Take $R = \mathbb{Z}$ and consider the $\mathbb{Z}$-modules $A = \mathbb{Z}$ and $B = \bigoplus_{n\in\mathbb{N}} \mathbb{Z}/p\mathbb{Z}$ for a prime $p \in \mathbb{Z}$. Take $M = A \oplus B$ with maps

$$\begin{aligned} f : \mathbb{Z} &\longrightarrow \mathbb{Z} \oplus B \\ a &\longmapsto pa \oplus 0 \end{aligned}$$

and

$$\begin{aligned} g : \mathbb{Z} \oplus \bigoplus_{n\in\mathbb{N}} \mathbb{Z}/p\mathbb{Z} &\longrightarrow \bigoplus_{n\in\mathbb{N}} \mathbb{Z}/p\mathbb{Z} \\ a \oplus b &\longmapsto (a + p\mathbb{Z}) \oplus b. \end{aligned}$$

In other words, f is multiplication by p composed with inclusion into the direct sum, and g is the map which quotients A onto the first component and shifts all the components in B to the right by one index.

Then f is inject and g is surjective with $\ker g = p\mathbb{Z} \oplus 0 = \operatorname{im} f$. Hence we have the following short exact sequence:

$$0 \longrightarrow A \xrightarrow{f} A \oplus B \xrightarrow{g} B \longrightarrow 0.$$

However, if there were a $\mathbb{Z}$-module homomorphism $q : B \to A \oplus B$ such that $g \circ q = \mathrm{id}_B$, it can be seen that q would need to map the first component of B to some part of A. In particular, given $(n + p\mathbb{Z}) \oplus 0 \in B$, we must be able to choose some $q((n + p\mathbb{Z}) \oplus 0) = a \oplus b \in A \oplus B$ such that

$$(n + p\mathbb{Z}) \oplus 0 = g(a \oplus b) = (a + p\mathbb{Z}) \oplus b,$$

which implies $b = 0$ and $a \in \mathbb{Z}$ is an integer with $a + p\mathbb{Z} = n + p\mathbb{Z}$. This means restricting q to the first component of B induces a $\mathbb{Z}$-module homomorphism $\mathbb{Z}/p\mathbb{Z} \to \mathbb{Z}$ where $n \mapsto a$. However, the only $\mathbb{Z}$-module homomorphism $\mathbb{Z}/p\mathbb{Z} \to \mathbb{Z}$ is the zero map. But in order to have $g \circ p = \mathrm{id}_B$, we must have q injective, but with the first component of B nonzero, this is not possible.

I believe that the formulation of (i) should also require that f and g act as inclusion and surjection. That is, not only do we need $M \cong A \oplus B$, but there must be an isomorphism $\varphi : M \to A \oplus B$ such that $\varphi \circ f : A \to A \oplus B$ is the natural inclusion and $g \circ \varphi^{-1} : A \oplus B \to B$ is the natural projection, i.e., the following diagram commutes with exact rows:

$$\begin{tikzcd} 0 \arrow[r] & A \arrow[r, "f"] \arrow[d, "\mathrm{id}_A"] & M \arrow[r, "g"] \arrow[d, "\varphi"] & B \arrow[r] \arrow[d, "\mathrm{id}_B"] & 0 \\ 0 \arrow[r] & A \arrow[r, hook] & A \oplus B \arrow[r, two heads] & B \arrow[r] & 0 \end{tikzcd}$$

(Where the bottom row uses the natural inclusion and projection.) The above example fails because the isomorphism $M \cong A \oplus B$ does not make the right square in this diagram commute.