Q1 Let K be a finite separable extension of F. Given $\alpha \in K$, the **norm** of α from K to F is defined as

$$N_{K/F}(\alpha) = \prod_{\varphi: K \to \overline{F},\, F\text{-embedding}} \varphi(\alpha).$$

(a) Show that $N_{K/F}(\alpha) \in F$.

(b) Suppose that $\alpha \in \overline{F}$ and $m_{\alpha,F}(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ is separable. Show that $N_{F(\alpha)/F}(\alpha) = (-1)^n a_0$.

Q2 §14.7 Problem 4.

Q3 §14.7 Problem 5.

Q4 §14.7 Problem 6. (Here $\varphi(n)$ is Euler's totient function. It counts the number of integers between 1 and n coprime to n. For $n = p_1^{k_1} p_2^{k_2} \cdots p_r^{k_r}$, $\varphi(n) = (p_1^{k_1} - p_1^{k_1-1})(p_2^{k_2} - p_2^{k_2-1}) \cdots (p_r^{k_r} - p_r^{k_r-1})$. You can use the fact that $[\mathbb{Q}(\zeta_n) : \mathbb{Q}] = \varphi(n)$.)

Q5 §14.7 Problem 18.