idk what is wanted, but imma fix a base field $\Bbbk$, which is likely $\mathbb{R}$ or $\mathbb{C}$. I think any subfield of $\mathbb{C}$ is fine.

A **vector space** (over $\Bbbk$) is a $\Bbbk$-module.

A **norm** on a vector space X is a function $\|-\| : X \to \mathbb{R}$ such that

- (positive definite) $\|x\| = 0$ implies $x = 0$, for all $x \in X$;
- (absolute homogeneity) $\|cx\| = |c|\|x\|$ for all $c \in \Bbbk$ and $x \in X$;
- (triangle inequality) $\|x + y\| \leq \|x\| + \|y\|$ for all $x, y \in X$.

One can check that these conditions also imply

- $\|x\| = 0$ if and only if $x = 0$, for all $x \in X$;
- $\|x\| \geq 0$ for all $x \in X$.

These slightly stronger conditions are sometimes added into the definition of a norm.

A **normed vector space** is a vector space X with a norm $\|-\|_X$.

Let X and Y be normed vector spaces.

A linear map $T : X \to Y$ is called **bounded** if there exists $C \in \mathbb{R}$ such that

$$\|Tx\|_Y \leq C\|x\|_X, \qquad \text{for all } x \in X.$$

(Necessarily, such a C would be positive.) We also call T a **bounded linear operator**.

If $T : X \to Y$ is a bounded linear operator, define

$$\|T\| := \inf\{C \in \mathbb{R} : \|Tx\|_Y \leq C\|x\|_X \text{ for all } x \in X\}.$$

One can check that

$$\|T\| = \sup_{\substack{x \in X \\ x \neq 0}} \frac{\|Tx\|_Y}{\|x\|_X} = \sup_{\substack{x \in X \\ 0 < \|x\|_X \leq 1}} \frac{\|Tx\|_Y}{\|x\|_X} = \sup_{\substack{x \in X \\ 0 < \|x\|_X < 1}} \frac{\|Tx\|_Y}{\|x\|_X} = \sup_{\substack{x \in X \\ \|x\|_X = 1}} \|Tx\|_Y$$

Let X and Y be normed vector spaces and $U \subseteq X$ be an open subset.

A function $f : U \to Y$ is called **differentiable at** $x \in U$ if there exists a bounded linear operator $T : X \to Y$ satisfying

$$\lim_{\substack{\|h\|_X \to 0 \\ h \in X \setminus \{0\}}} \frac{\|f(x + h) - f(x) - T(h)\|_Y}{\|h\|_X} = 0.$$

$$Q(f, x, h, T) = \frac{\|f(x + h) - f(x) - T(h)\|_Y}{\|h\|_X}$$

In which case, we say T is the* **derivative of f at x**, written $\mathrm{d}f_x = \mathrm{d}(f)_x = T$.

For any $V \subseteq U$, f is called **differentiable on V** if it is differentiable at every point of V.

f is called **differentiable** if it is differentiable on U.

Let $U \subseteq X$ be open and $f : U \to Y$ be differentiable.

$\mathcal{L}(X, Y)$.

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f is called **continuously differentiable at x**