



2021~2022学年第一学期期末考试试卷(A 卷)

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I. For each blank in the following statements, choose the best answer from the choices given below. (This problem contains 6 questions, 3 points for each question and 18 points in all)

1. $\lim_{x \rightarrow \infty} \frac{\sin x}{2x} = (\quad)$.
(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) does not exist
2. Assume that $f(x) = \begin{cases} (1+x)^{\frac{2}{x}} & \text{for } x \neq 0, \\ a & \text{for } x = 0, \end{cases}$ is continuous at $x = 0$, the value of a is (\quad) .
(A) e (B) 1
(C) e^2 (D) $2e$
3. Let $f(x) = 2^x$. Then $\lim_{\Delta x \rightarrow 0} \frac{f(1) - f(1+\Delta x)}{\Delta x} = (\quad)$.
(A) $\frac{\ln 2}{2}$ (B) $-\frac{\ln 2}{2}$ (C) $2 \ln 2$ (D) $-2 \ln 2$

4. $\int \ln x \, dx = (\quad)$.
(A) $x \ln x + x + C$ (B) $x \ln x - x + C$
(C) $-x \ln x + x + C$ (D) $-x \ln x - x + C$
5. The derivative of $f(x)$ is $\sin x$, find out which one in the following is an antiderivative of $f(x)$ (\quad) .
(A) $\sin x$ (B) $-\sin x$
(C) $\cos x$ (D) $-\cos x$
6. The improper integral $\int_1^2 \frac{1}{\sqrt{x-1}} dx$ is (\quad) .
(A) 2 (B) -2
(C) 1 (D) diverges

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II. Fill the correct answer in the blanks (This problem contains 6 questions, 3 points for each question and 18 points in all)

7. Assume that $f(x) = \frac{2x^2+3x+1}{4x^2+3}$. Then $\lim_{x \rightarrow \infty} f(x) = \underline{\hspace{2cm}}$.
8. Let $F(x) = \int_1^{x^2} \ln(1+t^2) dt$. Then $\frac{dF(x)}{dx} = \underline{\hspace{2cm}}$.
9. Assume that $f(x) = \begin{cases} x^3 \sin \frac{1}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$. Then $f'(0) = \underline{\hspace{2cm}}$.
10. If $y = 5^x + \sin(x^2) + \ln 2$, then $dy = \underline{\hspace{2cm}}$.
11. The definite integral $\int_{-1}^1 (x \sin^4 x + x^3 - x^2) dx = \underline{\hspace{2cm}}$.
12. The particular solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ with the initial condition $y(1) = 2$ is $\underline{\hspace{2cm}}$.

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III. Calculations (This problem contains 6 questions, 4 points for each question and 24 points in all)

13. Evaluate $\lim_{t \rightarrow 0} \frac{1 - \cos t}{2t^2}$.

14. Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{x+2} - \sqrt{2}}{x}$.

15. Let $f(x) = e^{x^2} \tan \sqrt{x}$. Find the $f'(x)$.

16. Evaluate $\int (x + 1)\sqrt{x^2 + 2x + 3} \, dx$

17. Evaluate $\int \sin^3 x \cos^2 x \, dx$

18. Evaluate $\int_1^4 \frac{1}{x + \sqrt{x}} \, dx$

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IV. Calculations (This problem contains 4 questions, 5 points for each question and 20 points in all

19. Find $\frac{dy}{dx}$ if $e^{xy} + x^2y = 4$.

20. Evaluate $\int_0^1 \frac{1}{\sqrt{x^2+2x+2}} \, dx$.

21. Let $\int xf(x)dx = \arcsin x + C$. Find $\int \frac{1}{f(x)}dx$.

22. Find the particular solution of

$$\frac{dy}{dx} + 3y = e^{2x}$$

that satisfies $y = 0$ when $x = 0$.

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20 points in all)

V. Application (This problem contains 4 question, 5 points for each question and

23. Let $f(x) = \frac{x}{x^2+4}$. Identify all local extrema of $f(x)$ and find all the inflection points of $f(x)$.

24. Find the area of the region between the curves $y = x^2$ and $y = 2x - x^2$.

25. Find the volume of the solid generated by revolving the region bounded by the parabolas $y = \frac{1}{2}x^2$ and $y^2 = 2x$ about the x-axis.

26. Assume that the function $f(x)$ is continuous on $[0, 1]$ and differentiable on $(0, 1)$, and

$$2 \int_0^{\frac{1}{2}} x f(x) dx = f(1).$$

Show that there exists at least $\xi \in (0, 1)$ such that $f(\xi) + \xi f'(\xi) = 0$.