theta(n^2)$. Show that a substitution proof with the assumption $T(n) \le cn^2$ fails. Then show how to subtract off a lower-order term to make a substitution proof work.

4.3-1

Show that the solution of $T(n) = T(n-1) + n$ is $O(n^2)$.

$T(n) = O(n^2)$

$T(n) \leq cn^2$

$T(1) + n - n^2 \leq cn^2$

Constante positiva

Supondo $T(1) = 1$: $T(1) \leq cn^2$

$T(1) \leq c$

$1 \leq c$ ✓

$T(n) = T(n-1) + n = T(n-2) + n + n = T(n-2) + 2n$

$T(n-1) = T(n-2) + n$

$T(n) = T(n-k) + kn$

$T(n) = T(n-n+1) + (1-n)n$

$T(n) = T(1) + n - n^2$

$n - k = 1$

$k = 1 - n$

$c(n^2 - 2n + 1) + n$

$T(n) \leq cn^2$

$T(n-1) \leq c(n-1)^2$

$T(n) \leq c(n-1)^2 + n$

$T(n) \leq cn^2 - 2cn + c + n$

$cn^2 - 2cn + c + n \leq cn^2$

$-2cn + c + n \leq 0$ $\quad c = 1$

$n(-2c+1) + c \leq 0$ ✓ $\quad n \geq 1$