MA 402: Project 5

Instructions:

- Detailed instructions regarding submission are available on the class website[1].
- The zip file should contain three files hw5.pdf, hw5.tex, classnotes.sty.

1) (20 points) Consider the function $f(x) = x$ in the interval $[0, 2\pi)$.

(a) Derive the Fourier coefficients c_k for $k = 0, \pm 1, \pm 2, \ldots$.

(b) Derive the Fourier coefficients a_0, a_k, b_k for $k = 1, 2, \ldots,$.

(c) Plot the partial Fourier series, along with the function f, by retaining $n = 1, 10, 50, 100$ terms in the summation (use the second form involving cosines and sines).

(d) Comment on the convergence of the partial Fourier series.

Note: you should submit only 1 plot for this problem.

2) (15 points) (Denoising a signal) Consider the function $f(x)$ defined as

$$f(x) = -\frac{1}{5}\left(\frac{x(2\pi - x)}{10}\right)^5 (x + 1.5)(x + 2.5)(x - 4) + 1.7 \qquad x \in [0, 2\pi).$$

(a) Sample this function at 512 evenly spaced points to obtain sample values $f_0, \ldots, f_{511}$. Add noise to this image as $\tilde{f}_j = f_j + \epsilon r_j$ where $r_j \sim \text{Normal}(0, 1)$ and $\epsilon = 10^{-1}$. Plot the sampled function values alongside the noisy function values.

(b) Denoise the signal as follows: set all the Fourier coefficients c_k to be zero except for lowest 4 frequencies. Plot the denoised signal with the original function.

(c) Repeat the previous part, but this time keeping only the lowest 10 frequencies.

Note: you should submit 3 plots for this problem.

3) (15 points) Consider the function $f(x)$ defined as

$$f(x) = 2\pi x - x^2 \qquad x \in [0, 2\pi).$$

(a) Compute the Fourier interpolant $p(x) = \sum_{k=-n/2}^{n/2-1} c_k e^{ikx}$. Plot the interpolant with the original function for $n = 8, 16, 32, 64$ points.

(b) Compute the derivative of the interpolant $p'(x)$. Plot this derivative against the true derivative for $n = 8, 16, 32, 64$ points.

(c) Comment on the accuracy of the Fourier series in parts (a) and (b).

Note: you should submit 2 plots for this problem.

[1] https://github.ncsu.edu/asaibab/ma402/blob/master/project.md