

1 Exercise 0.15 Enumerate all the subcomplexes of S^∞ , with the cell structure described in this section, having two cells in each dimension.

2 Exercise 0.16 Show that S^∞ is contractible.

3 Exercise 0.18 Show that $S^1 * S^1 = S^3$, and more generally $S^n * S^m = S^{n+m+1}$.

4 Exercise 0.19 Show that the space obtained from S^2 by attaching n 2-cells along any collection of n circles in S^2 is homotopy equivalent to the wedge sum of $n + 1$ 2-spheres.

5 Exercise 0.20 Show that the subspace $X \subseteq \mathbb{R}^3$ formed by a klein bottle intersecting itself in a circle is homotopy equivalent to $S^1 \vee S^1 \vee S^2$.

6 Exercise 0.23 Show that a CW complex is contractible if it is the union of two contractible subcomplexes whose intersection is also contractible.

7 Exercise 0.24 Let X and Y be CW complexes with 0-cells x_0 and y_0 . Show that the quotient spaces $X * Y / (X * \{y_0\} \cup \{x_0\} * Y)$ and $S(X \wedge Y) / S(\{x_0\} \wedge \{y_0\})$ are homeomorphic, and deduce that $X * Y \simeq S(X \wedge Y)$.

8 Exercise 0.25 If X is a CW complex with components X_α , show that the suspension SX is homotopy equivalent to $Y \bigvee_\alpha SX_\alpha$ for some graph Y . In the case that X is a finite graph, show that SX is homotopy equivalent to a wedge sum of circles and 2-spheres.