1 Exercise I.19 Let G be a finite group operating on a finite set S.

(a) For each $s \in S$ show that

$$\sum_{t \in G \cdot s} \frac{1}{|G \cdot t|} = 1.$$

(b) For each $x \in G$ define $f(x) =$ number of elements $s \in S$ such that $xs = s$. Prove that the number of orbits of G in S is equal to

$$\frac{1}{|G|} \sum_{x \in G} f(x).$$

2 Exercise I.21 Let G be a finite group and H a subgroup. Let P_H be a p-Sylow subgroup of H. Prove that there exists a p-Sylow subgroup P of G such that $P_H = P \cap H$.

3 Exercise I.22 Let H be a normal subgroup of a finite group G and assume that $|H| = p$. Prove that H is contained in every p-Sylow subgroup of G.

4 Exercise I.23 Let P, P' be a p-Sylow subgroups of a finite group G.

(a) If $P' \subseteq N(P)$ (normalizer of P), then $P' = P$.

(b) If $N(P') = N(P)$, then $P' = P$.

(c) We have $N(N(P)) = N(P)$.