

CS 480

Introduction to Artificial Intelligence

January 25, 2022

Announcements / Reminders

- Please follow the Week 02 To Do List instructions (if you haven't already)
- **In-person sessions start THIS week**
 - Please follow IIT COVID-19 guidelines
- Contribute to the discussion on Blackboard, please
- **Thursday: data structures, etc. review session**
- My office hours:
 - Tuesdays 11:30 AM - 01:30 PM in Stuart Building 217E or by appointment

Plan for Today

- **Problem Solving: Searching**

Designing the Searching Problem

**Analyze and
define the
Problem / Task**

**Model and build
the State Space**

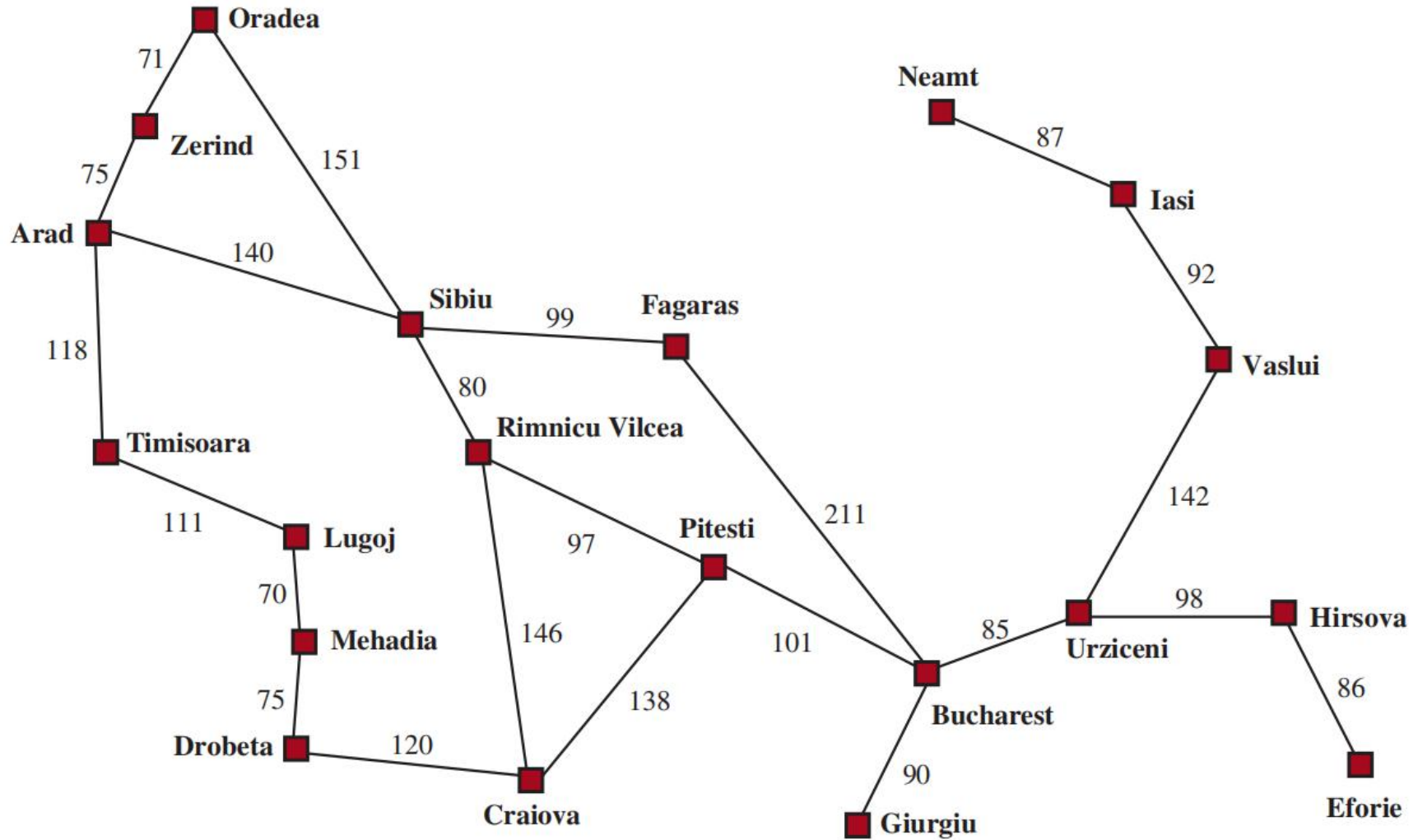
**Select searching
algorithm**

Search

Defining Search Problem

- Define a set of possible states: **State Space**
- Specify **Initial State**
- Specify **Goal State(s)** (there can be multiple)
- Define a FINITE set of possible **Actions** for EACH state in the State Space
- Come up with a **Transition Model** which describes what each action does
- Specify the **Action Cost Function**: a function that gives the cost of applying action a in state s

Sample Problem: Dracula's Roadtrip

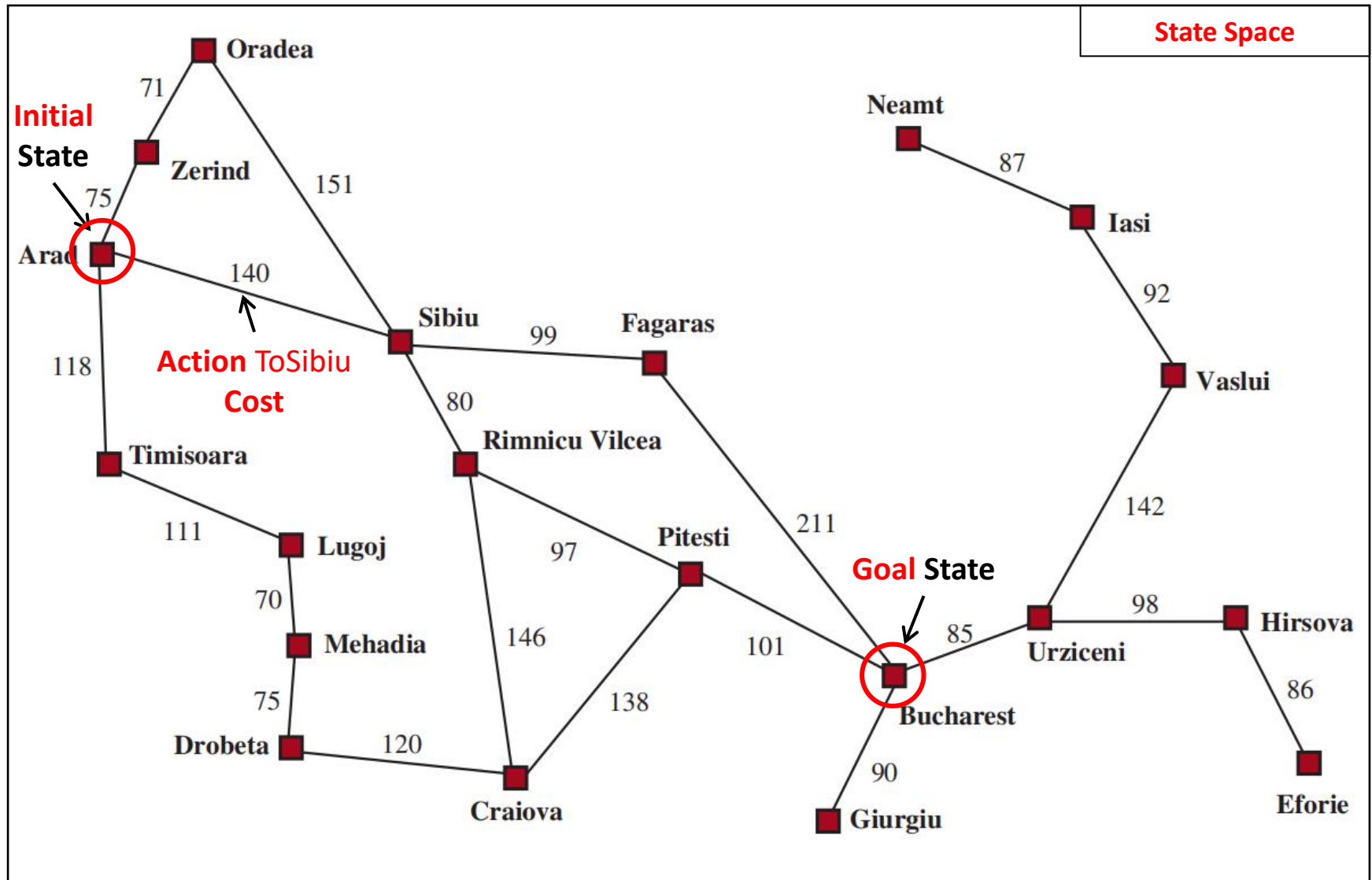


Problem: Get from Arad to Bucharest efficiently (for example: quickly or cheaply).

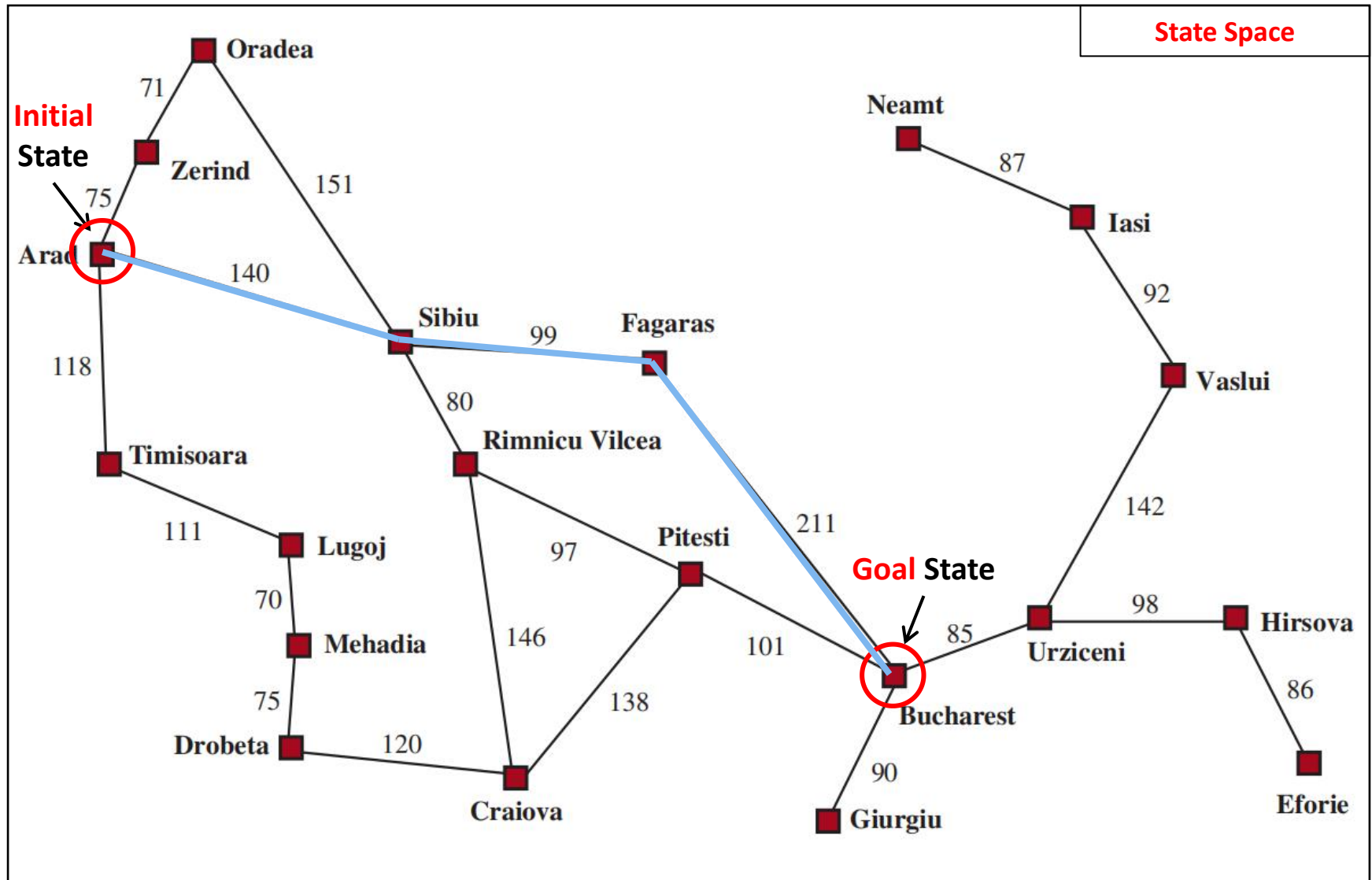
Search Problem: Dracula's Roadtrip

- State Space: **a map of Romania**
- Initial State: **Arad**
- Goal State: **Bucharest**
- Actions:
 - for example: **ACTIONS(Arad) = {ToSibiu, ToTimisoara, ToZerind}**
- Transition Model:
 - for example: **RESULT(Arad, ToZerind) = Zerind**
- Action Cost Function [**ActionCost(S_{current} , a, S_{next})**]
 - for example: **ActionCost(Arad, ToSibiu, Sibiu) = 140**

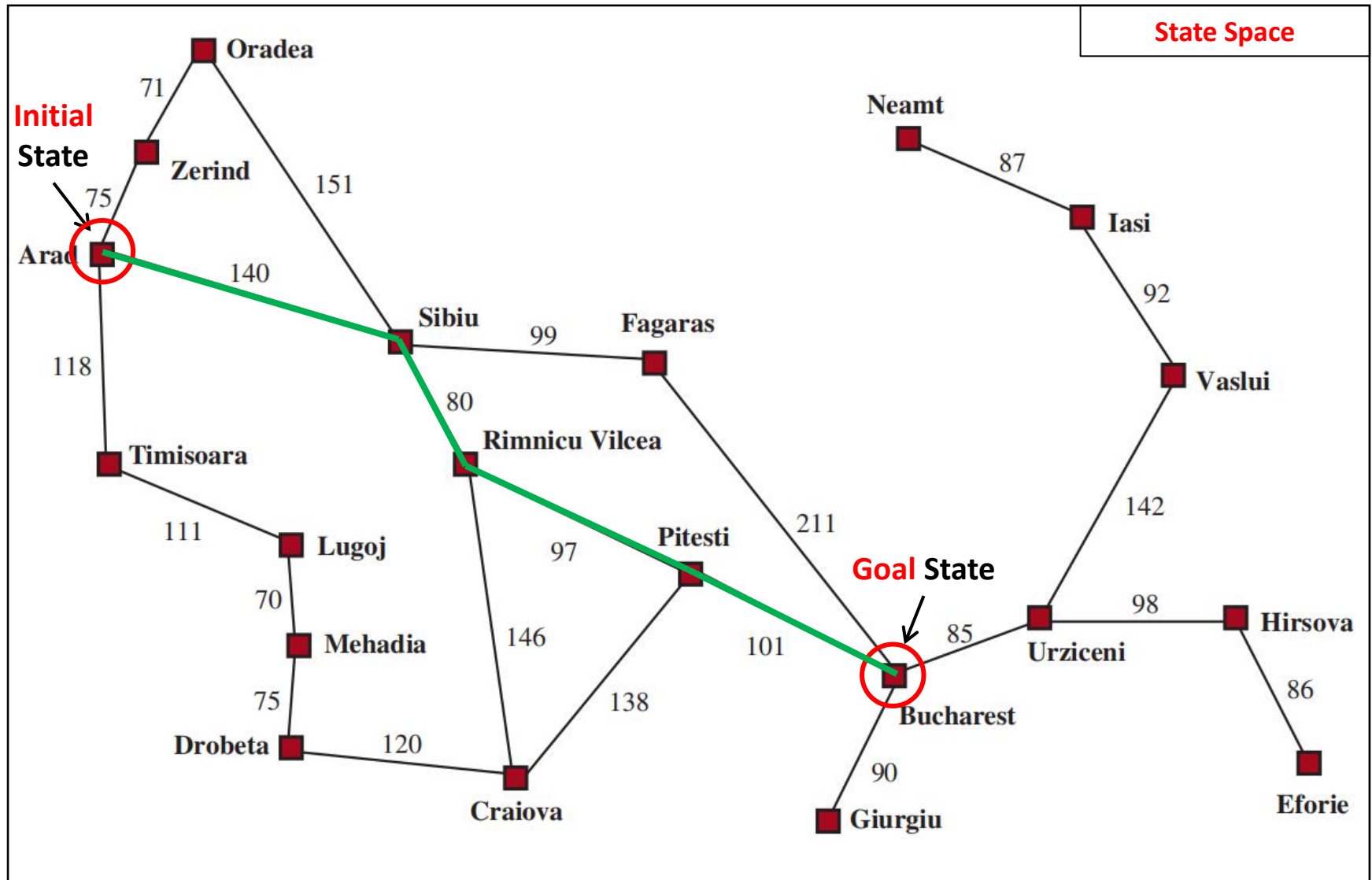
Sample Problem: Dracula's Roadtrip



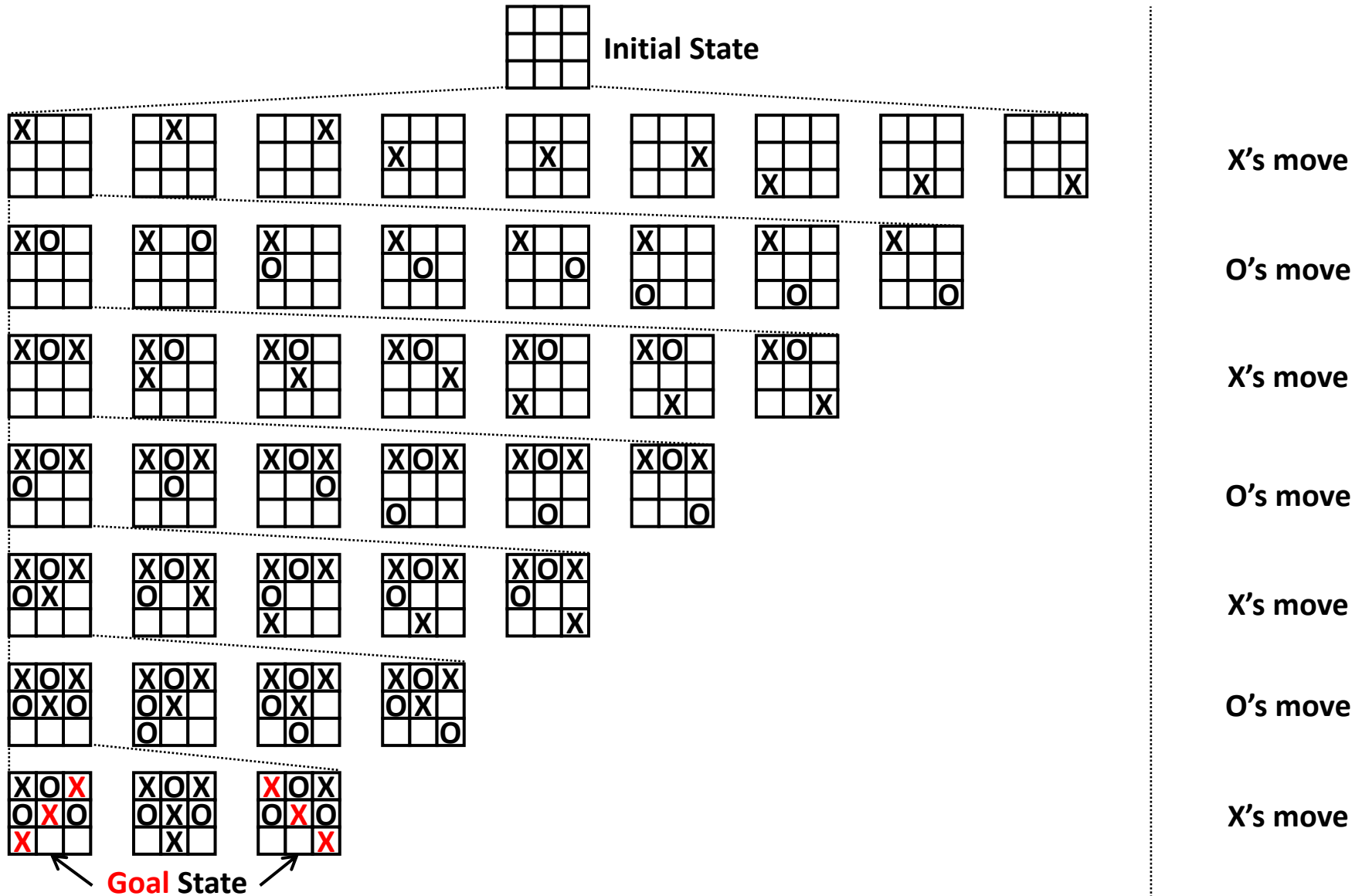
Dracula's Roadtrip: Potential Solution



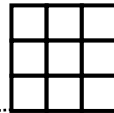
Dracula's Roadtrip: Potential Solution



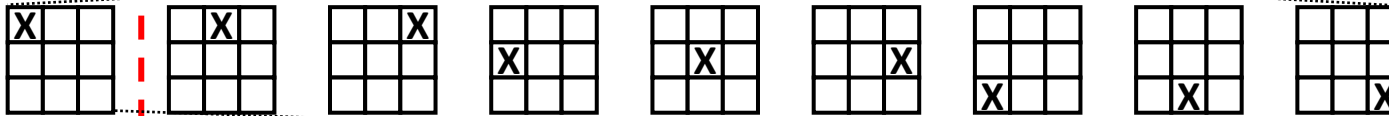
Tic Tac Toe: (Partial) State Space



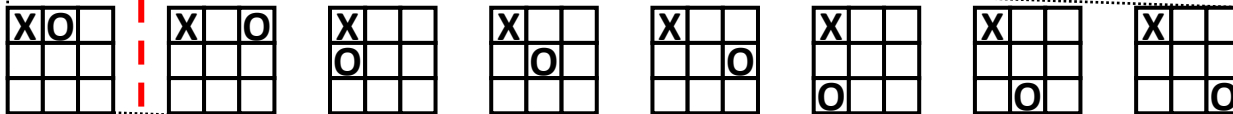
Tic Tac Toe: Solution



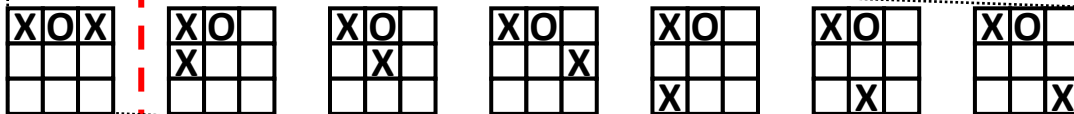
Initial State



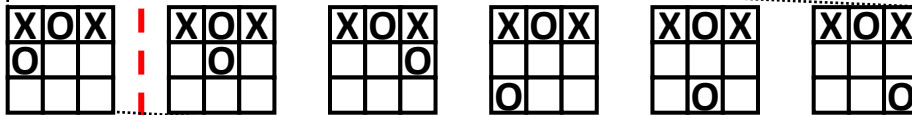
X's move



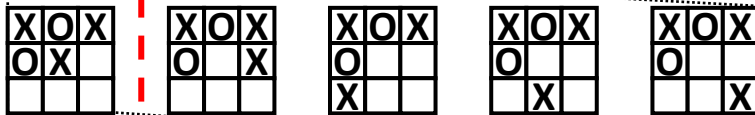
O's move



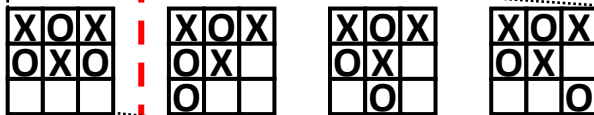
X's move



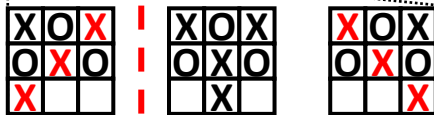
O's move



X's move



O's move



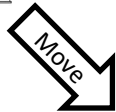
X's move

A **Solution** is a sequence of actions (a path) between the initial state and the goal state

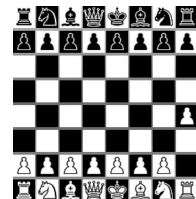
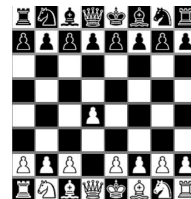
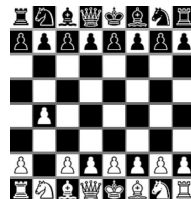
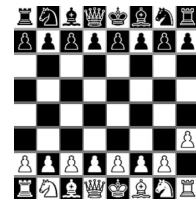
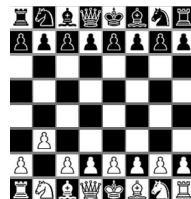
← Solution

Chess: (First Move) State Space

Initial
State



20 Possible **legal** first moves:
16 pawn moves
4 knight moves



Designing the Searching Problem

**Analyze and
define the
Problem / Task**

**Model and build
the State Space**

**Select searching
algorithm**

Search

State Space Model: A Graph

ACTIONS(A) = {toB, toC}

ACTIONS(B) = {toF}

ACTIONS(F) = {toE}

Action: toB

RESULT(A, toB) = B

$C(A, \text{toB}, B) = 1$

INITIAL
STATE
A

STATE
B

STATE
F

Action: toE

$C(F, \text{toE}, E) = 1$

Action: toF

$C(B, \text{toF}, F) = 1$

Action: toE

$C(D, \text{toE}, E) = 1$

Action: toB

$C(C, \text{toB}, B) = 1$

$C(A, \text{toC}, C) = 1$

Action: toC

STATE
C

STATE
D

GOAL
STATE
E

Action: toD

$C(C, \text{toD}, D) = 1$

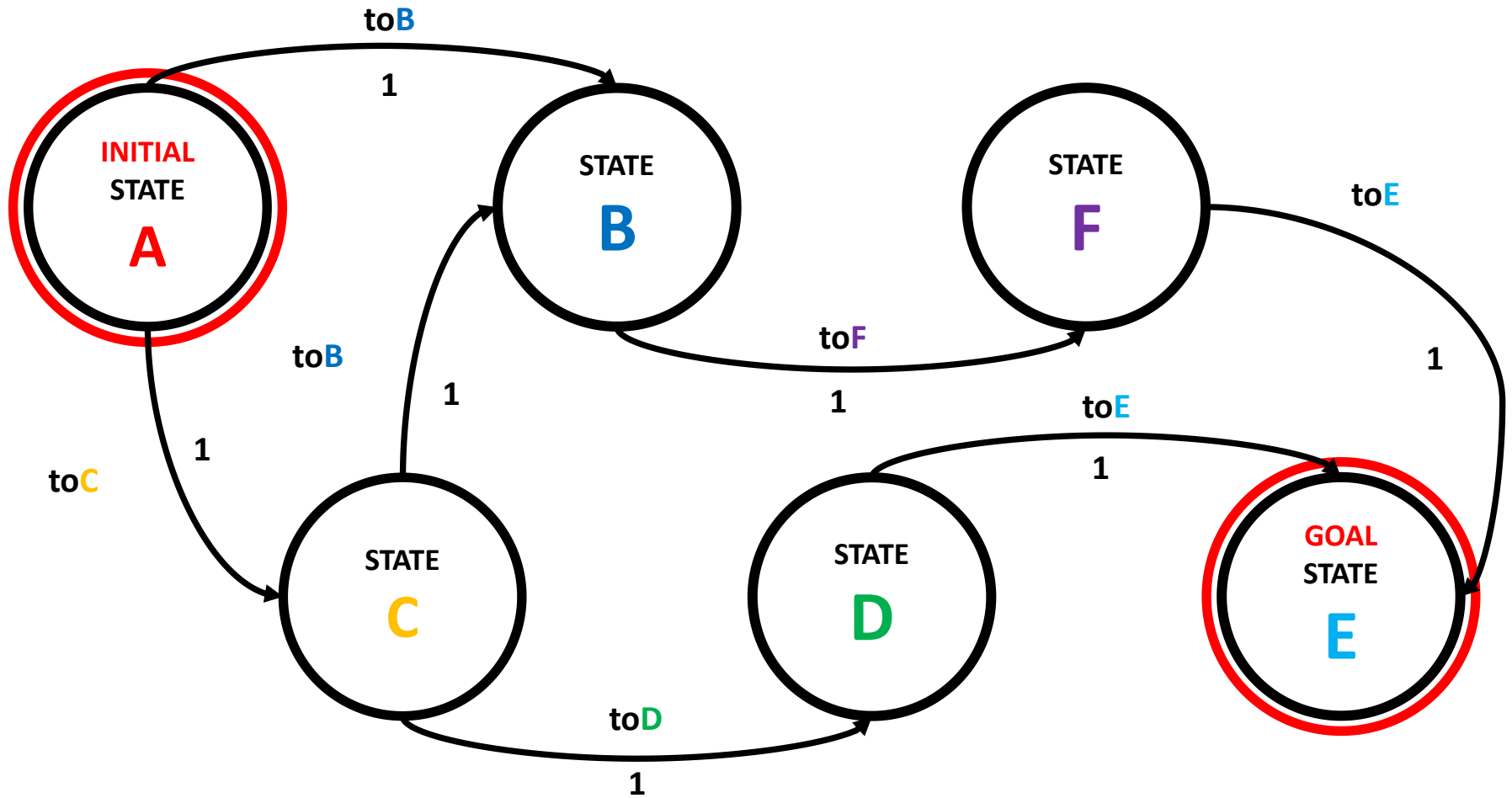
RESULT(A, toC) = C

ACTIONS(C) = {toB, toD}

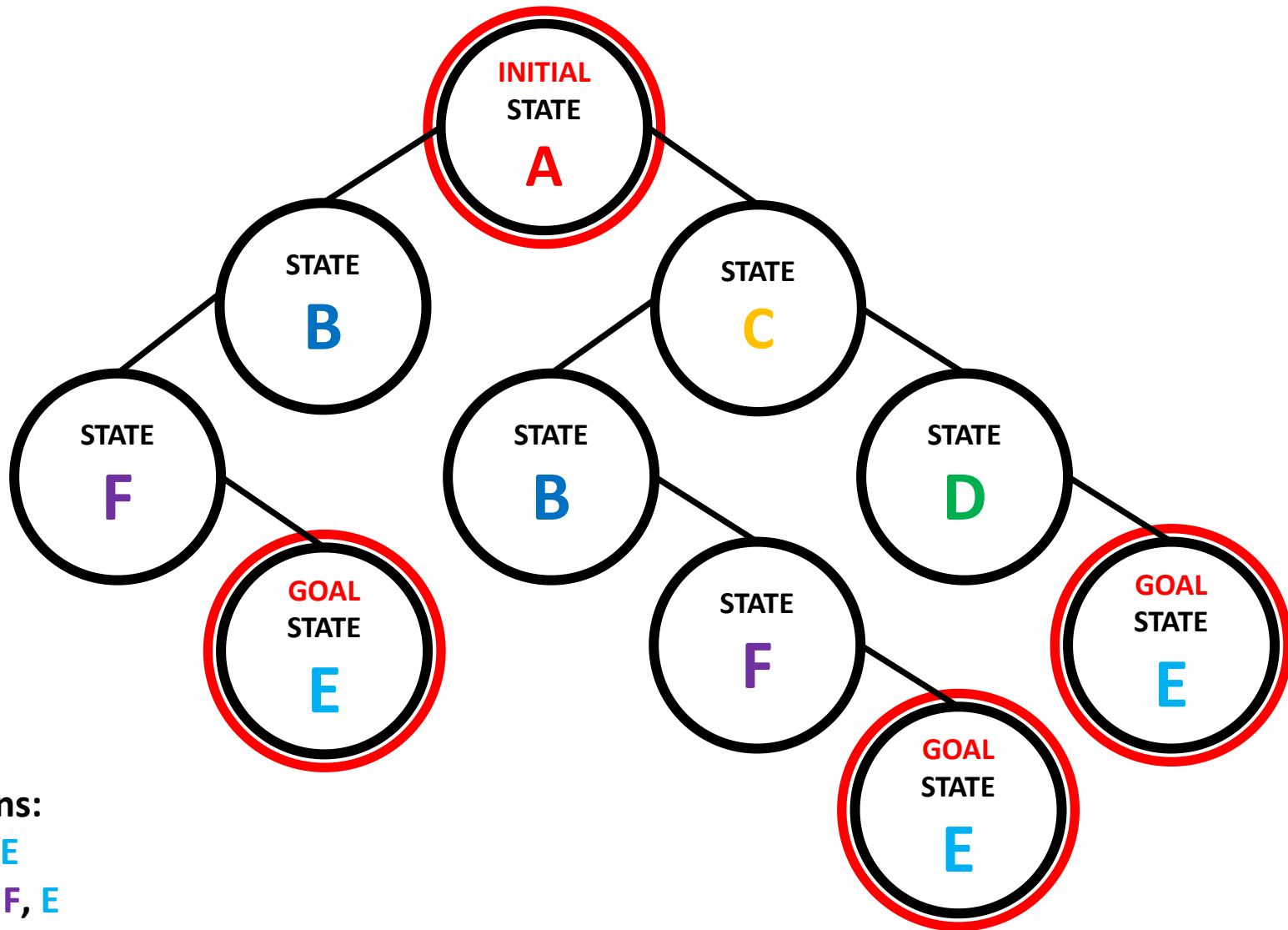
ACTIONS(D) = {toE}

ACTIONS(E) = \emptyset

State Space Model: A Graph



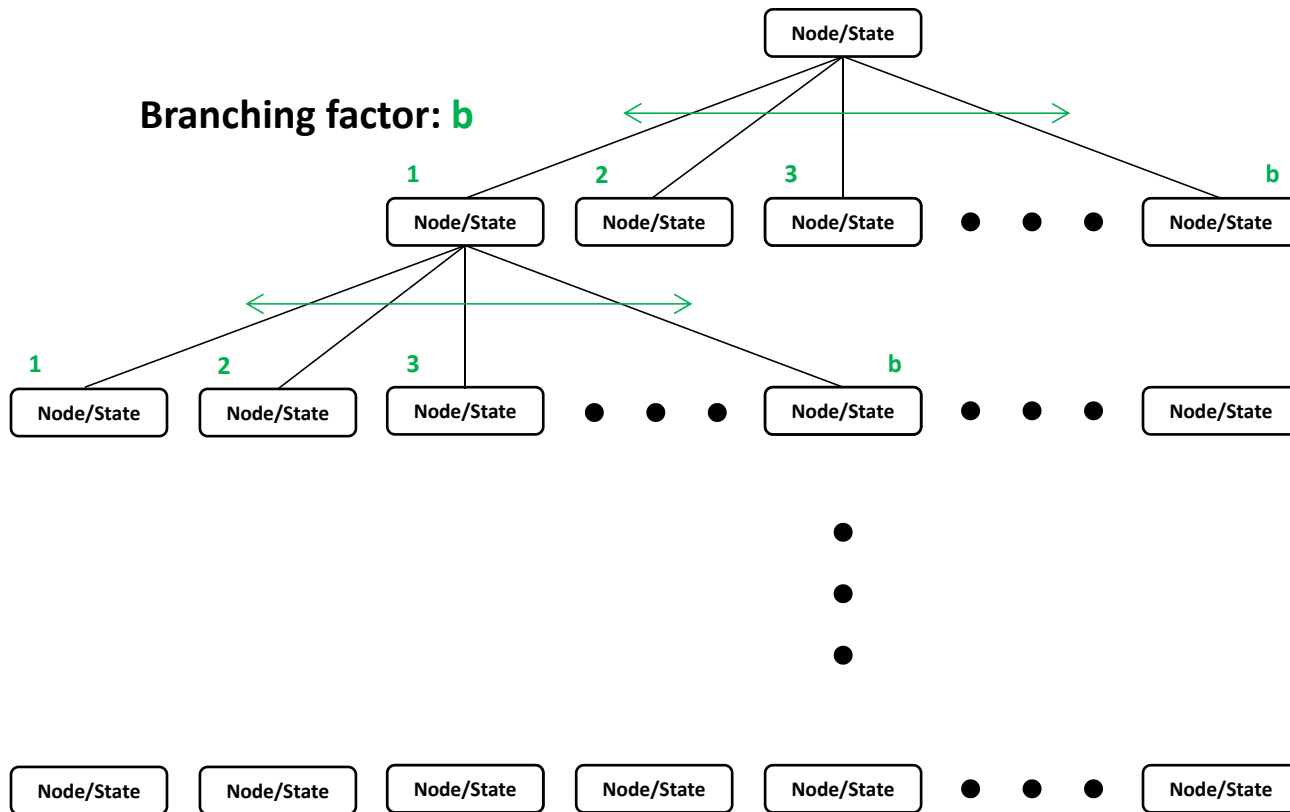
Searching State Space: Search Tree



Solutions:

A, B, F, E
A, C, B, F, E
A, C, D, E

Search Tree Challenges: Size



Depth: 0 | $N_0 = 1$

Depth: 1 | $N_1 = b$

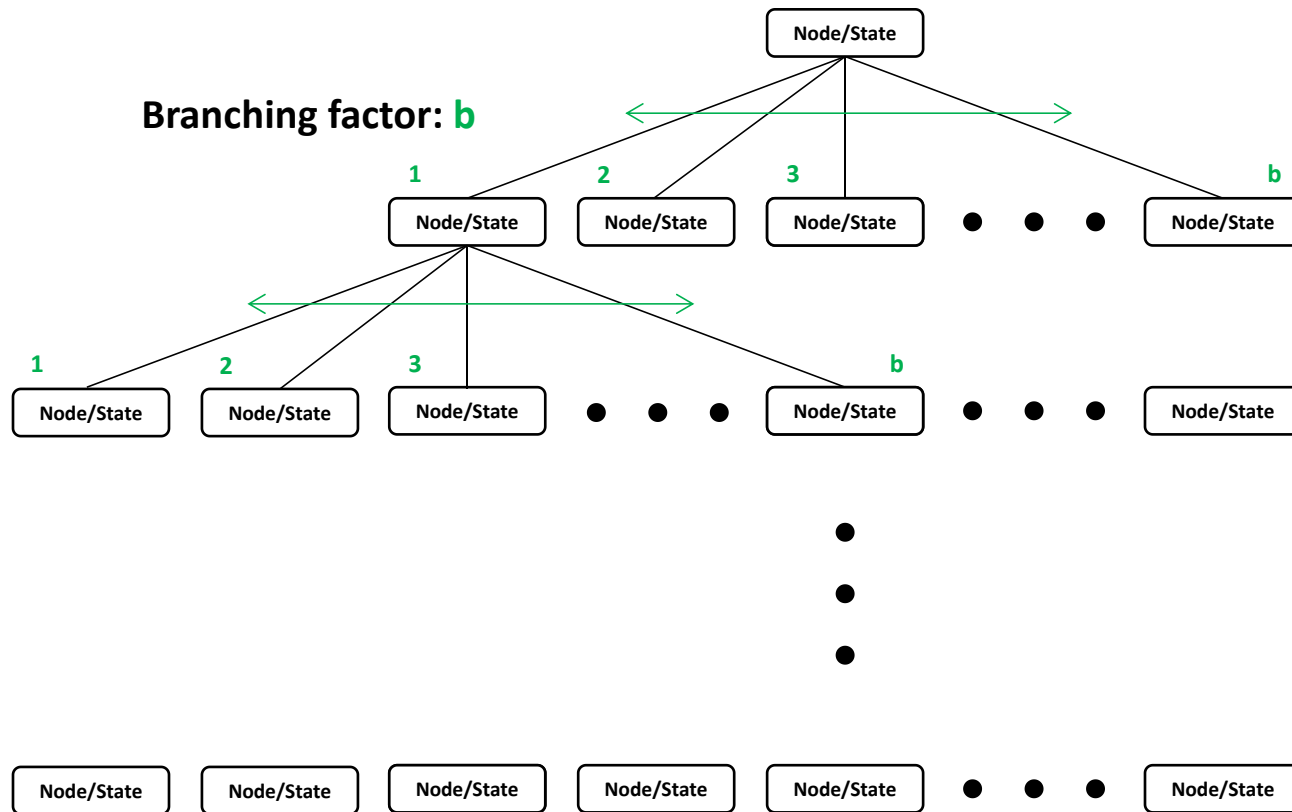
Depth: 2 | $N_2 = b^2$

Depth: d | $N_d = b^d$

Total number of nodes / states: $1 + b + b^2 + b^3 + \dots + b^d \rightarrow O(b^d)$

Quickly becomes unmanageable and impossible to search with brute force!

Search Tree Challenges: Infiniteness



Depth: 0

Depth: 1

Depth: 2

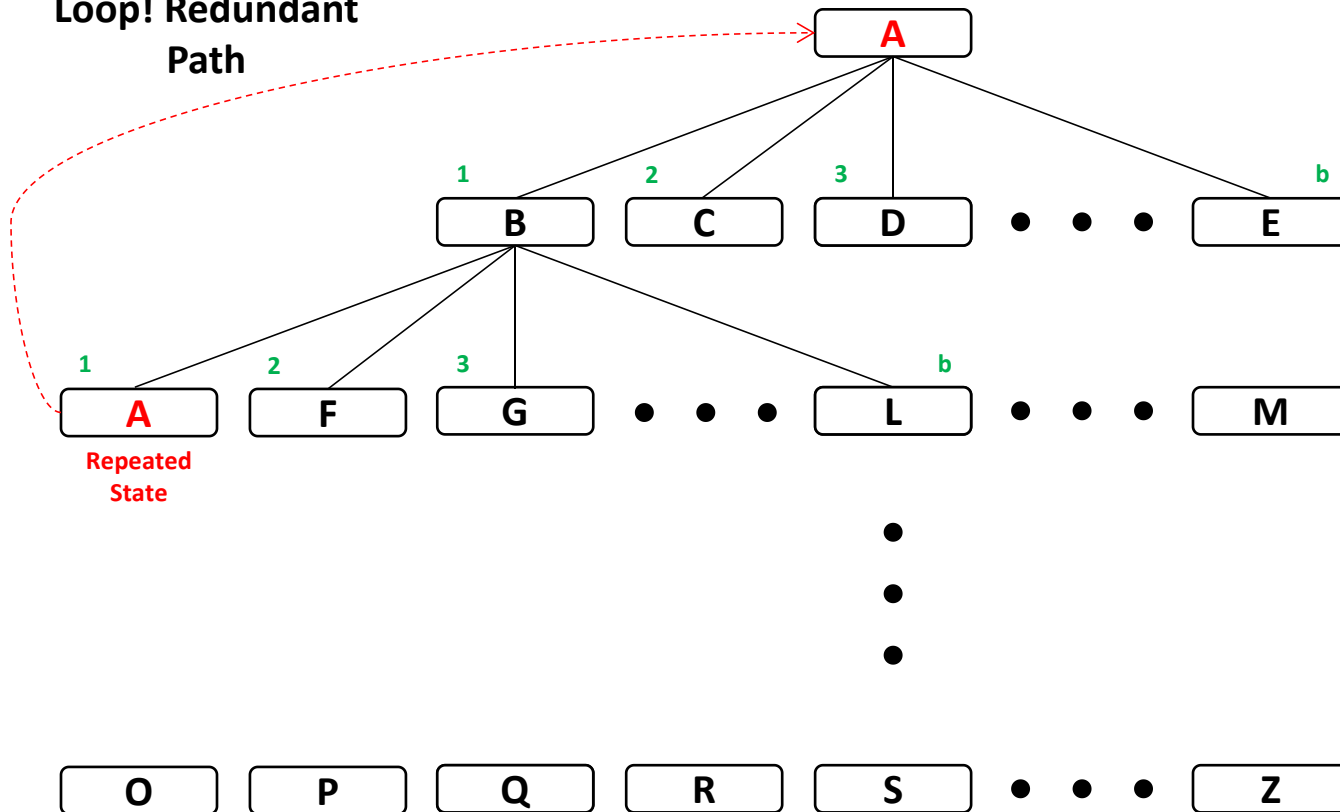
Depth: **d** $\rightarrow \infty$

Unmanageable and impossible to search with brute force!

Memory and time use grows quickly!

Search Tree Challenges: Loops

Loop! Redundant
Path



Depth: 0

Depth: 1

Depth: 2

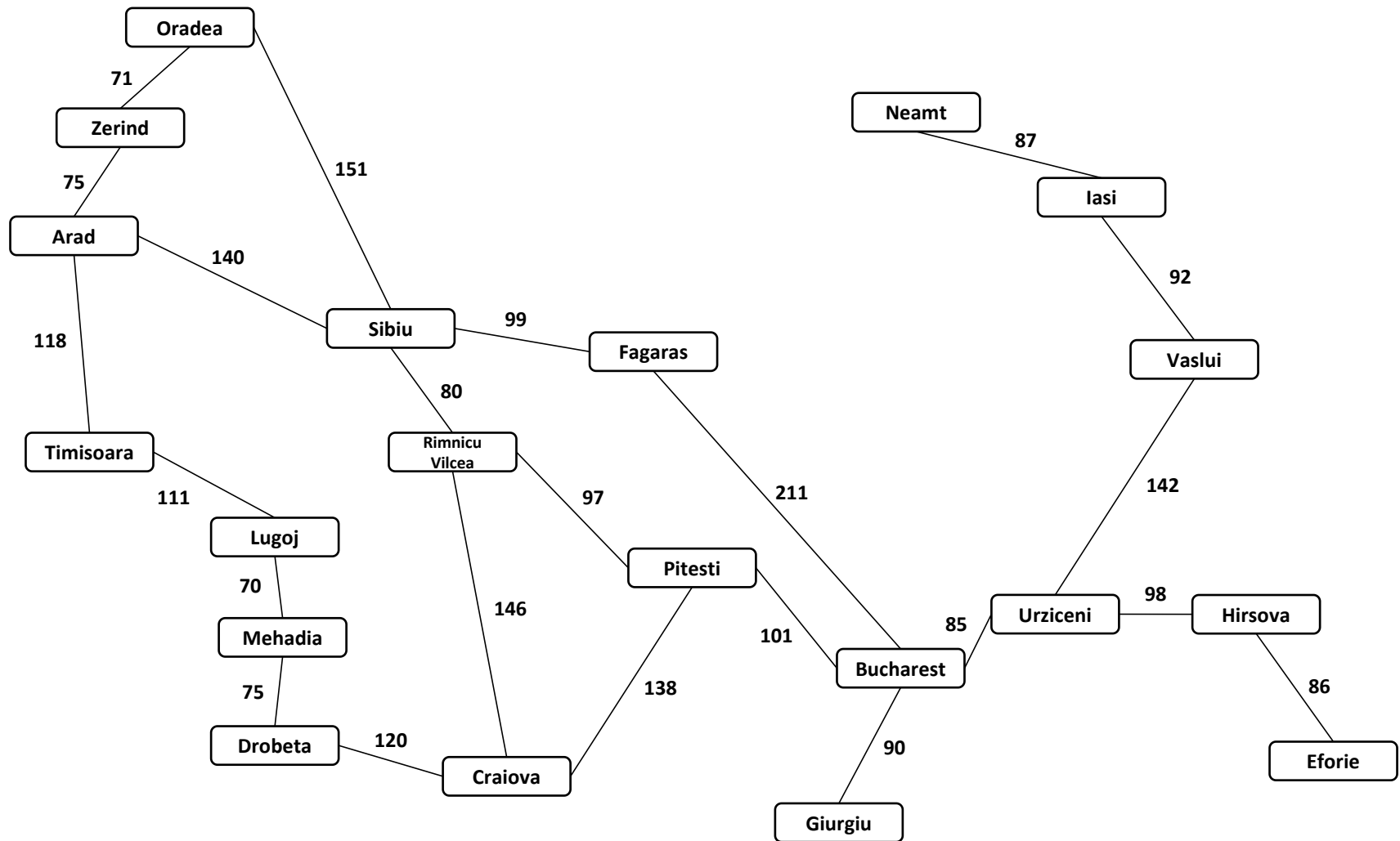
•
•
•

Depth: d

This would lead to an infinite state sequence repetition if not handled!

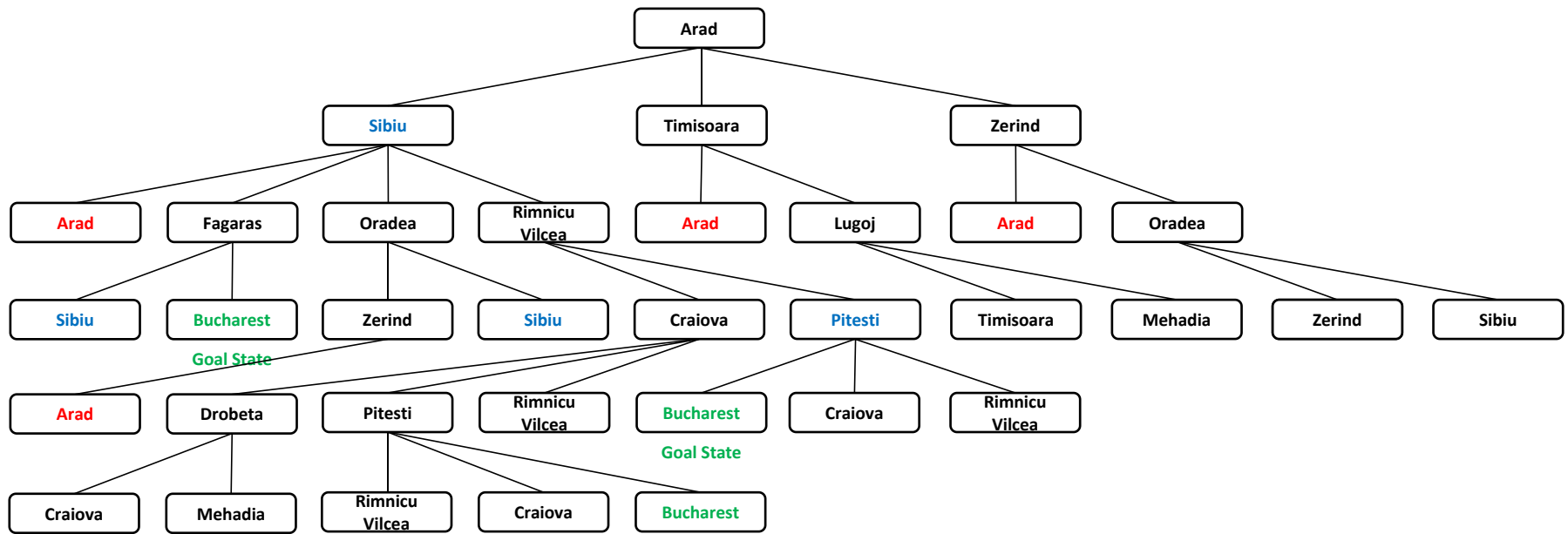
Memory and time use grows quickly!

Sample Problem: Dracula's Roadtrip



Problem: Get from Arad to Bucharest efficiently (for example: quickly or cheaply).

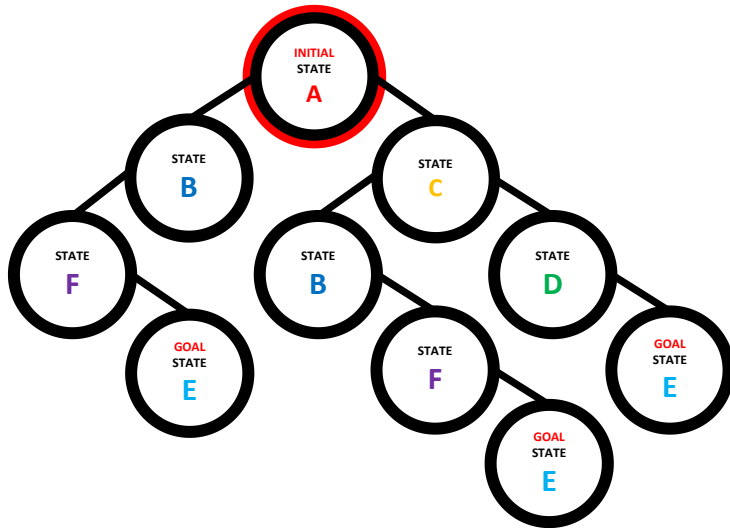
Dracula's Roadtrip as a Tree



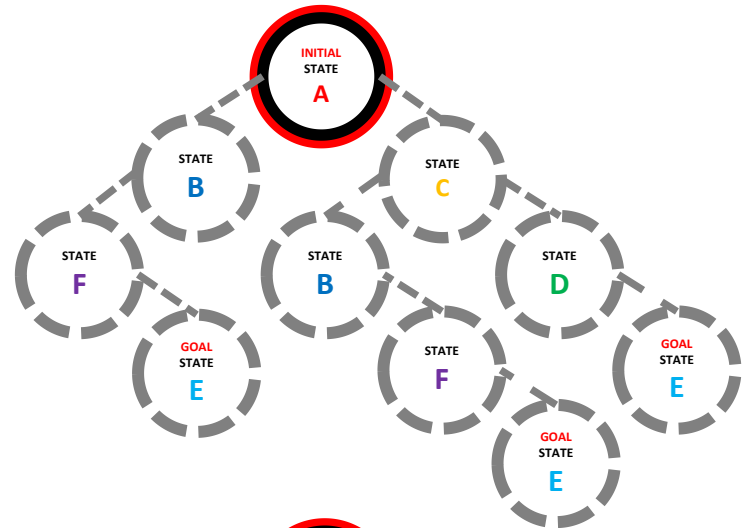
INCOMPLETE! I need to redraw it in smarter way

Search Tree: Implementations

Build entire search tree

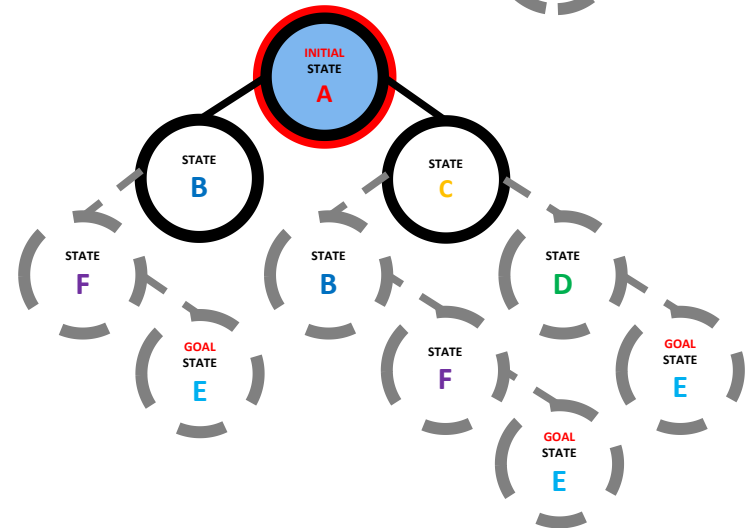


Expand/generate nodes as you go

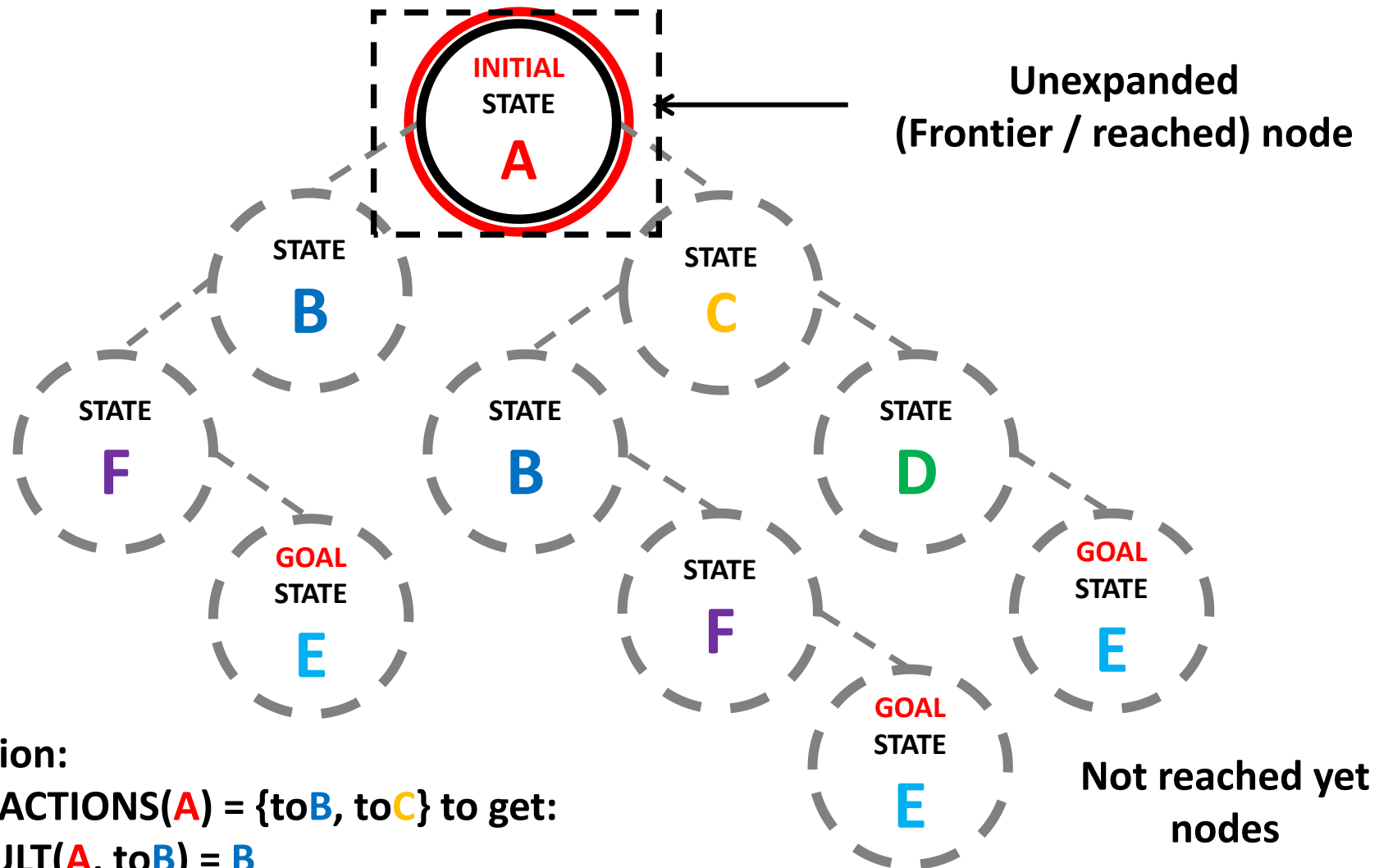


Challenges:

- memory requirements
- impossible for infinite number of states



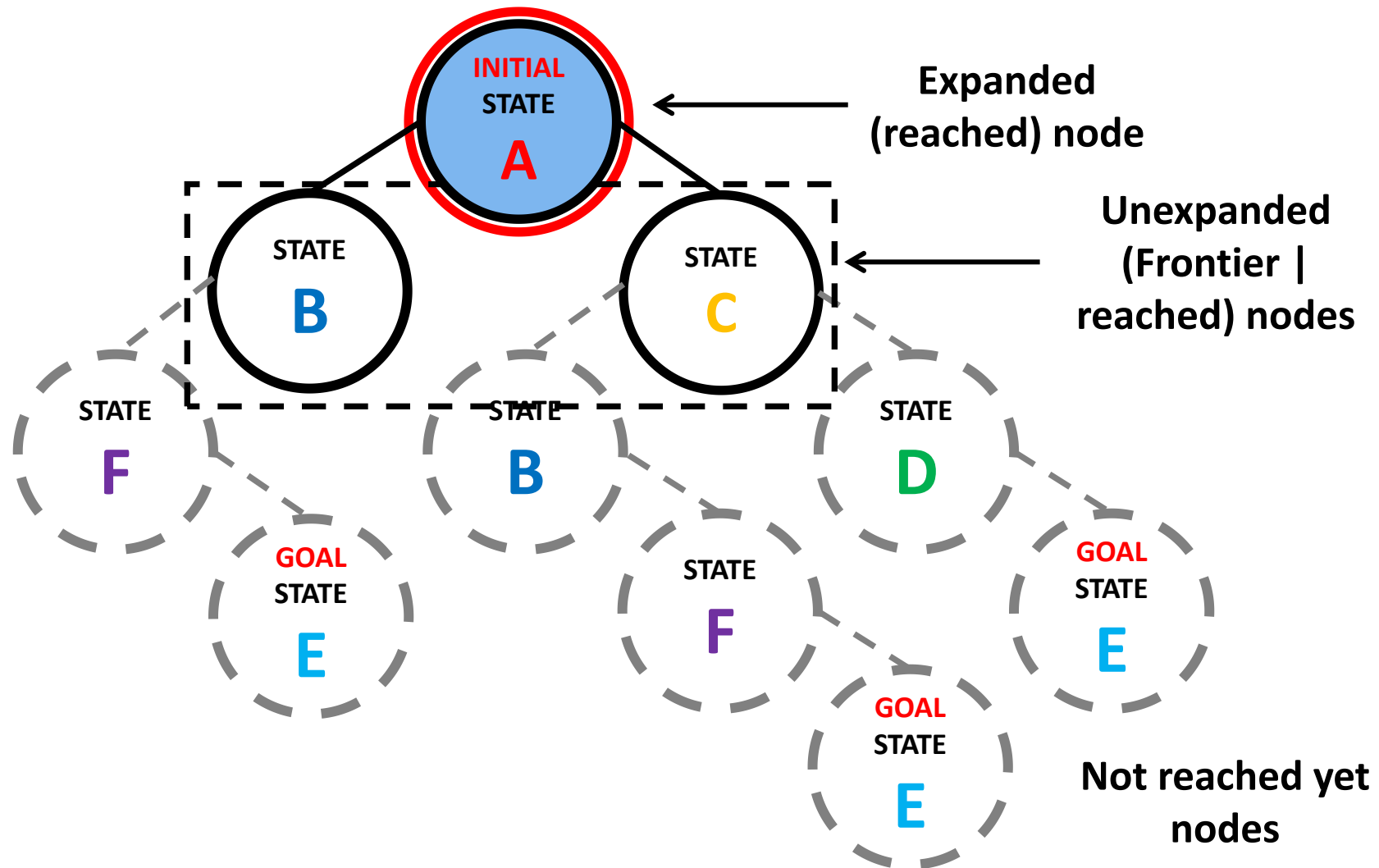
Search Tree: Node Expansion



Expansion:

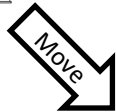
- Use $\text{ACTIONS}(\text{A}) = \{\text{toB}, \text{toC}\}$ to get:
- $\text{RESULT}(\text{A}, \text{toB}) = \text{B}$
- $\text{RESULT}(\text{A}, \text{toC}) = \text{C}$

Search Tree: Node Expansion

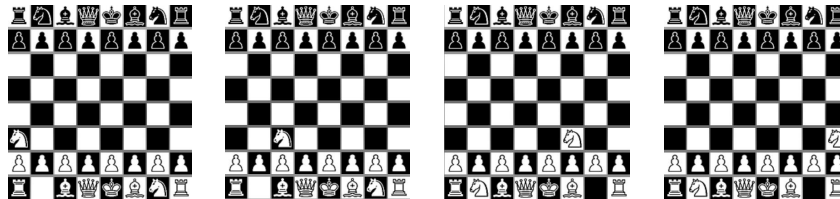


Chess: State Node Expansion

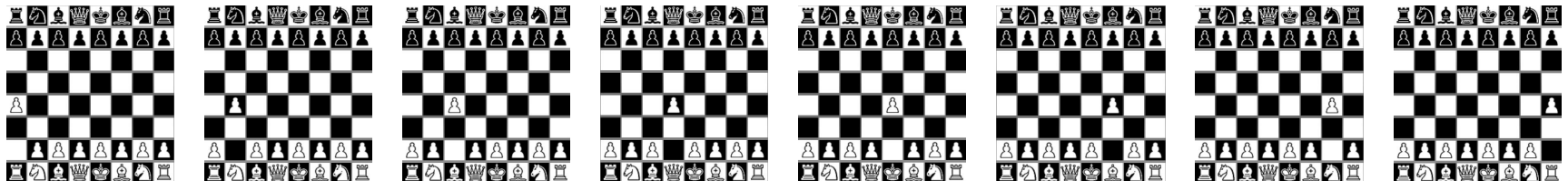
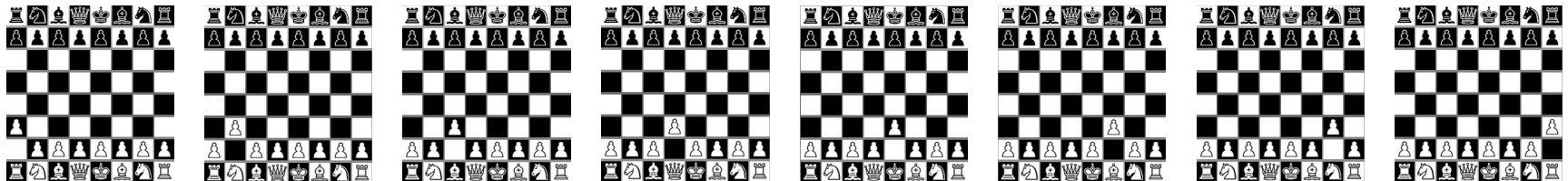
Initial
State



Use game rules to generate subsequent possible game
tree states / nodes!



20 Possible **legal** first moves:
16 pawn moves
4 knight moves



Designing the Searching Problem

**Analyze and
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**Select searching
algorithm**

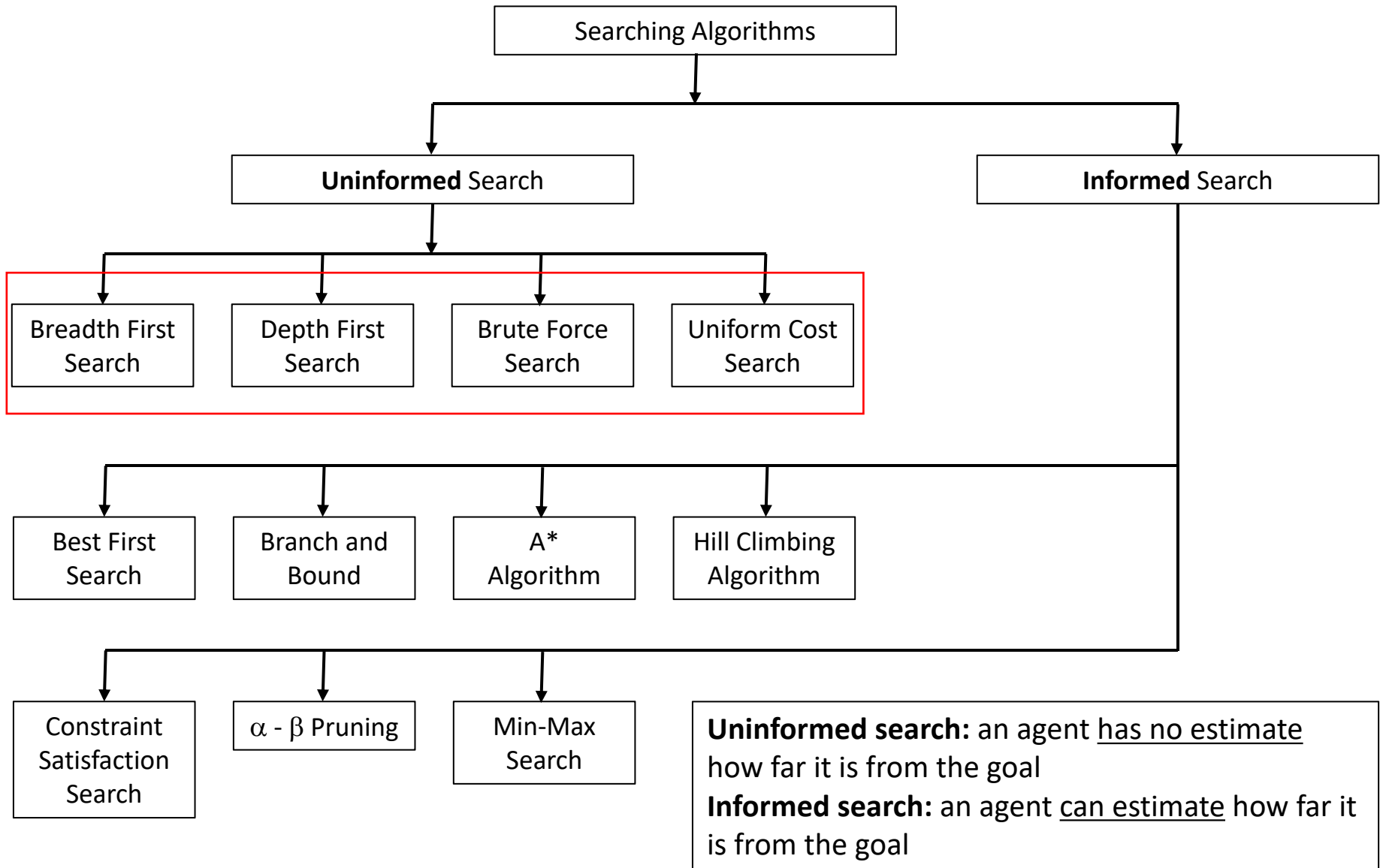
Search

Measuring Searching Performanc

Search algorithms can be evaluated in four ways:

- **Completeness**: Is the algorithm guaranteed to find a solution when there is one, and to correctly report failure when there is not?
- **Cost optimality**: Does it find a solution with the lowest path cost of all solutions?
- **Time complexity**: How long does it take to find a solution? (in seconds, actions, states, etc.)
- **Space complexity**: How much memory is needed to perform the search?

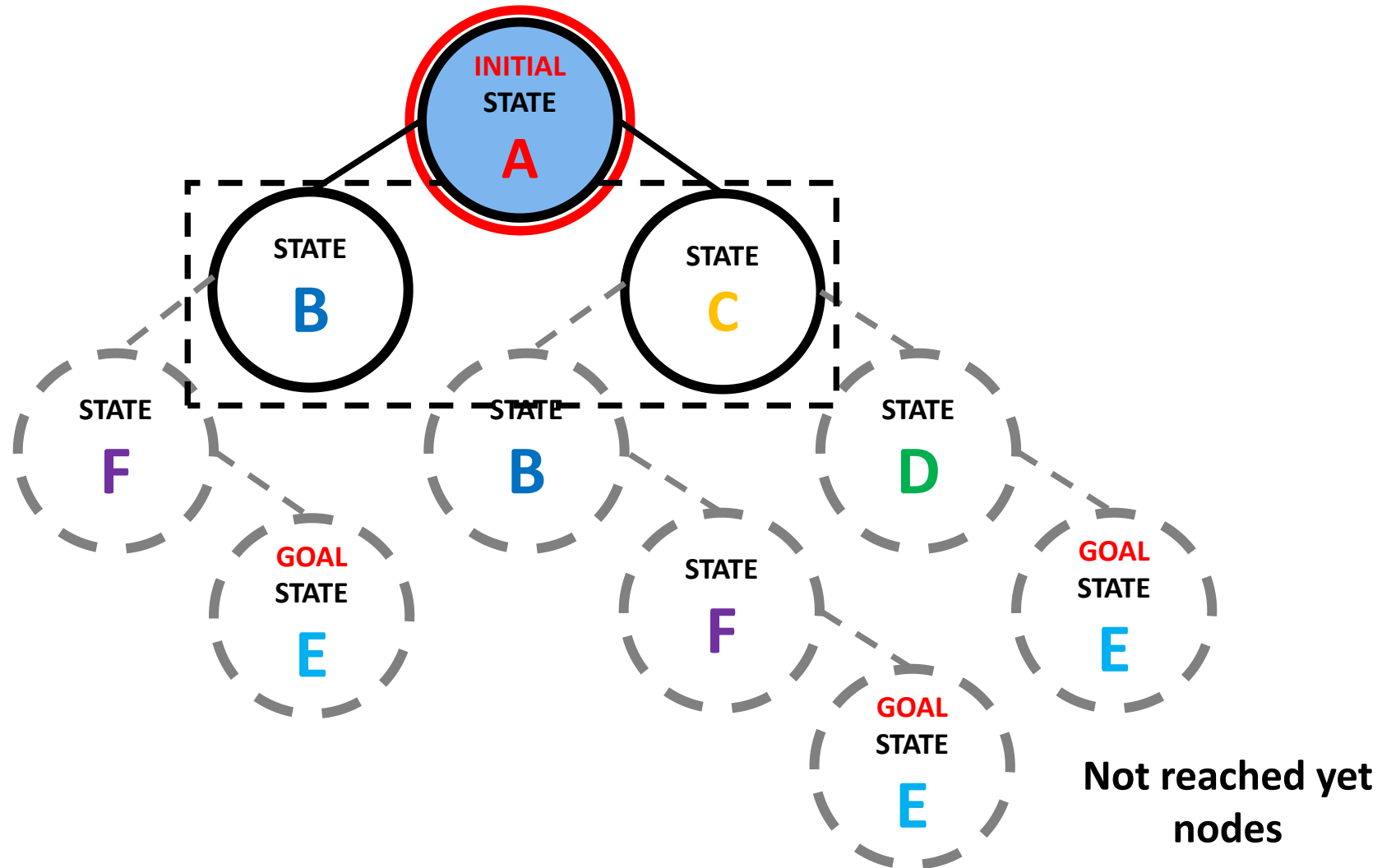
Selected Searching Algorithms



Uninformed Searching

- **Breadth First Search (BFS):**
 - Will find a solution with a minimal number of actions
 - Large memory requirement
 - Only relatively small problem instances are tractable
- **Depth First Search:**
 - May NOT find a solution with a minimal number of actions
 - Requires less memory than BFS (for tree search)
 - Backtracking (one child / successor generated at a time)
- **Brute Force Search:** depends on the approach -> bad
- **Uniform Cost Search:** minimize solution / path cost

Expansion: Which Node to Expand?



Evaluation function

Calculate / obtain:

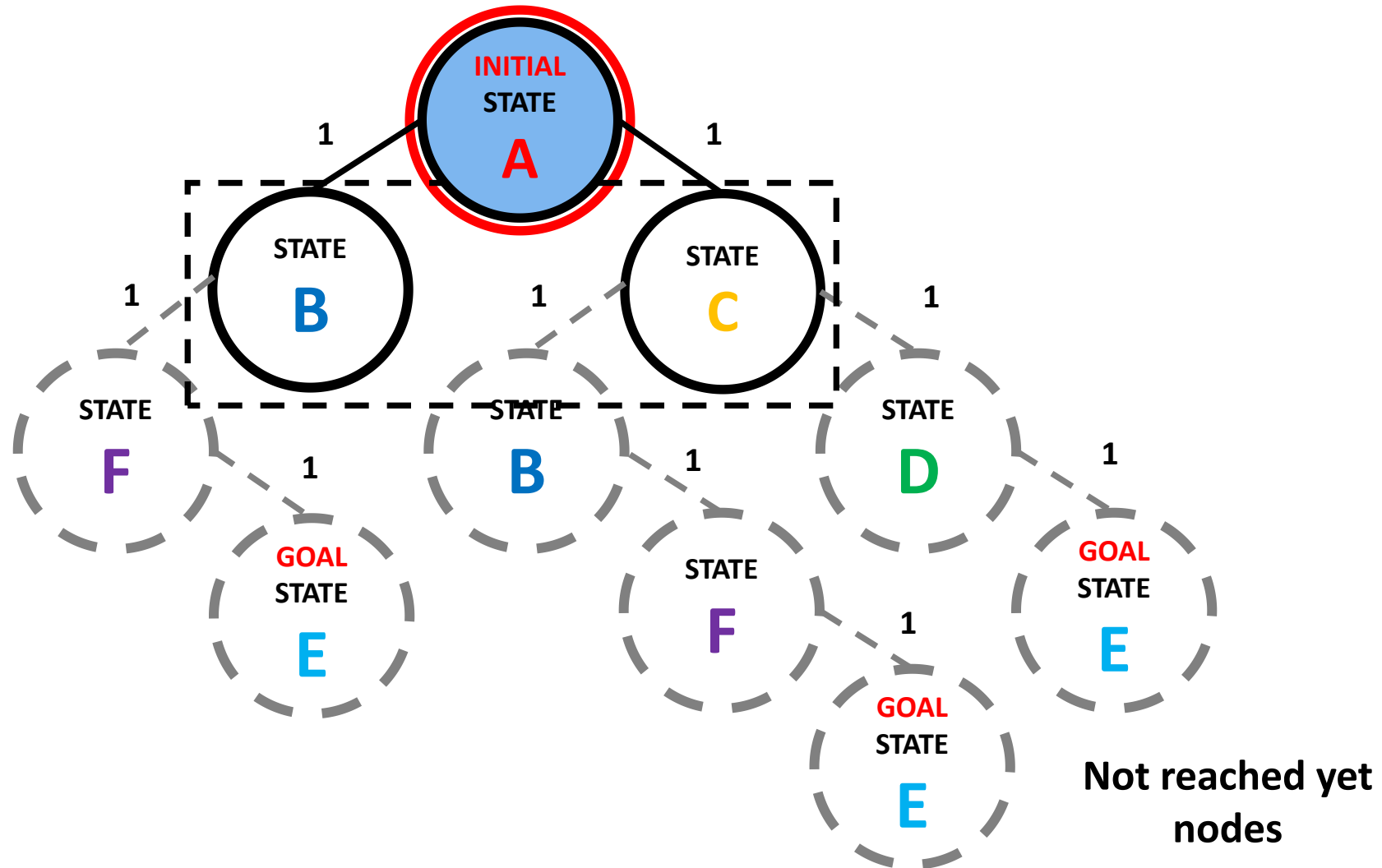
$$f(n) = f(\text{State } n)$$

$$f(n) = f(\text{relevant information about State } n)$$

**A state n with minimum $f(n)$ should be
chosen for expansion**

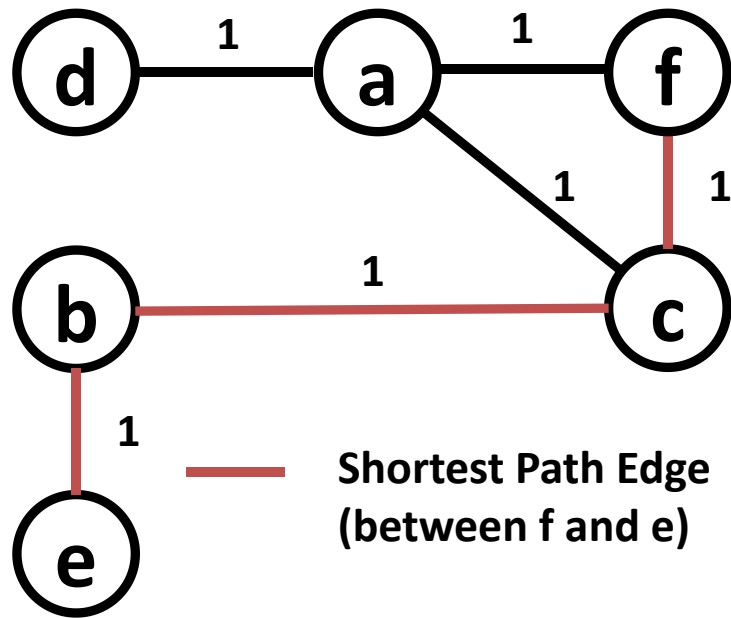
What about ties?

Search Tree: Uniform Action Cost



Uniform Cost Search | Dijkstra's Algo

Weighted Graph G



Shortest Path Edge
(between f and e)

Popular algorithms:

- Dijkstra's algorithm

Shortest Path Problem

Shortest path problem:

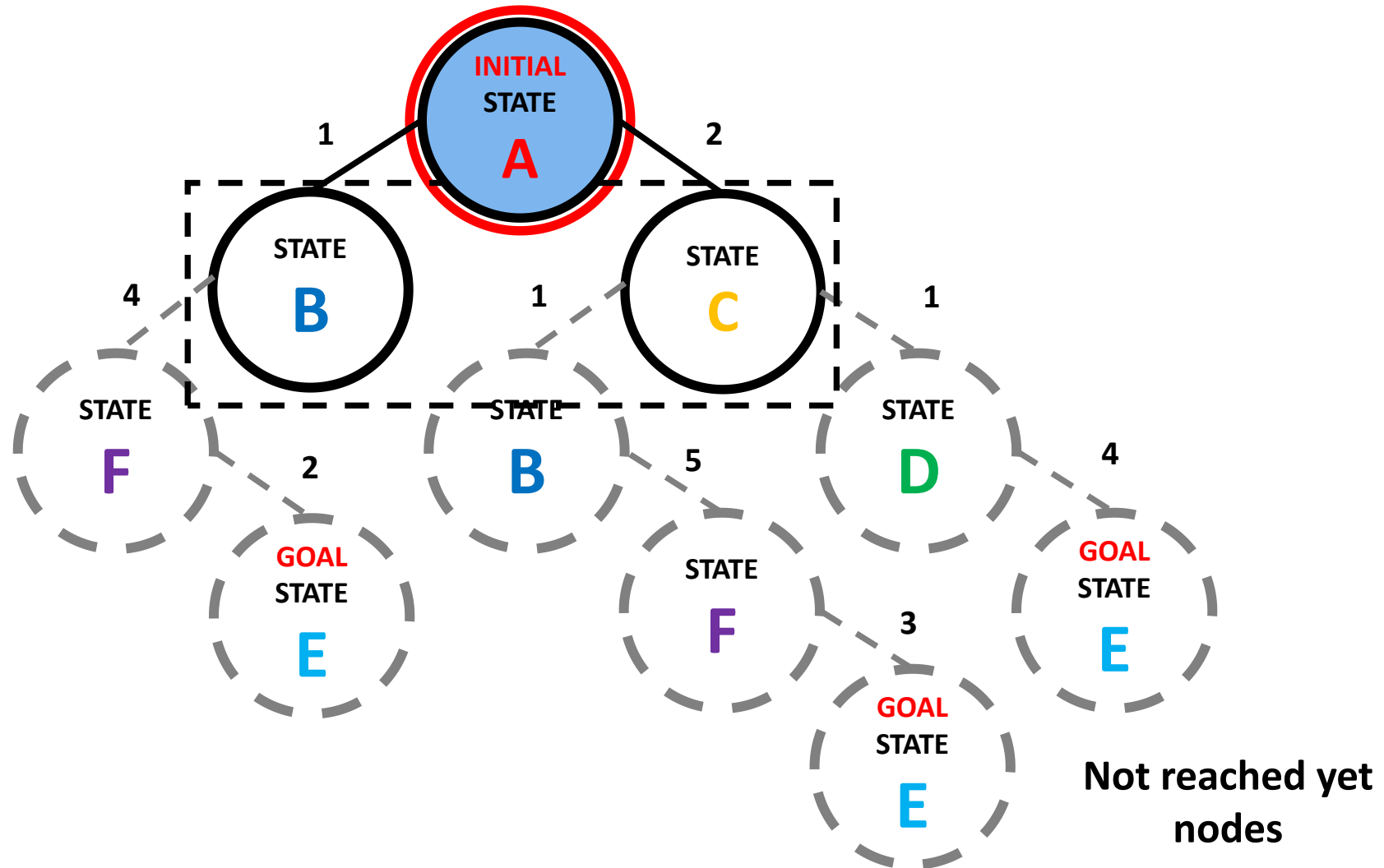
Given a weighted graph $G(V, E, w)$ and two vertices a, b in V , find the shortest path between vertices a and b (**all edge weights are equal**).

BFS and UCS: Pseudocode

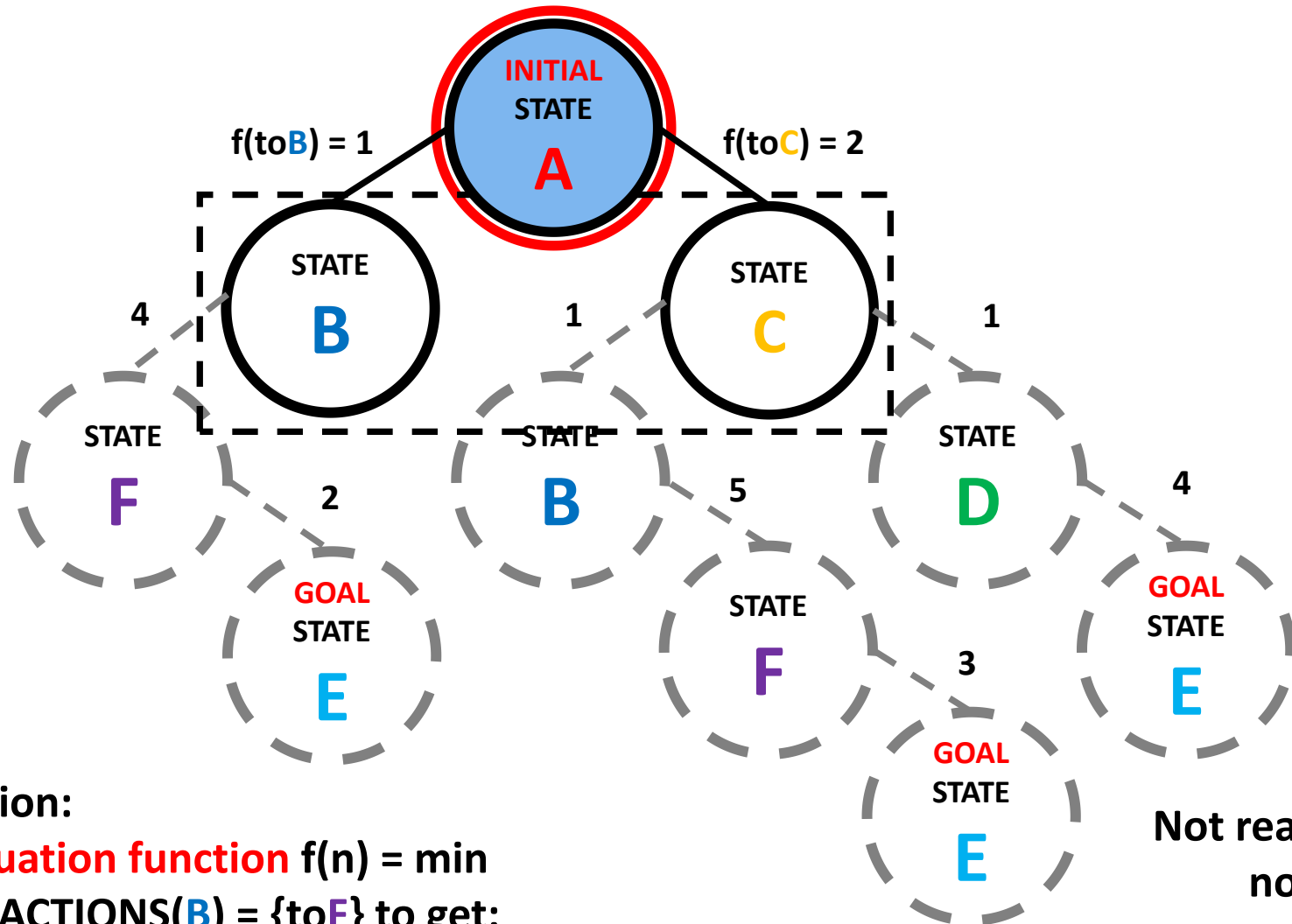
function BREADTH-FIRST-SEARCH(*problem*) **returns** a solution node or *failure*
 $node \leftarrow \text{NODE}(\text{problem.INITIAL})$
 if *problem.IS-GOAL*(*node.STATE*) **then return** *node*
 $frontier \leftarrow$ a FIFO queue, with *node* as an element
 $reached \leftarrow \{\text{problem.INITIAL}\}$
 while not IS-EMPTY(*frontier*) **do**
 $node \leftarrow \text{POP}(\text{frontier})$
 for each *child* **in** EXPAND(*problem*, *node*) **do**
 $s \leftarrow \text{child.STATE}$
 if *problem.IS-GOAL*(*s*) **then return** *child*
 if *s* is not in *reached* **then**
 add *s* to *reached*
 add *child* to *frontier*
 return *failure*

function UNIFORM-COST-SEARCH(*problem*) **returns** a solution node, or *failure*
 return BEST-FIRST-SEARCH(*problem*, PATH-COST)

Search Tree: Variable Action Cost



Search Tree: Variable Action Cost

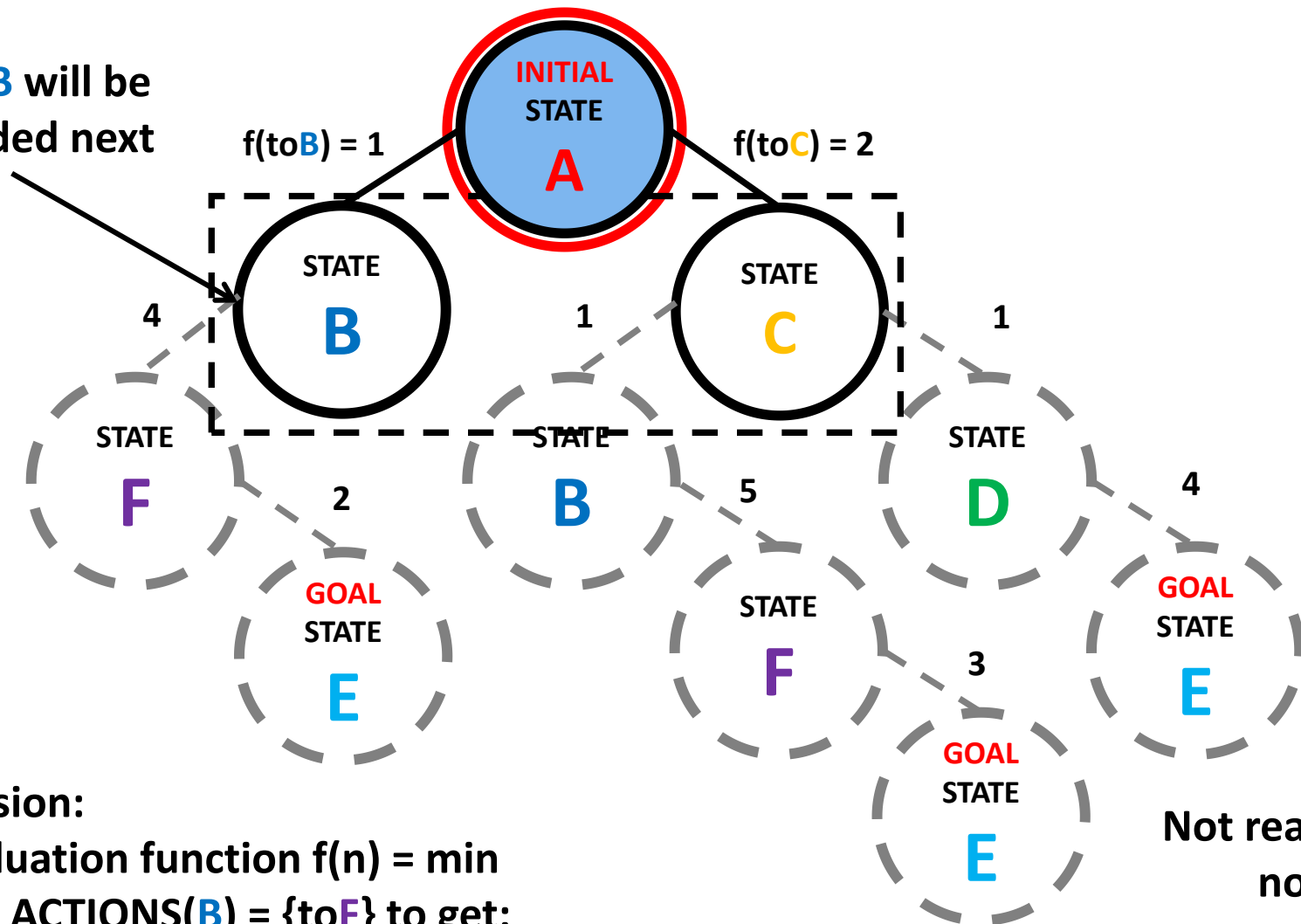


Expansion:

- **Evaluation function** $f(n) = \min$
- Use $\text{ACTIONS}(\text{B}) = \{\text{toF}\}$ to get:
- $\text{RESULT}(\text{B}, \text{toF}) = \text{F}$

Search Tree: Best-First Search

Node **B** will be expanded next



Expansion:

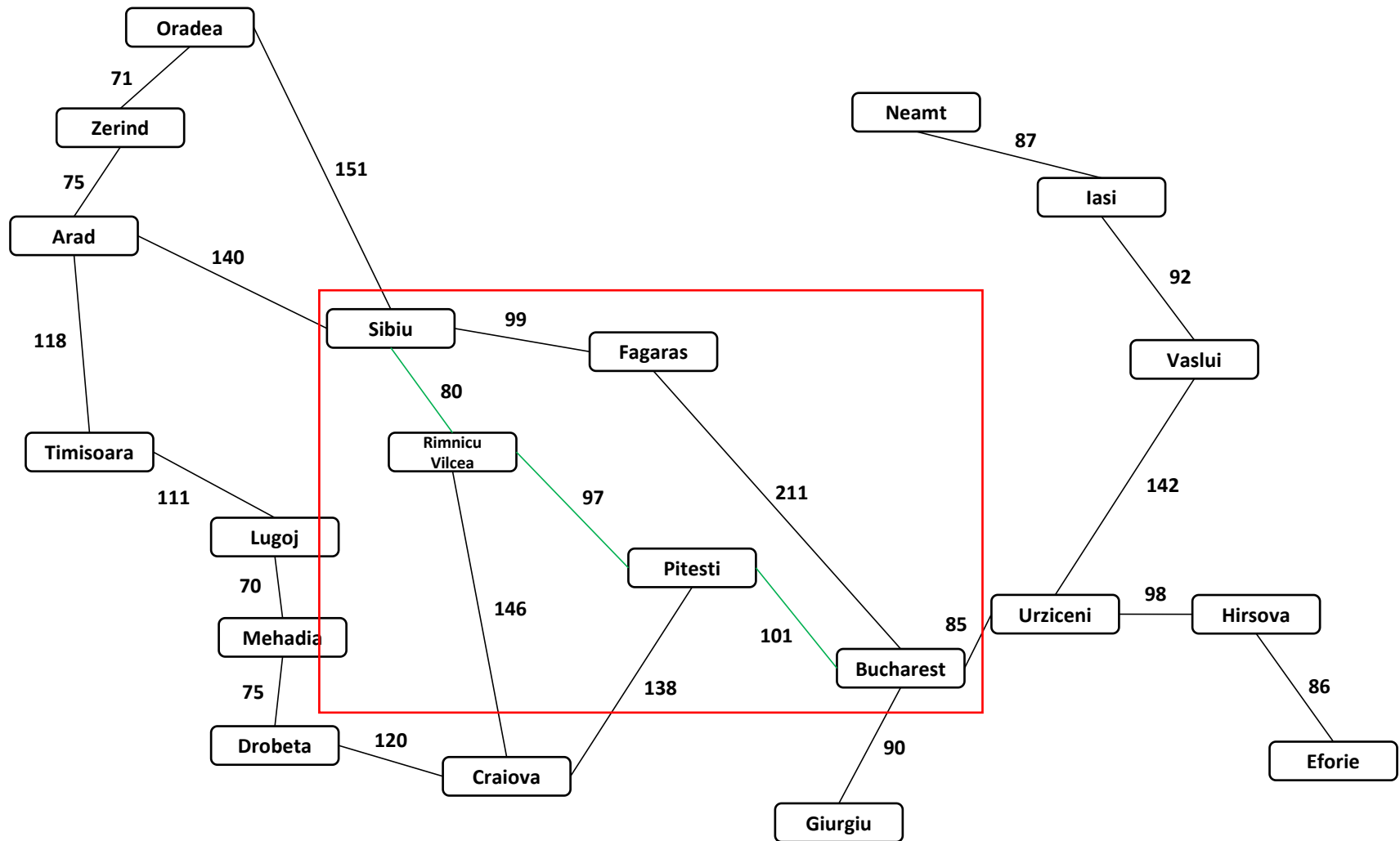
- Evaluation function $f(n) = \min$
- Use $\text{ACTIONS}(\mathbf{B}) = \{\text{toF}\}$ to get:
- $\text{RESULT}(\mathbf{B}, \text{toF}) = \mathbf{F}$

Best-First Search: Pseudocode

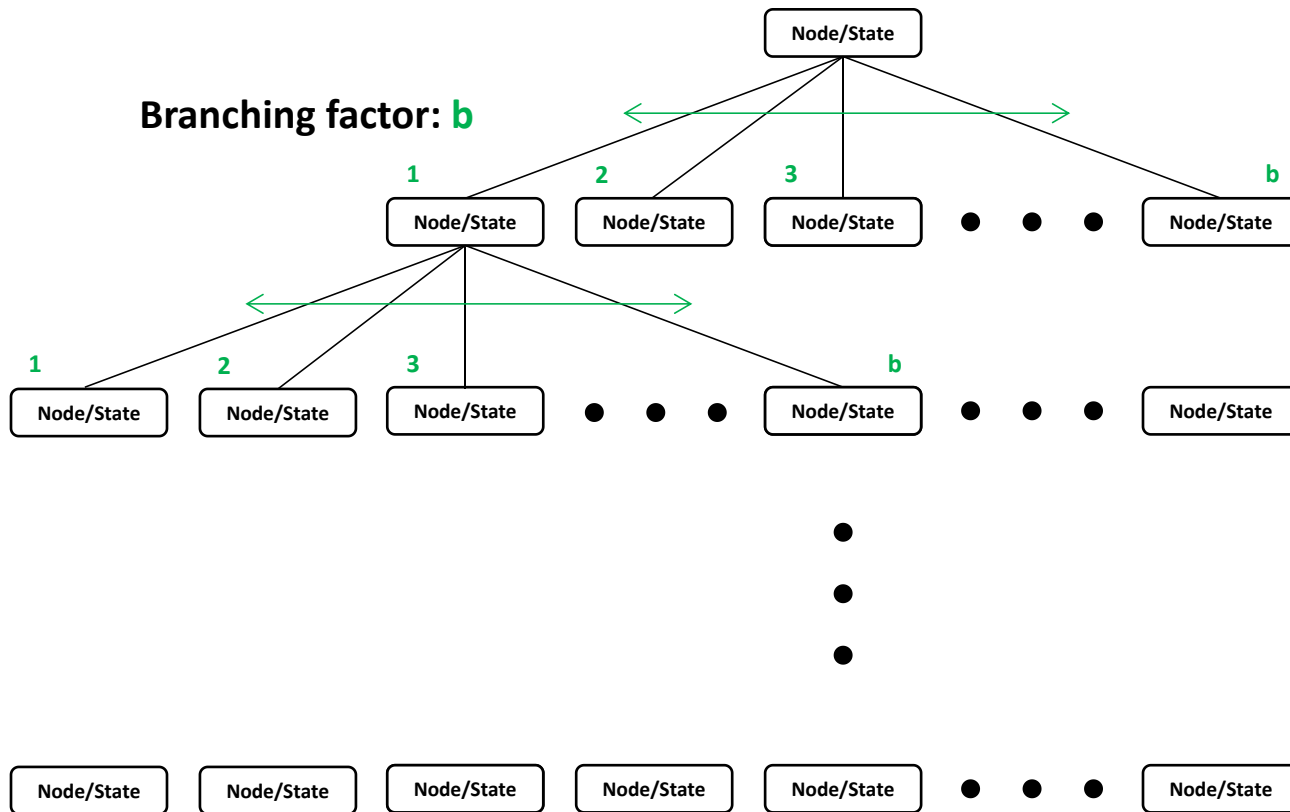
```
function BEST-FIRST-SEARCH(problem, f) returns a solution node or failure  
  node  $\leftarrow$  NODE(STATE=problem.INITIAL)  
  frontier  $\leftarrow$  a priority queue ordered by f, with node as an element  
  reached  $\leftarrow$  a lookup table, with one entry with key problem.INITIAL and value node  
  while not IS-EMPTY(frontier) do  
    node  $\leftarrow$  POP(frontier)  
    if problem.IS-GOAL(node.STATE) then return node  
    for each child in EXPAND(problem, node) do  
      s  $\leftarrow$  child.STATE  
      if s is not in reached or child.PATH-COST < reached[s].PATH-COST then  
        reached[s]  $\leftarrow$  child  
        add child to frontier  
  return failure
```

```
function EXPAND(problem, node) yields nodes  
  s  $\leftarrow$  node.STATE  
  for each action in problem.ACTIONS(s) do  
    s'  $\leftarrow$  problem.RESULT(s, action)  
    cost  $\leftarrow$  node.PATH-COST + problem.ACTION-COST(s, action, s')  
    yield NODE(STATE=s', PARENT=node, ACTION=action, PATH-COST=cost)
```

Best First Search: Issue



Let's Go Back to Depth First Search



Depth: 0 | $N_0 = 1$

Depth: 1 | $N_1 = b$

Depth: 2 | $N_2 = b^2$

Depth: d | $N_d = b^d$

Tree depth is an issue!

“Controlled” DFS: Pseudocode

function ITERATIVE-DEEPENING-SEARCH(*problem*) **returns** a solution node or *failure*
 for *depth* = 0 **to** ∞ **do**
 result \leftarrow DEPTH-LIMITED-SEARCH(*problem*, *depth*)
 if *result* \neq *cutoff* **then return** *result*

function DEPTH-LIMITED-SEARCH(*problem*, ℓ) **returns** a node or *failure* or *cutoff*
 frontier \leftarrow a LIFO queue (stack) with NODE(*problem*.INITIAL) as an element
 result \leftarrow *failure*
 while not IS-EMPTY(*frontier*) **do**
 node \leftarrow POP(*frontier*)
 if *problem*.IS-GOAL(*node*.STATE) **then return** *node*
 if DEPTH(*node*) > ℓ **then**
 result \leftarrow *cutoff*
 else if not IS-CYCLE(*node*) **do**
 for each *child* **in** EXPAND(*problem*, *node*) **do**
 add *child* to *frontier*
 return *result*

Iterative Deepening DFS: Illustration

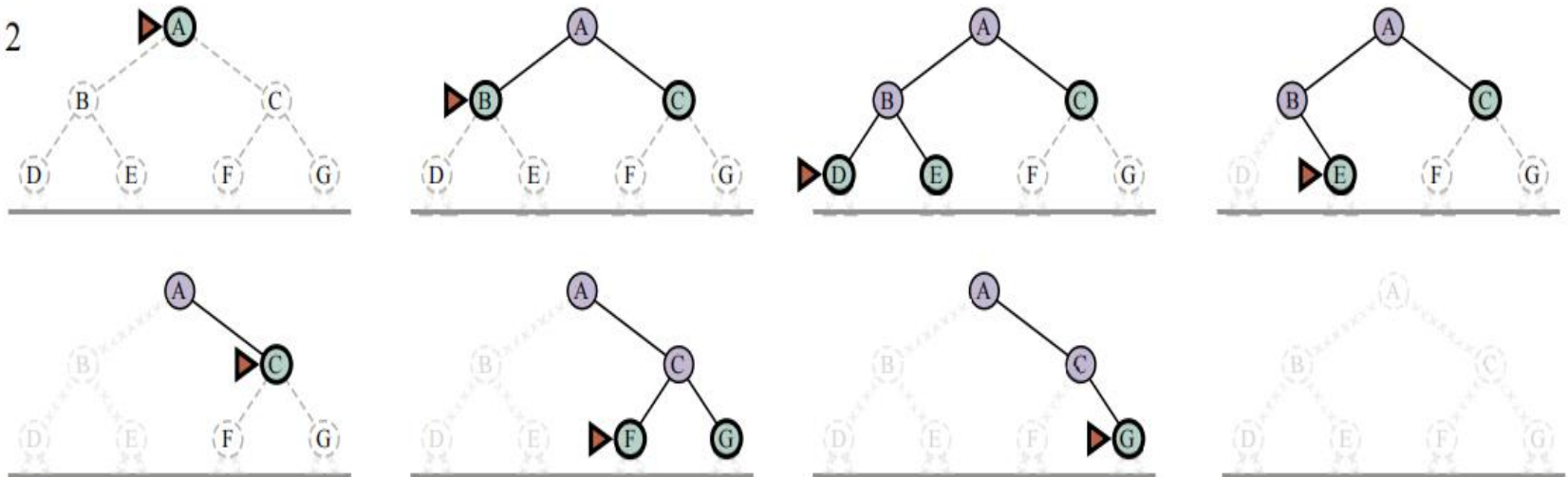
limit: 0



limit: 1

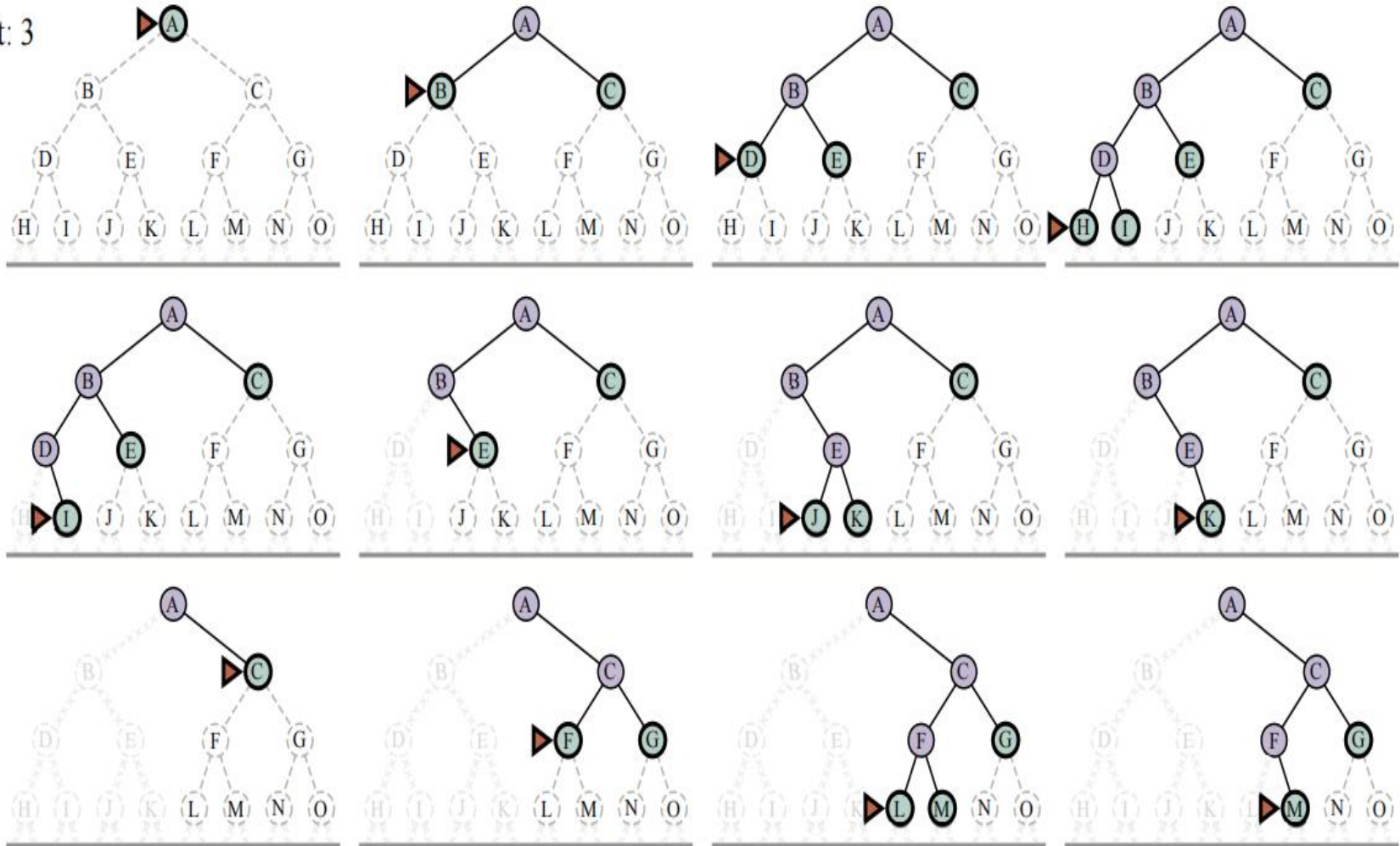


limit: 2



Iterative Deepening DFS: Illustration

limit: 3

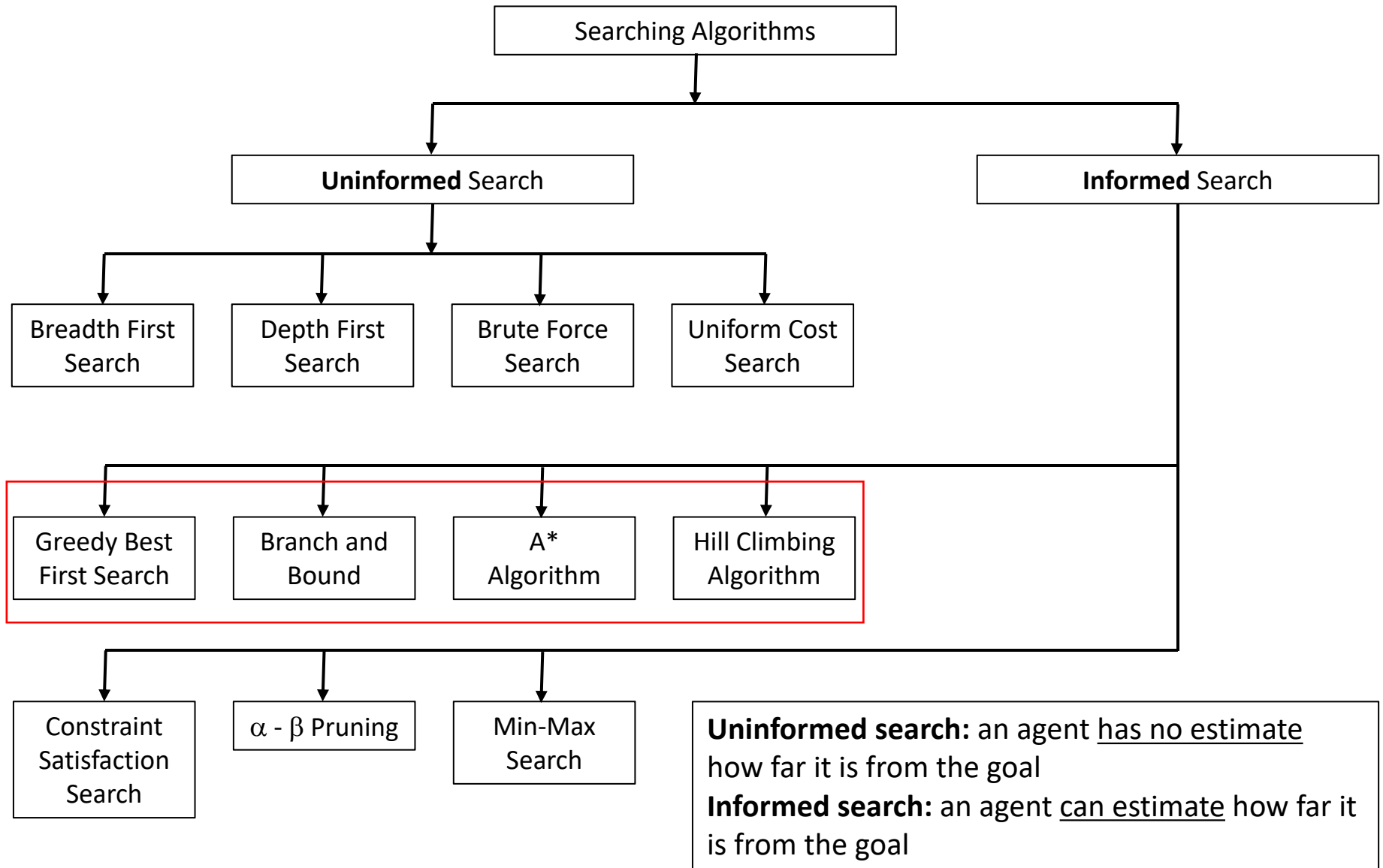


Uninformed Search Algorithms

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Complete?	Yes ¹	Yes ^{1,2}	No	No	Yes ¹	Yes ^{1,4}
Optimal cost?	Yes ³	Yes	No	No	Yes ³	Yes ^{3,4}
Time	$O(b^d)$	$O(b^{1+\lceil C^*/\epsilon \rceil})$	$O(b^m)$	$O(b^\ell)$	$O(b^d)$	$O(b^{d/2})$
Space	$O(b^d)$	$O(b^{1+\lceil C^*/\epsilon \rceil})$	$O(bm)$	$O(b\ell)$	$O(bd)$	$O(b^{d/2})$

Figure 3.15 Evaluation of search algorithms. b is the branching factor; m is the maximum depth of the search tree; d is the depth of the shallowest solution, or is m when there is no solution; ℓ is the depth limit. Superscript caveats are as follows: ¹ complete if b is finite, and the state space either has a solution or is finite. ² complete if all action costs are $\geq \epsilon > 0$; ³ cost-optimal if action costs are all identical; ⁴ if both directions are breadth-first or uniform-cost.

Selected Searching Algorithms



Informed Search and Heuristics

Informed search relies on **domain-specific knowledge / hints** that help locate the goal state.

$$h(n) = h(\text{State } n)$$

$$h(n) = n(\text{relevant information about State } n)$$

$h(n)$: heuristic function - estimated cost of the cheapest path from State n to the goal state