



If you create tables from functional dependencies you must be sure that there is no extra, or redundant, dependency information in the set of functional dependencies.

If there is you will have duplicated facts in your database.

That is, you must first make sure your dependencies are a **minimal cover.**



We can find a **minimal cover F_m of F and a minimal using a fairly simple algorithm.**

We present it here by example and will look at algorithms in more detail later in the course.

Apply decomposition rule so all dependencies have a single attribute for their right-hand side

Find a minimal cover of F

$$\begin{array}{lcl} F = \{ & & F = \{ \\ \quad ABD \rightarrow AC, & & \quad ABD \rightarrow A \quad // \text{decomposition} \\ \quad B \rightarrow E, & & \quad ABD \rightarrow C \quad // \text{decomposition} \\ \quad BA \rightarrow E, & = & \quad B \rightarrow E, \\ \quad C \rightarrow BE, & & \quad BA \rightarrow E, \\ \quad AD \rightarrow FB, & & \quad C \rightarrow B, \quad // \text{decomposition} \\ \quad C \rightarrow E & & \quad C \rightarrow E, \quad // \text{decomposition} \\ \} & & \quad AD \rightarrow F, \quad // \text{decomposition} \\ & & \quad AD \rightarrow B, \quad // \text{decomposition} \\ & & \quad C \rightarrow E \\ & & \} \end{array}$$

Remove any trivial dependencies
(reflexive rules) duplicate
dependencies or dependencies
implied directly by others.

$F = \{$		$F = \{$
$ABD \rightarrow A$ //reflexive		$ABD \rightarrow C$
$ABD \rightarrow C$		$B \rightarrow E,$
$B \rightarrow E,$		$C \rightarrow B,$
$BA \rightarrow E,$ //implied by //previous rule	=	$C \rightarrow E,$
$C \rightarrow B,$		$AD \rightarrow F,$
$C \rightarrow E,$		$AD \rightarrow B,$
$AD \rightarrow F,$		$\}$
$AD \rightarrow B,$		
$C \rightarrow E$ //duplicate		
$\}$		

Remove any unnecessary attributes from the left-hand side of dependencies.

Remove any dependencies that are transitively implied by others

$F = \{$		$F = \{$
$\quad \text{ABD} \rightarrow C$	//since $AD \rightarrow B$	$\quad AD \rightarrow C$
$\quad B \rightarrow E,$		$\quad B \rightarrow E,$
$\quad C \rightarrow B,$		$\quad C \rightarrow B,$
$\quad \text{C} \rightarrow E,$	//transitive	$\quad AD \rightarrow F,$
$\quad AD \rightarrow F,$	=	$\quad AD \rightarrow B,$
$\quad AD \rightarrow B,$		$\}$
$\}$		

Create a set of 3NF tables from the minimal cover by combining dependencies that have the same left-hand side into a single tables.

The left hand side forms the key.

$$F_m = \{AD \rightarrow C, AD \rightarrow F, B \rightarrow E, C \rightarrow B\}$$

3NF Tables:

ADCF

BE

CB

Exercise 1

Consider the set of attributes $R=\{A,B,C,D,E,F\}$ and the following set of functional dependencies proposed by the table designer.

$F_1 = \{ABD \rightarrow AC, B \rightarrow E, BA \rightarrow E, C \rightarrow BE, AD \rightarrow FB, C \rightarrow E\}$

A colleague suggests that they use the following dependency set instead

$F_2 = \{AD \rightarrow CF, C \rightarrow B, B \rightarrow E\}$

Determine if this is a reasonable suggestion

Approach 1

If we can show the sets are equivalent then F2 can be used instead of F1

F1 and F2 are equivalent if $F1^+ = F2^+$

To show this we must show that each functional dependency in F1 is implied by the set F2 and vice versa each functional dependency in F2 is implied by the set F1

Approach 1

As illustration we will show that:

ABD- \rightarrow AC from F1 is implied by set F2

and

AD- \rightarrow CF from F2 is implied by set F1

The same would have to be done for each functional dependency in each set, but for illustration here we only show the above two.

Approach 1

AD→CF from F2 is implied by set F1

Proof

AD→CF implied because :

AD→BF, given in F1

AD→F decomposition rule

ABD→AC given in F1

ABD→C decomposition rule

AD→B given in F1

AD→ABD augmentation rule

AD→C transitive rule : AD→ABD, ABD→C

AD→CF union rule, AD→C, AD→F

**$F_1 = \{$
ABD→AC,
B→E,
BA→E,
C→BE,
AD→BF,
C→E $\}$**

**$F_2 = \{$
AD→CF,
C→B,
B→E $\}$**

Approach 1

ABD- \rightarrow AC from F1 is implied by set F2

Proof

ABD \rightarrow AC implied because :

AD \rightarrow CF, given in F2

AD \rightarrow ACF augmentation rule (add A to both sides)

ABD \rightarrow ABCF augmentation rule (add B to both sides)

ABD \rightarrow AC decomposition rules (ABD \rightarrow AC, ABD \rightarrow BF)

**$F_1 = \{$
ABD \rightarrow AC,
B \rightarrow E,
BA \rightarrow E,
C \rightarrow BE,
AD \rightarrow FB,
C \rightarrow E $\}$**

**$F_2 = \{$
AD \rightarrow CF,
C \rightarrow B,
B \rightarrow E $\}$**

Approach 2

We can find a **minimal cover** F_{m1} of F_1 and a minimal cover F_{m2} of F_2 and show that these minimal covers are equivalent

Again we would have to show that $F_{m1}^+ = F_{m2}^+$ but the hope is that this would be trivial by inspection, or an easier problem than working with the “raw” dependency sets.

Approach 2

Find a minimal cover of F1

$$\begin{array}{lcl} F_1 = \{ & & F_1 = \{ \\ \quad ABD \rightarrow AC, & & \quad ABD \rightarrow A \quad // \text{decomposition} \\ \quad B \rightarrow E, & & \quad ABD \rightarrow C \quad // \text{decomposition} \\ \quad BA \rightarrow E, & = & \quad B \rightarrow E, \\ \quad C \rightarrow BE, & & \quad BA \rightarrow E, \\ \quad AD \rightarrow FB, & & \quad C \rightarrow B, \quad // \text{decomposition} \\ \quad C \rightarrow E & & \quad C \rightarrow E, \quad // \text{decomposition} \\ \} & & \quad AD \rightarrow F, \quad // \text{decomposition} \\ & & \quad AD \rightarrow B, \quad // \text{decomposition} \\ & & \quad C \rightarrow E \\ & & \} \end{array}$$

Approach 2

$$\begin{aligned} F_1 = \{ & \text{ABD} \rightarrow A \quad // \text{reflexive} \\ & \text{ABD} \rightarrow C \\ & B \rightarrow E, \\ & \text{BA} \rightarrow E, \quad // \text{implied by} \\ & \quad // \text{previous rule} \\ & C \rightarrow B, \\ & C \rightarrow E, \\ & AD \rightarrow F, \\ & AD \rightarrow B, \\ & C \rightarrow E \quad // \text{duplicate} \\ & \} \end{aligned} = \begin{aligned} F_1 = \{ & \text{ABD} \rightarrow C \\ & B \rightarrow E, \\ & C \rightarrow B, \\ & C \rightarrow E, \\ & AD \rightarrow F, \\ & AD \rightarrow B, \\ & \} \end{aligned}$$

Approach 2

$$\begin{aligned} F_1 = \{ & \text{ABD} \rightarrow C \quad // \text{since } AD \rightarrow B \\ & B \rightarrow E, \\ & C \rightarrow B, \\ & \text{C} \rightarrow E, \quad // \text{transitive} \\ & AD \rightarrow F, \\ & AD \rightarrow B, \\ & \} \end{aligned} = \begin{aligned} F_1 = \{ & AD \rightarrow C \\ & B \rightarrow E, \\ & C \rightarrow B, \\ & AD \rightarrow F, \\ & AD \rightarrow B, \\ & \} \end{aligned}$$

Approach 2

$$\begin{array}{l} F_1 = \{ \\ \quad AD \rightarrow C \\ \quad B \rightarrow E, \\ \quad C \rightarrow B, \\ \quad AD \rightarrow F, \\ \quad AD \rightarrow B, \text{ //transitive} \\ \} \end{array} = \begin{array}{l} F_1 = \{ \\ \quad AD \rightarrow C \\ \quad B \rightarrow E, \\ \quad C \rightarrow B, \\ \quad AD \rightarrow F, \\ \} \end{array}$$

$$F_{m1} = \{AD \rightarrow C, AD \rightarrow F, B \rightarrow E, C \rightarrow B\}$$

Approach 2

Find a minimal cover of F2

$$\begin{array}{lcl} F_2 = \{ & & F_2 = \{ \\ & AD \rightarrow CF, & AD \rightarrow C, \text{ //decomposition} \\ & C \rightarrow B, & AD \rightarrow F, \text{ //decomposition} \\ & B \rightarrow E & C \rightarrow B, \\ \} & = & B \rightarrow E \\ & & \} \end{array}$$

$$F_{m2} = \{ AD \rightarrow C, AD \rightarrow F, C \rightarrow B, B \rightarrow E \}$$

Approach 2

$$F_{m1} = \{AD \rightarrow C, AD \rightarrow F, B \rightarrow E, C \rightarrow B\}$$

$$F_{m2} = \{AD \rightarrow C, AD \rightarrow F, C \rightarrow B, B \rightarrow E\}$$

By inspection $F_{m1} = F_{m2}$ so the sets F_1 and F_2 are equivalent

Exercise 2

- For each of the following cases a relation R has been defined over attributes A,B,C,D,E,F along with a set of functional dependencies that apply to them.
- Find all the candidate keys for the relation
- State the highest normal form the table R=ABCDEF would currently satisfy
- Decompose the table until all resulting tables are in BCNF form

Exercise 2

$F = \{ AB \rightarrow CDEF, EF \rightarrow C \}$

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$F = \{ AB \rightarrow CDEF, EF \rightarrow C \}$

Candidate keys: AB

Current Normal Form: = 2nd NF since $EF \rightarrow C$ violates 3rd NF.

Decomposition:

ABDEF

EFC

Exercise 2

$F = \{ AB \rightarrow CDEF, EF \rightarrow B, D \rightarrow B \}$

Exercise 2

$F = \{ AB \rightarrow CDEF, EF \rightarrow B, D \rightarrow B \}$

Candidate keys: AB, AD, AEF

Current Normal Form: = 3rd NF since $D \rightarrow B$ violates BCNF.

Decomposition:

ADCEF

DB

(the dependency $EF \rightarrow B$ is lost going to BCNF)

Exercise 2

$F = \{ AB \rightarrow CDEF, BC \rightarrow D, \}$

Exercise 2

$F = \{ AB \rightarrow CDEF, BC \rightarrow D, \}$

Candidate keys: AB

Current Normal Form: = 2nd NF since $BC \rightarrow D$ violates 3rd NF.

(Recall: $Y \rightarrow A$ is a transitive dependency if Y is neither a superkey of R nor a proper subset of a key of R)

Decomposition:

ABCEF

BCD

Exercise 2

$F = \{ AB \rightarrow CDEF, B \rightarrow C, D \rightarrow C \}$

Exercise 2

$F = \{ AB \rightarrow CDEF, B \rightarrow C, D \rightarrow C \}$

Candidate keys: AB

Current Normal Form: = 1st NF since $B \rightarrow C$ violates 2nd NF.

Decomposition:

ABDEF

BC

DC