

Exercise 10.6 Let $f : X \rightarrow Y$ be a morphism of varieties, and let $a \in X$. Show that f induces a linear map $T_a X \rightarrow T_{f(a)} Y$ between tangent spaces.

Proof. The morphism $f : X \rightarrow Y$ corresponds to a K -algebra homomorphism $f^* : \mathcal{O}_{Y,f(a)} \rightarrow \mathcal{O}_{X,a}$.

□

Exercise 10.18 Let $X \subseteq \mathbb{P}^3$ be the degree 3 Veronese embedding of \mathbb{P}^1 as in Exercise 7.30. Of course, X must be smooth since it is isomorphic to \mathbb{P}^1 . Verify this directly using the projective Jacobi criterion of Exercise 10.13(b).