# 2020～2021学年第一学期期末考试试卷(**A 卷**)

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| **Scores** | |
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**I. For each blank in the following statements, choose the best answer from the choices given below. (This problem contains 6 questions, 3 points for each question and 18 points in all)**

1. Which of the following statements is true. (　　)

   (A) $\lim\limits_{x\to\infty}\frac{\sin x}{x}=1$　　(B) $\lim\limits_{x\to\infty}x\sin\frac{1}{x}=1$

   (C) $\lim\limits_{x\to\infty}(1-\frac{1}{x})^x=-e$　　(D) $\lim\limits_{x\to\infty}(1+\frac{1}{x})^{-x}=e$

2. Assume that

$$f(x)=\begin{cases}\ln(1+x^2) & \text{if} \quad x>1\\ x^2+b & \text{if} \quad x\le 1.\end{cases}$$

   is continuous everywhere. Then (　　)

   (A)$b=-1$.　　(B) $b=\ln 4-1$.

   (C) $b=\ln 3-1$.　　(D) $b=\ln 2-1$.

3. Assume that the function is determined by $e^{x+y}+\sin(xy)=1$. Then $y'(0)=$ (　　)

   (A) 0.　　(B)1.　　(C) $-1$.　　(D) $\frac{1}{2}$.

4. The antiderivative of $\int \arctan x\, dx$ is (    )

(A) $x\arctan x + \ln(1+x^2) + C$.    (B) $x\arctan x - \ln(1+x^2) + C$.

(C) $-x\arctan x + \frac{1}{2}\ln(1+x^2) + C$.    (D) $x\arctan x - \frac{1}{2}\ln(1+x^2) + C$.

5. Assume that $F(x) = \int_x^{x+2\pi} e^{\sin t}\sin t\, dt$. Then $F(x)$ is

(A) a positive constant.    (B) a negative constant.

(C) 0.    (D) not a constant.

6. Which of the following improper integrals is convergence (    )

(A) $\int_e^{+\infty} \frac{\ln^{2021} x}{x}\, dx$    (B) $\int_e^{+\infty} \frac{1}{x\ln x}\, dx$

(C) $\int_e^{+\infty} \frac{1}{x\ln^{\frac{1}{2021}} x}\, dx$    (D) $\int_e^{+\infty} \frac{1}{x\ln^{2021} x}\, dx$

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**II. Fill the correct answer in the blanks (This problem contains 6 questions, 3 points for each question and 18 points in all)**

7. Assume that $\lim\limits_{x\to+\infty} \frac{ax+\sin x}{x} = 2$. Then $a =$ __________________.

8. If $f(x)$ is determined by $f(x) = \int_1^u \cos^2 t\, dt$, where $u = \ln(x^2+x-1)$. Then $f'(1) =$ __________________.

9. $\int (e^{-2x}+1)\, dx =$ __________________________.

10. Assume that $f''(x)$ is continuous on $[0,2]$ and $f(2) = f(0) = 3$, $f'(2) = 5$. Then $\int_0^1 xf''(2x)\, dx =$ __________________.

11. The definite integral $\int_{-2}^{2} \left(\frac{x}{1+x^2} + \sqrt{4-x^2}\right) dx=$__________________________.

12. The particular solution the differential equation $\frac{dy}{dx} = \frac{x+3x^2}{y^2}$ with the initial condition $y(0) = 6$ is __________________________

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**III. Calculations（This problem contains 6 questions, 6 points for each question and 36 points in all）**

13. Find the Following limits.

(1) $\lim\limits_{x\to a^2} \frac{x-a^2}{\sqrt{x}-a}$, $\quad a>0$; (2) $\lim\limits_{x\to+\infty}(\sqrt{x^2+2x}-x)$; (3) $\lim\limits_{x\to 0^+}(x+1)^{\cot x}$.

14. Find the Differentials $dy$ of the dependent variable $y$ with $y=(x^2+1)^{\sin x^2}$.

15. If $\lim_{x\to\infty}\left(\frac{x+c}{x-c}\right)^x = \int_{-\infty}^{c} te^{2t}\,dt$, find the value of $c$.

16. Find the general antiderivative of

(1) $\int (5x^2+1)\sqrt{5x^3+3x-2}\,dx$;

(2) $\int \sin^2 x \cos^3 x\, dx$.

17. Let $f(x)$ be given by

$$f(x) = \begin{cases} xe^{-x} & \text{if} \quad x \le 0 \\ \sqrt{2x - x^2} & \text{if} \quad 0 < x \le 1. \end{cases}$$

Please evaluate the definite integral $\int_{-3}^{1} f(x)\, dx$.

18. Find the following improper integral:

$$\int_0^a \frac{x^3}{\sqrt{a^2 - x^2}}\, dx \quad (a > 0).$$

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**IV. Application（This problem contains 4 questions, 7 points for each question and 28 points in all）**

19. Let $R$ be the region bounded by $x = 0, y = e^x$ and the tangent line to $y = e^x$ that goes through the origin. Find

(a) the area of $R$;

(b) the volume of the solid obtained when $R$ is revolved about the $x$-axis.

20. Find the particular solution of

$$\frac{dy}{dx} - 3y = xe^{3x}$$

that satisfies $y = 4$ when $x = 0$.

21. Please identify the evaluation of the definite integral $\int_0^1 f(x)\,dx$ from the following equation:

$$2x\int_0^1 f(x)\,dx + f(x) = \ln(1+x^2).$$

22. Assume that the function $f(x)$ is continuous on $[a,b]$ and differentiable on $(a,b)$, and

$$\frac{1}{\lambda - a}\int_a^\lambda f(x)\,dx = f(b),$$

where $\lambda \in (a,b)$. Show that there exists at least $\xi \in (a,b)$ such that $f'(\xi) = 0$.