Q1 Prove the following statements.

(a) $\overline{\mathbb{Q}}/\mathbb{Q}$ is Galois.

(b) If F is a finite field, then every algebraic extension of F is Galois.

(c) $\overline{\mathbb{F}_p(t)}/\mathbb{F}_p(t)$ is not Galois.

Q2 Let K/F and L/K be algebraic extensions.

(a) Show that if L/F is normal, then L/K is normal.

(b) Show that if L/F is Galois, then L/K is Galois.

Q3 Let $\zeta_p = e^{2\pi i/p}$, a primitive p-th root of unity. Show that $\mathbb{Q}(\zeta_p)/\mathbb{Q}$ is Galois and $\mathrm{Gal}(\mathbb{Q}(\zeta_p)/\mathbb{Q}) \cong (\mathbb{Z}/p\mathbb{Z})^\times$.

Q4 Show that $\mathbb{Q}(\sqrt{2}+\sqrt{5})/\mathbb{Q}$ is Galois and $\mathrm{Gal}(\mathbb{Q}(\sqrt{2}+\sqrt{5})/\mathbb{Q}) \cong \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$.

Q5 §14.2 Problem 13.