CS 480

Introduction to Artificial Intelligence

March 29, 2022

Announcements / Reminders

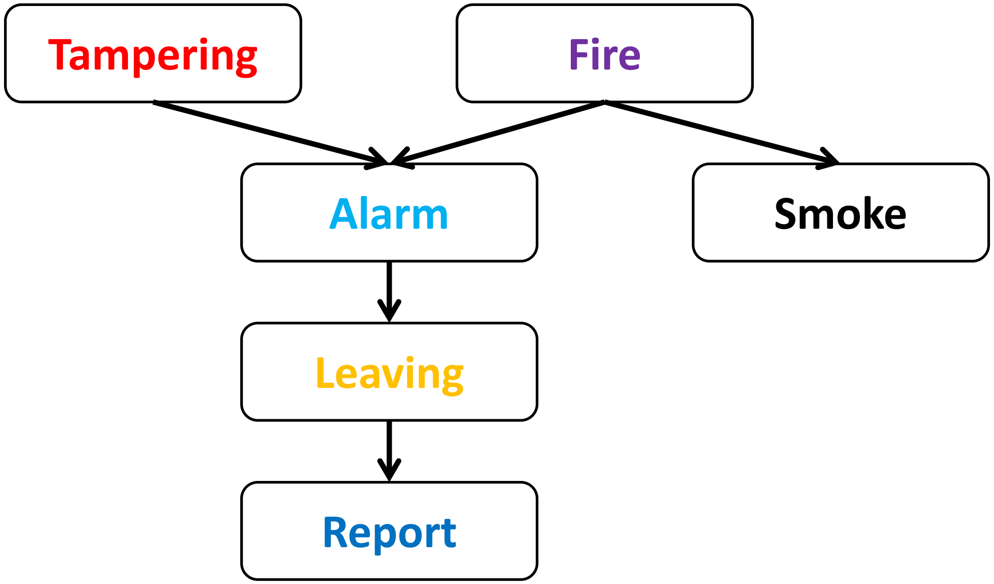
- Final Exam: April 28th!
  - Ignore Registrar date for CS 480
- Programming Assignment #02:
  - Posted
- Quiz #03: due on Sunday
- Written Assignment #03:
  - This week
- Grading TA assignment:

https://docs.google.com/spreadsheets/d/1Cav_GBTGC7fLGzxuBCAUmEuJYPeF-HMLCYvwPbq8Fus/edit?usp=sharing

Plan for Today

- **Quantifying and dealing with uncertainty**
- **Bayesian Networks**

Bayesian (Belief) Network: Example



Random Variables (Propositions):

- **Tampering: true if the alarm is tampered with**
- **Fire: true if there is a fire**
- **Alarm: true if the alarm sounds**
- **Smoke: true if there is smoke**
- **Leaving: true if people leaving the building at once**
- **Report: true if someone who left the building reports fire**

Domain for all variables: $\{\text{true}, \text{false}\}$

NOTE: RVs don't have to be Boolean

Full Joint Probability Distribution

H: grad	e: female	P(H, e) = P(H ∧ e): P(grad ∧ female)	Conditional probabilities
true	true	$P(H \mid e) * P(e) \approx 0.074$	$P(H \mid e) = \frac{P(e \mid H) * P(H)}{P(e)} = \frac{6/18 * 18/81}{13/81} \approx 0.462$
true	false	$P(H \mid \neg e) * P(\neg e) \approx 0.148$	$P(H \mid \neg e) = \frac{P(\neg e \mid H) * P(H)}{P(\neg e)} = \frac{12/18 * 18/81}{68/81} \approx 0.176$
false	true	$P(\neg H \mid e) * P(e) \approx 0.086$	$P(\neg H \mid e) = \frac{P(e \mid \neg H) * P(\neg H)}{P(e)} = \frac{7/63 * 63/81}{13/81} \approx 0.538$
false	false	$P(\neg H \mid \neg e) * P(\neg e) \approx 0.691$	$P(\neg H \mid \neg e) = \frac{P(\neg e \mid \neg H) * P(\neg H)}{P(\neg e)} = \frac{56/63 * 63/81}{68/81} \approx 0.824$
		SUM = 1	

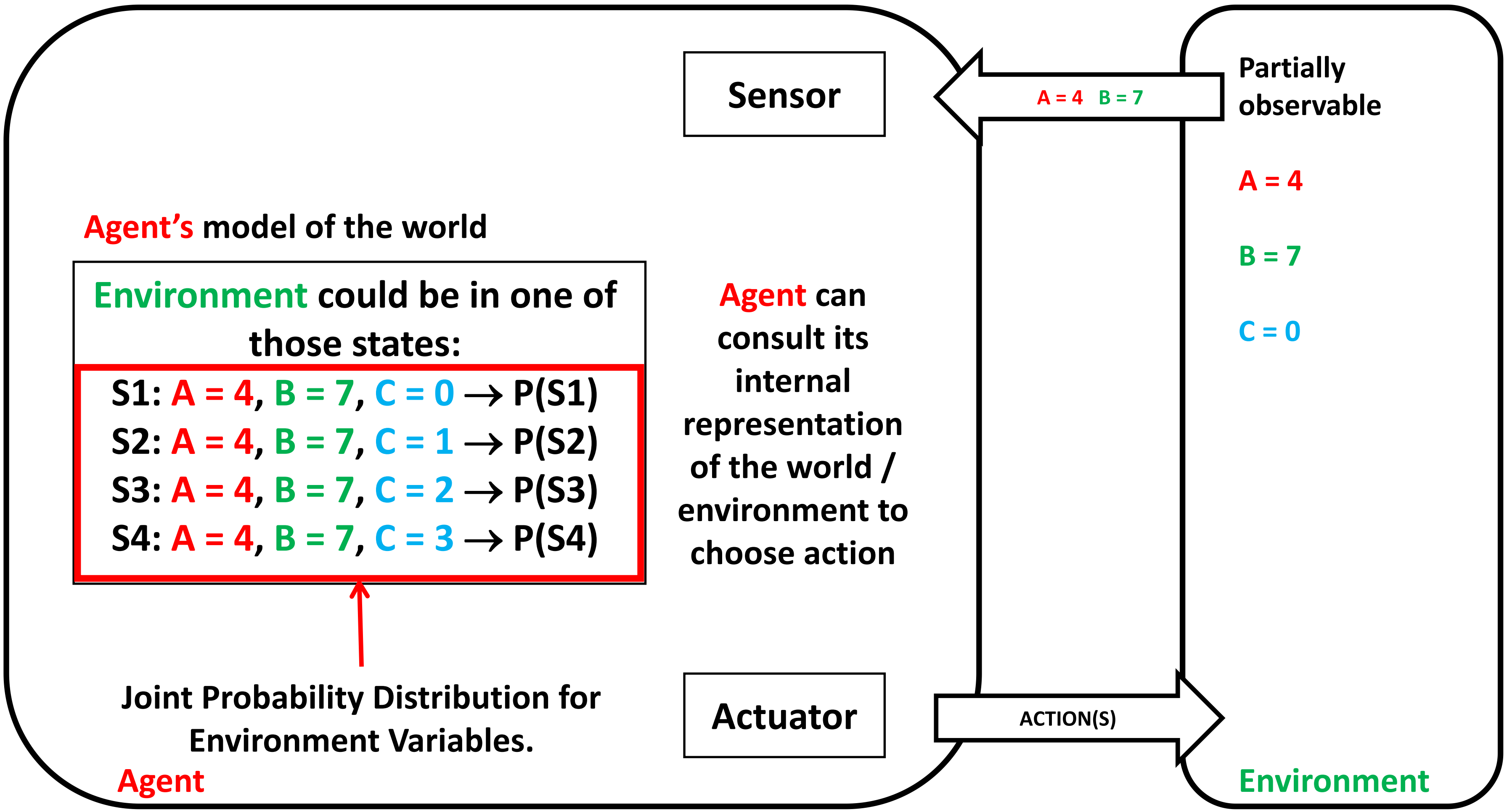
Joint probabilities calculated using the Product Rule:

$$P(A \wedge B) = P(A \mid B) * P(B)$$

Conditional probabilities calculated using Bayes' Rule:

$$P(A \mid B) = \frac{P(B \mid A) * P(A)}{P(B)}$$

Agents and Belief State



Assume: $D_C = \{0,1,2,3\}$

General Inference Procedure

Given:

- **a query involving a single variable X (in our example: Cavity),**
- **a list of evidence variables E (in our example: just Toothache),**
- **a list of observed values e for E,**
- **a list of remaining unobserved variables Y (in our example: just Catch),**

where X, E, and Y together are a COMPLETE set of variables for the domain, the probability P(X | E) can be evaluated as:

$$\boldsymbol{P}(X \mid \boldsymbol{e}) = \alpha * \boldsymbol{P}(X, \boldsymbol{e}) = \alpha * \sum_{\boldsymbol{y}} \boldsymbol{P}(X, \boldsymbol{e}, \boldsymbol{y})$$

where ys are all possible values for Ys, α - normalization constant.

P(X, e, y) is a subset of probabilities from the joint distribution

Complex Joint Distributions

Consider a complex joint probability distribution involving N random variables P_1, P_2, P_3, …, P_{N-1}, Pp_N .

2^N Possible Worlds (Models)

N Random Variables						Joint Probability
P_1	P_2	P_3	…	P_{N-1}	P_N	
true	true	true	…	true	true	false
true	true	true	…	true	false	true
true	true	false	…	false	true	false
…	…	…	…	…	…	…
false	false	true	…	true	false	true
false	false	true	…	false	true	true
false	false	false	…	false	false	false

2^N values

Non-binary / Non-Boolean RVs

Some Random Variables are going to have more than two possible, discrete, values:

- height **-> short, average, tall**
- size **-> S, M, L, XL**
- streetLight **-> green, orange, red**
- vision **-> 20/20, 15/15, etc.**
- continent **-> Africa, Antarctica, Asia, Australia, Europe, North America, South America**

Non-binary RVs increase the complexity.

This May Be Impossible to Manage!

N Random Variables

P_1	P_2	P_3	…	P_{N-1}	P_N	Joint Probability
true	true	true	…	true	true	false
true	true	true	…	true	false	true
true	true	false	…	false	true	false
…	…	…	…	…	…	…
false	false	true	…	true	false	true
false	false	true	…	false	true	true
false	false	false	…	false	false	false

2^N Possible Worlds (Models)

2^N values

Independent Variable

		Toothache		¬Toothache	
		Catch	¬Catch	Catch	¬Catch
¬Cloudy	Cavity	0.108	0.012	0.072	0.008
	¬Cavity	0.016	0.064	0.144	0.576
		Toothache		¬Toothache	
		Catch	¬Catch	Catch	¬Catch
Cloudy	Cavity	0.108	0.012	0.072	0.008
	¬Cavity	0.016	0.064	0.144	0.576

Let's introduce another random variable Cloudy **representing some weather conditions. It is difficult to imagine the other random variables here (**Toothache**,** Cavity, Catch**) being dependent on** Cloudy **and vice versa.**

Independent Variable

		Toothache		¬Toothache	
		Catch	¬Catch	Catch	¬Catch
¬Cloudy	Cavity	0.108	0.012	0.072	0.008
	¬Cavity	0.016	0.064	0.144	0.576
		Toothache		**¬Toothache**	
		Catch	**¬Catch**	**Catch**	**¬Catch**
Cloudy	Cavity	0.108	0.012	0.072	0.008
	¬Cavity	0.016	0.064	0.144	0.576

Let's try to calculate the following probability:

$$P(Toothache, Catch, Cavity, Cloudy)$$

using the Product Rule:

$$P(Toothache, Catch, Cavity, Cloudy) =$$
$$= P(Cloudy \mid Toothache, Catch, Cavity) * P(Toothache, Catch, Cavity)$$

Independent Variable

		Toothache		¬Toothache	
¬Cloudy		Catch	¬Catch	Catch	¬Catch
	Cavity	0.108	0.012	0.072	0.008
	¬Cavity	0.016	0.064	0.144	0.576
Cloudy		Toothache		¬Toothache	
		Catch	¬Catch	Catch	¬Catch
	Cavity	0.108	0.012	0.072	0.008
	¬Cavity	0.016	0.064	0.144	0.576

It's hard to imagine Cloudy influencing other variables, so:

$$P(Cloudy \mid Toothache, Catch, Cavity) = P(Cloudy)$$

and then:

$$P(Toothache, Catch, Cavity, Cloudy) =$$
$$= P(Cloudy) * P(Toothache, Catch, Cavity)$$

Independent Variable / Factoring

¬Cloudy		Toothache		¬Toothache	
		Catch	¬Catch	Catch	¬Catch
	Cavity	0.108	0.012	0.072	0.008
	¬Cavity	0.016	0.064	0.144	0.576

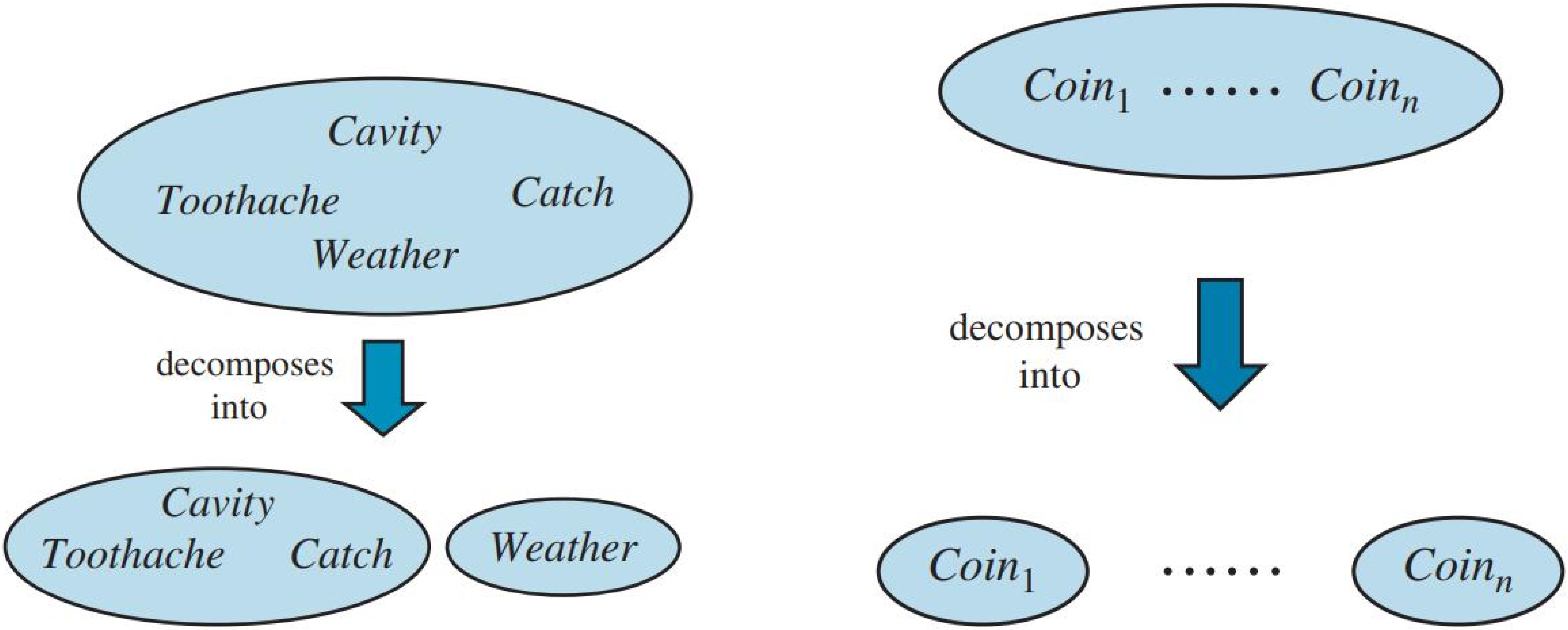
Cloudy		Toothache		¬Toothache	
		Catch	¬Catch	Catch	¬Catch
	Cavity	0.108	0.012	0.072	0.008
	¬Cavity	0.016	0.064	0.144	0.576

It's hard to imagine Cloudy influencing other variables, so:

$$P(Toothache, Catch, Cavity, Cloudy) =$$
$$= P(Cloudy) * P(Toothache, Catch, Cavity)$$

This shows that Cloudy **is INDEPENDENT of other variables and factoring can be applied.**

Factoring / Decomposition



Use Chain Rule To Decompose

N Random Variables						Joint Probability
P_1	P_2	P_3	…	P_{N-1}	P_N	
true	true	true	…	true	true	false
true	true	true	…	true	false	true
true	true	false	…	false	true	false
…	…	…	…	…	…	…
false	false	true	…	true	false	true
false	false	true	…	false	true	true
false	false	false	…	false	false	false

Chain Rule

Conditional probabilities can be used to decompose conjunctions using the chain rule. For any propositions $f_1, f_2, \ldots, f_n$ **:**

$$P(f_1 \wedge f_2 \wedge \ldots \wedge f_n) = \prod_{i=1}^{n} P(f_i \mid f_1 \wedge \ldots \wedge f_{i-1})$$

Full Joint Probability Distribution

H: grad	e: female	P(H, e) = P(H ∧ e): P(grad ∧ female)
true	true	$P(grad = true \wedge female = true) = P(H, e) = P(H \wedge e) = P(H) * P(e \mid H) \approx 0.074$
true	false	$P(grad = true \wedge female = false) = P(H, \neg e) = P(H) * P(\neg e \mid H) \approx 0.148$
false	true	$P(grad = false \wedge female = true) = P(\neg H, e) = P(\neg H) * P(e \mid \neg H) \approx 0.086$
false	false	$P(grad = false \wedge female = false) = P(\neg H, \neg e) = P(\neg H) * P(\neg e \mid \neg H) \approx 0.691$
		SUM = 1

Joint probabilities calculated using the Chain Rule:

$$P(f_1 \wedge f_2) = \prod_{i=1}^{2} P(f_i \mid f_1 \wedge \ldots \wedge f_{i-1})$$

$$P(f_1 \wedge f_2) = P(f_1) * P(f_2 \mid f_1)$$

so: $P(grad \wedge female) = P(H \wedge e) = P(H) * P(e \mid H)$

Full Joint Probability Distribution

H: grad	e: female	P(H, e) = P(H ∧ e): P(grad ∧ female)
true	true	$P(H, e) = P(H \wedge e) = P(H) * P(e \mid H) = 18 / 81 * 6 / 18 \approx 0.074$
true	false	$P(H, \neg e) = P(H) * P(\neg e \mid H) = 18 / 81 * 12 / 18 \approx 0.148$
false	true	$P(\neg H, e) = P(\neg H) * P(e \mid \neg H) = 63 / 81 * 7 / 63 \approx 0.086$
false	false	$P(\neg H, \neg e) = P(\neg H) * P(\neg e \mid \neg H) = 63 / 81 * 56 / 63 \approx 0.691$
		SUM = 1

Joint probabilities calculated using the Chain Rule:

$$P(f_1 \wedge f_2) = \prod_{i=1}^{2} P(f_i \mid f_1 \wedge \ldots \wedge f_{i-1})$$

$$P(f_1 \wedge f_2) = P(f_1) * P(f_2 \mid f_1)$$

so: $P(grad \wedge female) = P(H \wedge e) = P(H) * P(e \mid H)$

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		SUM = 1

Joint probabilities calculated using the Chain Rule:

$$P(f_1 \wedge f_2) = \prod_{i=1}^{2} P(f_i \mid parents(f_i))$$

$$P(f_1 \wedge f_2) = P(f_1) * P(f_2 \mid parents(f_i))$$

so: $P(H \wedge e) = P(H) * P(e \mid parents(e)) = P(H) * P(e \mid H)$

Full Joint Probability Distribution

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		SUM = 1

$$P(H \wedge e) = P(H) * P(e \mid parents(e)) = \boxed{P(H)} * \boxed{P(e \mid H)}$$

H: grad	¬H: ¬grad
18 / 81 ≈ 0.22	63 / 81 ≈ 0.78

H: grad	e: female	P(e | H)
true	true	6 / 18 ≈ 0.333
true	false	12 / 18 ≈ 0.667
false	true	7 / 63 ≈ 0.111
false	false	56 / 63 ≈ 0.889

Full Joint Probability Distribution

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Conditional Probability Table (CPT)

Bayesian (Belief) Network

A Bayesian belief network describes the joint probability distribution for a set of variables.

A Bayesian network is an acyclic, directed graph (DAG), where the nodes are random variables (propositions). There is an edge (arc) from each elements of $\mathrm{parents}(X_i)$ into X_i. Associated with the Bayesian network is a set of conditional probability distributions - the conditional probability of each variable given its parents (which includes the prior probabilities of those variables with no parents).

Consists of:

- a graph (DAG) with nodes corresponding to random variables
- a domain for each random variable
- a set of conditional probability distributions $P(X_i \mid \mathrm{parents}(X_i))$

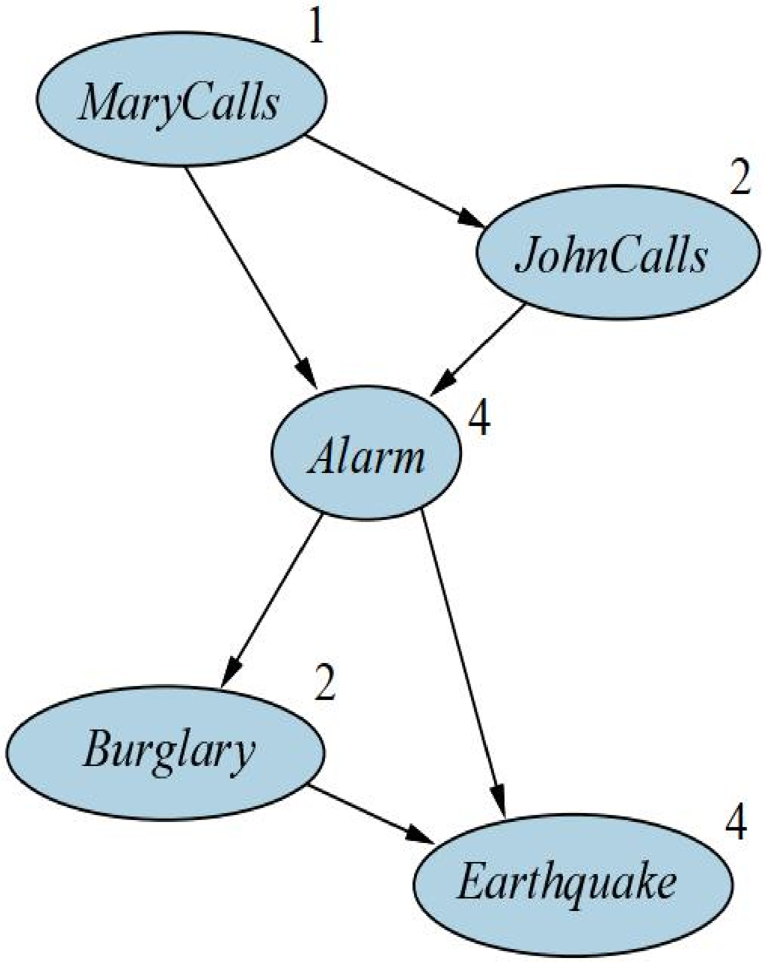
Building Bayesian (Belief) Network

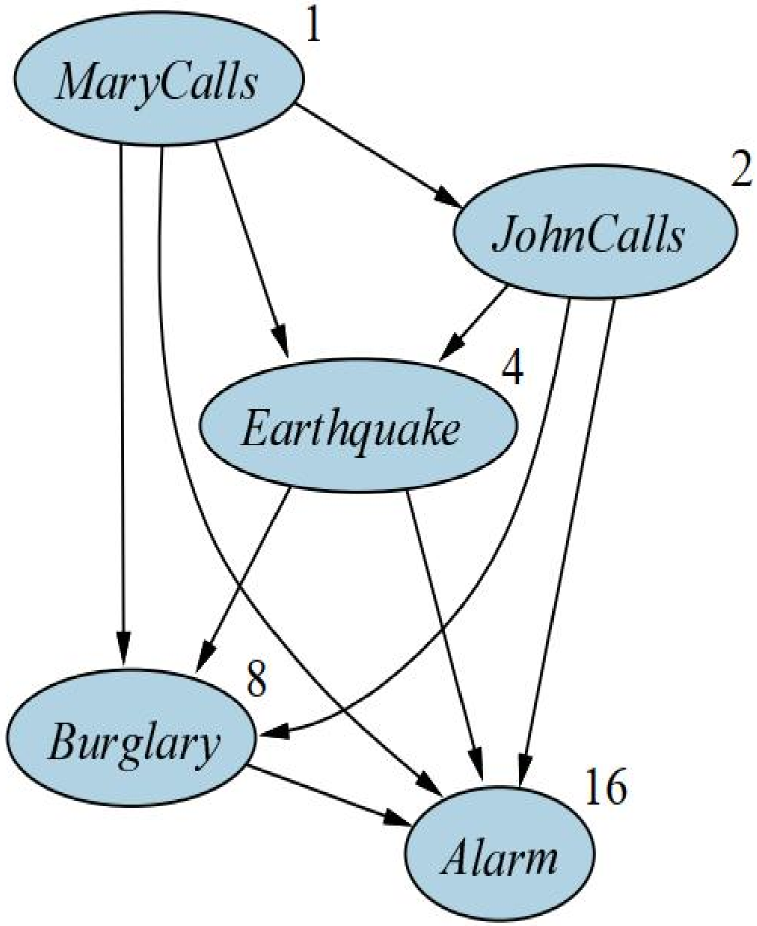
1. Order Random Variables (ordering matters!)
2. Create network nodes for each Random Variable
3. Add edges between parent nodes and children nodes
   - For every node node X_i:
     - choose a minimal set S of parents for X_i
     - for each parent node Y in S add an edge from Y to X_i

4. Add Conditional Probability Tables

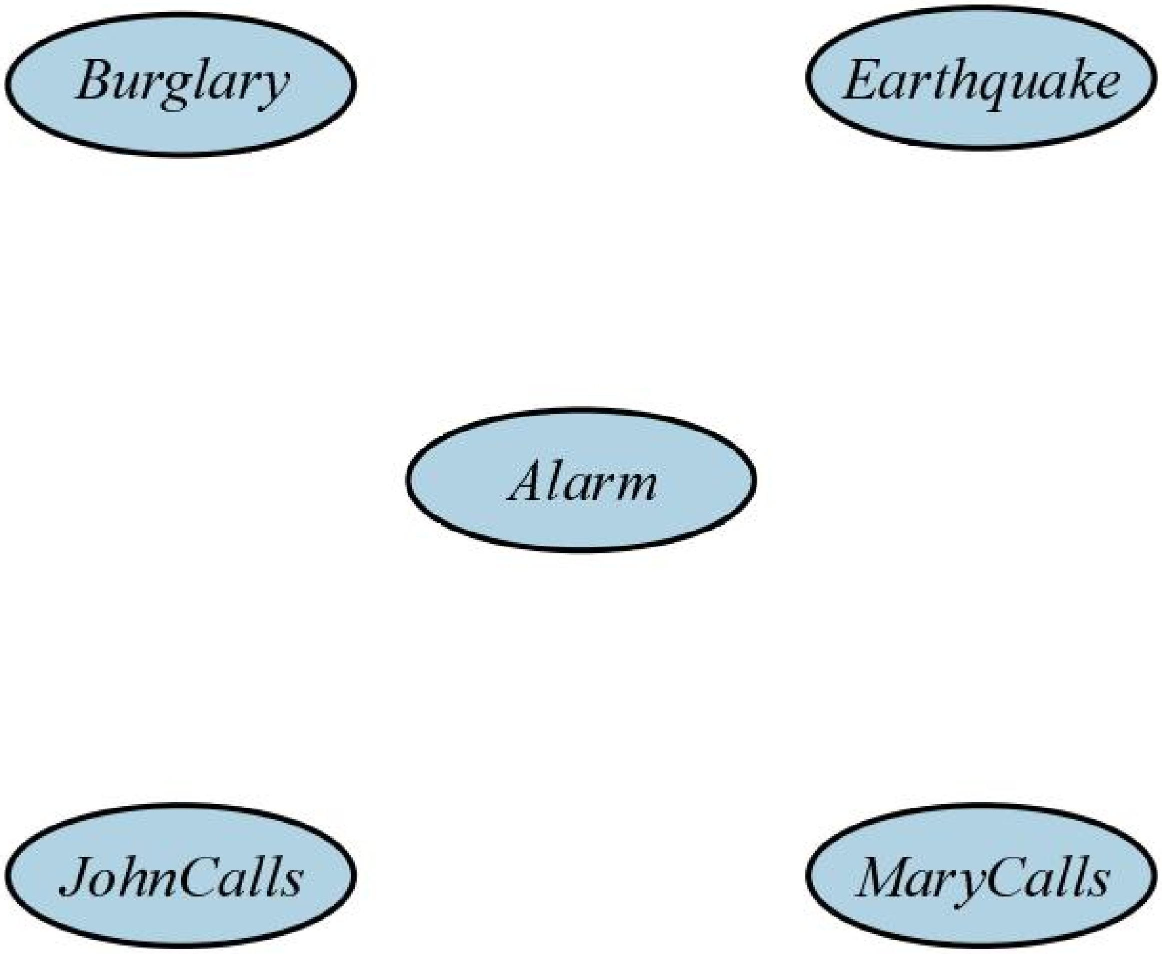
Make it compact / sparse: choose your Random Variable ordering wisely.

Ordering Matters!

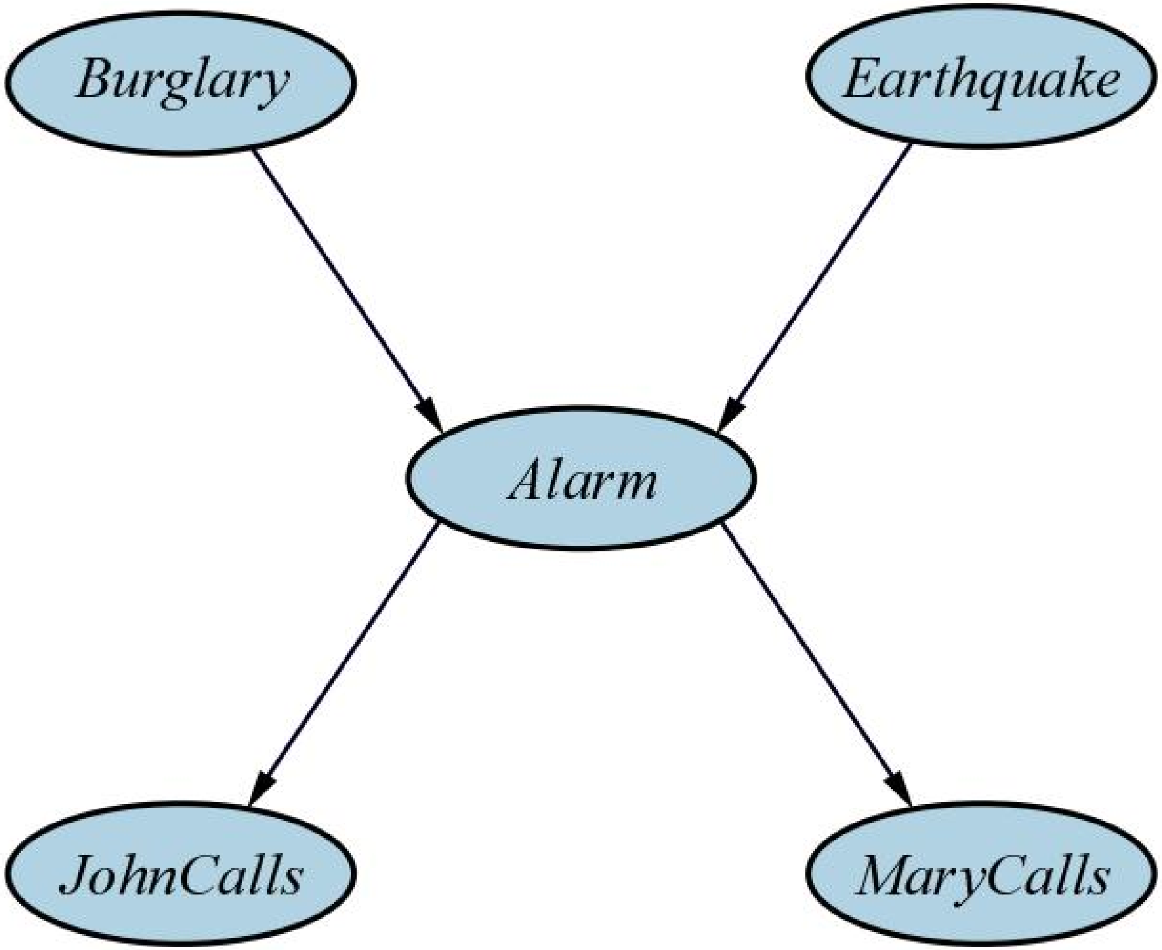




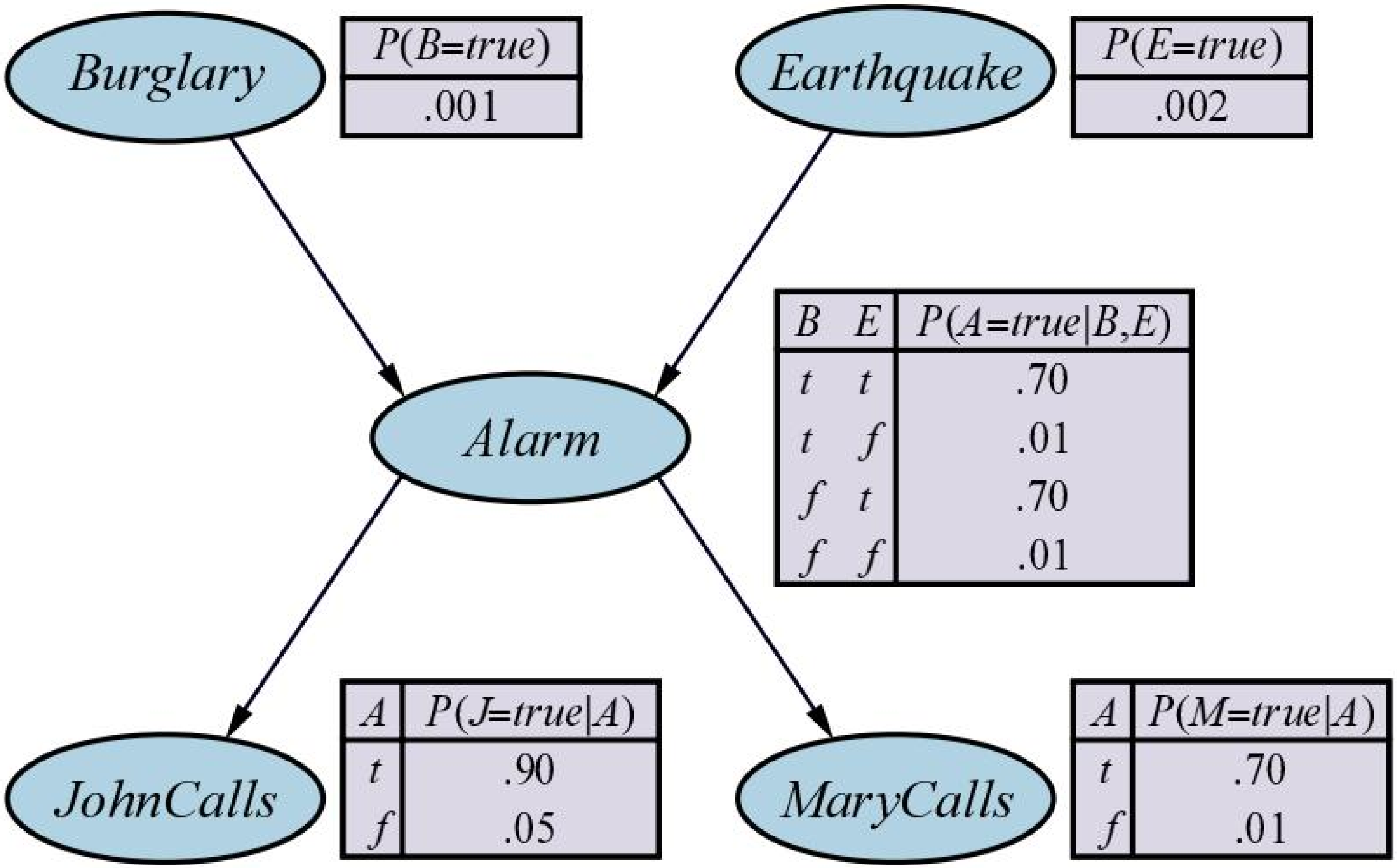
Create Vertices / Node / Random Vars



Add Edges



Add Conditional Probability Tables



$P(B=true)$
.001

$P(E=true)$
.002

B	E	$P(A=true \mid B,E)$
t	t	.70
t	f	.01
f	t	.70
f	f	.01

A	$P(J=true \mid A)$
t	.90
f	.05

A	$P(M=true \mid A)$
t	.70
f	.01

Full Joint Probability Distribution

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Conditional Probability Table (CPT)

Create Vertices / Node / Random Vars

Create Vertices / Node / Random Vars

Add Edges

Add Conditional Probability Tables

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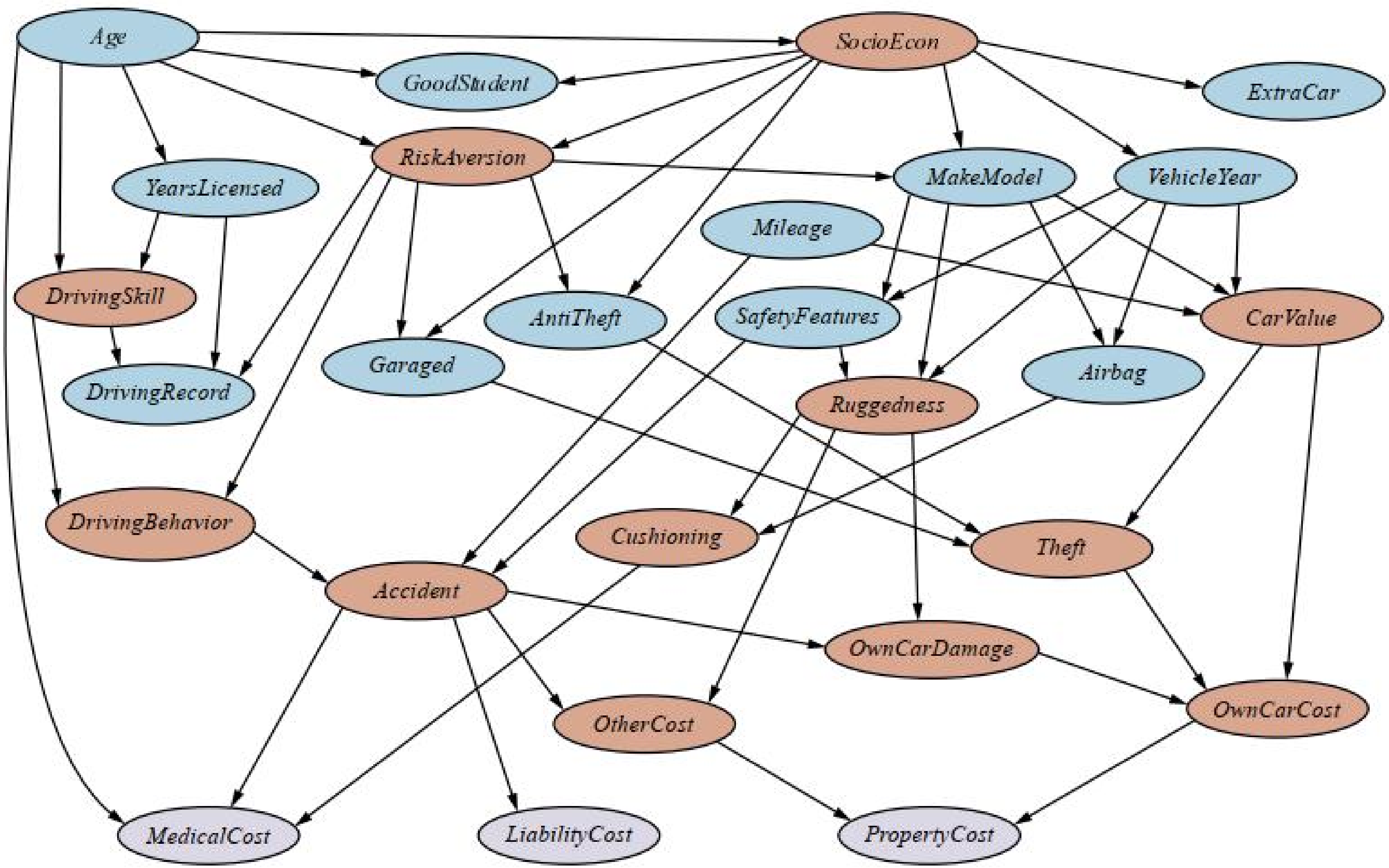
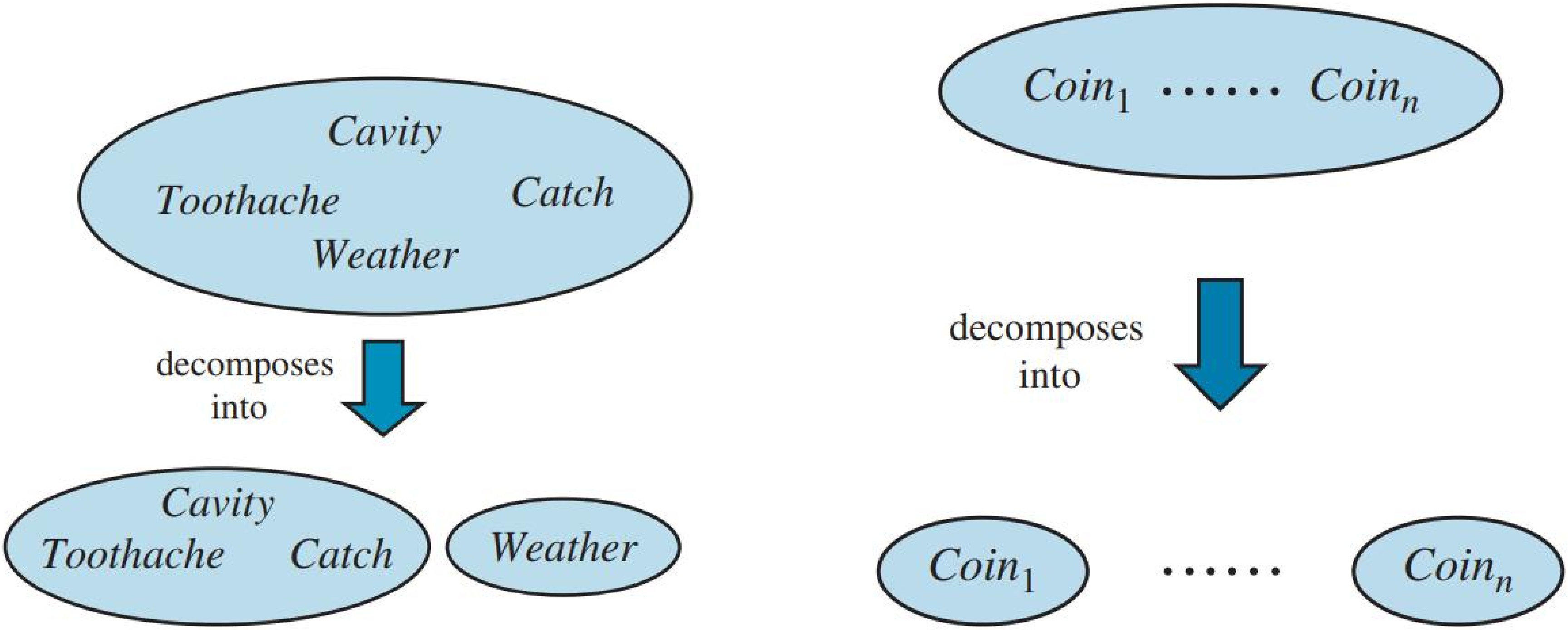
Bayesian Network: Car Insurance



Factoring / Decomposition



Full Joint Probability Distribution

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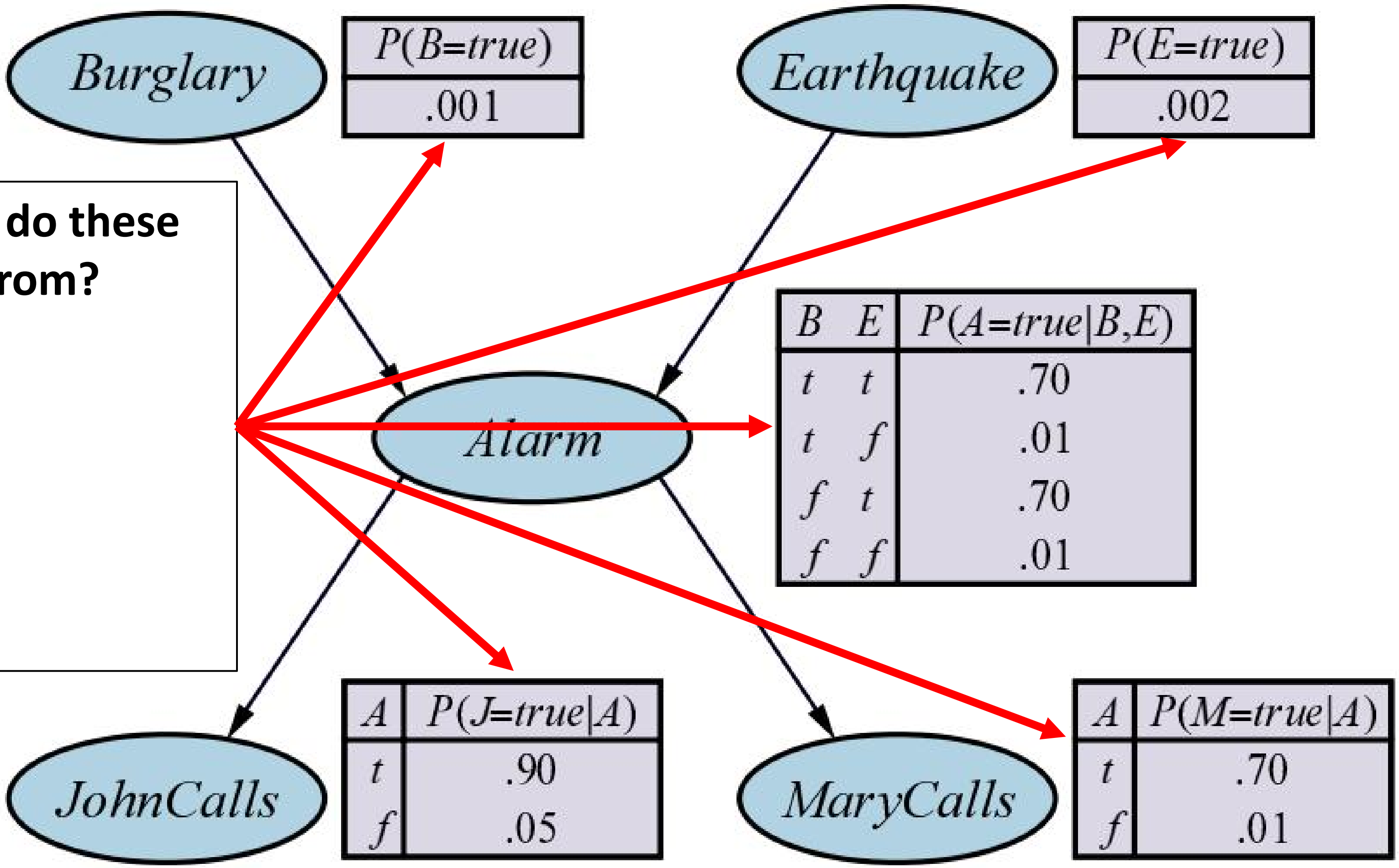
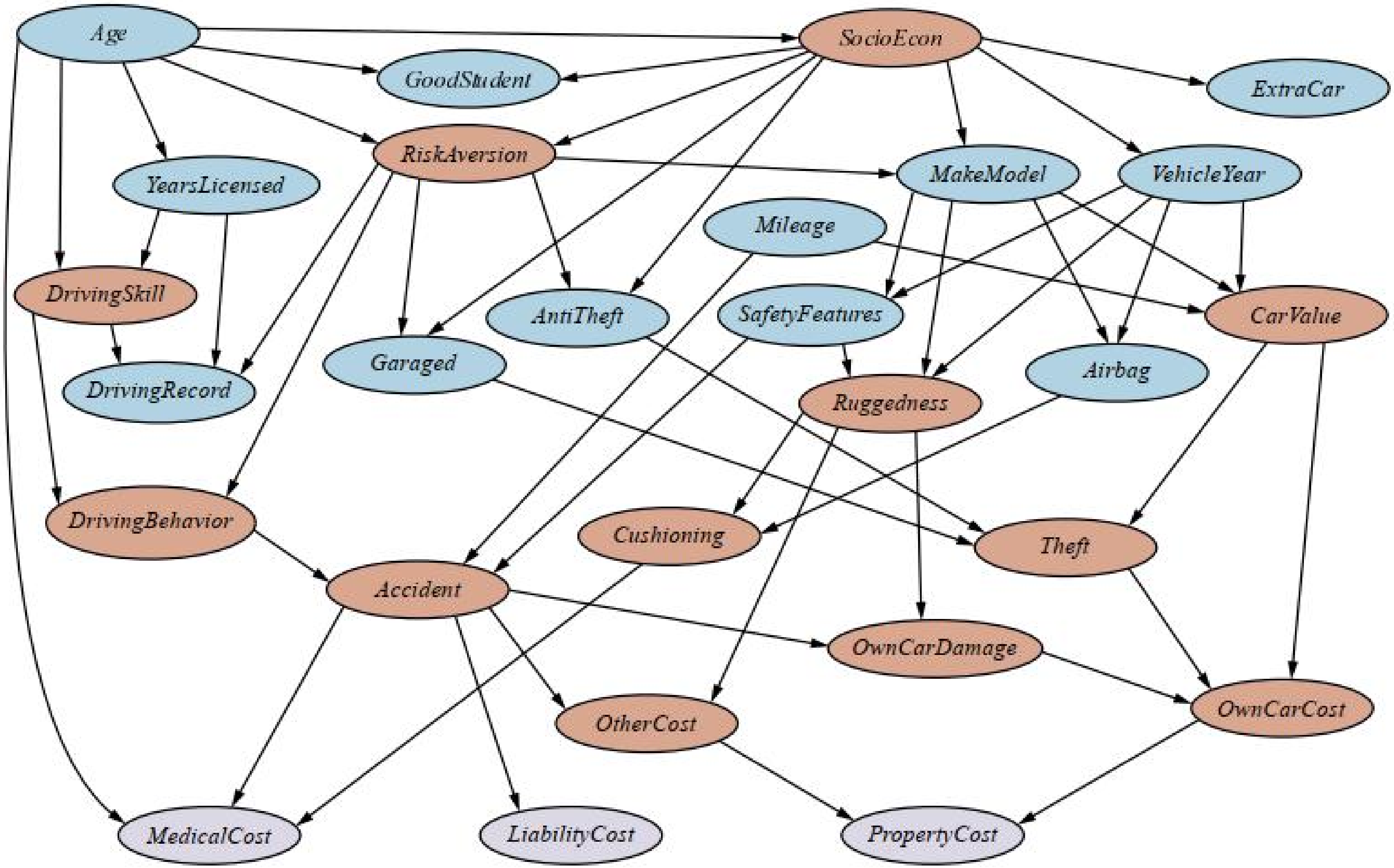
Conditional Probability Table (CPT)

Add Conditional Probability Tables



Where do these come from?

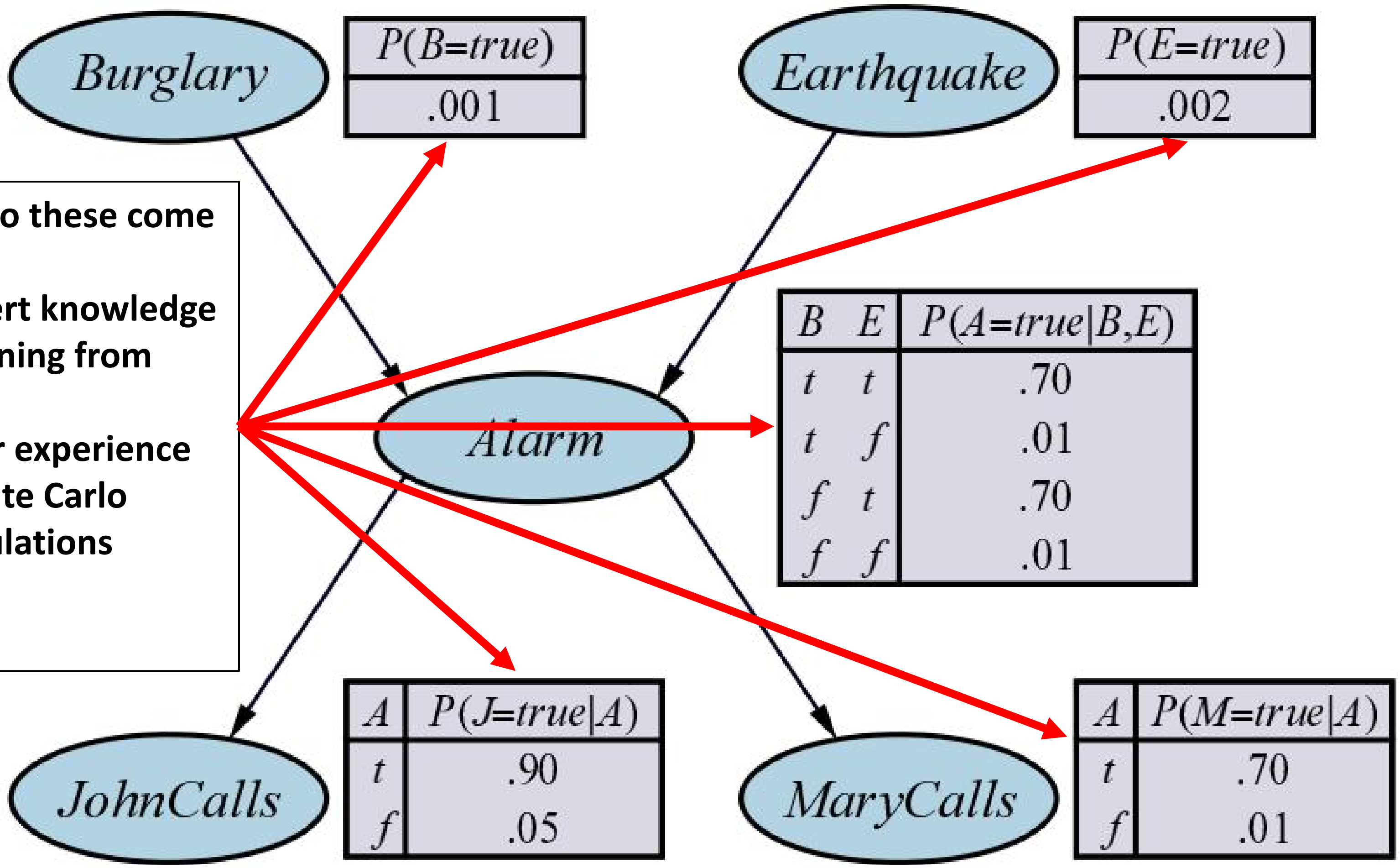
Bayesian Network: Car Insurance



Add Conditional Probability Tables

Where do these come from?

- Expert knowledge
- Learning from data
- Prior experience
- Monte Carlo simulations
- etc.



P(B=true)
.001

P(E=true)
.002

B	E	P(A=true|B,E)
t	t	.70
t	f	.01
f	t	.70
f	f	.01

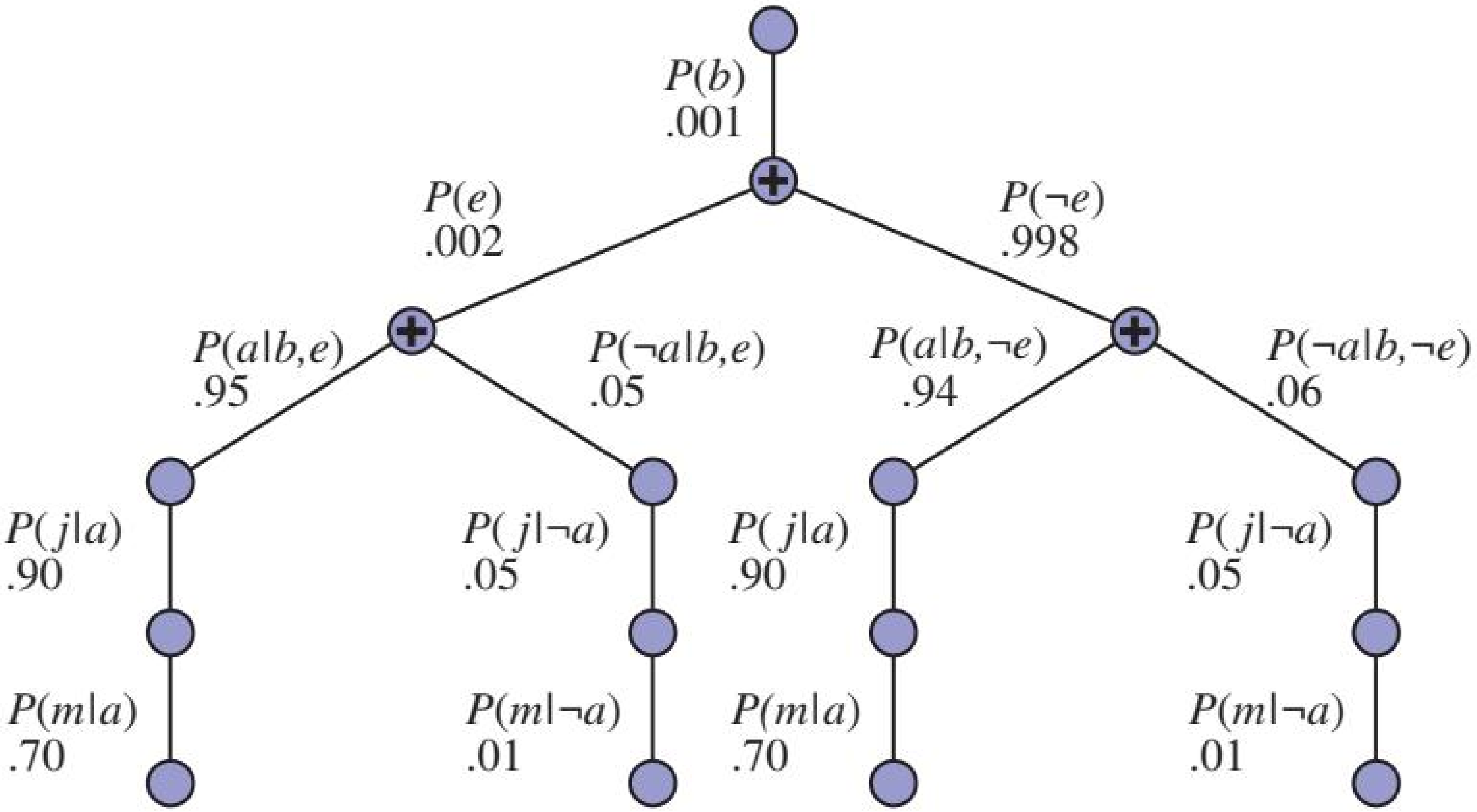
A	P(J=true|A)
t	.90
f	.05

A	P(M=true|A)
t	.70
f	.01

Inference by Enumeration: Example

Query:

$$\boldsymbol{P}(Burglary \mid JohnCalls = true \land MaryCalls = true)$$



General Inference Procedure

Given:

- **a query involving a single variable** X **(in our example:** Cavity**),**
- **a list of evidence variables E (in our example: just** Toothache**),**
- **a list of observed values** e **for E,**
- **a list of remaining unobserved variables Y (in our example: just** Catch**),**

where X**, E, and Y together are a COMPLETE set of variables for the domain, the probability** P(X | **E**) **can be evaluated as:**

$$\boldsymbol{P}(X \mid \boldsymbol{e}) = \alpha * \boldsymbol{P}(X, \boldsymbol{e}) = \alpha * \sum_{\boldsymbol{y}} \boldsymbol{P}(X, \boldsymbol{e}, \boldsymbol{y})$$

where y**s are all possible values for** Y**s,** α **- normalization constant.**

P(X**, e, y) is a subset of probabilities from the joint distribution**

Inference

Query:

$$P(Burglary \mid JohnCalls = true \wedge MaryCalls = true)$$

Given:

- **a query involving a single variable X**
- **a list of evidence variables K,**
- **a list of observed values k for K,**
- **a list of remaining unobserved variables Y**

the probability P(X | K) can be evaluated as:

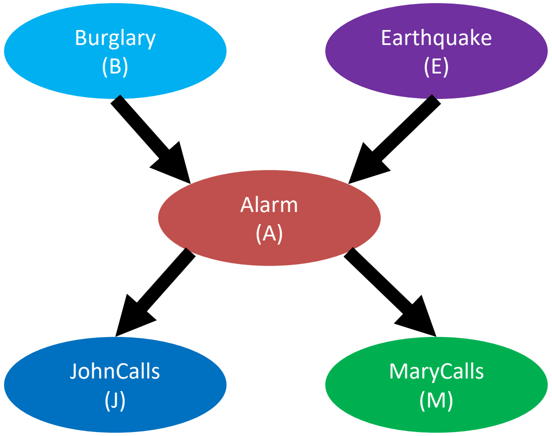
$$P(X \mid \boldsymbol{k}) = \alpha * P(X, \boldsymbol{k})$$

$$= \alpha * \sum_{\boldsymbol{y}} P(X, \boldsymbol{k}, \boldsymbol{y})$$

where ys are all possible values for Ys, α - normalization constant.

P(B)	P(¬B)
0.001	0.999

P(E)	P(¬E)
0.002	0.998



B	E	P(A|B,E)
t	t	0.95
t	f	0.94
f	t	0.29
f	f	0.001

A	P(J|A)
t	0.90
f	0.05

A	P(M|A)
t	0.70
f	0.01

Inference

Query:

$$P(Burglary \mid JohnCalls = true \wedge MaryCalls = true)$$

Given:

- **a query involving a single variable X:** *Burglary*
- **a list of evidence variables K:** *JohnCalls*, *MaryCalls*
- **a list of observed values k for K:** *johnCalls*, *maryCalls*
- **a list of remaining unobserved variables Y:** *Earthquake*, *Alarm*

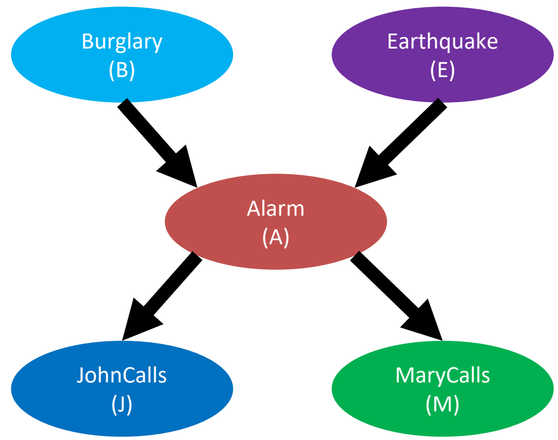
the probability $P(X \mid K)$ can be evaluated as:

$$P(X \mid k) = \alpha * \sum_{y} P(X, k, y)$$

where ys are all possible values for Ys, α - normalization constant.

P(B)	P(¬B)
0.001	0.999

P(E)	P(¬E)
0.002	0.998



B	E	P(A|B,E)
t	t	0.95
t	f	0.94
f	t	0.29
f	f	0.001

A	P(J|A)
t	0.90
f	0.05

A	P(M|A)
t	0.70
f	0.01

Inference

Query:

$$P(Burglary \mid JohnCalls = true \wedge MaryCalls = true)$$

Given:

- **a query involving a single variable** X**:** B
- **a list of evidence variables** $\mathbf{K}$**:** J, M
- **a list of observed values** $\mathbf{k}$ **for** $\mathbf{K}$**:** j, m
- **a list of remaining unobserved variables** $\mathbf{Y}$**:** E, A

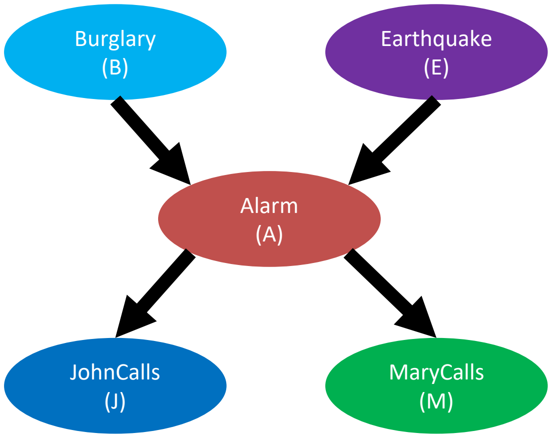
the probability $P(X \mid \mathbf{K})$ **can be evaluated as:**

$$\boldsymbol{P}(X \mid \boldsymbol{k}) = \alpha * \sum_{\boldsymbol{y}} \boldsymbol{P}(X, \boldsymbol{k}, \boldsymbol{y})$$

where $\mathbf{y}$**s are all possible values for** $\mathbf{Y}$**s,** α **- normalization constant.**

P(B)	P(¬B)
0.001	0.999

P(E)	P(¬E)
0.002	0.998



B	E	P(A|B,E)
t	t	0.95
t	f	0.94
f	t	0.29
f	f	0.001

A	P(J|A)
t	0.90
f	0.05

A	P(M|A)
t	0.70
f	0.01

Inference

Query:

$$P(Burglary \mid JohnCalls = true \wedge MaryCalls = true)$$

Given:

- **a query involving a single variable** B
- **a list of evidence variables** $\mathbf{K}$**:** J, M
- **a list of observed values** $\mathbf{k}$ **for** $\mathbf{K}$**:** j, m
- **a list of remaining unobserved variables** $\mathbf{Y}$**:** E, A

the probability $P(B \mid J, M)$ **can be evaluated as:**

$$\boldsymbol{P}(B \mid j, m) = \alpha * \sum_{e} \sum_{a} \boldsymbol{P}(B, j, m, e, a)$$

where $\mathbf{y}$**s are all possible values for** $\mathbf{Y}$**s,** α **- normalization constant.**

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0.001	0.999

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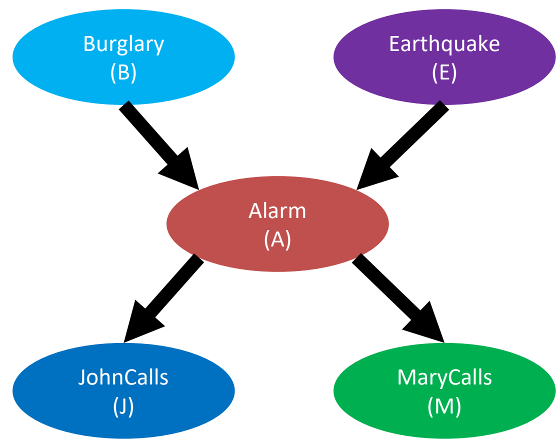

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t	0.90
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t	0.70
f	0.01

Inference

Query (let's change it a bit for simplicity):

$$\mathbf{P}(Burglary = true \mid JohnCalls = true \wedge MaryCalls = true)$$

Given:

- a query involving a single variable B
- a list of **evidence** variables $\mathbf{K}$: J, M
- a list of **observed** values $\mathbf{k}$ for $\mathbf{K}$: j, m
- a list of remaining **unobserved** variables $\mathbf{Y}$: E, A

the query can be evaluated as:

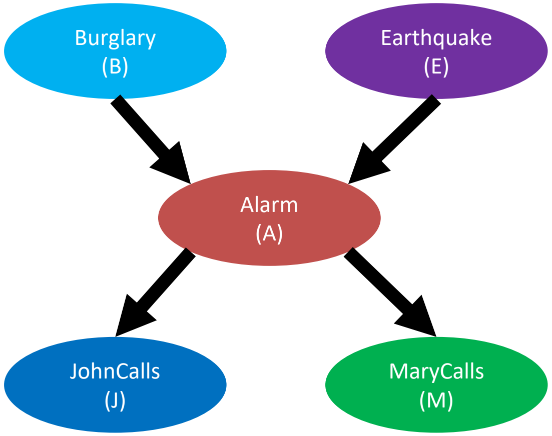
$$P(b \mid j, m) = \alpha * \sum_{e} \sum_{a} P(b, j, m, e, a)$$

By Chain rule:

$$P(b, j, m, e, a) = P(b) * P(e) * P(a|b,e) * P(j|a) * P(m|a)$$

P(B)	P(¬B)
0.001	0.999

P(E)	P(¬E)
0.002	0.998



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t	f	0.94
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A	P(J|A)
t	0.90
f	0.05

A	P(M|A)
t	0.70
f	0.01

Inference

Query (let's change it a bit for simplicity):

$P(Burglary\ =\ true\ |\ JohnCalls\ =\ true \wedge MaryCalls\ =\ true)$

Given:

- **a query involving a single variable** B
- **a list of evidence variables K:** J, M
- **a list of observed values k for K:** j, m
- **a list of remaining unobserved variables Y:** E, A

the query can be evaluated as:

$$P(b\,|\,j, m) = \alpha * \sum_{e} \sum_{a} P(b) * P(e) * P(a|b, e) * P(j|a) * P(m|a)$$

P(B)	P(¬B)
0.001	0.999

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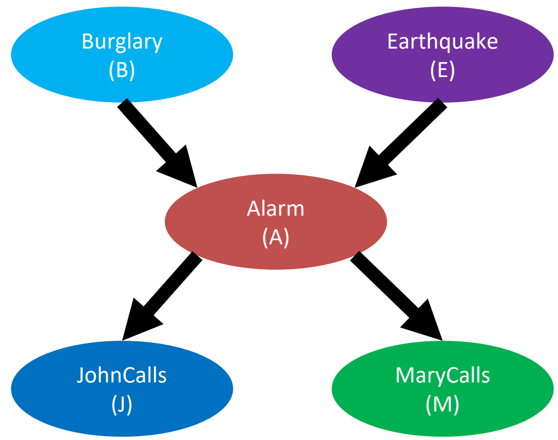

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A	P(M|A)
t	0.70
f	0.01

Inference

Query (let's change it a bit for simplicity):

$P(Burglary = true \mid JohnCalls = true \wedge MaryCalls = true)$

Given:

- **a query involving a single variable B**
- **a <u>list</u> of evidence variables $\mathbf{K}$: J, M**
- **a <u>list</u> of observed values $\mathbf{k}$ for $\mathbf{K}$: j, m**
- **a list of remaining unobserved variables $\mathbf{Y}$: E, A**

the query can be evaluated as:

$$P(b \mid j, m)$$
$$= \alpha * \sum_{e} \sum_{a} P(b) * P(e) * P(a|b,e) * P(j|a) * P(m|a)$$
$$= \alpha * P(b) * \sum_{e} P(e) * \sum_{a} P(a|b,e) * P(j|a) * P(m|a)$$

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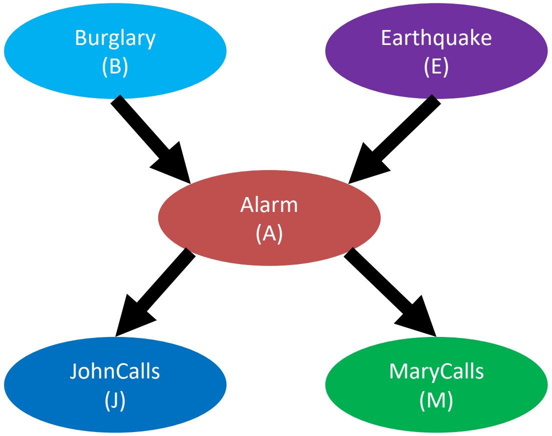

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Inference

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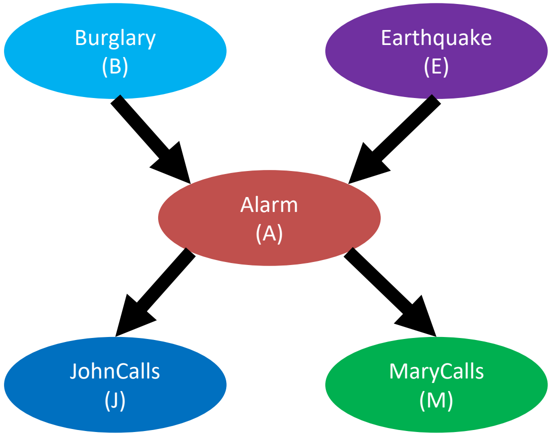

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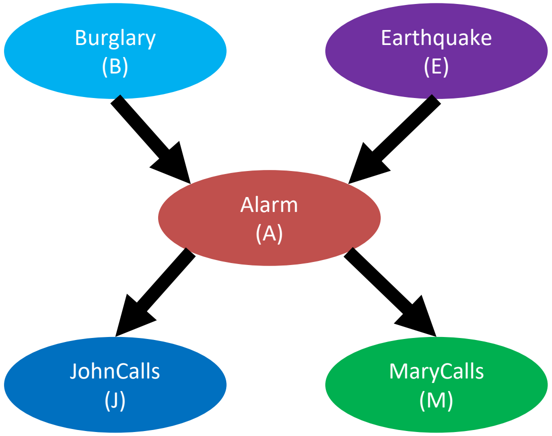

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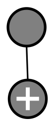
$P(b)$



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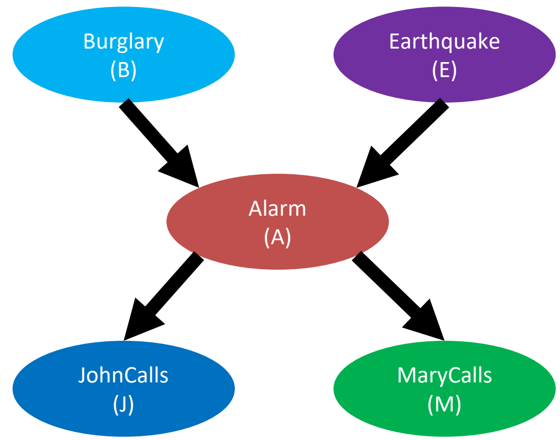

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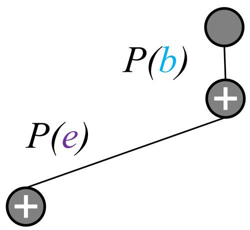

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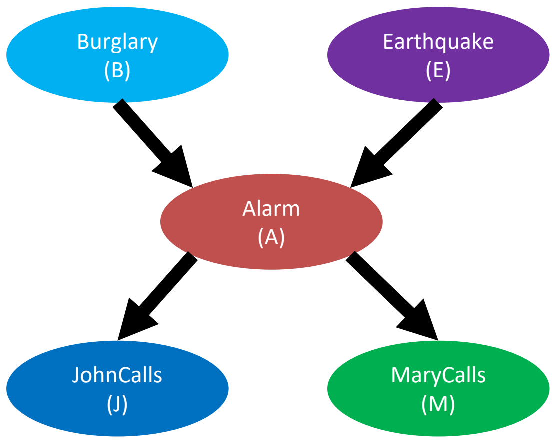

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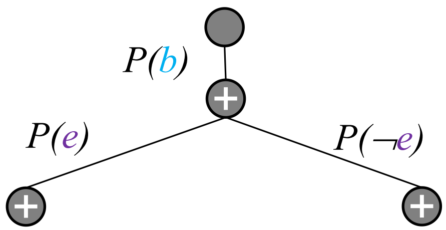

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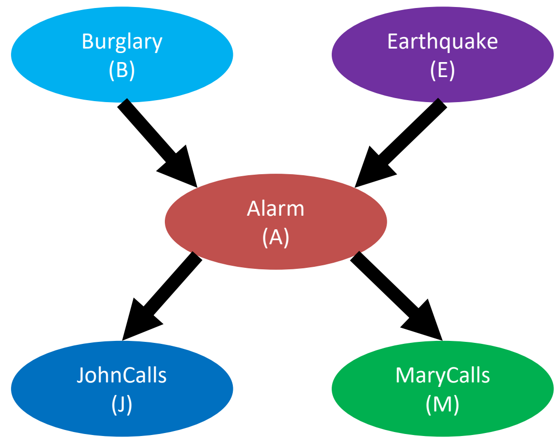

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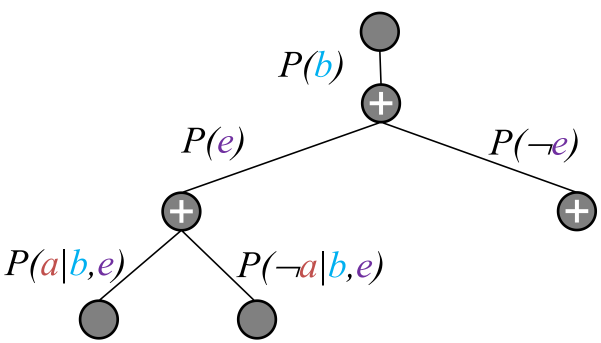

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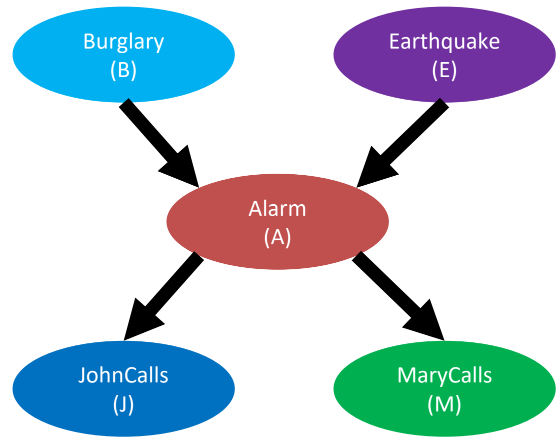

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Inference

Query (let's change it a bit for simplicity):

$P(Burglary = true \mid JohnCalls = true \land MaryCalls = true)$

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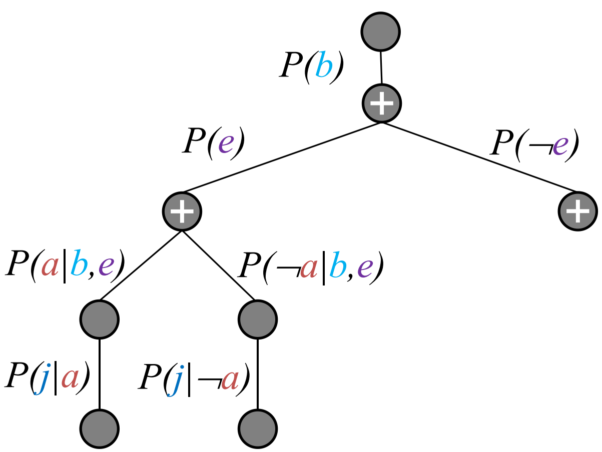

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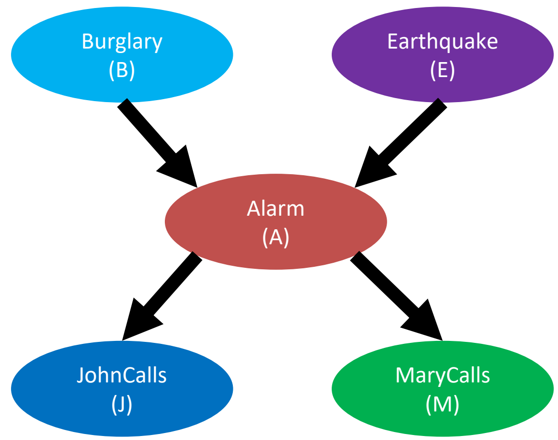

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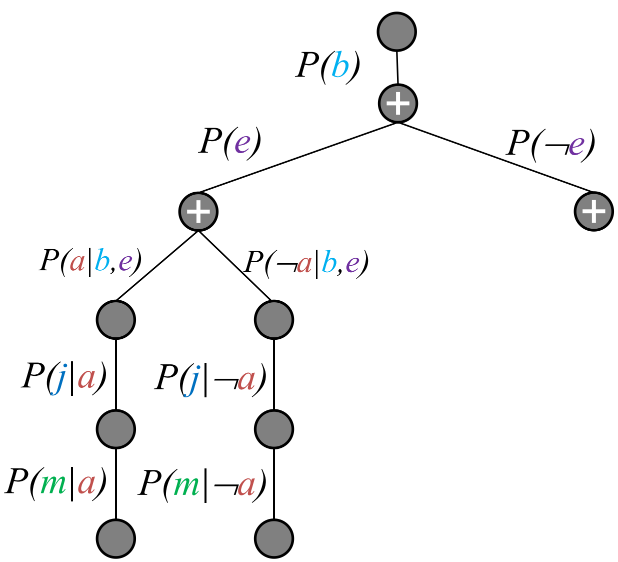

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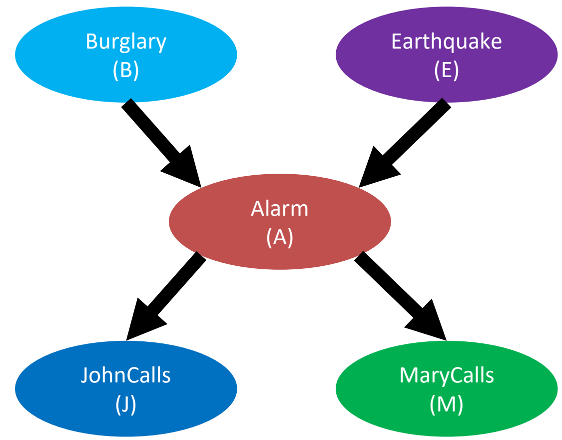

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Inference

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$$P(b \mid j, m)$$

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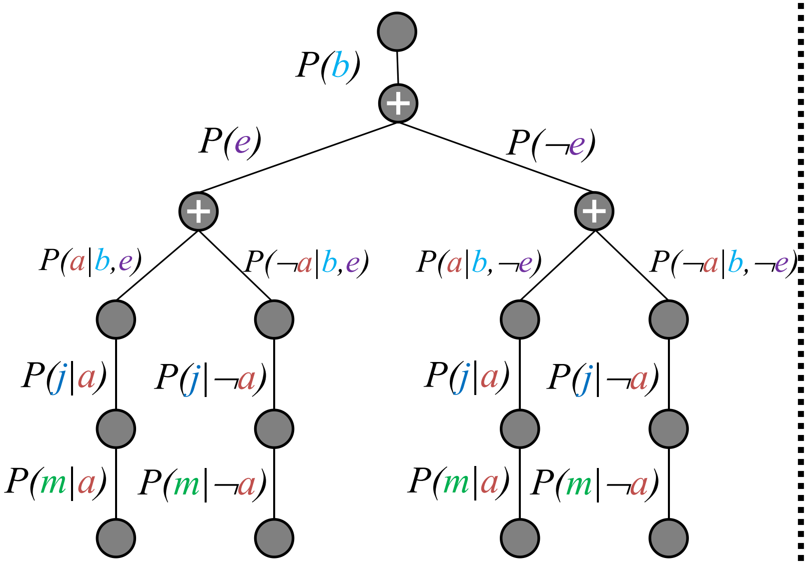

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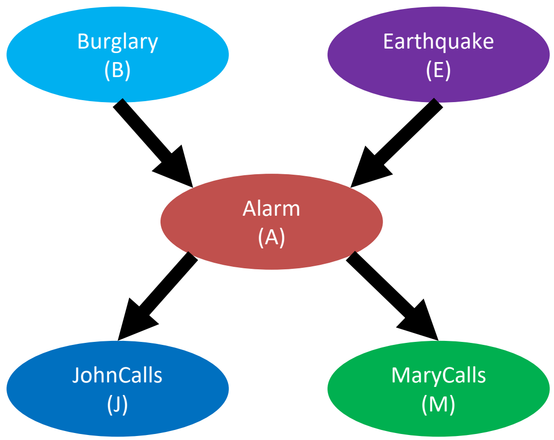

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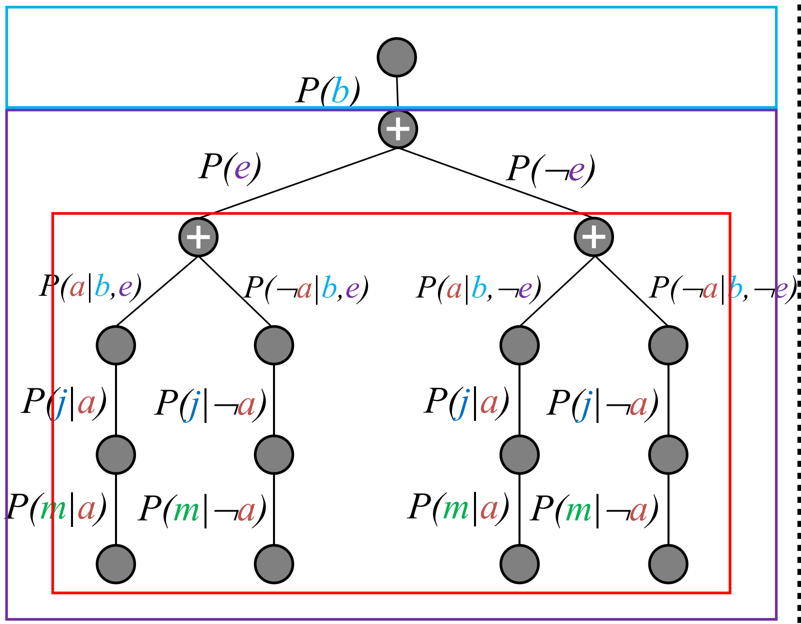

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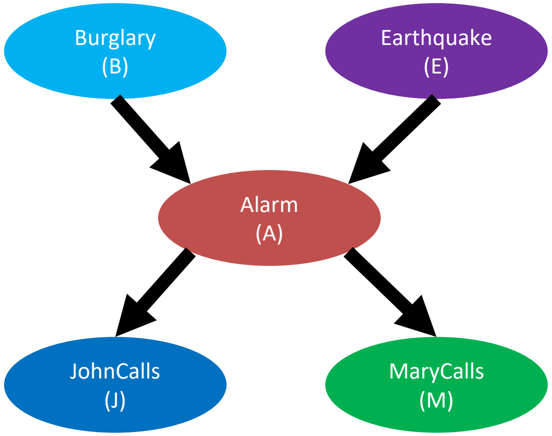

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Inference

Query (let's change it a bit for simplicity):

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Query rewritten:

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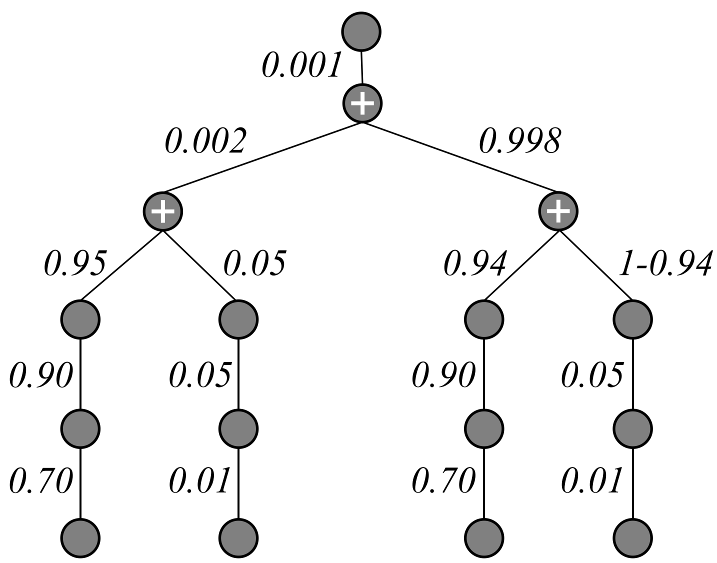

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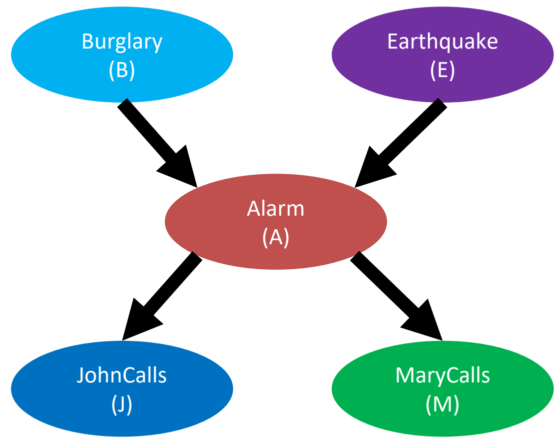

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A	P(M|A)
t	0.70
f	0.01

Inference

Query (let's change it a bit for simplicity):

$$P(Burglary = true \mid JohnCalls = true \wedge MaryCalls = true)$$

We can now calculate:

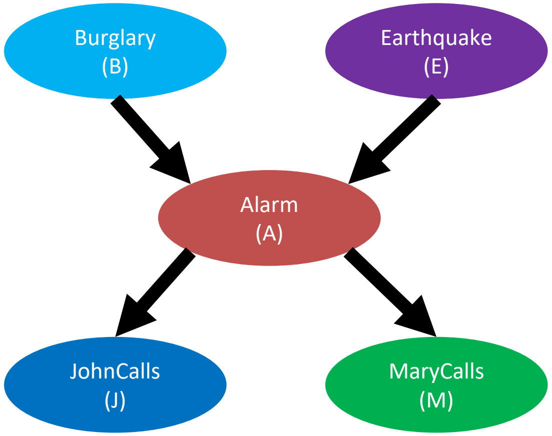
$$P(b \mid j, m) = \alpha * 0.00059224$$

And through similar process (not shown):

$$P(\neg b \mid j, m) = \alpha * 0.0014919$$

P(B)	P(¬B)
0.001	0.999

P(E)	P(¬E)
0.002	0.998



B	E	P(A|B,E)
t	t	0.95
t	f	0.94
f	t	0.29
f	f	0.001

A	P(J|A)
t	0.90
f	0.05

A	P(M|A)
t	0.70
f	0.01

Inference

Query (now we can get joint distribution):

$$\boldsymbol{P}(Burglary \mid JohnCalls = true \wedge MaryCalls = true)$$

We can now calculate:

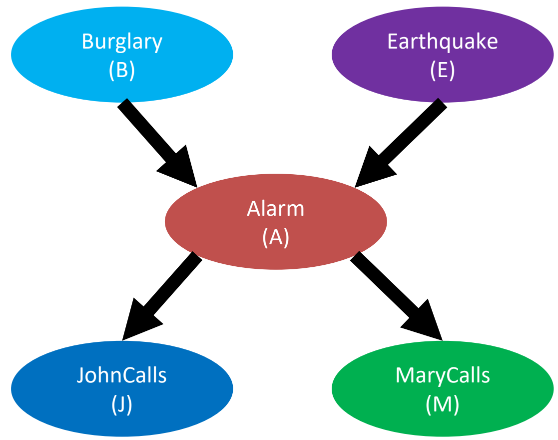
$$\boldsymbol{P}(B \mid j, m) = \alpha * \ < 0.00059224, 0.0014919 >$$

And after normalization:

$$\boldsymbol{P}(B \mid j, m) \approx < 0.284, 0.716 >$$

P(B)	P(¬B)
0.001	0.999

P(E)	P(¬E)
0.002	0.998



B	E	P(A|B,E)
t	t	0.95
t	f	0.94
f	t	0.29
f	f	0.001

A	P(J|A)
t	0.90
f	0.05

A	P(M|A)
t	0.70
f	0.01