Q1 Show that $[\overline{\mathbb{Q}} : \mathbb{Q}] = \infty$.

Q2 Let $F \subseteq K \subseteq L$ be fields. Show that if L/F is separable, then both K/F and L/K are separable.

Q3 Let F be a field and A be a subset of $F[x]$. An algebraic field extension K of F is called a *splitting field for A over F* if

(i) every polynomial in A splits completely in $K[x]$,

(ii) if $F \subseteq E \subseteq K$ and every polynomial in A splits completely in $E[x]$, then $E = K$.

(a) Suppose that $A = \{f_1(x), f_2(x), \ldots, f_n(x)\} \subseteq F[x]$. Let $f(x) = \prod_{j=1}^{n} f_j(x)$ and K be a splitting field of $f(x) \in F[x]$. Show that K is a splitting field of A over F.

(b) Let $S \subseteq \overline{F}$ be the subset consisting of roots of polynomials in A. Show that $F(S)$ is a splitting field of A over F.

(c) Suppose that K is a splitting field of A over F. Show that there exists a field isomorphism $\varphi : K \to F(S)$ such that $\varphi|_F = \mathrm{id}_F$.

(Hint: Prop 5 on Apr 13 can be useful.)

Q4 Let F be a field of characteristic p. Show that if K/F is a finite inseparable field extension, then $p \,\big|\, [K : F]$.