Q1 §13.1 Problem 1

Q2 §13.2 Problem 7

Q3 Let K/F be a field extension and $\alpha_1, \dots, \alpha_n \in K$. Show that

$$F(\alpha_1, \dots, \alpha_n) = \big(F(\alpha_1, \dots, \alpha_{n-1})\big)(\alpha_n).$$

(The LHS is the subfield generated by $\alpha_1, \dots, \alpha_n$ over F. The RHS is the subfield generated by α_n over the field $F(\alpha_1, \dots, \alpha_{n-1})$.)

Q4 Let K/F be a field extension and $\alpha, \beta \in K$. Suppose that $[F(\alpha) : F]$ and $[F(\beta) : F]$ are both finite.

(a) Show that $[F(\alpha) : F] \geq [F(\alpha, \beta) : F(\beta)]$.

(b) Show that $[F(\alpha, \beta) : F] \leq [F(\alpha) : F][F(\beta) : F]$, and the equality holds if $[F(\alpha) : F]$ and $[F(\beta) : F]$ are coprime.

(c) Given $\alpha_1, \dots, \alpha_n \in K$ with $[F(\alpha_j) : F]$, $1 \leq j \leq n$, all finite, show that

$$[F(\alpha_1, \dots, \alpha_n) : F] \leq [F(\alpha_1) : F][F(\alpha_2) : F] \cdots [F(\alpha_n) : F].$$