

- Q1** §14.2 Problem 15.  
(The Klein 4-group is a group isomorphic to  $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} \cong D_4$ . )
- Q2** Let  $K/F$  be a separable finite extension. Show that  $K$  has finitely many subfields containing  $F$ .
- Q3** Let  $K$  be the Galois closure of  $\mathbb{Q}(\sqrt{1+\sqrt{3}})$ .
- (a) Show that  $[K : \mathbb{Q}] = 8$ .
  - (b) Show that  $\text{Gal}(K/\mathbb{Q})$  is not commutative.
  - (c) Show that  $\text{Gal}(K/\mathbb{Q})$  has a normal subgroup isomorphic to  $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$ .
  - (d) Show that  $\text{Gal}(K/\mathbb{Q}) \cong D_8$ .
- Q4** Show that if  $K/\mathbb{Q}$  is a finite Galois extension with  $\text{Gal}(K/\mathbb{Q}) \cong S_3$ , then  $K$  is the splitting field for some irreducible cubic polynomial in  $\mathbb{Q}[x]$ .