Z_2 Class

時間反転演算子 $\hat{T}$

$\hat{K}$ を波動関数の共役をとる演算子とする. $\frac{1}{2}$ スピン系で

$$\hat{T} = -i\sigma_2 \hat{K} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \hat{K} \tag{1}$$

$$\hat{T}^2 = -1 \tag{2}$$

$$\left\langle \hat{T}\psi \middle| \hat{T}\phi \right\rangle = \langle \phi | \psi \rangle \tag{3}$$

$$\left\langle \psi \middle| \hat{T}\phi \right\rangle = -\left\langle \phi \middle| \hat{T}\psi \right\rangle \tag{4}$$

$$\begin{aligned} \left\langle \hat{T}\psi \middle| \hat{T}\hat{A}\hat{T}^{-1} \middle| \hat{T}\phi \right\rangle &= \left\langle \hat{T}\psi \middle| \hat{T}\hat{A}\phi \right\rangle \\ &= \left\langle \hat{A}\phi \middle| \psi \right\rangle = \langle \phi | \hat{A}^\dagger | \psi \rangle \end{aligned} \tag{5}$$

時間反転対称な系

$$\hat{T}^{-1}\hat{H}(k)\hat{T} = \hat{H}(-k) \tag{6}$$

$T\,|u_n(k)\rangle$ は $\hat{H}(-k)$ の固有状態である

$$\hat{H}(-k)\hat{T}\,|u_n(k)\rangle = \hat{T}\hat{H}(k)\,|u_n(k)\rangle = E_n(k)\left(\hat{T}\,|u_n(k)\rangle\right) \tag{7}$$

w 行列

$$\hat{T}\left|u_\beta(k)\right\rangle = w_{\gamma\beta}(k)\left|u_\gamma(-k)\right\rangle \tag{8}$$

と定める.

$$w_{\alpha\beta}(k) = \left\langle u_\alpha(-k)\right|\hat{T}\left|u_\beta(k)\right\rangle \tag{9}$$

w 行列の性質

Unitary

$$\begin{aligned}\left(w(k)^\dagger w(k)\right)_{\alpha\beta} &= w^*_{\gamma\alpha}(k)w_{\gamma\beta}(k)\\ &= \left(\left\langle u_\gamma(-k)\right|\hat{T}\left|u_\alpha(k)\right\rangle\right)^*\left(\left\langle u_\gamma(-k)\right|\hat{T}\left|u_\beta(k)\right\rangle\right)\\ &= \left\langle \hat{T}u_\alpha(k)\middle|u_\gamma(-k)\right\rangle\left\langle u_\gamma(-k)\middle|\hat{T}u_\beta(k)\right\rangle\\ &= \left\langle \hat{T}u_\alpha(k)\middle|\hat{T}u_\beta(k)\right\rangle\\ &= \left\langle u_\beta(k)\middle|u_\alpha(k)\right\rangle = \delta_{\alpha\beta}\end{aligned} \tag{10}$$

$w(-k)w(k) = -1$

$$\begin{aligned}\hat{T}^2\left|u_\alpha(k)\right\rangle &= -\left|u_\alpha(k)\right\rangle\\ &= w_{\beta\alpha}(k)\hat{T}\left|u_\beta(-k)\right\rangle\\ &= w_{\beta\alpha}(k)w_{\gamma\beta}(-k)\left|u_\gamma(k)\right\rangle\end{aligned} \tag{11}$$

よって$w_{\gamma\beta}(-k)w_{\beta\alpha}(k) = -\delta_{\alpha\gamma}$

$w(k) = -w(-k)^\intercal$

$$\begin{aligned}w_{\alpha\beta}(k) &= \left\langle u_\alpha(-k)\right|\hat{T}\left|u_\beta(k)\right\rangle\\ &= -\left\langle u_\beta(k)\right|\hat{T}\left|u_\alpha(-k)\right\rangle\\ &= -w_{\beta\alpha}(-k)\end{aligned} \tag{12}$$

TRIM Λ では $w(\Lambda)$ は反対称行列