



You may use a calculator throughout this module.

## Words to remember

<i>ascending</i>	From smallest to largest. E.g. 2, 4, 5, 12.
<i>data set</i>	A collection of measurements or observations. E.g. the height of students in Year 7.
<i>difference</i>	Subtracting two numbers gives the difference between them. E.g. The difference between 7 and 5 is 2, since $7 - 5 = 2$ .
<i>midpoint</i>	The middle number between two numbers. E.g. the midpoint of 10 and 14 is 12.
<i>summary statistic</i>	A number describing a data set. This is useful to understand and compare data sets.

## Gromo's Fun Fact: History of Statistics



The first statistics book was written in the 9th century by Al-Kindi (801 – 873) and it was called "Manuscript on Deciphering Cryptographic Messages".

Modern statistics started in the late 19th century and is a branch of mathematics that looks at collecting and analysing data.

## Questions

1. Questions 1 and 2 are about the summary statistic, the range:

Below are three examples finding the range of each data set.

1. Calculate the range of: 2, 5, 7, 9. The range is 7.	2. Calculate the range of: 12, 5, 6, 7, 1. The range is 11.	3. Calculate the range of: 2, 3, 17, 22, 3. The range is 20.
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What do you think the range is for the following data sets?

a) 2, 5, 6, 7

b) 4, 6, 2, 8, 10, 13, 26, 5, 2

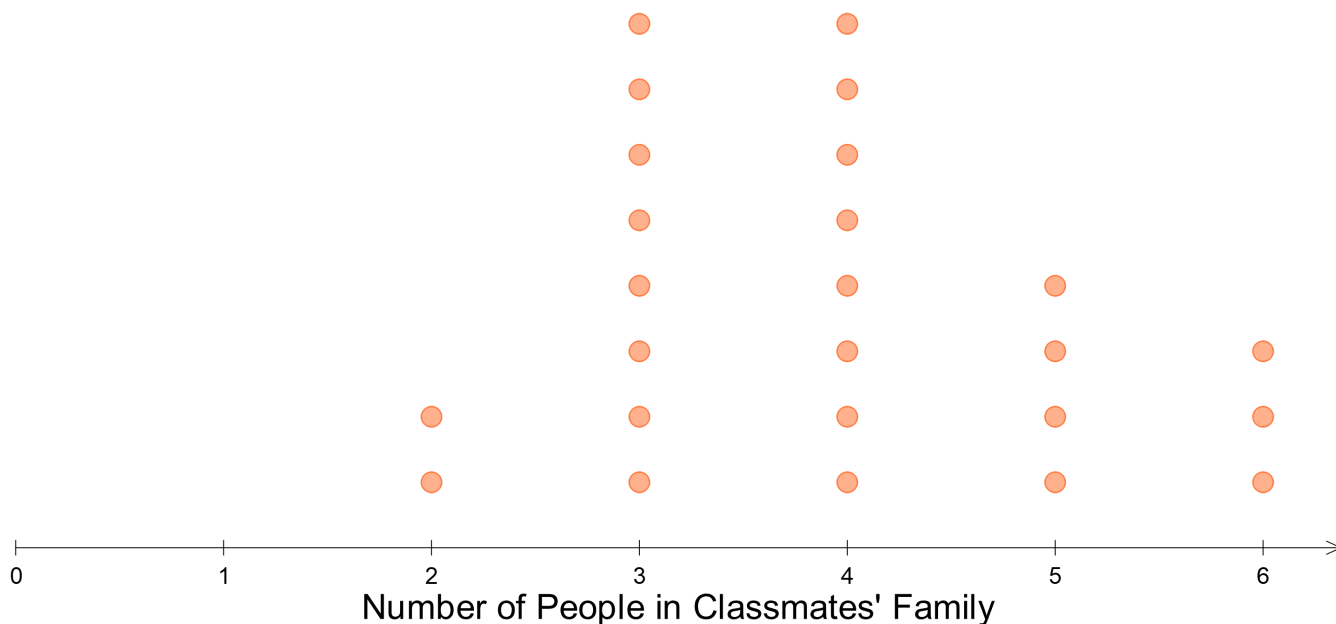
2. Calculate the range for the following data sets.

a) 234, 65, 2, 14, 0, 43

b) Maximum temperatures in Astoria this week: (in degrees Celsius):

Mon	Tue	Wed	Thu	Fri	Sat	Sun
25	31	17	21	23	27	20

c) Dot plot showing the number of people Emmanuel's classmates' have in their family.



3. Question 3 is about the summary statistic, the **mode**.

a) Which is the most frequent value (occurs the most)?

1, 6, 7, 3, 7, 7, 3, 5

b) The most frequent value is called the **mode**. What is the mode of the data in part a)?

c) Which is the most frequent value in the data set below?

3, 5, 6, 6, 7, 7, 8

**d)** You can have more than one value as the mode. The mode in part c) is 6 and 7.

Find the mode of the following data:

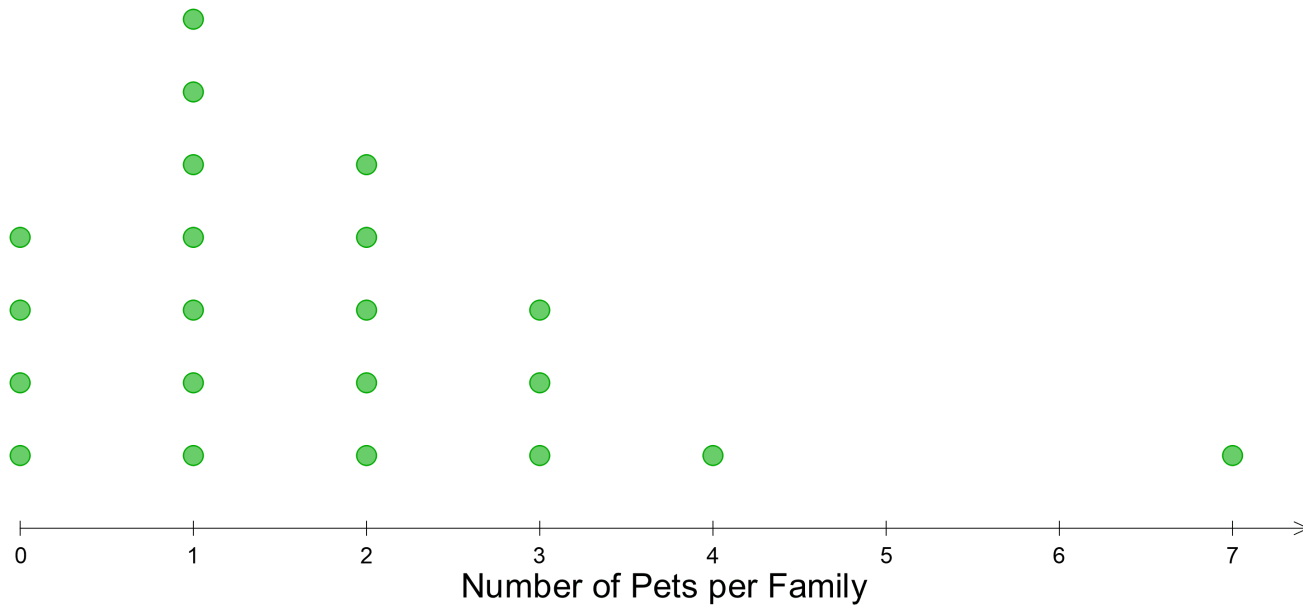
**i)** 3, 4, 2, 1, 1, 7

**ii)** 23, 102, 34, 98, 45, 23, 109

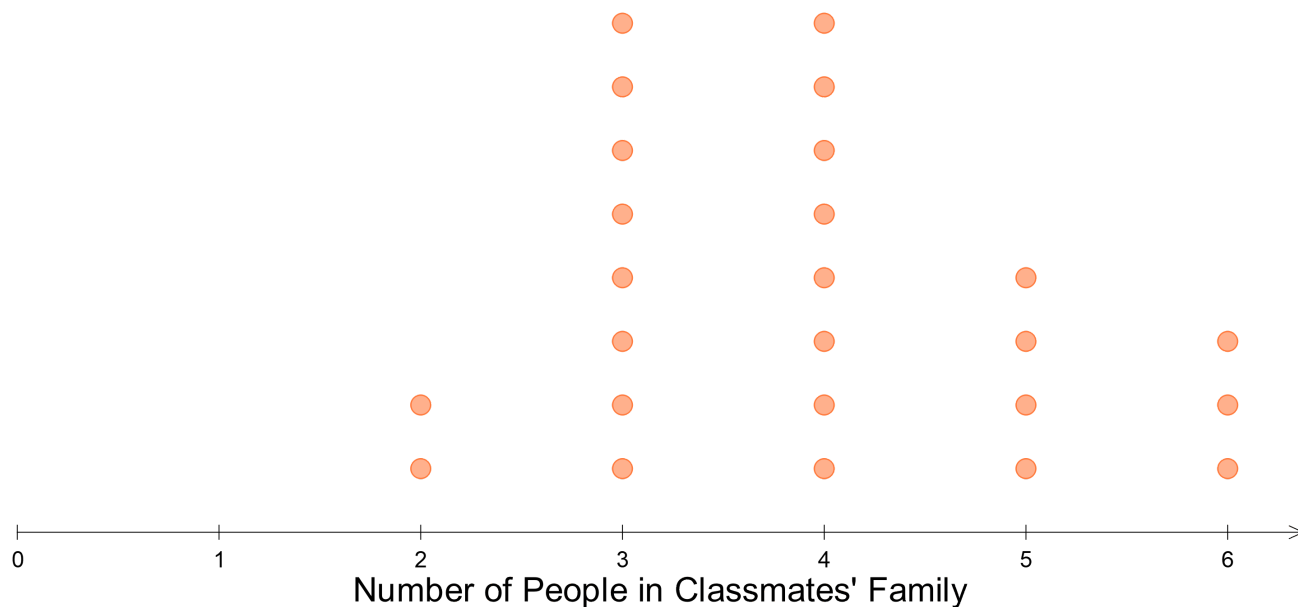
**iii)** 0, 3, 3, 3, 0, 9, 0, 2, 1, 1, 1, 3, 7, 0, 1

**iv)** 7, 8, 2, 3

**v)**



**vi)**



## Gromo's Fun Fact

A person who is an expert in statistics is called a Statistician.

An expert in statistics could also have other job titles, such as:

Mathematician, Operations Research Analyst, Data Scientist, Actuary, Security Analyst or Computer Systems Analyst.

These jobs are often rated as the best jobs, as they have high job security and salaries!



### 4. Questions 4-8 are about the summary statistic, the **median**.

Write the following data sets in ascending order (from smallest to largest).

The first one has been done for you.

**a)** 67, 45, 0, 14, 34, 45, 124, 3  
0, 3, 14, 34, 45, 45, 67, 124

**b)** 6, 2, 1, 7, 3, 4

**c)** 345, 23, 6789, 2, 1, 56, 231

**d)** 12.06, 12.0311, 11, 13.3, 13.09

### 5. Below are three examples finding the median of each data set.

<p>1. Calculate the median of 2, 5, 7, 9, 10.</p> <p>The median is 7.</p>	<p>2. Calculate the median of 12, 5, 6, 7, 1.</p> <p>1, 5, 6, 7, 12</p> <p>The median is 6.</p>	<p>3. Calculate the median of 2, 3, 17, 22, 3.</p> <p>2, 3, 3, 17, 22</p> <p>The median is 3.</p>
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**a)** What do you think the median is for the following data sets?

**i)** 2, 5, 6, 12, 45

**ii)** 0, 5, 5, 6, 7

**iii)** 8, 56, 12

**b)** The middle value in part a) is called the **median**.

Find the median by sorting the data in ascending order and then finding the middle value.

**i)** 345, 23, 6789, 2, 1, 56, 231

**ii)** 12.06, 12.0311, 11, 13.3, 13.09

6.

a) Can you find one middle number to circle for the following data set?

23, 24, 26, 32, 56, 65

b) The numbers in part c) are already ordered in ascending order. Circle the **two** middle numbers.

23, 24, 26, 32, 56, 65

c) What is the midpoint of 26 and 32?

7. For the data sets below, put them in ascending order and then circle the two middle numbers.

a) 4, 6, 7, 2, 4, 1

b) 32, 16, 75, 28, 45, 12

c) 1.2, 1.3, 0.6, 2.3, 0.4, 0.1, 0.4, 0.2

d) 345, 12, 200, 150

8. For an *even number of data values*, the **median** is the *midpoint* of the two middle numbers.

By finding the midpoint of the circled numbers, find the median of the data sets from the last question.

a) 4, 6, 7, 2, 4, 1

b) 32, 16, 75, 28, 45, 12

c) 1.2, 1.3, 0.6, 2.3, 0.4, 0.1, 0.4, 0.2

d) 345, 12, 200, 150

9. Kadijah, Hani and Ayaan calculate the **range**, **mode** and **median** of the following data set:

1, 3, 0, 5, 6, 6, 0

Khadijah's working out:

1, 3, 0, 5, 6, 6, 0  
~~0, 0, 1, 3, 5, 6, 6~~  
 Range = 6  
 Mode = 0, 6  
 Median = 3

Hani's working out:

1, 3, 0, 5, 6, 6, 0  
 mode = 6  
 range = 6 - 1 = 5  
 median = 5  
~~1, 3, 5, 6, 6~~

Ayaan's working out:

~~1, 3, 0, 5, 6, 6, 0~~  
 mode = 6  
 median = 5  
 range = 1

Who has the correct working out? What mistakes did the others make?

## Range, mode and median definitions

In this module we looked at three different types of summary statistics. These are their definitions:

**range:** The difference between the largest and smallest data value. E.g. in the data set: ①, 5, 8, 9, ⑨, the range is 8.

**mode:** The most frequent value in a data set. E.g. in the data set: 1, 5, 8, ⑨, ⑨, the mode is 9.

There can be more than one mode. E.g. in the data set: 1, 5, 8, 8, 9, 9 the mode is 8 and 9.

There can also be no mode. E.g. in the data set: 1, 5, 8, 9 there is no mode, as all the numbers appear the same amount of times.

**median:** The middle value in an ordered data set. E.g. 1, 5, ⑧, 9, 9, the median is 8.

If there are an **even** number of data values, the median is the midpoint of the two middle values in the data set. E.g. in the data set: 1, 5, 8, ⑨, ⑪, 13, 15, 28 the median is 10.

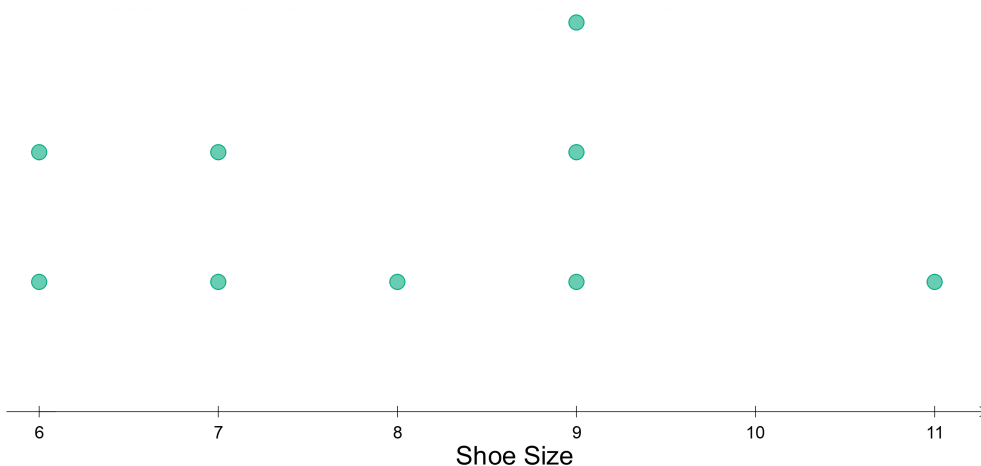
In future modules, you will learn about other summary statistics, such as the **mean** and **interquartile range**.

10. Find the range, mode and median of the following:

a) 12, 1, 4, 13, 16, 2, 3, 2, 2

b) 22, 17, 4, 13, 16, 5, 3, 5, 3, 15

c)



11. What is the same when finding the median from a data set with an **odd** versus **even** number of data values. What is different?

12. Gromo has a test. He needs to remember the definitions of the words range, mode and median and how to calculate them. Come up with a way to help Gromo remember.

## Answers

### 1. Questions 1 and 2 are about the summary statistic, the range:

Below are three examples finding the range of each data set.

1. Calculate the range of: 2, 5, 7, 9. The range is 7.	2. Calculate the range of: 12, 5, 6, 7, 1. The range is 11.	3. Calculate the range of: 2, 3, 17, 22, 3. The range is 20.
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What do you think the range is for the following data sets?

a) 2, 5, 6, 7

The data set is ordered from smallest to largest.

smallest value = minimum = 2

largest value = maximum = 7

range = maximum – minimum =  $7 - 2 = 5$

The **range** is the difference between the maximum and minimum value.

Therefore, the range is 5.

b) 4, 6, 2, 8, 10, 13, 26, 5, 2

maximum = 26

minimum = 2

range = maximum – minimum =  $26 - 2 = 24$

The range is 24.

### 2. Calculate the range for the following data sets.

a) 234, 65, 2, 14, 0, 43

maximum = 234

minimum = 0

range =  $234 - 0 = 234$

The range is 234.

b) Maximum temperatures in Astoria this week: (in degrees Celsius):

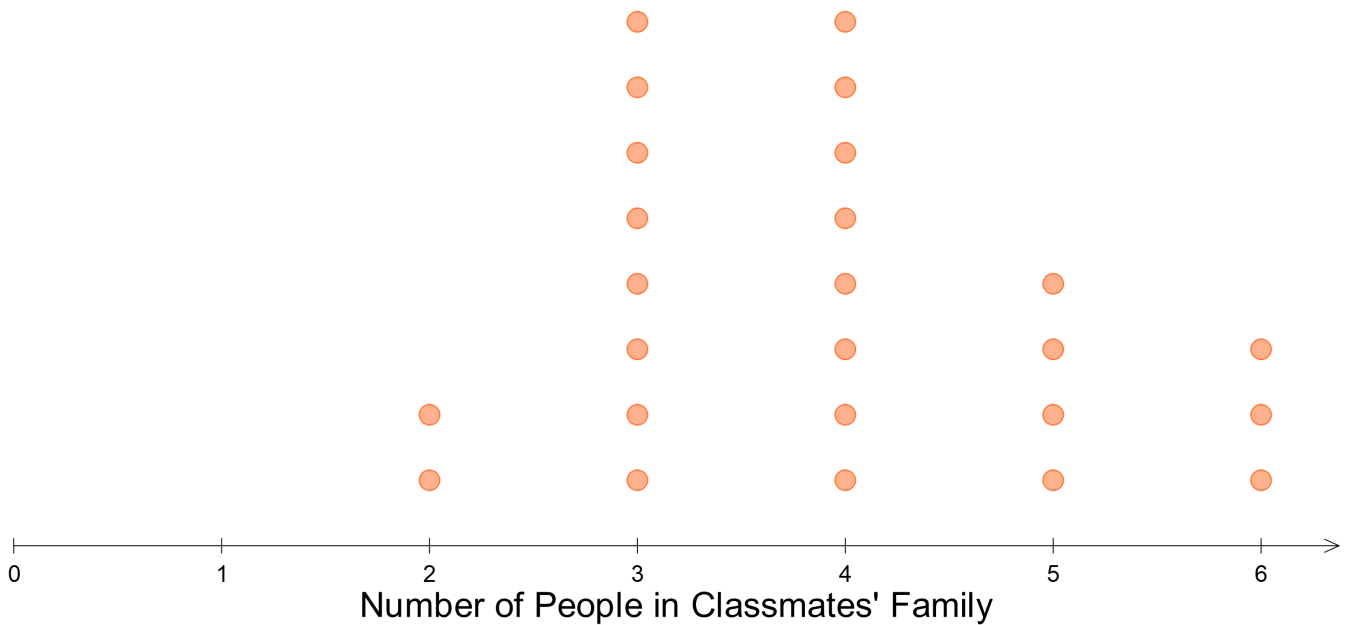
Mon	Tue	Wed	Thu	Fri	Sat	Sun
25	31	17	21	23	27	20

minimum = 17

maximum = 31

range = maximum – minimum =  $31 - 17 = 14$

c) Dot plot showing the number of people Emmanuel's classmates' have in their family.



**maximum = 6**

**minimum = 2**

**range = maximum – minimum = 6 – 2 = 4**

**The range is 4.**

3. Question 3 is about the summary statistic, the **mode**.

a) Which is the most frequent value (occurs the most)?

1, 6, 7, 3, 7, 7, 3, 5

**7, as it appears 3 times. This is more than any other number appears.**

b) The most frequent value is called the **mode**. What is the mode of the data in part a)?

**The mode is 7.**

c) Which is the most frequent value in the data set below?

3, 5, 6, 6, 7, 7, 8

**6 and 7 both appear twice.**



**d)** You can have more than one value as the mode. The mode in part c) is 6 and 7.

Find the mode of the following data:

**i)** 3, 4, 2, 1, 1, 7

**mode = 1**

**ii)** 23, 102, 34, 98, 45, 23, 109

**mode = 23**

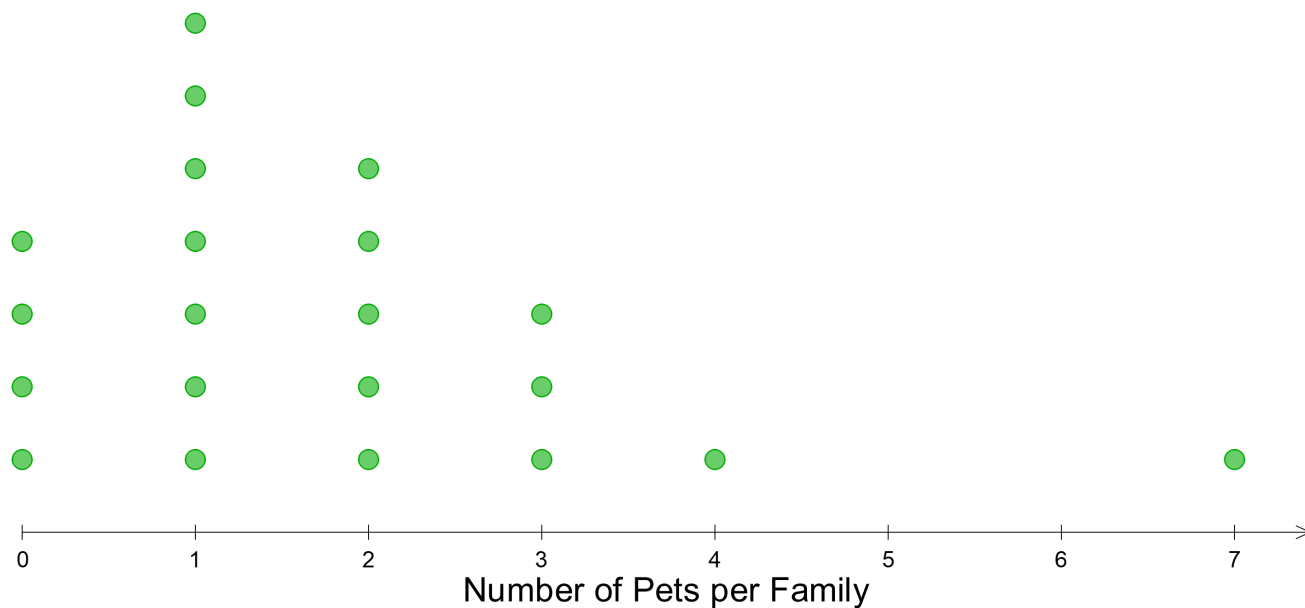
**iii)** 0, 3, 3, 3, 0, 9, 0, 2, 1, 1, 1, 3, 7, 0, 1

**mode = 0, 1, 3**

**iv)** 7, 8, 2, 3

**There is no mode as no number appears more than any other.**

**v)**



**mode = 1**

**vi)**

**4. Questions 4-8 are about the summary statistic, the *median*.**

Write the following data sets in ascending order (from smallest to largest).

The first one has been done for you.

**a)** 67, 45, 0, 14, 34, 45, 124, 3

0, 3, 14, 34, 45, 45, 67, 124

0

1

2

3

4

5

6

Number of People in Classmates' Family

**c)** 345, 23, 6789, 2, 1, 56, 231.

1, 2, 23, 56, 231, 345, 6789

**b)** 6, 2, 1, 7, 3, 4

1, 2, 3, 4, 6, 7

**d)** 12.06, 12.0311, 11, 13.3, 13.09

11, 12.0311, 12.06, 13.09, 13.3

**5. Below are three examples finding the median of each data set.**

<p>1. Calculate the median of 2, 5, 7, 9, 10.</p> <p>The median is 7.</p>	<p>2. Calculate the median of 12, 5, 6, 7, 1.</p> <p>1, 5, 6, 7, 12</p> <p>The median is 6.</p>	<p>3. Calculate the median of 2, 3, 17, 22, 3.</p> <p>2, 3, 3, 17, 22</p> <p>The median is 3.</p>
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**a) What do you think the median is for the following data sets?**

**i)** 2, 5, 6, 12, 45

The median is 6 because it is the middle number.

**ii)** 0, 5, 5, 6, 7

The median is 5 because it is the middle number.

**iii)** 8, 56, 12

If you sort the data in order you get: 8, 12, 56.

The middle number is 12, so the median is 12.

**b)** The middle value in part a) is called the **median**.

Find the median by sorting the data in ascending order and then finding the middle value.

**i)** 345, 23, 678, 2, 1, 56, 231

Sort the numbers in ascending order:

1, 2, 23, 56, 231, 345, 678

Cancel the first number on the left:

~~1~~, 2, 23, 56, 231, 345, 678

Then cancel a number from the right:

~~1~~, 2, 23, 56, 231, ~~345~~ ~~678~~

Continue cancelling until you have one number left in the middle:

~~1~~, ~~2~~, 23, 56, 231, ~~345~~ ~~678~~

~~1~~, ~~2~~, 23, 56, 231, ~~345~~ ~~678~~

~~1~~, ~~2~~, ~~23~~, 56, 231, ~~345~~ ~~678~~

~~1~~, ~~2~~, ~~23~~, 56, ~~231~~, ~~345~~ ~~678~~

Notice there is now only one value in the middle. Put a circle around it:

~~1~~, ~~2~~, ~~23~~, 56, ~~231~~, ~~345~~ ~~678~~

The median is 56.

**ii)** 12.06, 12.0311, 11, 13.3, 13.09

Sort the numbers in ascending order:

11, 12.0311, 12.06, 13.09, 13.3

Cross out one number from each side until there is one number in the middle:

~~11~~, ~~12.0311~~, 12.06, ~~13.09~~, ~~13.3~~

Circle the middle value:

~~11~~, ~~12.0311~~, 12.06, ~~13.09~~, ~~13.3~~

The median is 12.06.

**6.**

**a)** Can you find one middle number to circle for the following data set?

23, 24, 26, 32, 56, 65

No, if you cross one off from either side, there won't be one middle number left. This is because there are an even number of data values.

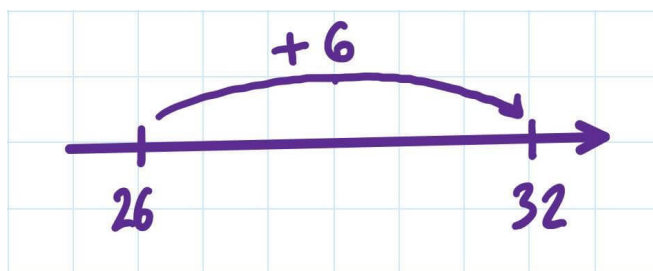
**b)** The numbers in part c) are already ordered in ascending order. Circle the **two** middle numbers.

23, 24, 26, 32, 56, 65

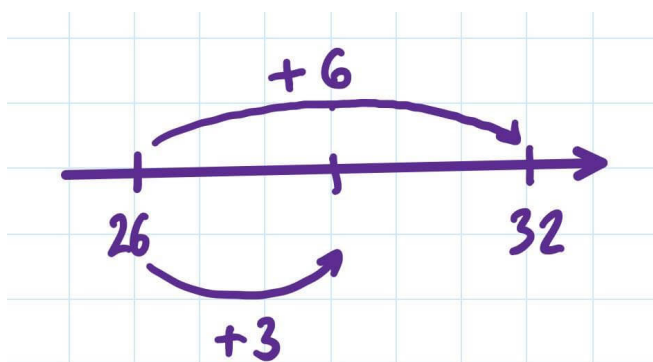
23, 24, 26 32, 56, 65

c) What is the midpoint of 26 and 32?

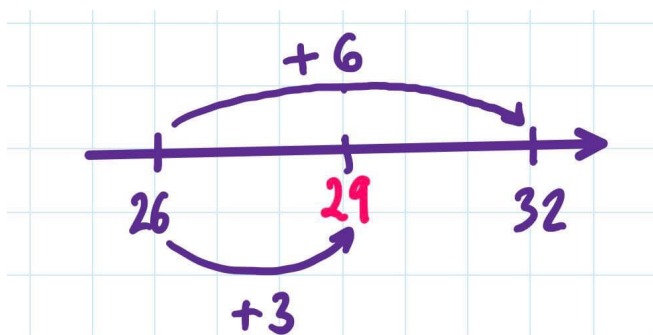
To jump from 26 to 32, you need to add 6.



To go half way, add 3.



Starting from 26, if you add 3, you get 29.



So the midpoint of 26 and 32 is 29.

7. For the data sets below, put them in ascending order and then circle the two middle numbers.

a) 4, 6, 7, 2, 4, 1

1, 2, 4, 4, 6, 7

1, 2, (4)(4), 6, 7

b) 32, 16, 75, 28, 45, 12

12, 16, 28, 32, 45, 75

12, 16, (28)(32), 45, 75

- c) 1.2, 1.3, 0.6, 2.3, 0.4, 0.1, 0.4, 0.2  
 0.1, 0.2, 0.4, 0.4, 0.6, 1.2, 1.3, 2.3  
 0.1, 0.2, 0.4, (0.4) (0.6), 1.2, 1.3, 2.3

- d) 345, 12, 200, 150  
 12, 150, 200, 345  
 12, (150) (200), 345

8. For an even number of data values, the **median** is the *midpoint* of the two middle numbers. By finding the midpoint of the circled numbers, find the median of the data sets from the last question.

- a) 4, 6, 7, 2, 4, 1  
 1, 2, (4) (4), 6, 7  
 The midpoint of 4 and 4 is just 4.  
 Therefore, the median = 4.

- b) 32, 16, 75, 28, 45, 12  
 12, 16, (28) (32), 45, 75  
 The midpoint of 28 and 32 is 30.  
 Therefore, the median = 30.

- c) 1.2, 1.3, 0.6, 2.3, 0.4, 0.1, 0.4, 0.2  
 0.1, 0.2, 0.4, (0.4) (0.6), 1.2, 1.3, 2.3  
 The midpoint of 0.4 and 0.6 is 0.5.  
 Therefore, the median = 0.5.

- d) 345, 12, 200, 150  
 12, (150) (200), 345  
 The midpoint of 150 and 200 is 175.  
 Therefore, the median = 175.

9. Kadijah, Hani and Ayaan calculate the **range**, **mode** and **median** of the following data set: 1, 3, 0, 5, 6, 6, 0

Khadija's working out:

1, 3, 0, 5, 6, 6, 0  
~~0~~, ~~0~~, 1, (3), ~~5~~, ~~6~~, ~~6~~  
 Range = 6  
 Mode = 0, 6  
 Median = 3

Hani's working out:

1, 3, 0, 5, 6, 6, 0  
 mode = 6  
 range = 6 - 1 = 5  
 median = 5  
~~1~~, ~~3~~, (5), ~~6~~, ~~6~~

Who has the correct working out? What mistakes did the others make?

Khadija is correct.

Hani has incorrectly ignored the zero values in the data set.

Ayaan did not order the data set from smallest to largest, so she is also incorrect.

**10.** Find the range, mode and median of the following:

**a)** 12, 1, 4, 13, 16, 2, 3, 2, 2

**Range:**

**minimum** = 1

**maximum** = 16

**range** = **maximum** – **minimum** =  $16 - 1 = 15$

The range is 15.

**Mode:**

Two is the most frequent value. It occurs three times.

The mode is 2.

**Median:**

12, 1, 4, 13, 16, 2, 3, 2, 2

1, 2, 2, 2, 3, 4, 12, 13, 16

1, 2, 2, 2, (3), 4, 12, 13, 16

The median is 3.

**b)** 22, 17, 4, 13, 16, 5, 3, 5, 3, 15

**Range:**

**minimum** = 3

**maximum** = 22

**range** = **maximum** – **minimum** =  $22 - 3 = 19$

The range is 19.

**Mode:**

Three and five are the most frequent values. They both occur twice.

The mode is 3 and 5.

**Median:**

22, 17, 4, 13, 16, 5, 3, 5, 3, 15

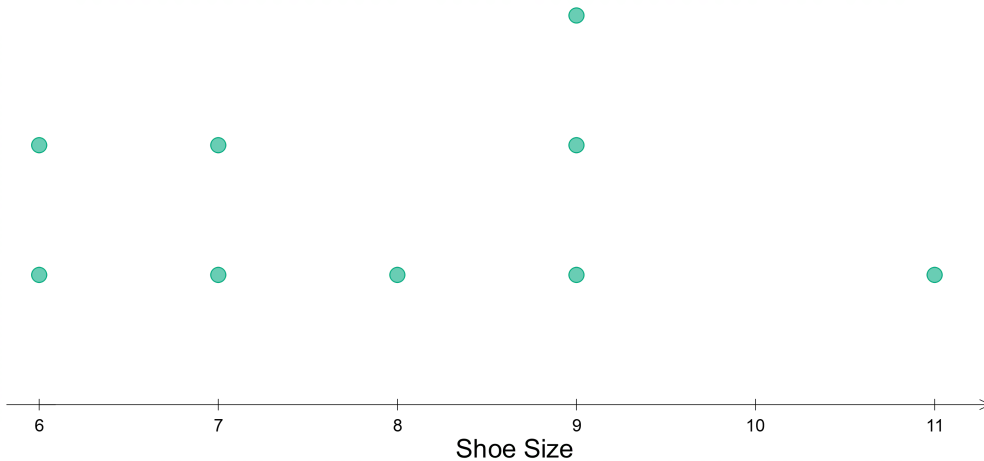
3, 3, 4, 5, 5, 13, 15, 16, 17, 22

3, 3, 4, 5, (5), (13), 15, 16, 17, 22

The midpoint of 5 and 13 is 9.

The median is 9.

c)



**Range:**

**minimum = 6**

**maximum = 11**

**range = maximum – minimum = 11 – 6 = 5**

**The range is 5.**

**Mode:**

**Nine is the most frequent value. It occurs three times.**

**The mode is 9.**

**Median:**

**6, 6, 7, 7, 8, 9, 9, 9, 11**

**6, 6, 7, 7, (8), 9, 9, 9, 11**

**The median is 8.**

- 11.** What is the same when finding the median from a data set with an **odd** versus **even** number of data values. What is different?

**Same:**

- **You always have to order the data from smallest to largest first.**

**Different:**

- **For an odd number of data values, the median is the middle number in the ordered data set.**
- **For an even number of data values, the median is the midpoint of the two middle numbers in the ordered data set.**

- 12.** Gromo has a test and he is stressing out! He needs to remember the definitions of the words range, mode and median and how to calculate them. Can you help? Come up with a way to help Gromo remember.

**There is no one correct answer here. Check your answer by using your method.**

**Here is one example, a haiku poem:**

*Range, max minus min,  
Median is the middle  
Mode is most common*

**If you are stuck, ask a friend or your teacher.**

**P.S. Gromo says thank you!**