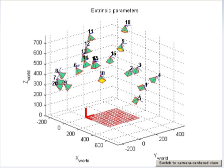
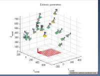


Geometric Camera Calibration

based on “A Flexible New Technique for Camera Calibration” by Zhengyou Zhang

Alexander Reuter

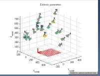
2.12.2009



Motivations

What we know so far

- Pinhole cameras are ideal cameras
- There doesn't exist a pinhole camera actually
- So there doesn't exist images obtained by an ideal camera



Motivations

What is camera calibration?

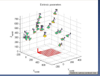
Find the parameters of a camera that produced several images

Application field

- Transform the image to one obtained by an ideal camera
- Find the global position and orientation of a camera

Available techniques

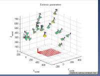
- Self-calibration
- Photogrammetric calibration



Overview

Overview

- Pinhole Camera
  - Intrinsic parameters
  - Extrinsic parameters
- Zhangs Algorithm
- Degenerate configurations



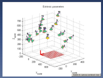
Homography

Properties

- Linear projection between two planes
- Described by a $\mathbb{R}^{3\times3}$ matrix

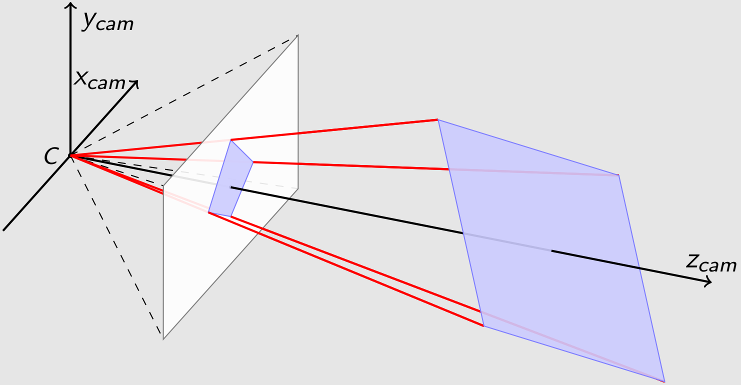
Advantages

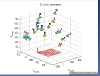
- Transformation from a rectangle plane in world space to a quadriliteral in image plane can be described as a homography problem
- Important tool for camera calibration



Homography

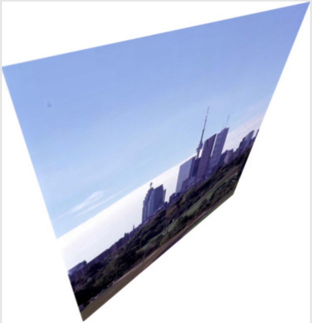
Homography between image plane and object plane

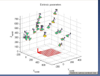




Homography

example

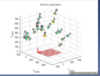




Developing intrinsic camera matrix

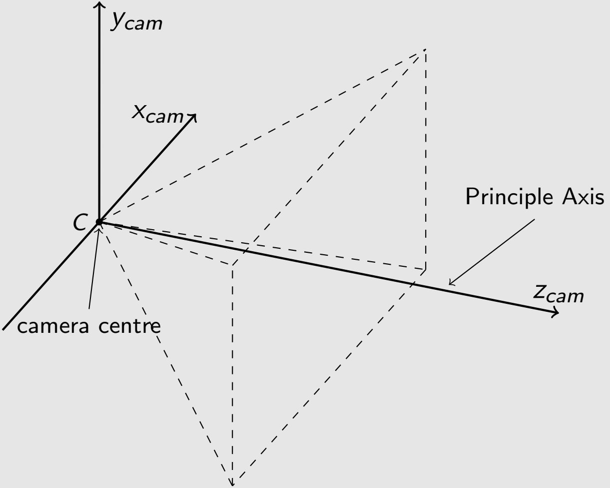
Motivation

- definition of camera properties required to apply camera calibration
- different parameter definitions make introduction necessary
- assume naive pinhole camera model



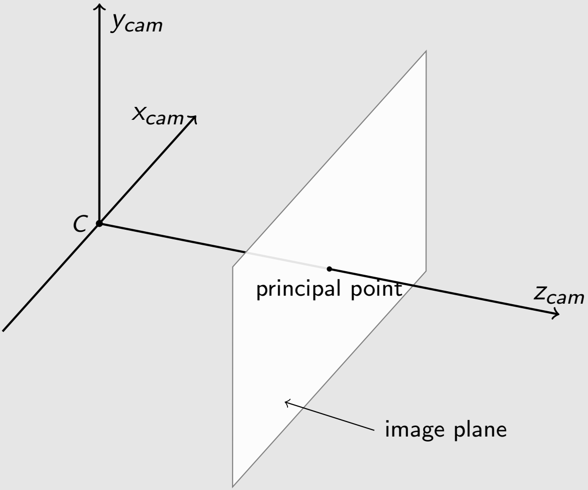
Pinhole camera

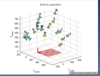
From raw model to an initial transformation matrix



Pinhole camera

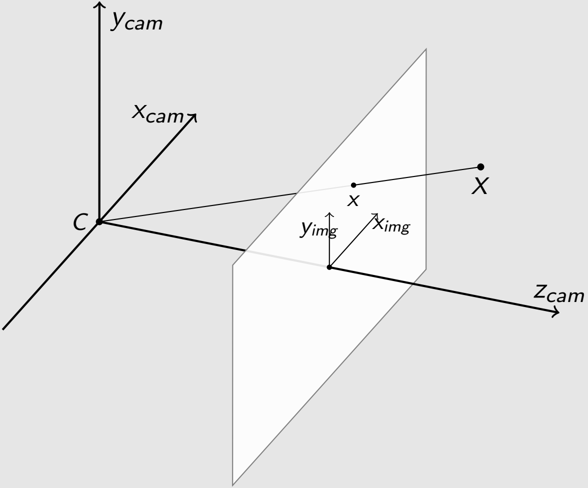
From raw model to an initial transformation matrix

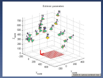




Pinhole camera

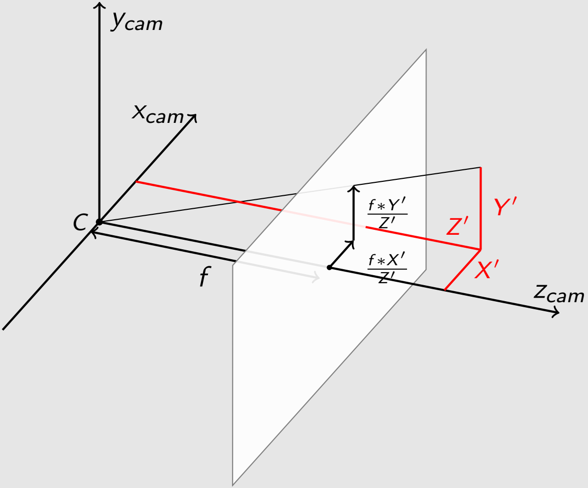
From raw model to an initial transformation matrix

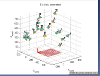




Pinhole camera

From raw model to an initial transformation matrix





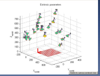
Development of intrinsic camera matrix

Applying focal length [1]

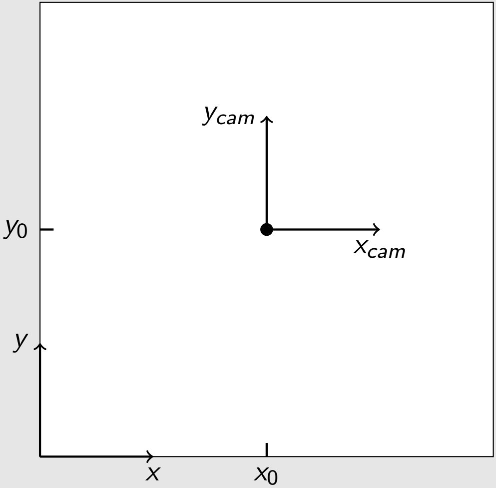
$$\begin{pmatrix} X' \\ Y' \\ Z' \\ 1 \end{pmatrix} \longmapsto \begin{pmatrix} fX' \\ fY' \\ Z' \end{pmatrix} = \begin{bmatrix} f & & & 0 \\ & f & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X' \\ Y' \\ Z' \\ 1 \end{pmatrix}$$

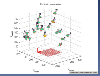
Focal length examples

Figure: 40mm, 180mm and 600mm focal length



Principal point offset





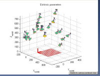
Development of intrinsic camera matrix

Principal point offset

$$\begin{pmatrix} X' \\ Y' \\ Z' \\ 1 \end{pmatrix} \longmapsto \begin{pmatrix} fX' + Z'x_0 \\ fY' + Z'y_0 \\ Z' \end{pmatrix} = \begin{bmatrix} f & & x_0 & 0 \\ & f & y_0 & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X' \\ Y' \\ Z' \\ 1 \end{pmatrix}$$

Camera calibration matrix

$$K := \begin{bmatrix} f & & x_0 \\ & f & y_0 \\ & & 1 \end{bmatrix}$$



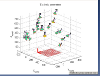
Independent scaling

Unequal scale factors

$$K := \begin{bmatrix} \alpha_x & & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$

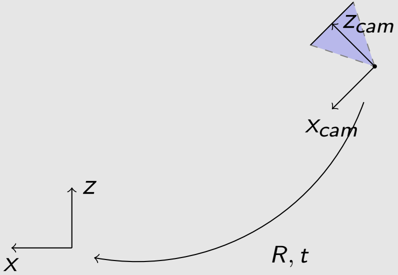
Skew parameter

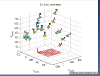
$$K := \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$



Development of extrinsic camera matrix

From world to camera coordinates





Development of extrinsic camera matrix

Camera rotation and translation

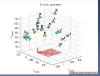
X_{cam} := object coordinates depending on camera orientation $R \in \mathbb{R}^{3x3}$ and origin $C \in \mathbb{R}^{3x1}$

Getting X_{cam}

$$
\begin{aligned}
X_{cam} &= \begin{bmatrix} R & -RC \\ 0 & 1 \end{bmatrix} \begin{pmatrix} X' \\ Y' \\ Z' \\ 1 \end{pmatrix} = \begin{bmatrix} R & -RC \\ 0 & 1 \end{bmatrix} X \\
&= R[I|-C]X
\end{aligned}
$$

Putting K and X_{cam} together

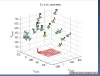
$$x = KX_{cam} = KR[I|-C]X \stackrel{t:=-RC}{=} K[R|t]$$



Finite projective camera

Finite projective camera

- $P \in \mathbb{R}^{3\times 4}$
- $P := K[R|t] = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix}$
- $x' = PX$, where $X = \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$ and $x' = \begin{pmatrix} x \\ y \\ w \end{pmatrix}$
- $\Rightarrow x = \frac{x'}{w}$

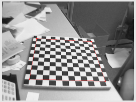


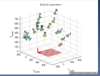
Zhangs Algorithm

Prework

- Take several images of a planar checkboard

- Find checkboard in each image

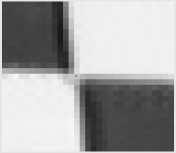




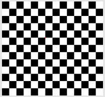
Zhangs Algorithm

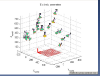
Prework

- Find subpixel corners of each checkboard



- Get the checkboard coordinate in 2D worldspace of each checkboard corner





Zhangs Algorithm

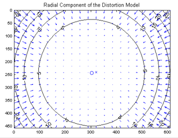
Algorithm

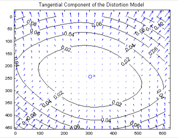
- Estimate a homography for each image
- Estimate intrinsic matrix K from the set of homographies
- Estimate extrinsic parameters for each checkboard
- Estimate coefficients of radial distortion

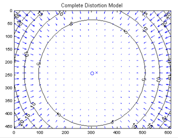
Distortion

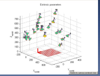
Two important distortion types

- Radial distortion
- Tangential distortion





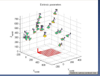




Estimate coefficients of radial distortion

Distortion in Zhangs Algorithm

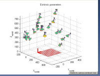
- Only radial distortion is handled
- Estimation provided by minimization



Zhangs Algorithm

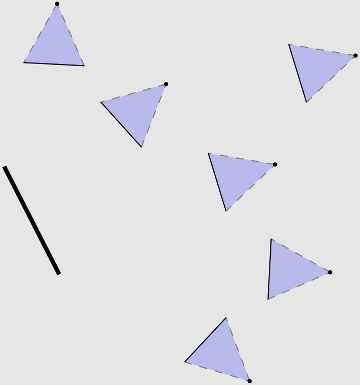
Method Summary

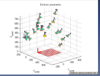
- Print a pattern
- Take images
- Detect feature points
- Estimate intrinsic & extrinsic parameters
- Estimate coefficients of radial distortion
- Refine by minimizing



Multiple cameras

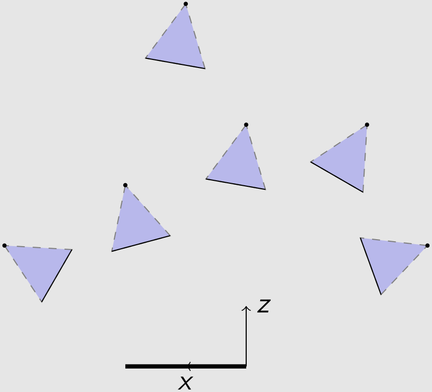
Scene with several camera positions and orientations

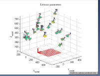




Multiple cameras

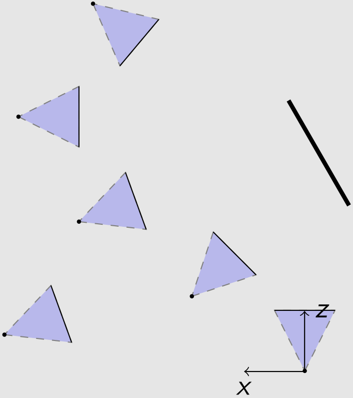
World centered scene

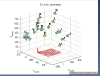




Multiple cameras

Scene centered by a selected camera





Degenerate configurations

Degenerate configurations

- Rotation between two images needed to get homography
- Long focal length / small working volume lead to non reliable results
- Outliers in feature point sets not acceptable (RANSAC)
- Subpixel accuracy for feature points required

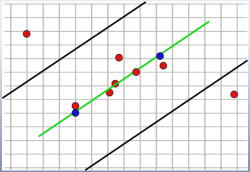
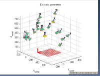
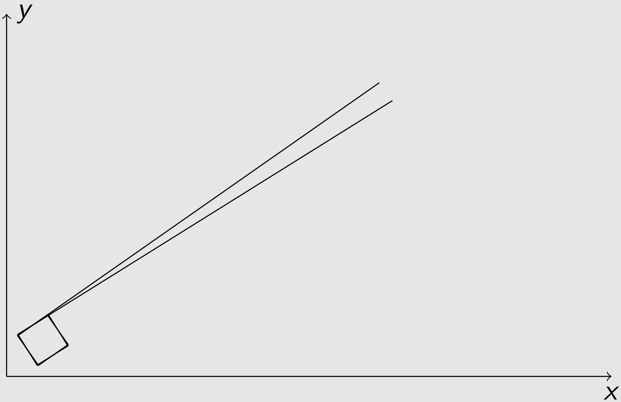


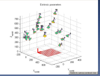
Figure: RANSAC algorithm



Degenerate configurations

Subpixel & Working volume

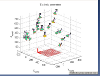




Conclusion

Summary

- Camera calibration obtains intrinsic and extrinsic parameters of a camera
- Plane-to-plane transformations, called Homographies are used to map from object plane to image plane
- Many approaches (like Zhangs) estimate distortions
- Relative position and orientation of each camera based on the properties of a selected camera can be obtained



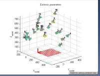
Homogeneous coordinates

Cartesian coordinates

- Standard coordinate system
- Standard matrix operation allows rotation, scaling and shearing

Homogeneous coordinates

- Enhanced coordinate system
- One matrix operation allows translation and perspective transformations additionally



Homogeneous coordinates

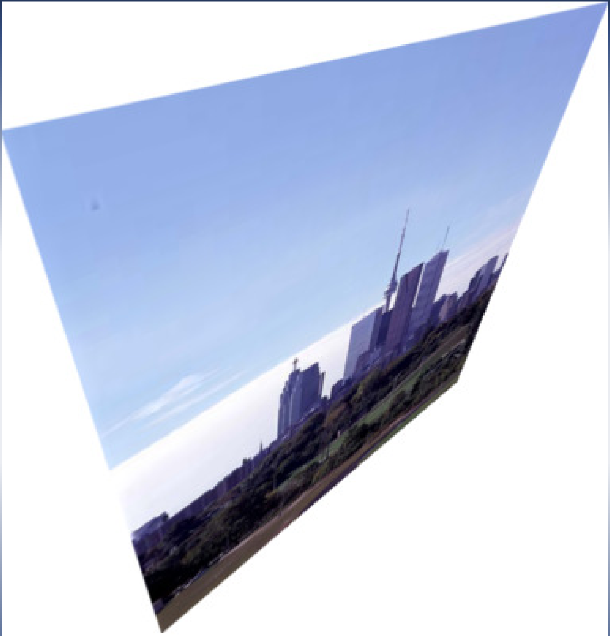
From Cartesian coordinates to Homogeneous coordinates

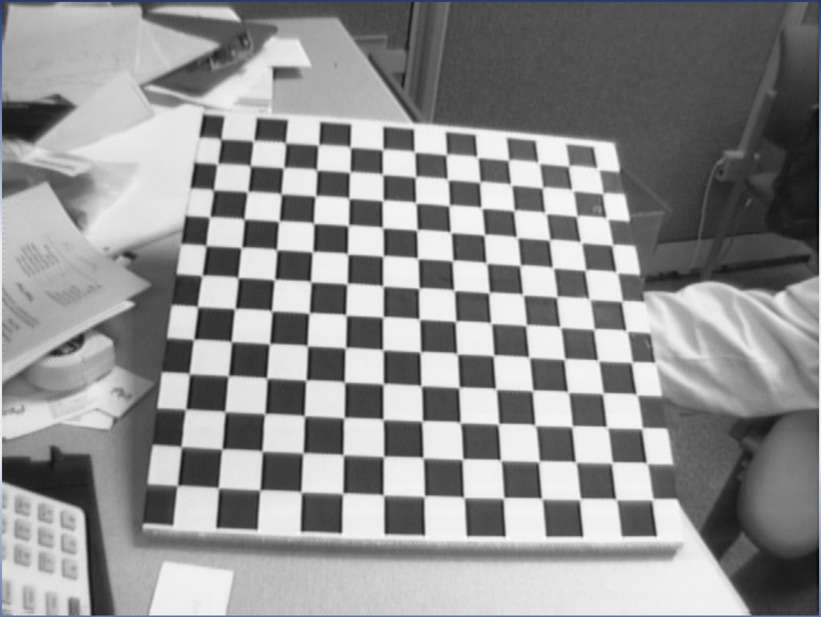
- $\begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$ and $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$

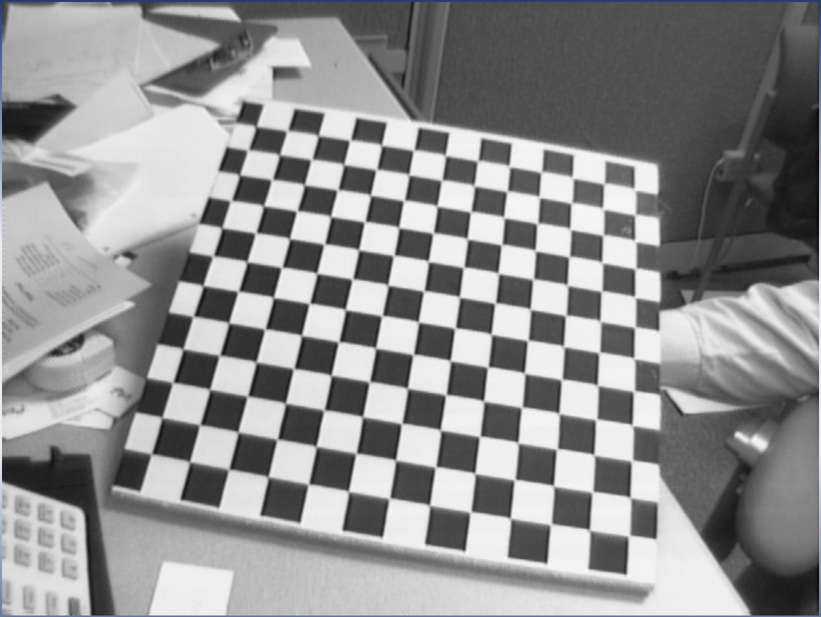
From Homogeneous coordinates to Cartesian coordinates

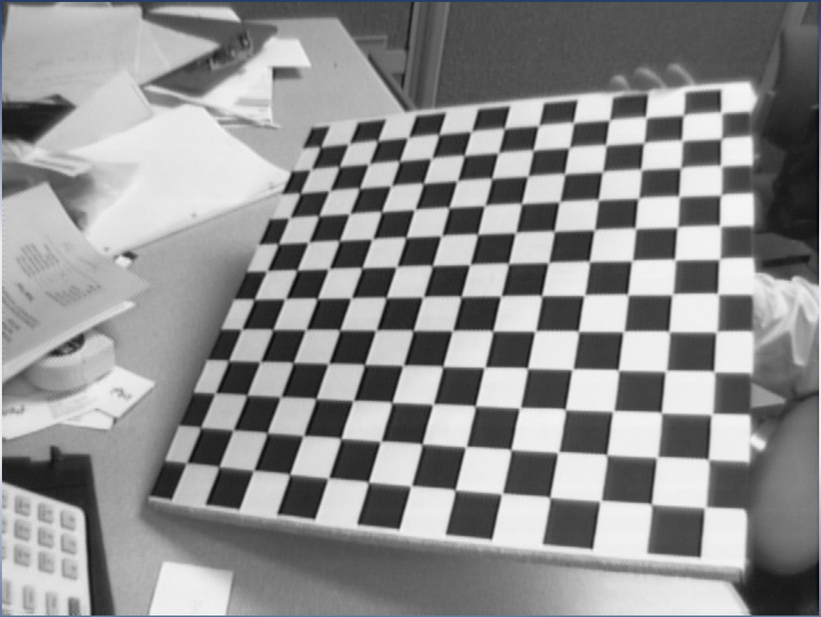
- $\begin{pmatrix} x \\ y \\ w \end{pmatrix} \rightarrow \begin{pmatrix} \frac{x}{w} \\ \frac{y}{w} \end{pmatrix}$ and $\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \rightarrow \begin{pmatrix} \frac{x}{w} \\ \frac{y}{w} \\ \frac{z}{w} \end{pmatrix}$

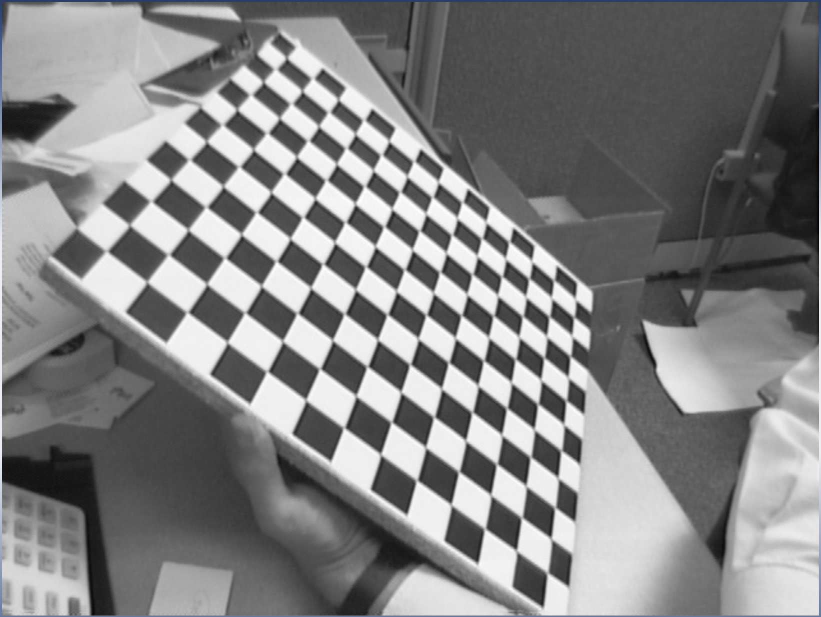
10

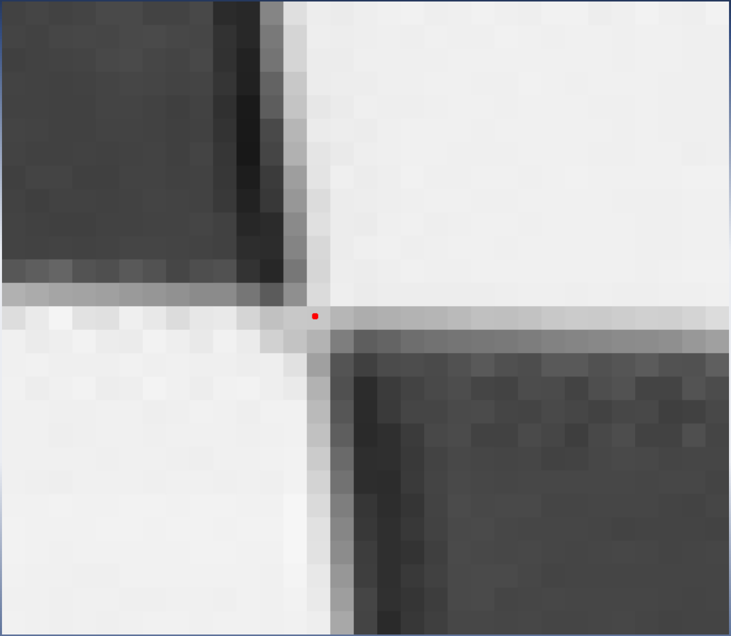


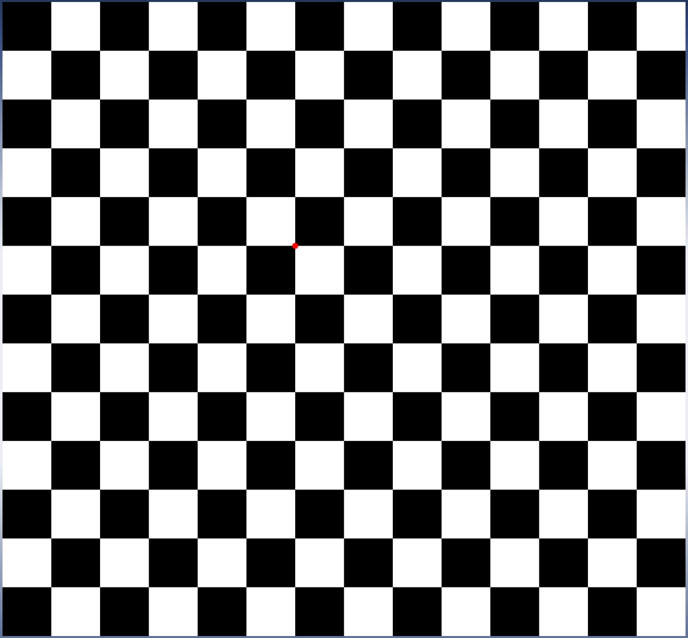


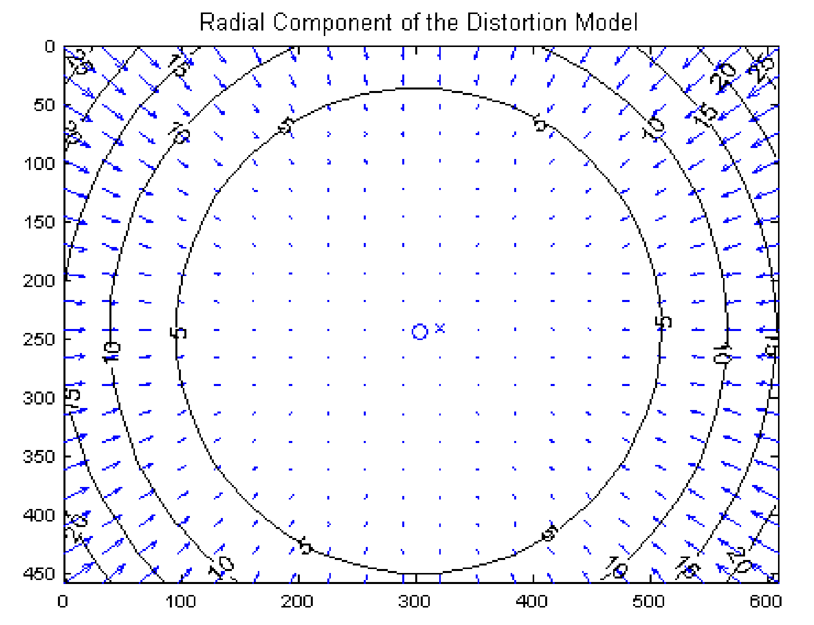


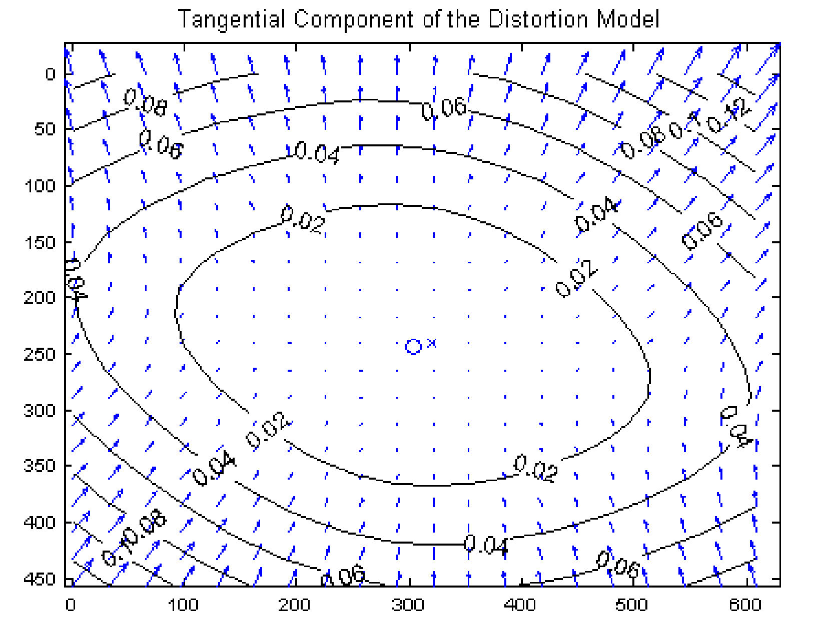









Radial Component of the Distortion Model
0
50
100
150
200
250
300
350
400
450
0
100
200
300
400
500
600
5
10
15
20
25


Tangential Component of the Distortion Model
0.08
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0.04
0.02
0.1
0.12
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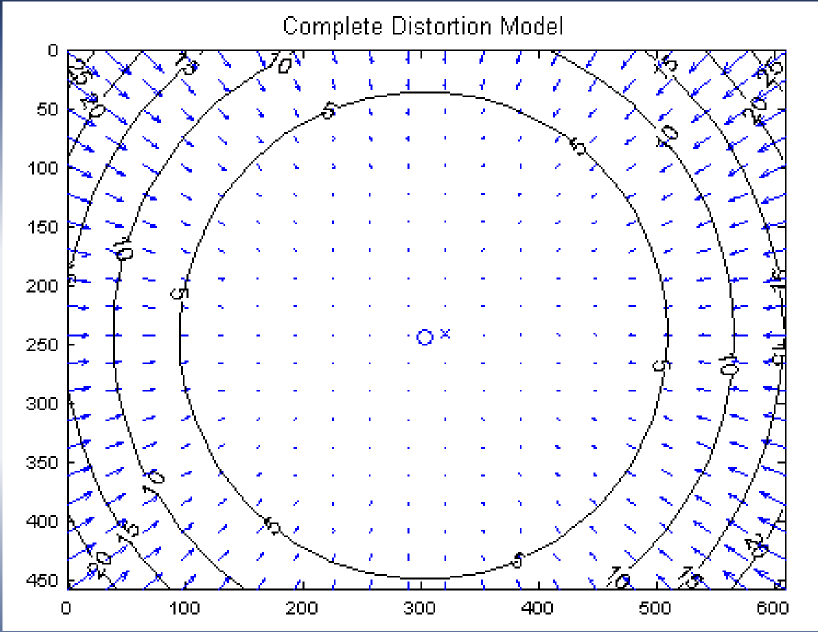
50
100
150
200
250
300
350
400
450
0
100
200
300
400
500
600


Complete Distortion Model
0
50
100
150
200
250
300
350
400
450
0
100
200
300
400
500
600