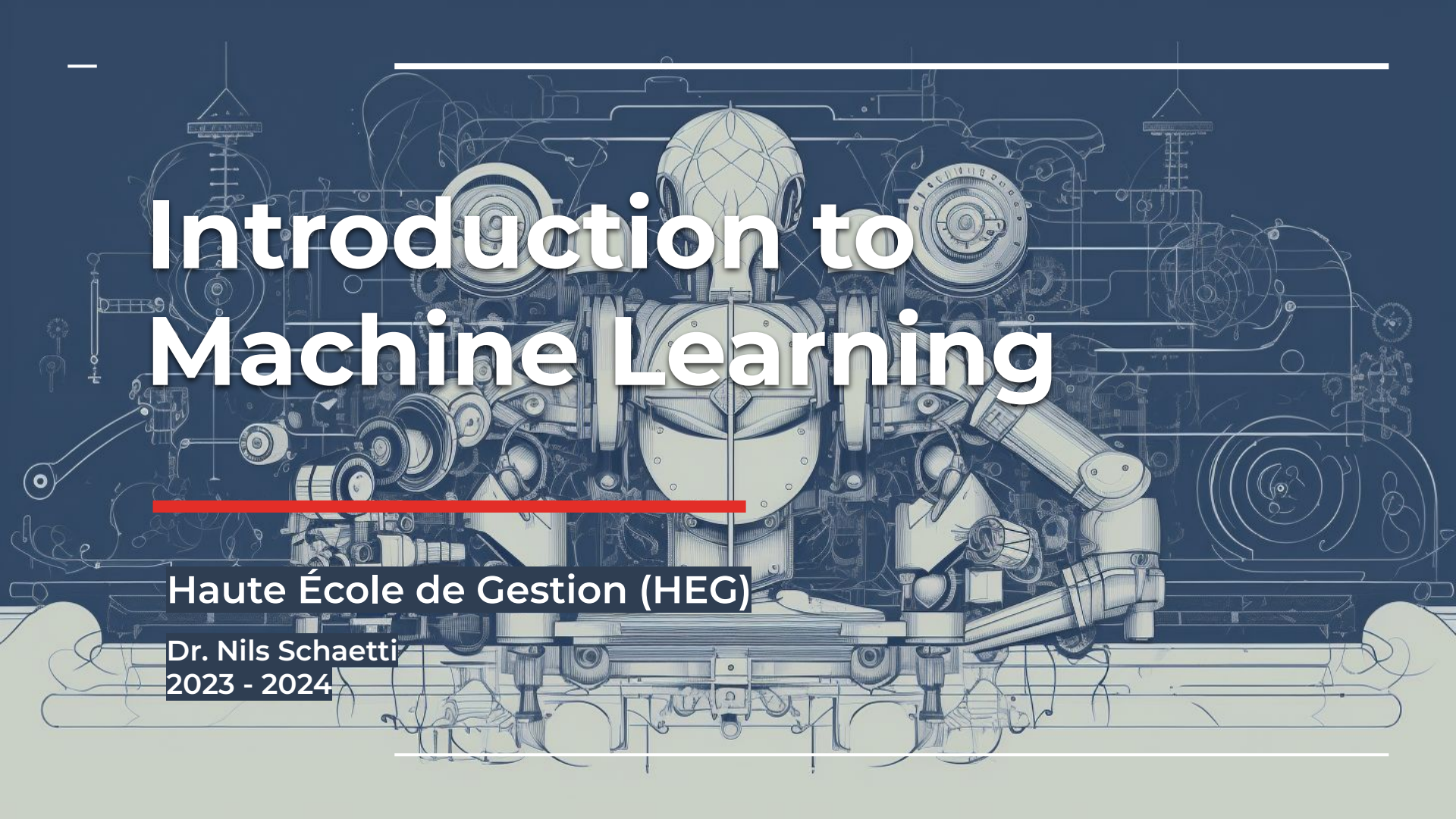


Introduction to Machine Learning

Haute École de Gestion (HEG)

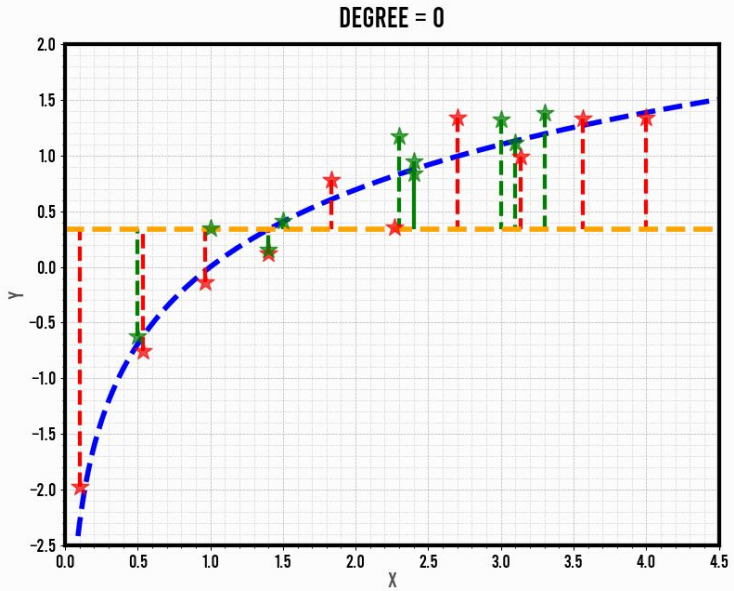
Dr. Nils Schaetti

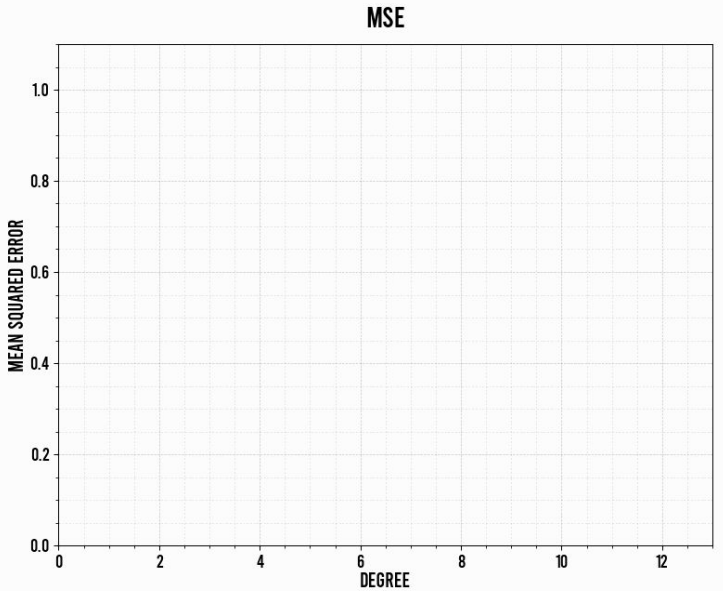
2023 - 2024

Bias and variance

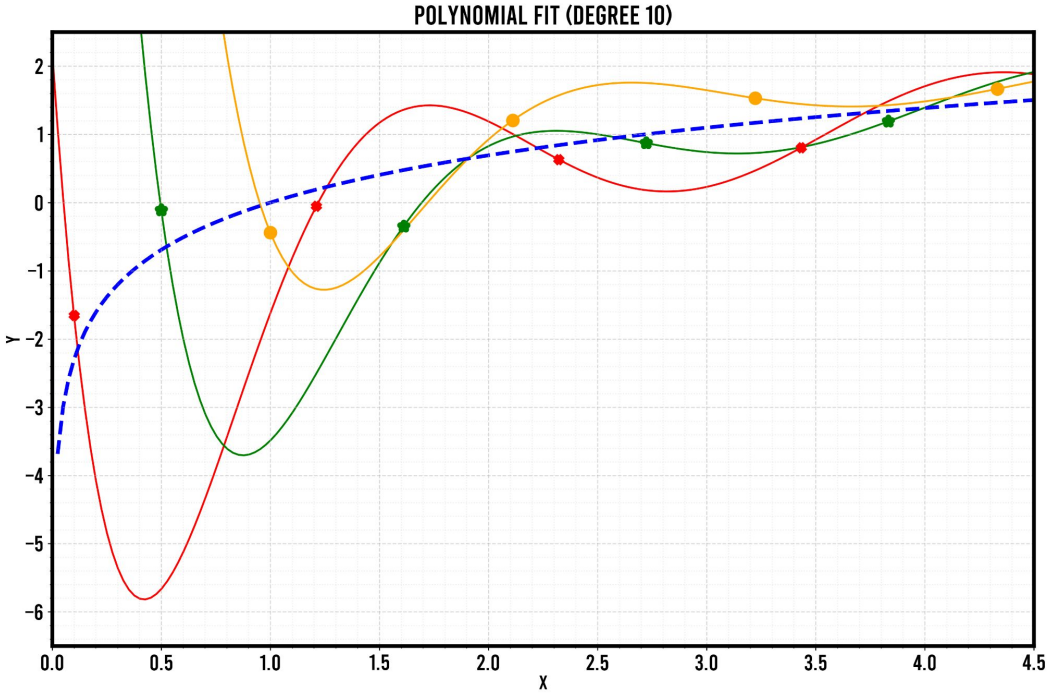
Principles and application

Bias and Variance



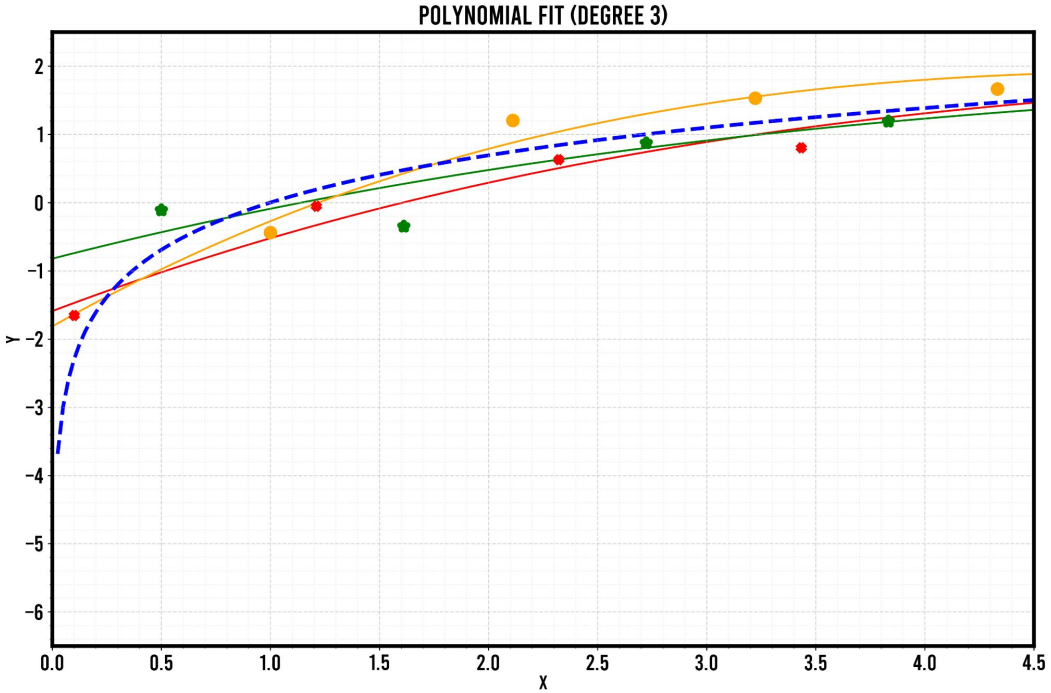


Model variance



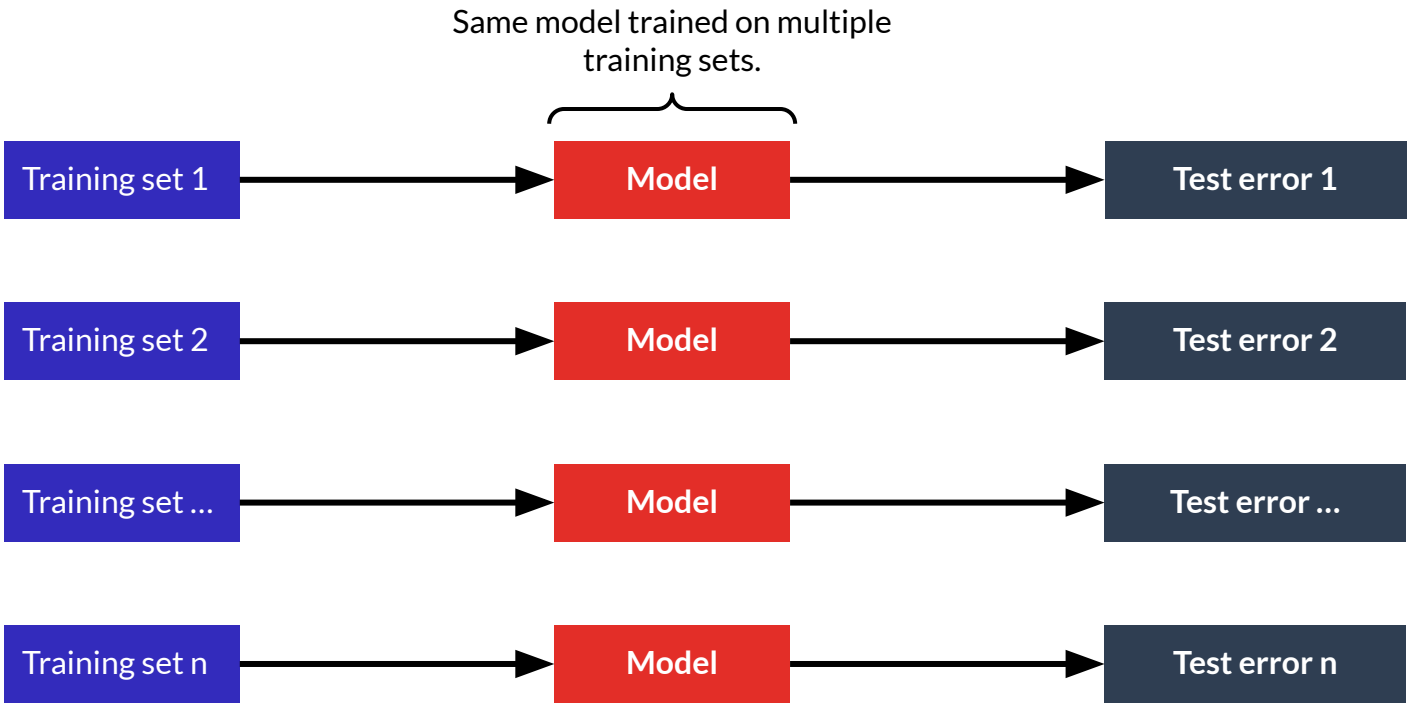
- We have 3 set of training points coming from the same background function.
- We fit each set with polynomials of degree 8.
- The learned curves are completely different but should be similar (same background phenomenon).
- **Variance**: lot of variability in the predicted values !

Model variance

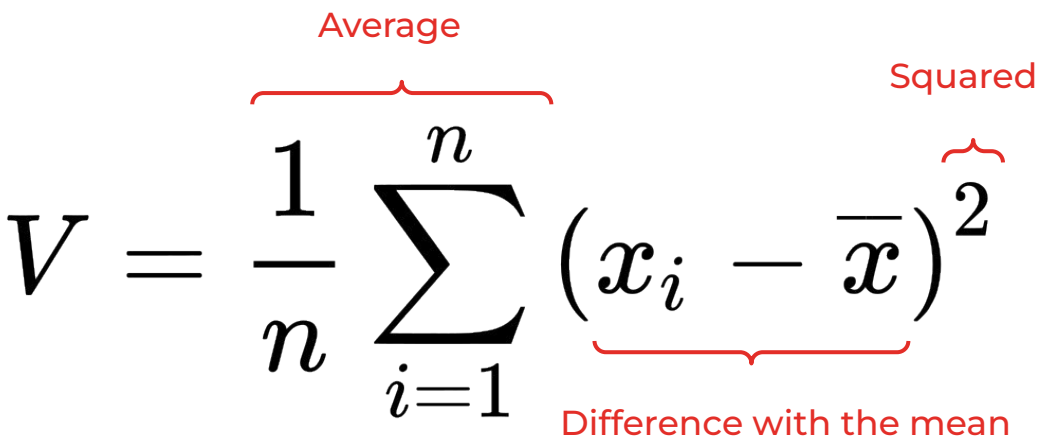


- The predicted models are more similar with polynomials of degree-3.
- **Variance**: less variability in the predicted values !

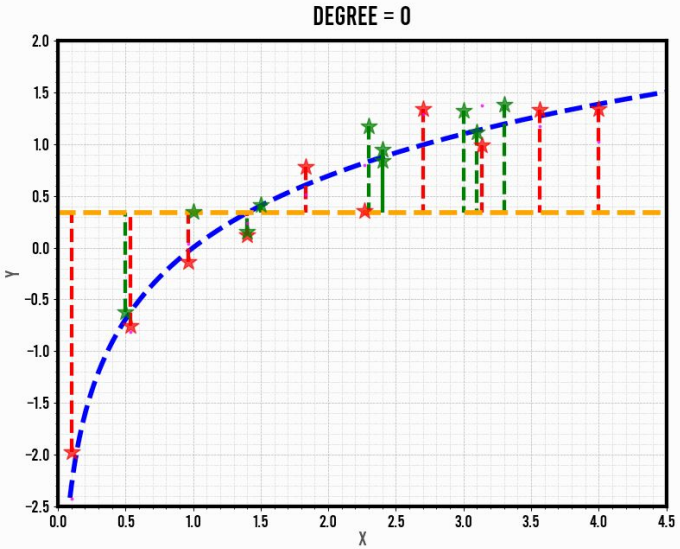
Model variance

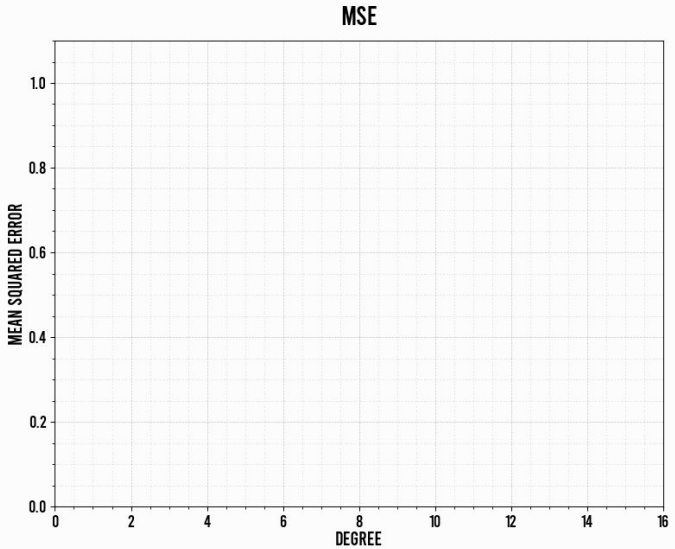


Model variance

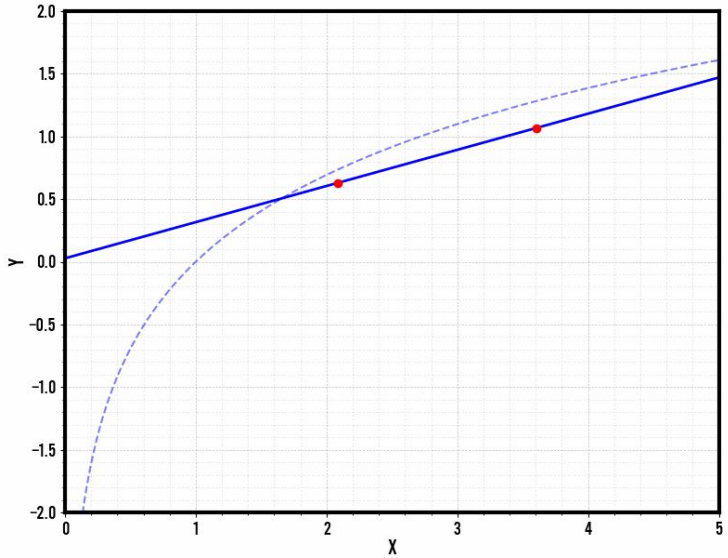


Model variance





Model bias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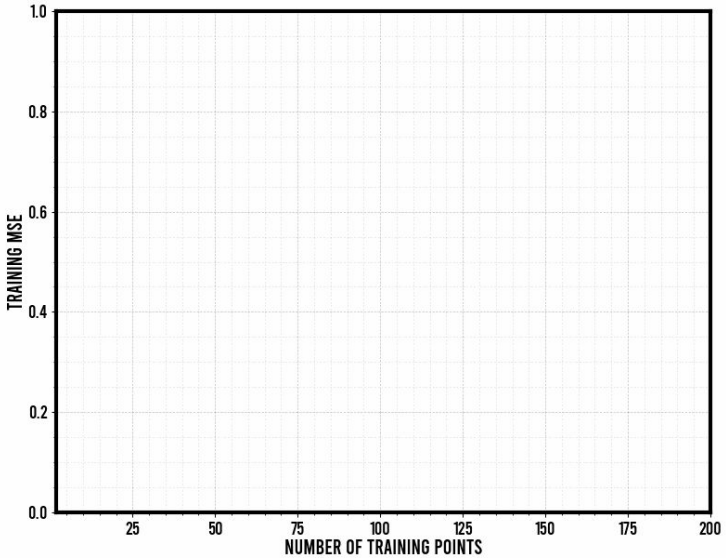
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TRAINING MSE
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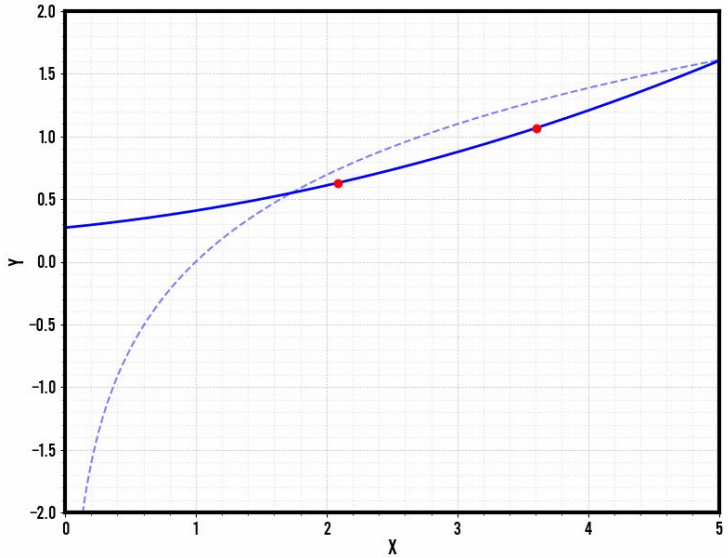
NUMBER OF TRAINING POINTS

Model bias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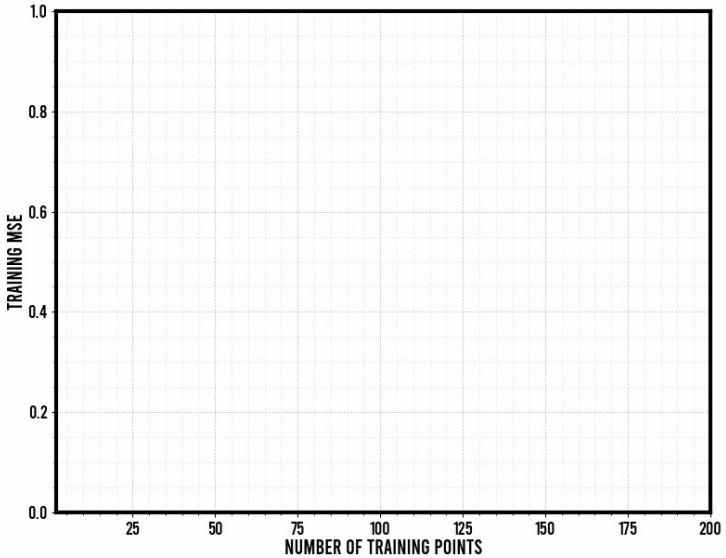
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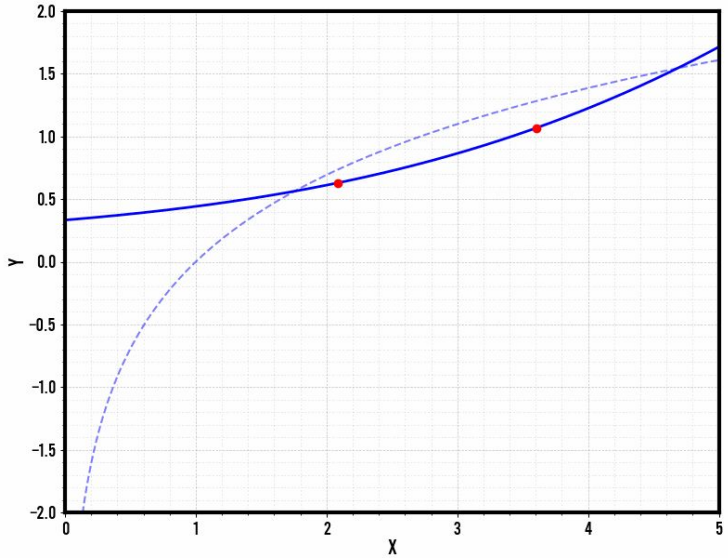
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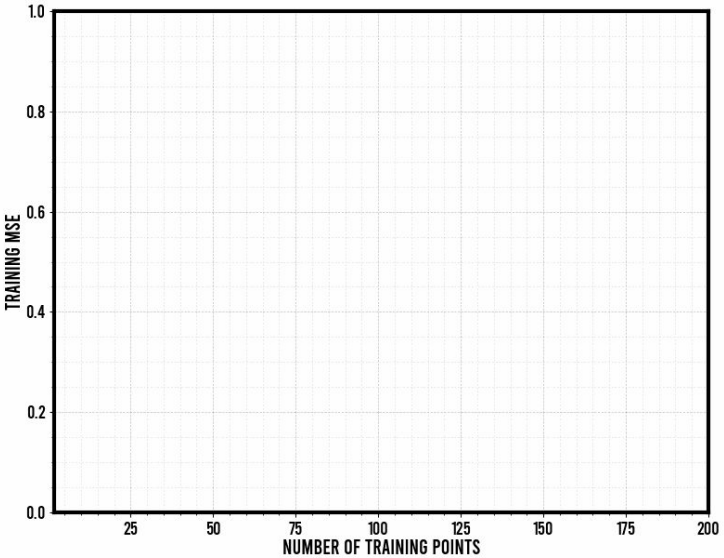
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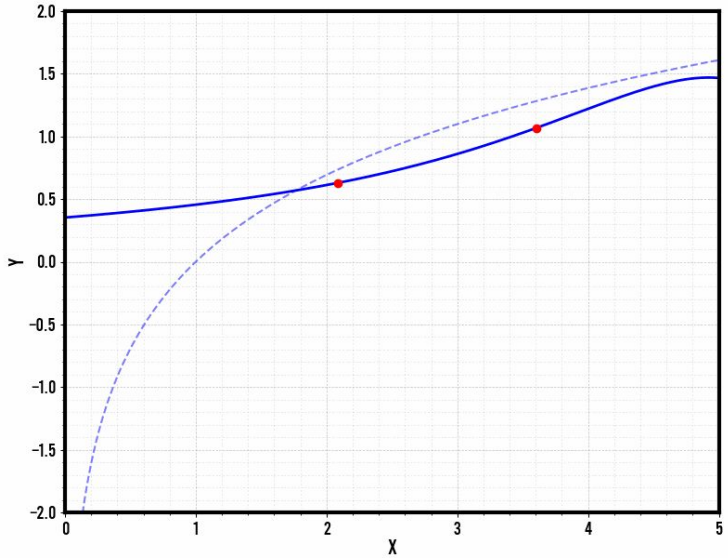
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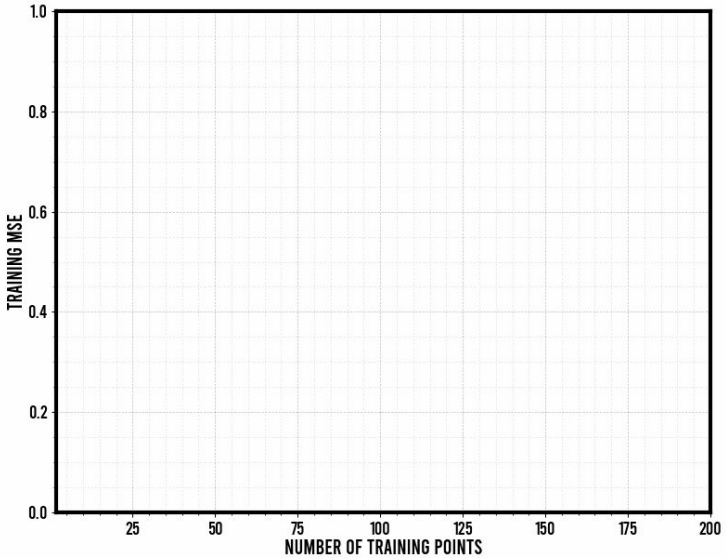
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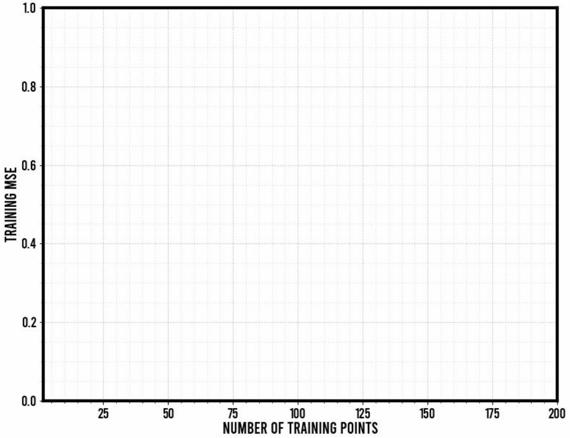
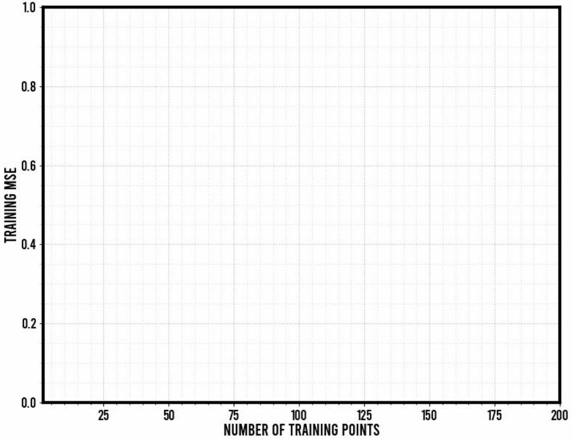
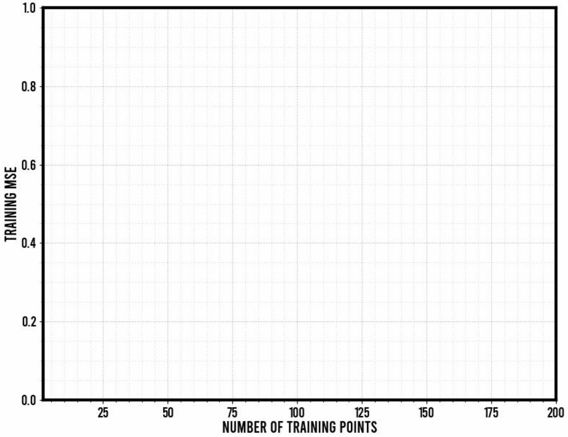
Model bias


TRAINING MSE
NUMBER OF TRAINING POINTS


TRAINING MSE
NUMBER OF TRAINING POINTS


TRAINING MSE
NUMBER OF TRAINING POINTS

Bias-variance tradeoff

Variance

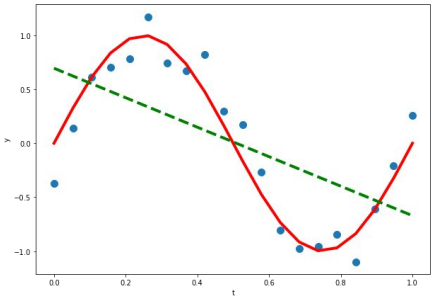
- Capture how much the classifier changes if you train on a different training set.
- How “over-specialized” is the classifier to a particular training set (overfitting)
- Considering the best possible model for a training data, how far are we from the average classifier ?

Bias

- The inherent error that we can get with the classifier even with infinite training data.
- The classifier is biased to a particular kind of solution (ex. linear regression).
- The bias is inherent to the model.
- The model is not robust enough to produce accurate prediction.

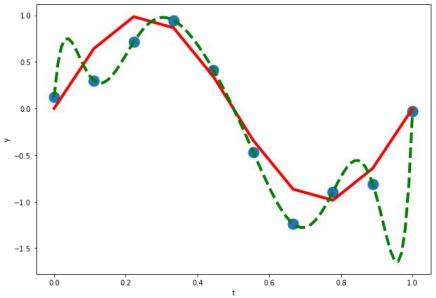
Bias-variance tradeoff

Order 1 (M=1)



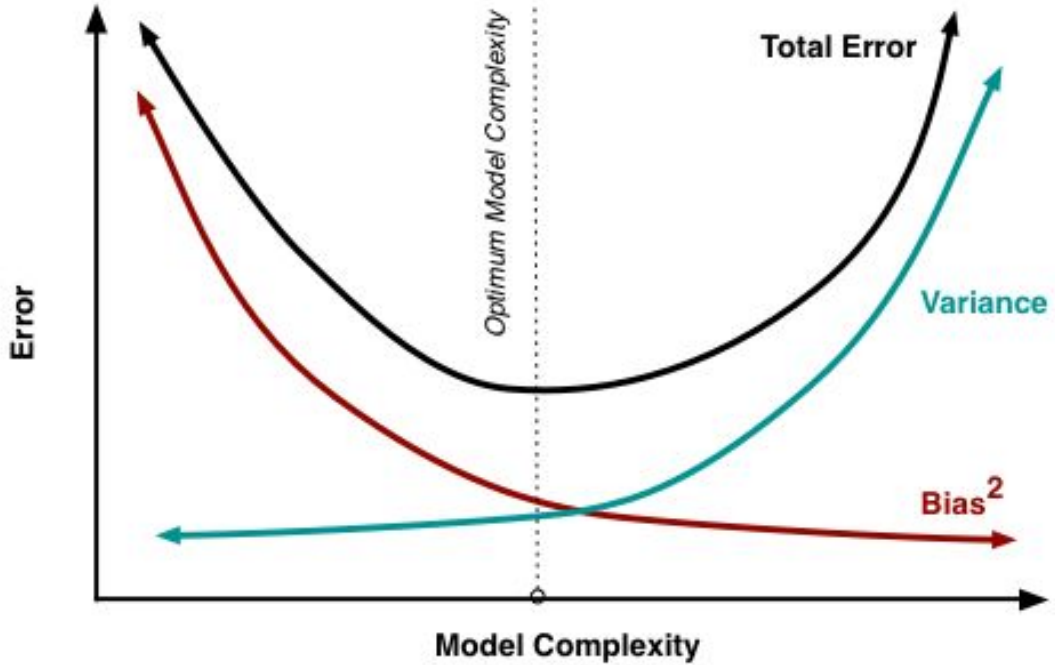
- **Capacity decreases** or **regularization increases**.
- Fit the data less on average which increases the bias term.
- h does no vary a lot with training data.
- But is inherently unable to predict accurately.

Order 9 (M=9)



- **Capacity increase** or **regularization decreases**.
- h varies a lot with training data which increases the variance term.
- Mean of predicted value converge to the mean of the target.
- But the prediction varies more across training datasets.

Bias-variance tradeoff



Bias-variance tradeoff