CS 480

Introduction to Artificial Intelligence

March 8, 2021

Announcements / Reminders

- **Programming Assignment #01:**
  - **due: March 13th, 11:00 PM CST**

- **Grading TA assignment:**

  https://docs.google.com/spreadsheets/d/1Cav_GBTGC7fLGzxuBCAUmEuJYPeF-HMLCYvwPbq8Fus/edit?usp=sharing

Plan for Today

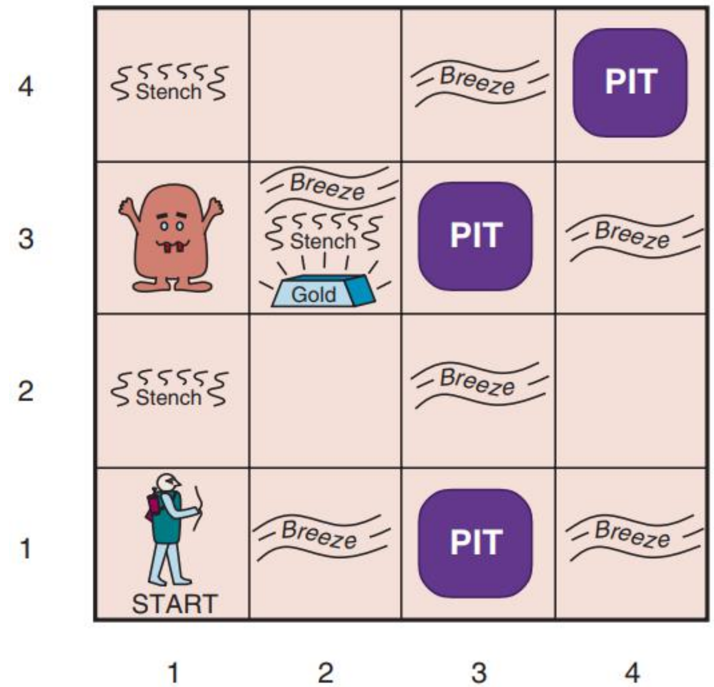
- **Predicate / First-Order Logic**

Propositional Logic Weaknesses

Wumpus World is represented by several variables in propositional logic:

- $B_{x,y}$: **there is a breeze in square (x,y)**
- $P_{x,y}$: **there is a pit in square (x,y)**
- $S_{x,y}$: **there is a stench in square (x,y)**
- $W_{x,y}$: **there is a wumpus in square (x,y)**
- $G_{x,y}$: **there is gold in square (x,y)**
- $L_{x,y}$: **agent is located in square (x, y)**

Either one of those can be either true **or** false**. There is 4 x 4 variables for every type.**



What if this Wumpus World was a N x N grid and N was a really large number. How many variables we would have?

Propositional Logic Weaknesses

Similarly, consider the following English sentence:

"robot R514 is standing at coordinates (123, 4501)"

It could be represented by a following propositional logic variable:

robot_R514_is_standing_at_coordinates_(123_4501)

and a true / false **value. What's the problem?**

Propositional Logic Weaknesses

How about relationships such as this one:

“robot R514 is to the left of robot R89”

It could be represented by a following propositional logic variable:

robot_R514_is_to_the_left_of_R89

and a true **/** false **value. What’s the problem?**

Propositional Logic Weaknesses

How about relationships such as this one:

"robot R514 is to the left of robot R89"

It would be much easier to represent it in a form similar to this one:

toTheLeftOf(R514, R89)

Propositional Logic Weaknesses

Propositional logic:

- **CAN represent facts**
- **CANNOT represent objects**
- **CANNOT represent relationships between objects**
- **is overall not very expressive, for example it is impossible to transform this sentence into propositional logic:**

“Some people live in Illinois.”

A More Expressive Formal Language

A more expressive formal language could be obtained by augmenting propositional logic with:

- **objects:**
  - **people, houses, humbers, theories, colors, basketball games, wars, etc.**
- **relations:**
  - **unary relations (properties): red, round, bogus, etc.**
  - **n-ary relations: brother of, bigger than, inside, part of, etc.**
- **functions:**
  - **relations in which there is only one “value” for a given “input”**
  - **father of, best friend, one more than, beginning of, etc.**

Predicate / First-Order Logic

Predicate Logic is an extension of Propositional Logic. It is a formal language in which propositions are expressed in terms of predicates, variables, and quantifiers.

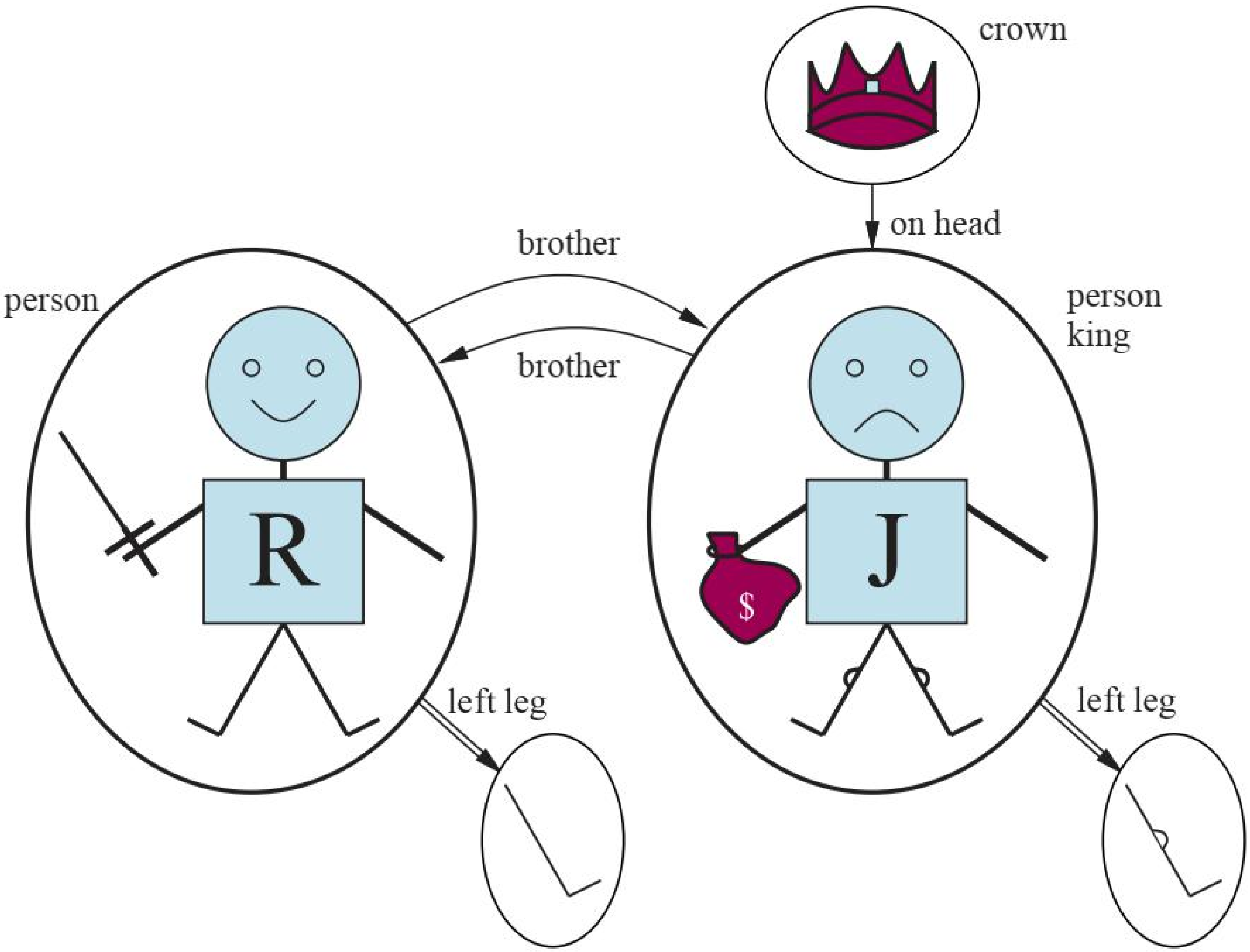
Predicates

In logic, a predicate is a symbol which represents a property or a relation. For example:

- **in the sentence $P(a)$, the symbol P is a predicate which applies to the individual constant a,**
- **in the formula $R(a,b)$ the symbol R is a predicate which applies to the individual constants a and b.**

In the semantics of logic, predicates are interpreted as relations.

Predicate Logic: Model of the World



Predicate Logic: Model and Objects

Domain $\mathbb{D}$

lukeSkywalker | **gun**

genghisKhan | **sword**

wyattEarp | **lightSaber**

The domain of a model $\mathbb{D}$ is a nonempty set of domain elements it contains. Objects may be related.

Symbols: Refresher

Symbol	Name	Alternative symbols*	Should be read
$\neg$	Negation	$\sim$, !	not
$\wedge$	(Logical) conjunction	$\bullet$, &	and
$\vee$	(Logical) disjunction	$+$, $\|\|$	or
$\Rightarrow$	(Material) implication	$\rightarrow$, $\supset$	implies
$\Leftrightarrow$	(Material) equivalence	$\leftrightarrow$, $\equiv$, iff	if and only if
$\top$	Tautology	T, 1, $\blacksquare$	truth
$\bot$	Contradiction	F, 0, $\square$	falsum empty clause
$\forall$	Universal quantification		for all; for any; for each
$\exists$	Existential quantification		there exist

* you can encounter it elsewhere in literature

Predicate Logic Syntax: Symbols

Predicate calculus symbols include:

- **truth symbols: true and false**
- **constant symbols**
- **variable symbols**
- **predicate symbols**
- **function symbols**

Syntax: Constant

A constant c (c - symbol) represent objects

- **for example:**
  - lightSaber
  - kingJohn
  - lukeSkywalker
  - box
  - table
  - crown

Predicate Logic: Model and Objects

Domain D

lukeSkywalker

genghisKhan

wyattEarp

gun

sword

lightSaber

Existing constants in this representation are

D = {lukeSkywalker, genghisKhan, wyattEarp, gun, sword, lightSaber}

Syntax: Variable

A variable v (v - symbol) can be used to represent classes of objects or properties in the world.

- **for example:**
  - weapon
  - character
  - person
  - height
  - distance

Predicate Logic: Model and Objects

Domain $\mathbb{D}$

lukeSkywalker	gun
genghisKhan	sword
wyattEarp	lightSaber

Possible variables for this model: person and weapon

D_{person} = {lukeSkywalker, genghisKhan, wyattEarp} D_{weapon} = {gun, sword, lightSaber}

Syntax: Predicate Symbols

A predicate p (p - symbol) maps object(s) to a boolean value.

$p(argument_1, argument_2, ..., argument_N)$ **or just p**

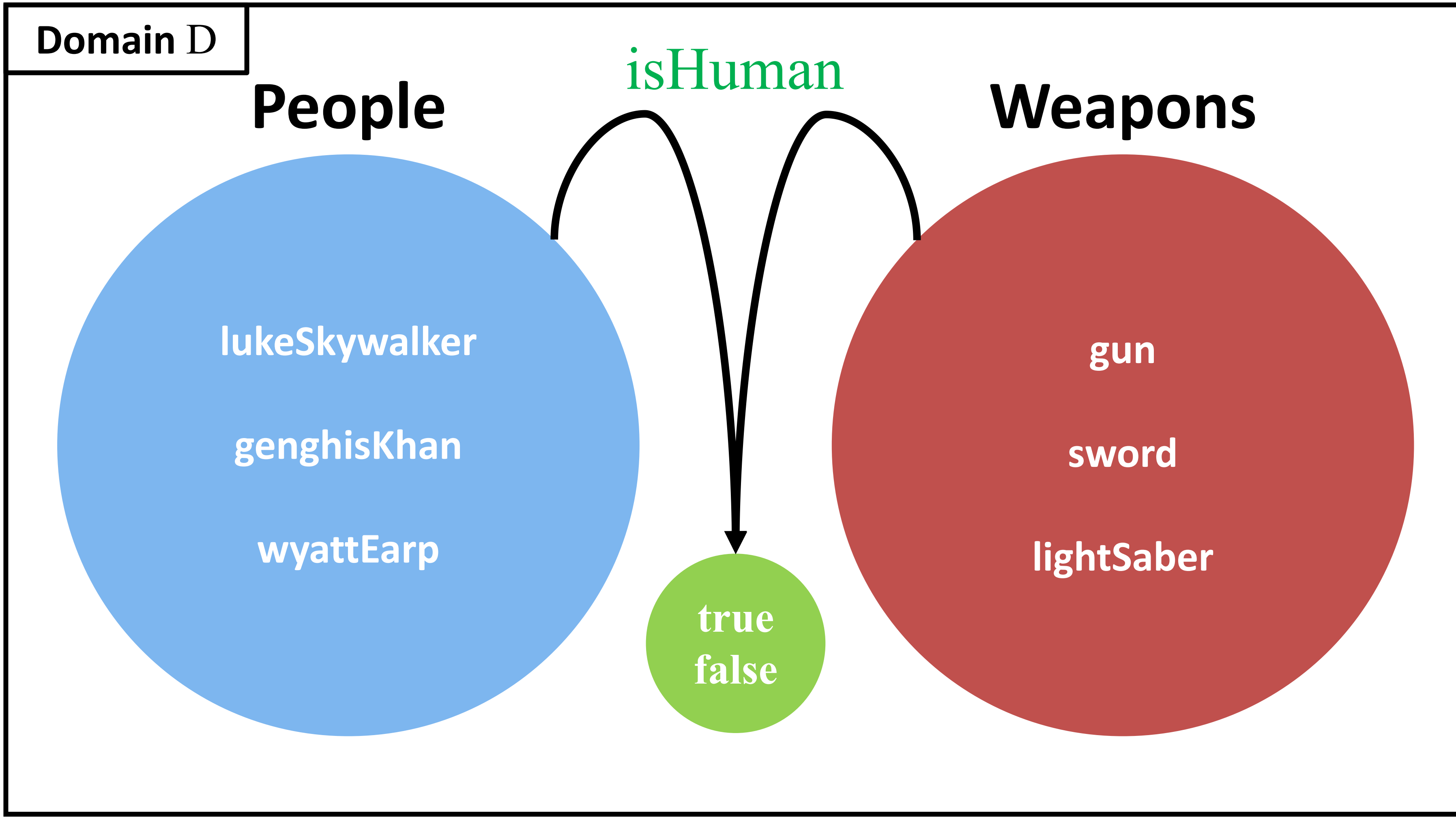
Predicates:

- **take objects as arguments**
- **output boolean values** true / false
- **for example:**

happy(student) **would output value** false **or** true

wasRaining(today) **would output value** true

Predicates: Example



isHuman(lukeSkywalker) = true

isHuman(sword) = false

Syntax: Function Symbols

A function f (f - symbol) maps object(s) to an object.

$$f(\text{argument}_1, \text{argument}_2, \ldots, \text{argument}_N)$$

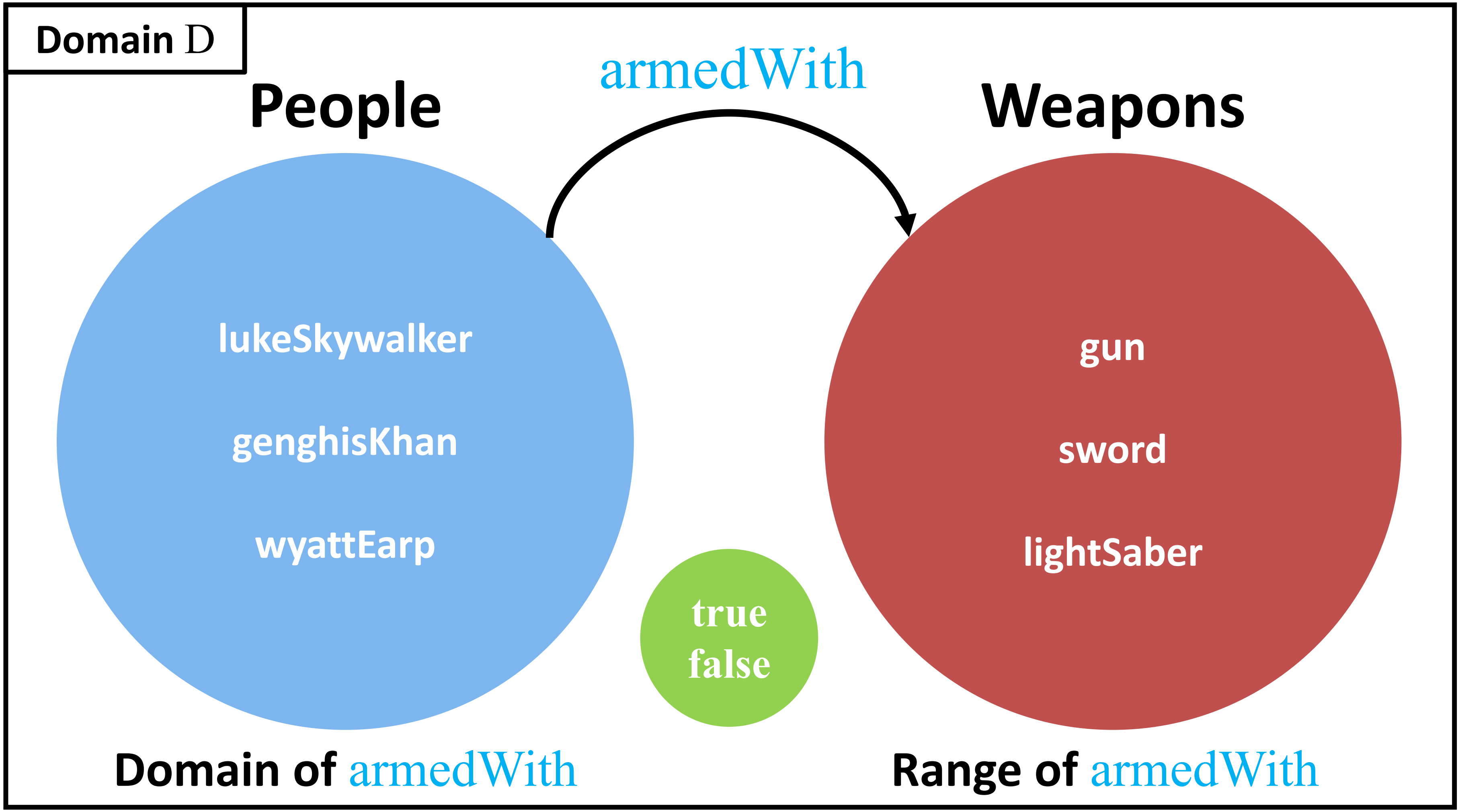
Unlike predicates, functions:

- **take objects as arguments**
- **output objects**
- **for example:**

currentCourse(student) **would output object** cs480

instructor(cs480) **would output object** Jacek

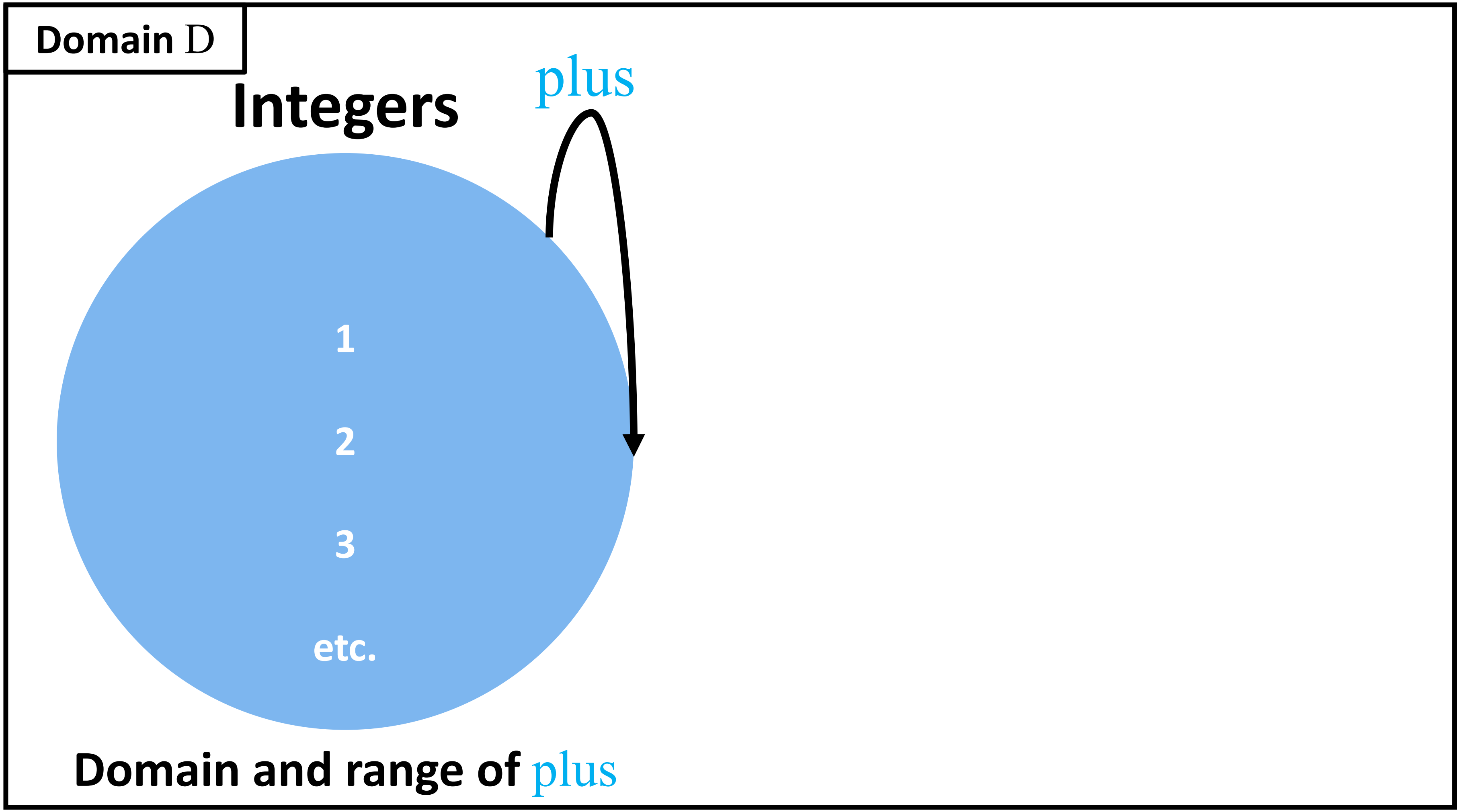
Functions: Example



armedWith(lukeSkywalker) = lightSaber

armedWith(genghisKhan) = sword

Functions: Example



$$\text{plus}(2,3) = 5$$

Syntax: Terms

A term is a logical expression that refers to an object. Terms can be:

- **constants:** lukeSkywalker
- **variables:** son
- **complex term - a function symbol followed by a parenthesized list of terms as arguments:**

father(lukeSkywalker) **or**

father(brother(princessLeia))

Both output an OBJECT: darthVader

Syntax: Complex Sentences

Complex sentences can be constructed using logical connectives (just like in propositional logic). For example:

$$\neg brother(leftLeg(Richard), John)$$

$$brother(Richard, John) \wedge brother(John, Richard)$$

$$king(Richard) \vee king(John)$$

$$\neg king(Richard) \Rightarrow king(John)$$

Syntax: Universal Quantifier

Quantifiers allow expressing properties of collections of objects.

Universal quantifier ("for all") indicates that a sentence is true for all possible values of the variable. For example:

$$\forall x \ likes(x, cake)$$

- $\forall$ is the universal quantifier symbol
- x is a variable
- $likes(x, cake)$ is a sentence

Syntax: Existential Quantifier

Quantifiers allow expressing properties of collections of objects.

Existential quantifier ("there exists") indicates that a sentence is true for <u>at least one value</u> of the the variable. For example:

$$\exists x \; likes(x, cake)$$

- $\exists$ is the existential quantifier symbol
- x is a variable
- $likes(x, cake)$ is a sentence

Syntax: Equality

In predicate logic we can ise the equality symbol to indicate that two terms refer to the same object.

For example:

$$\text{father}(\text{John}) = \text{Henry}$$

This sentence means that the output of function father(John) **is the same as object (constant)** Henry**.**

Syntax: Summary

$$
\begin{array}{rcl}
Sentence & \rightarrow & AtomicSentence \mid ComplexSentence \\
AtomicSentence & \rightarrow & Predicate \mid Predicate(Term, \ldots) \mid Term = Term \\
ComplexSentence & \rightarrow & (\ Sentence\ ) \\
& \mid & \neg\ Sentence \\
& \mid & Sentence \land Sentence \\
& \mid & Sentence \lor Sentence \\
& \mid & Sentence \Rightarrow Sentence \\
& \mid & Sentence \Leftrightarrow Sentence \\
& \mid & Quantifier\ Variable, \ldots\ Sentence \\
\\
Term & \rightarrow & Function(Term, \ldots) \\
& \mid & Constant \\
& \mid & Variable \\
\\
Quantifier & \rightarrow & \forall \mid \exists \\
Constant & \rightarrow & A \mid X_1 \mid John \mid \cdots \\
Variable & \rightarrow & a \mid x \mid s \mid \cdots \\
Predicate & \rightarrow & True \mid False \mid After \mid Loves \mid Raining \mid \cdots \\
Function & \rightarrow & Mother \mid LeftLeg \mid \cdots \\
\textsc{Operator Precedence} & : & \neg, =, \land, \lor, \Rightarrow, \Leftrightarrow
\end{array}
$$

Predicate Calculus: Interpretation

Let the domain D be a non-empty set. An *interpretation* I over D is an assignment of the entities in D to each of the constant, variable, predicate, and function symbols of a predicate calculus expression, such that:

- **each constant is assigned an element of D,**
- **each variable is assigned to non-empty subset of D; these are the allowable substitutions for that variable,**
- **each function f of arity m is defined on m arguments of D and defines mapping from D^m into D,**
- **each predicate p of arity n is defined on n arguments from D and defines a mapping from D^n into $\{\text{true}, \text{false}\}$.**

Predicate Calculus: Sentences

Predicate calculus sentences are created according to following rules:

- **every atomic sentence** s **is a sentence**
- **if** s **a sentence, so it its negation** $\neg s$**,**
- **if** s_1 **and** s_2 **are sentences, then so is their conjunction** $s_1 \wedge s_2$**,**
- **if** s_1 **and** s_2 **are sentences, then so is their disjunction** $s_1 \vee s_2$**,**
- **if** s_1 **and** s_2 **are sentences, then so is their implication** $s_1 \Rightarrow s_2$**,**
- **if** s_1 **and** s_2 **are sentences, then so is their equivalence** $s_1 \Rightarrow s_2$**,**
- **if** x **is a variable** s **a sentence, then** $\forall x$ **is a sentence,**
- **if** x **is a variable** s **a sentence, then** $\exists x$ **is a sentence.**

Predicate Calculus: Evaluation

Assume a sentence E and an interpretation I for E over a non-empty domain D. The truth value for E is determined by:

- **the value of each constant is the element of D it is assigned to by I,**
- **the value of a variable is the set of elements of D it is assigned to by I,**
- **the value of a function expression is that element of D obtained by evaluating the function for the parameter values I assigned ,**
- **the value of "true" is true and "false" is false,**
- **The value of an atomic sentence is either true or false, as determined by the interpretation I.**

Predicate Calculus: Evaluation

- **the value of negation of a sentence is** true **if the value of the sentence is** false **(is false if the value of the sentence is** true**),**
- **the value of the conjunction of two sentences is** true **if the value of both sentences is** true **and is** false **otherwise**
- **similarly, the truth value of expressions using** $\vee$**,** $\Rightarrow$**, and** $\Leftrightarrow$ **is determined using related truth tables,**

- **The value of** $\forall$x s **is** true **if** s **is** true **for all assignments to** x **under** I**, and it is** false **otherwise,**
- **The value of** $\exists$x s **is** true **if** s **is** true **if there is an assignment to** x **under** I**, and it is** false **otherwise,**

First-Order Logic Sentences: Examples

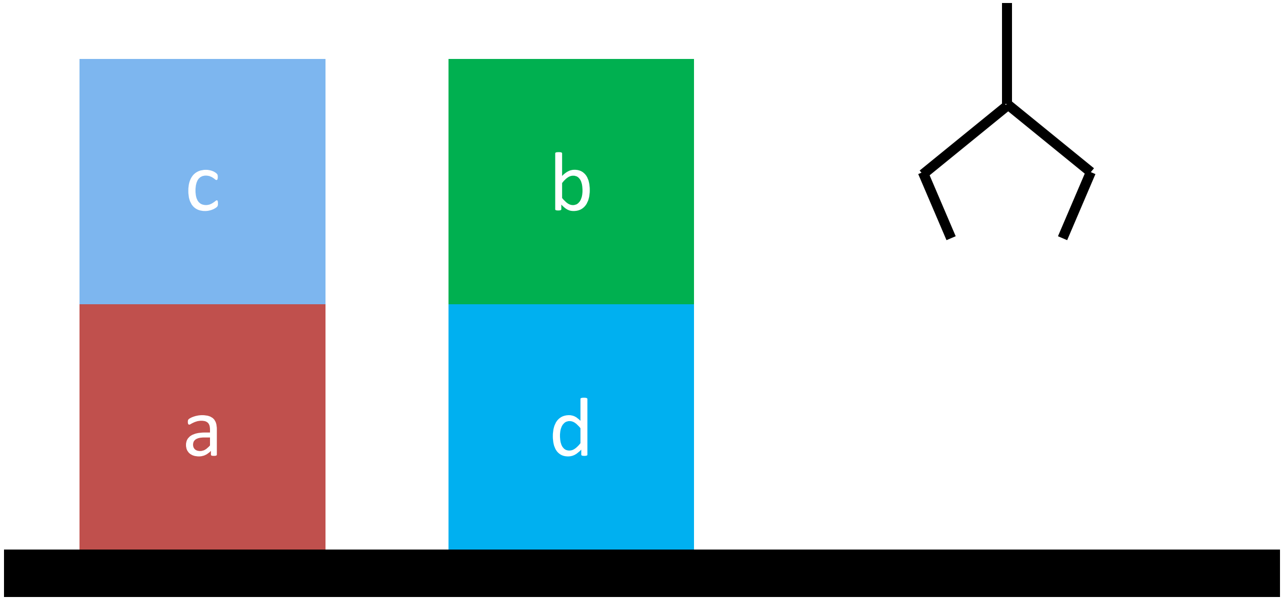
Sentence in First-Order Logic	Sentence in English
$\forall x\ frog(x) \Rightarrow green(x)$	All frogs are green. (For all xs being a frog implies being green)
$\forall x\ frog(x) \wedge brown(x) \Rightarrow big(x)$	All brown frogs are big. (For all xs being a frog and brown implies being big)
$\forall x\ likes(x, cake)$	Everyone likes cake. (All xs like cake.)
$\neg\forall x\ likes(x, cake)$	Not everyone likes cake. (Not all xs like cake.)
$\exists x\ \forall y\ likes(y, x)$	There is something that everyone likes. (There exists something x that all ys like.)
$\exists x\ \forall y\ likes(x, y)$	There is someone that likes everyone. (There exists an x that is likes all ys.)
$\forall x\ \exists y\ likes(y, x)$	Everything is liked by someone. (For all xs there exists a y that likes them.)
$\forall x\ \exists y\ likes(x, y)$	Everyone likes something. (For all xs there exists a y that is being liked by x.)

First-Order Logic Sentences: Examples

Sentence in First-Order Logic	Sentence in English
$\forall x\ customer(x) \Rightarrow likes(bob, x)$	Bob likes every customer. (For all xs being a customer implies being liked by Bob.)
$\exists x\ customer(x) \wedge likes(x, bob)$	There is a customer who likes Bob. (There exist an x who is a customer and likes Bob.)
$\exists x\ baker(x) \wedge \forall y\ customer(y) \Rightarrow likes(x, y)$	There is a baker who likes all customers. (There exist an x who is a baker such that for all ys being a customer, implies that y is liked by x.)
$\forall x\ older(mother(x), x)$ *	Every mother is older than her child. (For all xs, mother of x is older than x.)
$\forall x\ older(mother(mother(x)), x)$ *	Every grandmother is older than her daugther’s child. (For all xs, mother of mother of x is older than x.)
$brother(x, bob) \Rightarrow \exists y\ parent(y, x) \wedge parent(y, bob)$	If x is Bob’s brother, x and Bob must have the same parent.
$\forall x \forall y \forall z\ f(x, y) \wedge f(y, z) \Rightarrow f(x, z)$	f is a transitive relation.

* mother is function symbol here

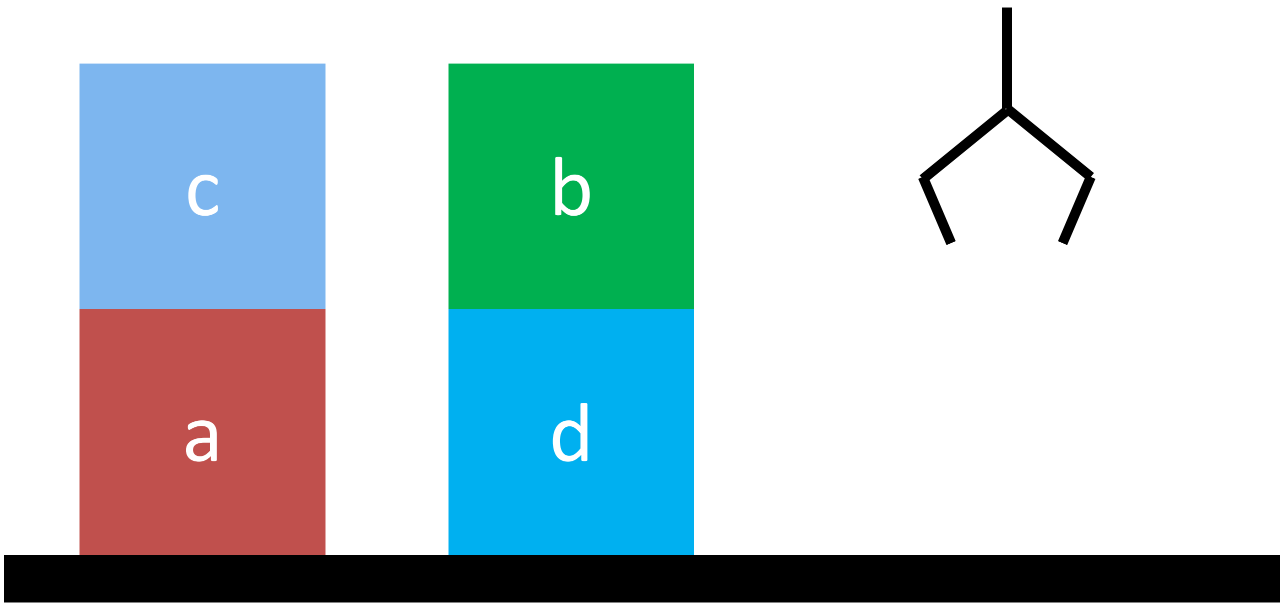
Predicate Calculus: Model Example



Predicate calculus description of the world:

on(c, a)
on(b, d)
onTable(a)
onTable(d)
clear(b)
clear(c)
manipulatorEmpty

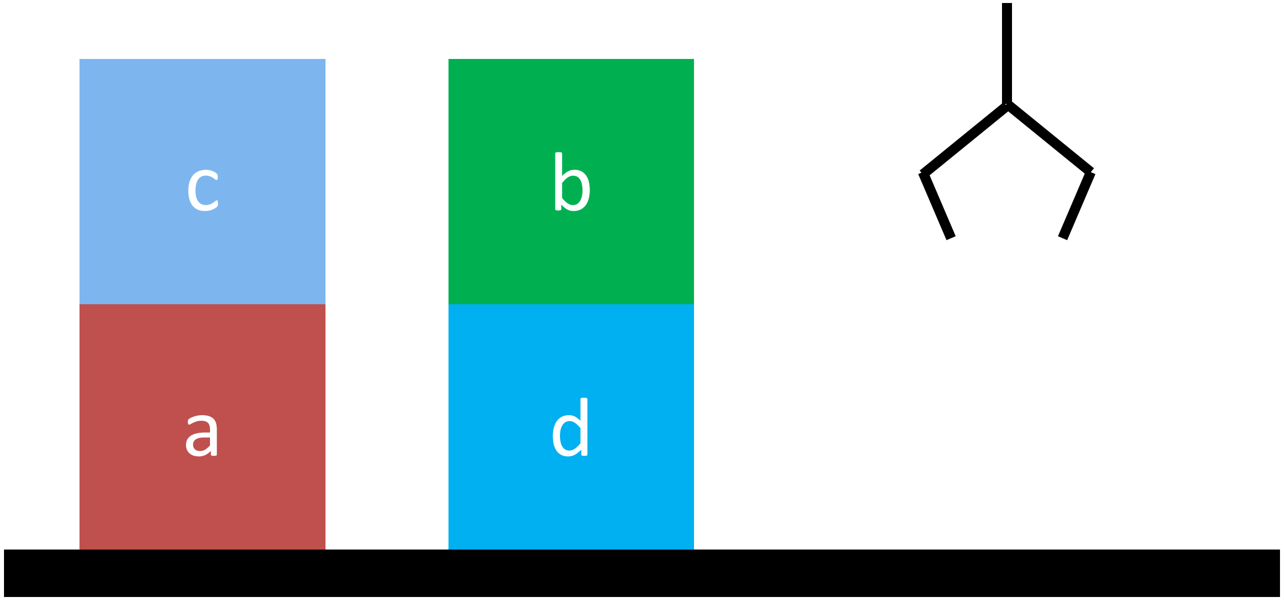
Predicate Calculus: Objects



Objects:

table
blocks a, b, c, d
manipulator

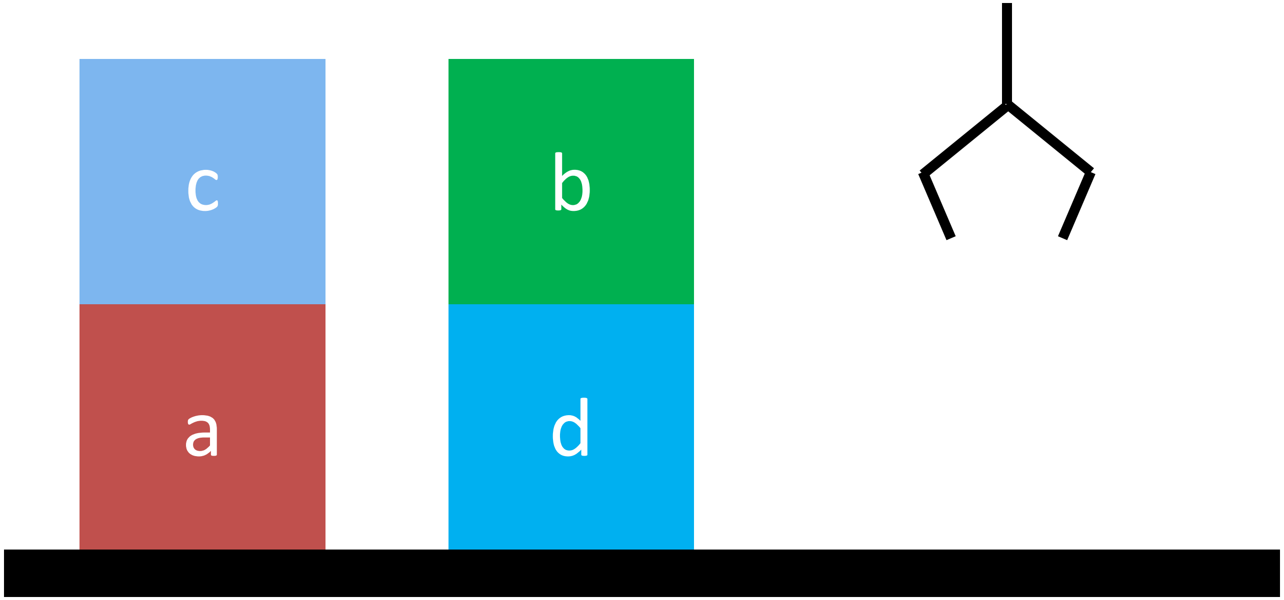
Predicate Calculus: Predicates



Predicates:

on(x, y)	**asserts that object x is on top of y**
onTable(x)	**asserts that object x is on the table**
clear(x)	**asserts that no objects is on top of x**
manipulatorEmpty	**asserts the status of the manipulator**

Model of the World



Predicate calculus description of the world (model):

on(c, a)
on(b, d)
onTable(a)
onTable(d)

clear(b)
clear(c)
manipulatorEmpty

Quantifier Nesting

Quantifiers can be nested to obtain more complex expressions. For example:

$$\forall x\ \forall y\ \text{brother}(x, y) \Rightarrow \text{sibling}(x, y)$$

means "Brothers are siblings". Here

$$\forall x\ \forall y\ \text{sibling}(y, x) \Leftrightarrow \text{sibling}(x, y)$$

a symmetric relationship is expressed.

Quantifier Nesting: Ordering

When quantifiers are nested it is important to pay attention to ordering. For example:

$$\forall x \, \exists y \; loves(x, y)$$

means “Everybody loves somebody”. Here

$$\exists x \, \forall y \; loves(y, x)$$

we have “There exists someone who is loved by everyone”.

Quantifier Nesting (Use Parentheses)

Quantifiers can be nested to obtain more complex expressions. For example:

$$\forall x\ (\forall y\ \mathrm{brother}(x, y) \Rightarrow \mathrm{sibling}(x, y))$$

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Quantifier Nesting (Use Parentheses)

When quantifiers are nested it is important to pay attention to ordering. For example:

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Quantifier Nesting: Variable Names

Certain quantified sentences may be confusing:

$$\forall x\ (crown(x) \lor (\exists x\ brother(Richard, x))$$

Variable x **is used twice:**

- **universally quantified** x **in** $\forall x\ (crown(x) \lor$
- **existentially quantified** x **in** $\exists x\ brother(Richard, x)$
- x **and** x **are NOT the same (different "context")**

Rule:

variable belongs to innermost quantifier that mentions it.

Universal Quantifier: Conjuctions

Universal quantifier ("for all") indicates that a sentence is true for all possible values of the variable. For example:

$$\forall x \; likes(x, cake)$$

is true if $likes(x, cake)$ **is** true **for all interpretations of variable** x**. Assuming that**

$$x \in \{x_1, x_2, \ldots, x_n\}$$

we can rewrite $\forall x \; likes(x, cake)$ **as:**

$$likes(x_1, cake) \wedge likes(x_2, cake) \wedge \ldots \wedge likes(x_n, cake)$$

Existential Quantifier: Disjunctions

Existential quantifier ("there exists") indicates that a sentence is true for <u>at least one value</u> of the the variable. For example:

$$\exists x \text{ likes}(x, \text{cake})$$

is true if $\text{likes}(x, \text{cake})$ **is** true **for at least one interpretation of variable** x**. Assuming that**

$$x \in \{x_1, x_2, \ldots, x_n\}$$

we can rewrite $\exists x \text{ likes}(x, \text{cake})$ **as:**

$$\text{likes}(x_1, \text{cake}) \vee \text{likes}(x_2, \text{cake}) \vee \ldots \vee \text{likes}(x_n, \text{cake})$$

Universal/Existential Quantifiers

We assumed that $x \in \{x_1, x_2, \ldots, x_n\}$ **and then we rewrote** $\forall x\ likes(x, cake)$ **as:**

$$likes(x_1, cake) \land likes(x_2, cake) \land \ldots \land likes(x_n, cake)$$

and $\exists x\ likes(x, cake)$ **as:**

$$likes(x_1, cake) \lor likes(x_2, cake) \lor \ldots \lor likes(x_n, cake)$$

From De Morgan's rules we can obtain the following equivalence:

$$\forall x\ likes(x, cake) \equiv \neg\exists x\ \neg likes(x, cake)$$

"Everyone likes cake" ≡ "Nobody dislikes cake"

Universal/Existential Q. Equivalences

Selected equivalences:

$$\forall x\ (P(x) \land Q(x)) \equiv \forall x\ (P(x)) \land \forall x\ (Q(x))$$

$$\exists x\ (P(x) \lor Q(x)) \equiv \exists x\ (P(x)) \lor \exists x\ (Q(x))$$

$$\neg[\exists x\ (N(x))] \equiv \forall x\ (\neg N(x))$$

$$\neg[\forall x\ (N(x))] \equiv \exists x\ (\neg N(x))$$

Quantifiers: Scope of Quantification

Consider the following sentence:

$$\forall x\ \underbrace{(P(x) \wedge Q(x))}_{\text{Scope of quantification for variable } x}$$

Variable x is universally quantified in both $P(x)$ and $Q(x)$.

In this sentence:

$$\exists x\ \underbrace{(P(x) \vee Q(y) \Rightarrow R(x))}_{\text{Scope of quantification for variable } x}$$

Variable x is existentionally quantified in both $P(x)$ and $R(x)$.