Note

$$\hat{\mathcal{H}}_k = e^{-ik\cdot\hat{r}}\mathcal{H}e^{ik\cdot\hat{r}}$$

$$\hat{\mathcal{H}}_k \left|u_n(k)\right\rangle = E_n(k)\left|u_n(k)\right\rangle$$

$$
\begin{aligned}
Q_n^{ij}(k) &= \left\langle\partial_i u_n(k)\right|\left(1 - \left|u_n(k)\right\rangle\left\langle u_n(k)\right|\right)\left|\partial_j u_n(k)\right\rangle \\
&= \sum_{m\neq n}\frac{\left\langle u_n(k)\right|\partial_i H_k\left|u_m(k)\right\rangle\left\langle u_m(k)\right|\partial_j H_k\left|u_n(k)\right\rangle}{(E_m - E_n)^2} \\
&= g_n^{ij}(k) - \frac{i}{2}\Omega_n^{ij}(k)
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{d}s^2 &:= 1 - \left|\left\langle u_n(k)|u_n(k+\mathrm{d}k)\right\rangle\right|^2 \\
&= g_{ij}(k)\mathrm{d}k_i\mathrm{d}k_j
\end{aligned}
$$

$$\gamma = \int_S \mathrm{d}\boldsymbol{S}\cdot\boldsymbol{\Omega}_n(k)$$

$$a_{mn}^i(k) = i \langle u_m(k) | \partial_i u_n(k) \rangle$$

$$\Omega_n^{ij}(k) = \partial_i a_{nn}^j(k) - \partial_j a_{nn}^i(k)$$

P Symmetry : 適切な Gauge をとると

$$P |u_n(k)\rangle = |u_n(-k)\rangle$$

$$a_{mn}^i(-k) = -a_{mn}^i(k)$$

$$\Omega_n^{ij}(-k) = \Omega_n^{ij}(k)$$

T Symmetry :

$$T |u_n(k)\rangle = |u_n(-k)\rangle$$

$$a_{mn}^i(-k) = a_{nm}^i(k)$$

$$\Omega_n^{ij}(-k) = -\Omega_n^{ij}(k)$$

PT Symmetry :

$$PT |u_n(k)\rangle = |u_n(k)\rangle$$

$$a_{mn}^i(k) = -a_{nm}^i(k)$$

$$\Omega_n^{ij}(k) = 0$$

$$(\langle T\phi | T\psi \rangle = \langle \psi | \phi \rangle)$$