

1 Redo #1 from Homework 1, but this time use the snake lemma: Let A be a Δ -complex and build a new Δ -complex X by adding a single new n -simplex D . (**Clarification:** D is glued to A via a continuous map from the boundary of D to A .) Using simplicial homology, compute the difference between $H_*(A)$ and $H_*(X)$. (This will depend heavily on the chain ∂D .)

2 Hatcher 2.1.12 Show that chain homotopy of chain maps is an equivalence relation.

3 Hatcher 2.1.14 Determine whether there exists a short exact sequence $0 \rightarrow \mathbb{Z}_4 \rightarrow \mathbb{Z}_8 \oplus \mathbb{Z}_2 \rightarrow \mathbb{Z}_4 \rightarrow 0$. More generally, determine which abelian groups A fit into a short exact sequence $0 \rightarrow \mathbb{Z}_{p^m} \rightarrow A \rightarrow \mathbb{Z}_{p^n} \rightarrow 0$ with p prime. What about the case of short exact sequences $0 \rightarrow \mathbb{Z} \rightarrow A \rightarrow \mathbb{Z}_n \rightarrow 0$?

4 Hatcher 2.1.16

(a) Show that $H_0(X, A) = 0$ iff A meets each path-component of X .

(b) Show that $H_1(X, A) = 0$ iff $H_1(A) \rightarrow H_1(X)$ is surjective and each path-component of X contains at most one path-component of A .

5 Hatcher 2.1.18 Show that for the subspace $\mathbb{Q} \subseteq \mathbb{R}$, the relative homology group $H_1(\mathbb{R}, \mathbb{Q})$ is free abelian and find a basis.