1 Vakil Exercise 21.7.K Suppose C is an irreducible smooth projective curve over an algebraically closed field $k = \overline{k}$ of characteristic 0, of genus $g \geq 2$. Suppose that G is a finite group of automorphisms of C.

(a) Let C' be the smooth projective curve corresponding to the field extension $K(C)^G$ of k (via Theorem 17.4.3). (($K(C)^G$ means the G-invariants of $K(C)$.) Describe a morphism $\pi : C \to C'$ of degree $|G|$, as well as a faithful G-action on C that commutes with π.

(b) Show that above each branch point of π, the preimages are all ramified to the same order (as G acts transitively on them). Suppose there are n branch points and the ith one has ramification r_i (each $|G|/r_i$ times).

(c) Use the Riemann-Hurwitz formula to show that

$$(2g-2) = |G|\left(2g(C') - 2\sum_{i=1}^{n}\frac{r_i - 1}{r_i}\right)$$

2 Gathamnn Exercise 7.8.9 Show that any smooth projective curve of genus 2...

(i) can be realized as a curve of degree 5 in $\mathbb{P}^3$,

(ii) admits a two-to-one morphism to $\mathbb{P}^1$. How many ramification points does such a morphism have?

3 Gathamnn Exercise 7.8.10 Let X be a smooth projective curve, and let $P \in X$ be a point. Show there is a rational function on X that is regular everywhere except at P.