Institute of Technology of Cambodia

Mechanical and Industrial Engineer

Class: Construction Mechanic

Report: kinematic of KUKA KR 10 R1100-2

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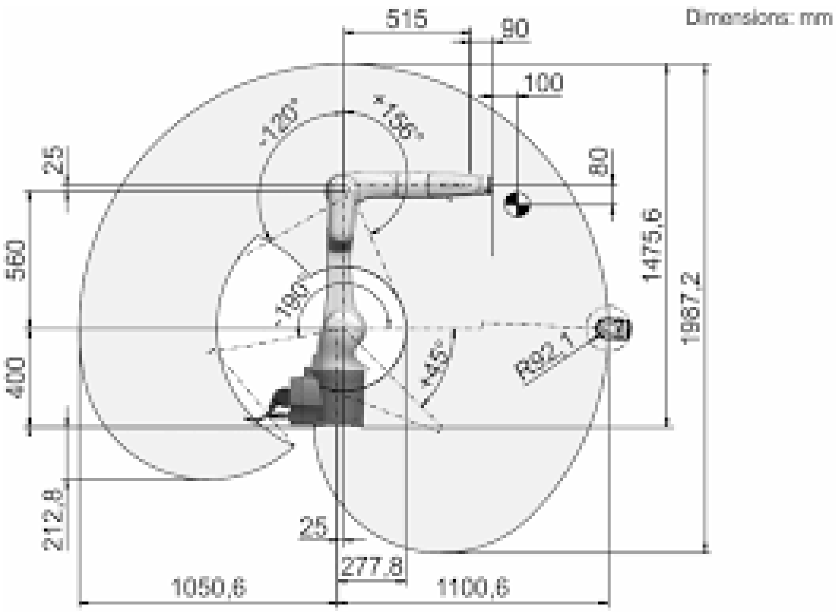
I. *Introduction*

In this report we study about the movement of the KUKA KR 10 R1100-2. Then we would need to learn about the forward kinematic and the inverse kinematic of the robot. For forward kinematic we can find position in the end of robot arm (x, y, z). Inverse kinematic we can find the start point (angle and distance) int this robot arm.

II. *The work pieces*

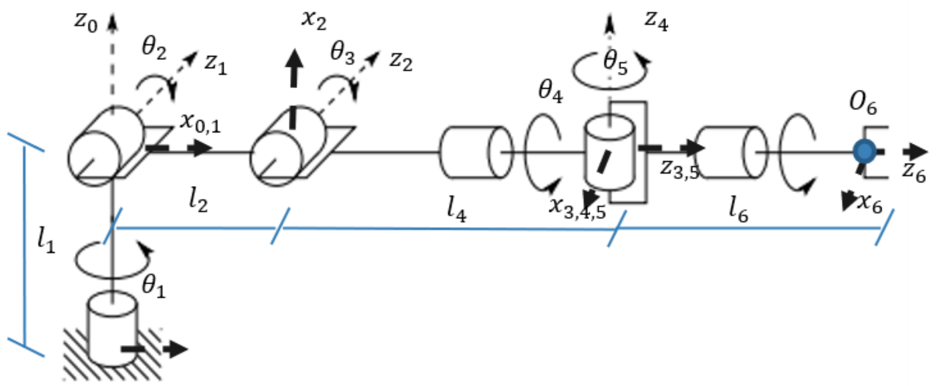
The robot **The work space**



The D-H parameters of the robot

Link	θ_i	α_i	a_i	d_i
1	θ_1	90	a_1	d_1
2	θ_2	0	a_2	0
3	$\theta_3 - 90$	90	a_3	0
4	θ_4	-90	0	d_4
5	θ_5	90	0	0
6	$\theta_6 + 180$	0	0	d_6

III. ***Forward Kinematic of the robot***



4 parameters of Denavit-Hartenberg: θ, d, a *and* α:

a_i: link length (displacement along x_{i-1} from z_i-1 to z_i)

α_i: link twist (rotation around x_{i-1} from z_{i-1} to z_i)

d_i: link offset (displacement along z_i from $x_{i-1}$to x_i)

θ_i: joint angle (rotation around z_i from $x_{i-1}$to x_i)

$${}^{i-1}_{\ \ i}T = \mathrm{Rot}(z, \theta_i) trans(0,0,d_i) trans(a_i,0,0) Rot(x,\alpha_i)$$

$$= \begin{bmatrix} \cos\theta_i & -sin\theta_i cos\alpha_i & sin\theta_i sin\alpha_i & a_i cos\theta_i \\ sin\theta_i & cos\theta_i cos\alpha_i & -cos\theta_i sin\alpha_i & a_i sin\theta_i \\ 0 & sin\alpha_i & cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then for the 6-DOF robot as KUKA KR 10 R1100-2, then the overall would be:

$${}^0_6T = {}^0_1T\,{}^1_2T\,{}^2_3T\,{}^3_4T\,{}^4_5T\,{}^5_6T = \begin{bmatrix} l_x & m_x & n_x & P_x \\ l_y & m_y & n_y & p_y \\ l_z & m_z & n_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then

$${}^0_1T = \begin{bmatrix} cos\theta_1 & 0 & sin\theta_1 & a_1 cos\theta_1 \\ sin\theta_1 & 0 & -\cos\theta_1 & a_1 sin\theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1_2T = \begin{bmatrix} cos\theta_2 & -sin\theta_2 & 0 & a_2 cos\theta_2 \\ sin\theta_2 & cos\theta_2 & 0 & a_2 sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$ {}_3^2T = \begin{bmatrix} cos\theta_3 & 0 & sin\theta_3 & 0 \\ sin\theta_3 & 0 & -cos\theta_3 & 0 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_4^3T = \begin{bmatrix} cos\theta_4 & 0 & -\sin\theta_4 & 0 \\ \sin\theta_4 & 0 & \cos\theta_4 & 0 \\ 0 & -1 & 0 & \mathrm{d}_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_5^4T = \begin{bmatrix} cos\theta_5 & 0 & sin\theta_5 & 0 \\ sin\theta_5 & 0 & -cos\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_6^5T = \begin{bmatrix} cos\theta_6 & -sin\theta_6 & 0 & 0 \\ sin\theta_6 & cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Then by the overall calculation of matrix

Let $cos\theta = c$
$sin\theta = s$
$\sin(\theta_2 + \theta_3) = s_{23}$
$\cos(\theta_2 + \theta_3) = c_{23}$

$$ l_x = s_1(s_4c_5c_6 + c_4s_6) + c_1\left(-s_{23}s_5c_6 + c_{23}(c_4c_5c_6 - s_4s_6)\right) $$
$$ l_y = -c_1(s_4c_5c_6 + c_4s_6) + s_1(-s_{23}s_5c_6 + c_{23}(c_4c_5c_6 - s_4s_6)) $$
$$ l_z = -c_6(s_{23}c_4c_5 + c_{23}s_5) + s_{23}s_5s_6 $$
$$ m_x = c_6(s_1c_4 - c_1c_{23}s_4) - s_6(s_1s_4c_5 + c_1(c_{23}c_4c_5 - s_{23}s_5)) $$
$$ m_y = c_1(-c_4c_6 + c_5s_4s_6) - s_1(-s_{23}s_5s_6 + c_{23}(s_4c_6 + c_4c_5s_6)) $$
$$ m_z = s_{23}s_4c_6 + s_6(s_{23}c_4c_5 + c_{23}s_5)) $$
$$ n_x = -s_1s_4s_5 - c_1(s_{23}c_5 + c_{23}c_4s_5) $$
$$ n_y = -s_1s_{23}s_5 + s_5(-s_1c_{23}c_4 + c_1s_4) $$
$$ n_z = -c_{23}c_5 + c_4s_{23}s_5 $$
$$ p_x = -d_6s_1s_4s_5 + c_1\left(a_1 + a_2c_2 - s_{23}(d_4 + d_6c_5) + c_{23}(a_3 - d_6c_4s_5)\right) $$
$$ p_y = d_6c_1s_4s_5 + s_1(a_1 + a_2c_2 - s_{23}(d_4 + d_6c_5) + c_{23}(a_3 - d_6c_4s_5)) $$
$$ p_z = d_1 - c_{23}(d_4 + d_6c_5) - a_2s_2 + s_{23}(-a_3 + d_6c_4s_5) $$

if we want to translate from rotation matrix to Euler angles then:

$R_{RPY}(\gamma,\beta,\alpha) = R_z(\alpha).R_y(\beta).R_x(\gamma)$

$$= \begin{bmatrix} cos\alpha cos\beta & cos\alpha sin\beta sin\gamma - sin\alpha cos\gamma & cos\alpha sin\beta cos\gamma + sin\alpha sin\gamma \\ sin\alpha cos\beta & sin\alpha sin\beta sin\gamma + cos\alpha cos\gamma & sin\alpha sin\beta cos\gamma - cos\alpha sin\gamma \\ -sin\beta & cos\beta sin\gamma & cos\beta cos\gamma \end{bmatrix}$$

$$= \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

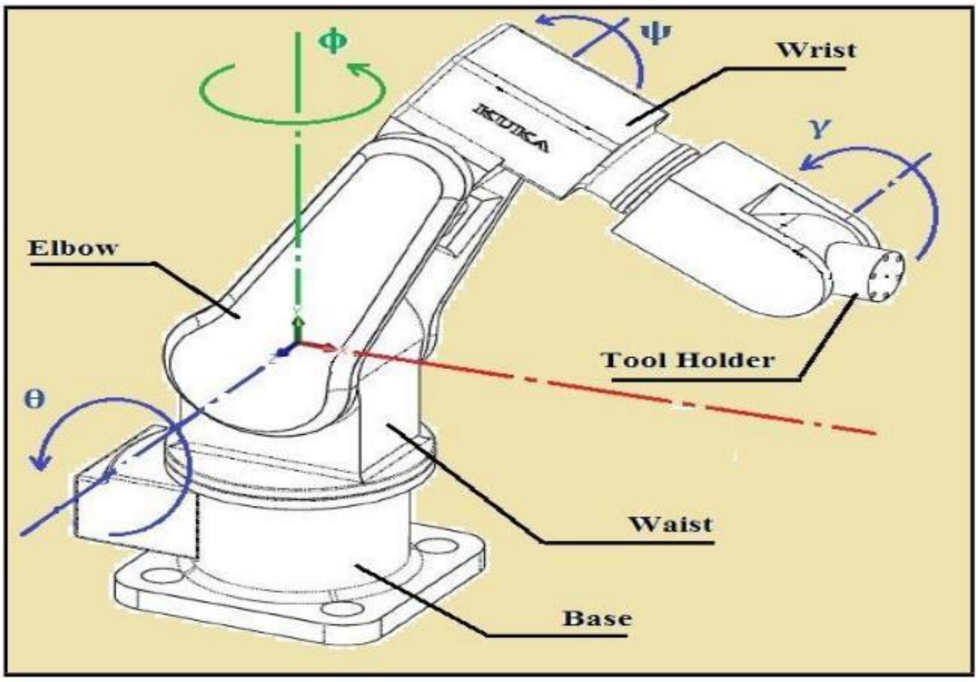
Then

$$\beta = Atan2\left(-r_{31}, \sqrt{r_{11}^2 + r_{21}^2}\right)$$
$$\alpha = Atan2(r_{21}, r_{11})$$

$$\gamma = Atan2(r_{32}, r_{33})$$

IV. <u>Inverse Kinematic of the robot</u>

Inverse kinematic analysis is the opposite of the forward kinematic analysis. The inverse kinematic analysis is done by multiplying each inverse matrix of T matrices on the left side of above equation and then equalizing the corresponding elements of the equal matrices of both ends. With this solution we can determine the order to place the arm and it desired position.



To solve for the angles, we will multiply the two matrices with the ${}^0_6T$ $starting\ with$ ${}^1_0T$

Then

$$=> \begin{bmatrix} C_{23}(C_4C_5C_6 - S_4C_6) - S_{23}S_5C_6 & C_{23}(-C_4C_5C_6 - S_4C_6) - S_{23}S_5C_6 & C_{23}C_4S_5 + S_{23}C_5 & a_2C_2 \\ S_{23}(C_4C_5C_6 - S_4S_6) - C_{23}S_5C_6 & S_{23}(-C_4C_5C_6 - S_4S_6) - C_{23}S_5S_6 & S_{23}C_4S_5 - C_{23}C_5 & a_2S_2 \\ S_4C_5C_6 + C_4S_6 & -S_4C_5S_6 + C_4C_6 & S_5S_4 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

From elements (2,4)

$$\mathrm{P_z} = a_2S_2 \qquad (1)$$

$$S_2 = \frac{P_z}{a_2} \qquad (2)$$

$$\theta_2 = \sin^{-1}(\frac{P_z}{a_2}) \qquad (3)$$

From elements (3,4)

$$P_xS_1 - P_yC_1 = d_3 \qquad (4)$$

$$S_1 = \frac{d_3 + P_yC_1}{P_x} \qquad (5)$$

From elements (1,4)

$$P_xC_1 + P_yS_1 - a_1 = a_2C_2 \qquad (6)$$

By equation (5) and (6)

$$C_1 = \frac{P_x(a_2C_2 + a_1) - P_yd_3}{P_x^2 + P_y^2} \qquad (7)$$

$$\theta_1 = \cos^{-1}(\frac{P_x(a_2C_2 + a_1) - P_yd_3}{P_x^2 + P_y^2}) \qquad (8)$$

Next step is to remultiply by the inverse of A_1 through A_3

$$A_3^{-1}A_2^{-1}A_1^{-1}\ x \begin{bmatrix} l_x & m_x & n_x & P_x \\ l_y & m_y & n_y & P_y \\ l_z & m_z & n_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_4A_5A_6$$

From element (1,4) and (3,4)

$$\left(P_x C_1 + P_y S_1 - a_1\right) C_{23} + p_Z S_{23} - a_2 C_3 = 0 \qquad (9)$$

$$(P_X C_1 + P_Y S_1 - a_1) S_{23} - P_Z C_{23} - a_2 S_3 = 0 \qquad (10)$$

From equation (9) and (10)

$$C_{23} = \sqrt{\frac{P_Z^2 - a_2^2 + \left[P_x C_1 + P_y S_1 - a_1\right]^2}{2P_Z^2}} \qquad (11)$$

$$\theta_{23} = \cos^{-1} \sqrt{\frac{P_Z^2 - a_2^2 + \left(P_x C_1 + P_y S_1 - a_1\right)^2}{2P_Z^2}} \qquad (12)$$

$$\theta_3 = \theta_{23} - \theta_2 \qquad (13)$$

From element (1,3) and (2,3)

$$a_x S_1 - a_y C_1 = S_4 S_5 \qquad (14)$$
$$a_x C_1 C_{23} + a_y S_1 C_{23} + a_y S_1 C_{23} + a_z S_{23} = C_4 S_5 \qquad (15)$$

From equation (14) and (15)

$$\theta_4 = \tan^{-1}\left[\frac{a_x S_1 - a_y C_1}{a_x C_1 C_{23} + a_y S_1 S_{23} + a_z S_{23}}\right] \qquad \mathbf{(}16)$$

From element (3,1) and (3,2)

$$m_x C_1 S_{23} + m_y S_1 S_{23} - m_z = S_5 S_6 \qquad (17)$$

$$l_x C_1 S_{23} + l_y S_1 S_{23} - l_z C_{23} = -S_5 S_6 \qquad (18)$$

From equation (17) and (18)

$$\boldsymbol{\theta}_6 = \tan^{-1}\left[\frac{m_z C_{23} - m_x C_1 S_{23} - m_y S_1 S_{23}}{l_x C_1 S_{23} + l_y S_1 S_{23} - l_z C_{23}}\right] \qquad (19)$$

Since

$$A_4^{-1} A_3^{-1} A_2^{-1} A_1^{-1}\ x \begin{bmatrix} l_x & m_x & n_x & P_x \\ l_y & m_y & n_y & P_y \\ l_z & m_z & n_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_5 A_6$$

From element (1,3) and (2,3)

$$a_x C_1 C_{23} C_4 + a_y S_1 S_{23} C_4 + a_2 S_{23} C_4 + a_x S_1 S_4 - a_y C_1 S_4 = S_5 \qquad (20)$$

$$-a_x S_{23} - a_y S_1 S_{23} + a_z C_{23} = -C_5 \qquad (21)$$

From equation (20) and (21)

$$\theta_5 = -\tan^{-1}\left[\frac{a_z C_1 C_{23} C_4 + a_y S_1 S_{23} C_4 + a_z S_{23} C_4 + a_z S_1 S_4 - a_y C_1 S_4}{-a_z S_{23} - a_y S_1 S_{23} + a_z C_{23}}\right]$$

As the Summary

i. **First Position: arm link**

$$\theta_1 = \cos^{-1}\left(\frac{P_x(a_2 C_2 + a_1) - P_y d_3}{P_x^2 + P_y^2}\right)$$

$$\theta_2 = \sin^{-1}\left(\frac{P_z}{a_2}\right)$$

$$\theta_3 = \theta_{23} - \theta_2$$

But $\theta_{23} = \cos^{-1}\sqrt{\frac{P_z^2 - a_2^2 + \left(P_x C_1 + P_y S_1 - a_1\right)^2}{2P_z^2}}$

$$==> \theta_3 = \cos^{-1}\sqrt{\frac{P_z^2 - a_2^2 + \left(P_x C_1 + P_y S_1 - a_1\right)^2}{2P_z^2}} - \sin^{-1}\left(\frac{P_z}{a_2}\right)$$

ii. **Second position: Wrist link**

$$\theta_4 = \tan^{-1}\left[\frac{a_x S_1 - a_y C_1}{a_x C_1 C_{23} + a_y S_1 S_{23} + a_z S_{23}}\right]$$

$$\theta_5 = -\tan^{-1}\left[\frac{a_z C_1 C_{23} C_4 + a_y S_1 S_{23} C_4 + a_z S_{23} C_4 + a_z S_1 S_4 - a_y C_1 S_4}{-a_z S_{23} - a_y S_1 S_{23} + a_z C_{23}}\right]$$

$$\boldsymbol{\theta}_6 = \tan^{-1}\left[\frac{m_z C_{23} - m_x C_1 S_{23} - m_y S_1 S_{23}}{l_x C_1 S_{23} + l_y S_1 S_{23} - l_z C_{23}}\right]$$

IV. ***Conclusion***

After studying the movement of the robot. We have gained and much deeper understanding of forward and inverse kinematic of the robot. Know more clearly about KUKA KR 10 R1100-2 robot arm.