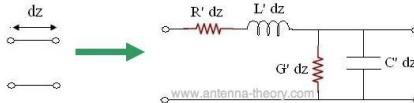


Phase velocity: Speed of wave, affected by materials (ex. Coax is not speed of light)

Magnetic field =  $H$

Parameters	Permittivity	Permeability
Definition	Material capacity to polarize in presence of an electric field	Material capacity to magnetize in response to an magnetic field
Principles Involved	Polarization of electric charges	Magnetization
Represented as	$\epsilon$	$\mu$
Formula	$\epsilon = D/E$	$\mu = B/H$
SI Unit	Farad/meter (or) $F m^{-1}$	Henries/meter (or) $H m^{-1}$
Field	Electric Field	Magnetic Field
Free Space	Free Space Permittivity is $\epsilon_0$ which is called as absolute permittivity. $\epsilon_0 = 8.85 \cdot 10^{-12} F m^{-1}$	Free Space Permeability is $\mu_0$ which is called as absolute permeability. $\mu_0 = 4\pi \cdot 10^{-7} H/m$
Application	High Permittivity is used in Dielectric Capacitors	High Permeability is used in Transformer core & Inductors

Transmission line:



$$V_0 \cos(\omega t) \text{ at source}$$

$$\text{Along line: } V = V_0 \cos(\omega t - \beta z)$$

where  $z$  is distance along line,  $\beta$  is speed,  $-$  means travelling from source.

$$\beta = \frac{2\pi}{\lambda} = \frac{\omega}{u_p}$$

$$V_{load} = V_0 \cos(\omega t - \beta e), e \text{ is length}$$

**Rule of thumb:** If  $e/\lambda < 0.01$

"lumped system", otherwise "distributed".

For tiny segment of each line:  $R'$  in  $\Omega/m$ , and also inductance  $L'$  in  $H/m$ .

Between conductors:  $C'$  in  $F/m$ ,

leakage resistance  $G'$  in  $S/m$ . **2-port network**.

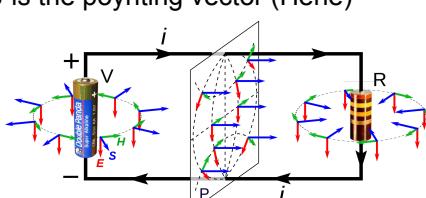
**TEM (everything perpendicular, IMPORTANT):**

Electric, Magnetic, Direction (all perp).  $E \times H = S$ , units:

$$V/m * A/m = W/m^2 \text{ (cool).}$$

Now if you integrate over cross section you get power.

$S$  is the poynting vector (Hehe)



Similar for coax, except everything is perfectly contained

Two plates  $d$  apart,  $w$  is width facing each other

$$L' = \mu \frac{d}{w} = \mu_0 \frac{d}{w}$$

$$G' = \sigma_{dielectric} \frac{w}{d}$$

$$C' = \epsilon \frac{w}{d} = \epsilon_0 \epsilon_r \frac{w}{d}$$

$$R' = R_s \frac{2}{w},$$

$$R_s = \sqrt{(unreadable)} \text{ TODO, skin}$$

effect surface resistance

Skin effect: At high frequencies most current density is at the regions of the plates facing each other.

$$L'C' = \mu \epsilon, u_p \text{ (phase velocity)}$$

$$= \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{L'C'}}$$

$$\frac{G'}{C'} = \frac{\sigma_{dielectric}}{\epsilon}$$

Parameter	Coaxial	Two-Wire	Parallel-Plate	Unit
$R'$	$\frac{R_s}{2\pi} \left( \frac{1}{a} + \frac{1}{b} \right)$	$\frac{2R_s}{\pi d}$	$\frac{2R_s}{w}$	$\Omega/m$
$L'$	$\frac{\mu}{2\pi} \ln(b/a)$	$\frac{\mu}{\pi} \ln \left[ (D/d) + \sqrt{(D/d)^2 - 1} \right]$	$\frac{\mu h}{w}$	$H/m$
$G'$	$\frac{2\pi\sigma}{\ln(b/a)}$	$\frac{\pi\sigma}{\ln \left[ (D/d) + \sqrt{(D/d)^2 - 1} \right]}$	$\frac{\sigma w}{h}$	$S/m$
$C'$	$\frac{2\pi\epsilon}{\ln(b/a)}$	$\frac{\pi\epsilon}{\ln \left[ (D/d) + \sqrt{(D/d)^2 - 1} \right]}$	$\frac{\epsilon w}{h}$	$F/m$

Notes: (1) Refer to Fig. 2-4 for definitions of dimensions. (2)  $\mu, \epsilon, \sigma$  pertain to the insulating material between the conductors. (3)  $R_s = \sqrt{\pi f \mu_0 / \epsilon_0}$ . (4)  $\mu_c$  and  $\sigma_c$  pertain to the conductors.

(5) If  $(D/d)^2 \gg 1$ , then  $\ln \left[ (D/d) + \sqrt{(D/d)^2 - 1} \right] \approx \ln(2D/d)$ .

$a$  = outer radius,  $b$  = inner radius

**Loss tangent**  $\tan(\Delta)$ ,  $\Delta$  is the angle on an impedance phasor from vertical to the right to the impedance complex value. No loss where  $\Delta = 0$

Quality factor ( $Q$ )

$$= \frac{|X|}{R} = \frac{|B|}{G} = \frac{1}{\tan(\Delta)} X \text{ is reactance,}$$

$B$  is susceptence. Capacitors have  $Q=1000 \rightarrow 2000$ , Inductors are much lower.

**How do we calculate TL impedance:**

Transmission line block with:

$R'$ ,  $L'$ ,  $C'$ , and  $G'$

Apply  $I_{in}(Z)$  to input.

Apply KVL:

$$V_{in}(Z) = (R' + j\omega L' \Delta Z) + V_{out}(Z + \Delta Z)$$

$$- (R' + j\omega L')I = \frac{V_{out}(Z+\Delta Z) - V_{out}(Z)}{\Delta Z} = \frac{dV}{dZ}$$

<- Looks like a derivative limit

$$I(Z) = V_{out}(j\omega C' \Delta Z + G' \Delta Z) + I(Z + \Delta Z)$$

$$- (G' + j\omega C')V = \frac{I(Z+\Delta Z) - I(Z)}{\Delta Z} = \frac{dI}{dZ}$$

$$\frac{d^2V}{dZ^2} = (R' + j\omega L')(G' + j\omega C')V,$$

$$(R' + j\omega L')(G' + j\omega C') = \alpha^2$$

$$V = Ae^{-\alpha Z}$$

Gamma ( $\gamma$ ) "encodes" the TL parameters

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$

**TL Equations:**

$$\frac{d^2V}{dZ^2} - \gamma^2 V = 0$$

$$\frac{d^2I}{dZ^2} - \gamma^2 I = 0$$

**Impedance:**

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

**Assume lossless:** (like short PCB traces), in this case frequency doesn't really matter

$$R' \rightarrow 0, G' \rightarrow 0$$

This also makes

$$\gamma \rightarrow j\omega \sqrt{L'C'} = \frac{j\omega}{\mu_p} \text{ <- Purely}$$

imaginary, no attenuation

**Note about coax  $Z$ :**

$$Z_0 = \frac{60}{\sqrt{\epsilon_r}} \ln(b/a)$$

$$\text{Derivation: } L' = \frac{\mu_0}{2\pi} \ln(b/a),$$

$$C' = \frac{2\pi\epsilon_0}{\ln(b/a)},$$

$$Z_0 = \sqrt{L'/C'} = \frac{1}{2\pi} \sqrt{\mu_0/\epsilon_0} \frac{1}{\sqrt{\epsilon_0}} \ln(b/a)$$

**Free space wave impedance (**

$$\sqrt{\mu_0/\epsilon_0}) 377 \text{ ohms VERY}$$

**IMPORTANT,  $377/(2\pi) = 60$**

**Reflections:**

$$V_0^- = (\frac{Z_L - Z_0}{Z_L + Z_0}) V_0^+$$

Return loss: ("RL")

$$= -20 \log(\Gamma_L) [\text{dB}]$$

**Summary 1:**

1.  $D = \epsilon_0 \epsilon_r E, E = ? / Q_2$  and

$$B = \mu_0 \mu_r H$$

2. Waves:

$$y(z, t) = Ae^{-\alpha z} \cos(\omega t - \beta z), \alpha [\text{Np/m}] \text{ or } [\text{dB/m}], \beta = 2\pi/\lambda = \omega/u_p$$

[rad/s]

$$u_p = c/\sqrt{\epsilon_r}$$

3. Phasors:  $Z = |Z|e^{j\theta} = x + jy$

4. TL Stuff: Infinitesimal model:  $L', R', C', G'$

5. Reflections:

$$\Gamma_L = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}, \text{ quantitatively:}$$

$$V_0^- = \Gamma_L V_0^+$$

$$\frac{R'}{L'} = \frac{G'}{C'} \quad (\text{criteria for distortionless line})$$

$$Z_0 = \sqrt{\frac{R'}{G'}} = \sqrt{\frac{L'}{C'}} \quad (\text{distortionless line})$$

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$$\gamma = \alpha + j\beta = \sqrt{R'G'} \left( 1 + \frac{j\alpha C'}{G'} \right) \quad \text{propagation constant for distortionless lines}$$

$$\alpha = \sqrt{R'G'}$$