

$$1. D = \{0, 1, 3, 5.5\} \quad k=2$$

$$C_1: m_1 = -1 \quad C_2: m_2 = 7.5$$

Distance from D to centroid:

Point	C_1	C_2
0	1	7.5
1	2	6.5
3	4	4.5
5.5	6.5	2

New centroid

$$C'_1: 1.33$$

$$C'_2: 5.5$$

Point	C'_1	C'_2
0	1.33	5.5
1	0.33	4.5
3	1.67	2.5
5.5	4.17	0

New centroid

$$C_1'' : 1.33$$

$$C_2'' : 5.5$$

$$2. \text{ data : } \begin{matrix} x & \begin{bmatrix} -1 & -1 & 0 & 2 & 0 \end{bmatrix} \\ y & \begin{bmatrix} -2 & 0 & 0 & 1 & 1 \end{bmatrix} \end{matrix}$$

step 1: Standardize the data

$$x\text{-mean} = (-1 + (-1) + 0 + 2 + 0) / 5 = 0$$

$$y\text{-mean} = (-2 + 0 + 0 + 1 + 1) / 5 = 0$$

Data Adjust

$$x : -1 \quad -1 \quad 0 \quad 2 \quad 0$$

$$y : -2 \quad 0 \quad 0 \quad 1 \quad 1$$

Step 2. Find covariance matrix

$$\text{Cov}(X, Y) = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{n-1}$$

$$Cov = \begin{pmatrix} 1.5 & 1 \\ 1 & 1.5 \end{pmatrix}$$

Step 3 Calculate the eigen values and eigen vectors

$$\text{eigen values} = \begin{pmatrix} 0.5 \\ 2.5 \end{pmatrix}$$

$$\text{eigen vectors} = \begin{pmatrix} -0.7071 & , & 0.7071 \\ 0.7071 & , & 0.7071 \end{pmatrix}$$

Step 4: Mapping

$$Y = \begin{bmatrix} -0.7071, -0.7071, 0, 1.4142, 0 \\ -1.4142, 0, 0, 0.7071, 0.7071 \end{bmatrix}$$