Notation

Denote $I = [0, 1] \subseteq \mathbb{R}$ with the usual topology.

Terminology

Let X and Y be (topological) spaces.

A **function** $X \to Y$ (from X to Y) is used to mean the most general sort of function of the underlying sets.

A **map** $X \to Y$ (from X to Y) is used to mean a continuous function.

Let X and Y be (topological) spaces.

A **homotopy** from X to Y is a map $F : X \times I \to Y$.

A **homotopy** from X to Y is a family of functions $\{f_t : X \to Y\}_{t \in I}$ such that the associated function $X \times I \to Y$ sending $(x, t) \mapsto f_t(x)$ is continuous. (Footnote: In particular, each f_t will be continuous.)

We say that the maps f_0 and f_1 are **homotopic**, written $f_0 \simeq f_1$.

Let $A \subseteq X$ be a subspace.

For a map $f : A \to Y$, an **extension** of f to X is a map $F : X \to Y$ such that $F|_A = f$.

A **retraction** of X onto A is a map $r : X \to X$ such that $r(X) = A$ and $r|_A = \mathbf{1}_A$.

A **retraction** of X onto A is a map $r : X \to A$ such that $r|_A = \mathbf{1}_A$.

A **retraction** of X onto A is an extension of $\mathbf{1}_A$ to X.

When such a retraction exists, we say A is a **retract** of X.

A **retraction** of X is a map $r : X \to X$ such that $r^2 = r$. (Then $r(X)$ is the retract.)

A **deformation retraction** of X onto A is a homotopy $f_t : X \to X$ such that $f_0 = \mathbf{1}_X$, $f_t|_A = \mathbf{1}_A$ for all $t \in I$, and $f_1(X) = A$. (Footnote: f_1 is a retraction of X onto A.)

In which case, say A is a **deformation retract** of X.

Given a homotopy $f_t : X \to Y$ such that $f_t|_A = f_0|_A$ for all $t \in I$ is called a **homotopy relative** to A, or a homotopy $\operatorname{rel} A$.

A **deformation retraction** of X onto A is a homotopy rel A from $\mathbf{1}_X$ to a retraction of X onto A.

Given subspace $A \subseteq X$ and map $f : A \to Y$. Construct

$$X \sqcup_f Y = X \sqcup Y / \{a \sim f(a) : a \in A\}$$

$$X \sqcup_f Y = \frac{X \sqcup Y}{a \sim f(a) : a \in A}$$

$$X \sqcup_f Y = X \sqcup Y / \Gamma(f) \quad \text{where} \quad \Gamma(f) = \{(x, f(x)) : x \in \operatorname{dom} f\}$$