Note

$$\hat{\mathcal{H}}_k = e^{-ik\cdot\hat{r}}\mathcal{H}e^{ik\cdot\hat{r}}$$

$$\hat{\mathcal{H}}_k \left|u_n(k)\right\rangle = E_n(k)\left|u_n(k)\right\rangle$$

$$\begin{aligned} Q_n^{ij}(k) &= \left\langle \partial_i u_n(k)\right| \left(1 - \left|u_n(k)\right\rangle \left\langle u_n(k)\right|\right) \left|\partial_j u_n(k)\right\rangle \\ &= \sum_{m\neq n} \frac{\left\langle u_n(k)\right| \partial_i H_k \left|u_m(k)\right\rangle \left\langle u_m(k)\right| \partial_j H_k \left|u_n(k)\right\rangle}{(E_m - E_n)^2} \\ &= g_n^{ij}(k) - \frac{i}{2}\Omega_n^{ij}(k) \end{aligned}$$

$$\begin{aligned} \mathrm{d}s^2 &\coloneqq 1 - \left|\left\langle u_n(k)|u_n(k+\mathrm{d}k)\right\rangle\right|^2 \\ &= g_{ij}(k)\mathrm{d}k_i\mathrm{d}k_j \end{aligned}$$

$$\gamma = \int_S \mathrm{d}\boldsymbol{S}\cdot\boldsymbol{\Omega}_n(k)$$

$$a^i_{mn}(k) = i \langle u_m(k) | \partial_i u_n(k) \rangle$$

$$\Omega^{ij}_n(k) = \partial_i a^j_{nn}(k) - \partial_j a^i_{nn}(k)$$

P Symmetry : 適切な Gauge をとると

$$P |u_n(k)\rangle = |u_n(-k)\rangle$$

$$a^i_{mn}(-k) = -a^i_{mn}(k)$$

$$\Omega^{ij}_n(-k) = \Omega^{ij}_n(k)$$

T Symmetry :

$$T |u_n(k)\rangle = |u_n(-k)\rangle$$

$$a^i_{mn}(-k) = a^i_{nm}(k)$$

$$\Omega^{ij}_n(-k) = -\Omega^{ij}_n(k)$$

PT Symmetry :

$$PT |u_n(k)\rangle = |u_n(k)\rangle$$

$$a^i_{mn}(k) = -a^i_{nm}(k)$$

$$\Omega^{ij}_n(k) = 0$$

$$(\langle T\phi | T\psi \rangle = \langle \psi | \phi \rangle)$$