My thought is to prove and apply Bezóut's theorem. The Shafarevich text seems to discuss it as a sort of elementary motivation for projective spaces. Gathmann's notes "Plane Algebraic Curves", which is referenced in our text, also cover the theorem. The basic result is for curves in the projective plane, but can also be generalized and applied to higher dimensions, etc. There appears to be no shortage of applications of Bezóut's theorem, so if it turns out to be too easy/difficult to prove, I can reevaluate the scope of my project while remaining in the same general area.