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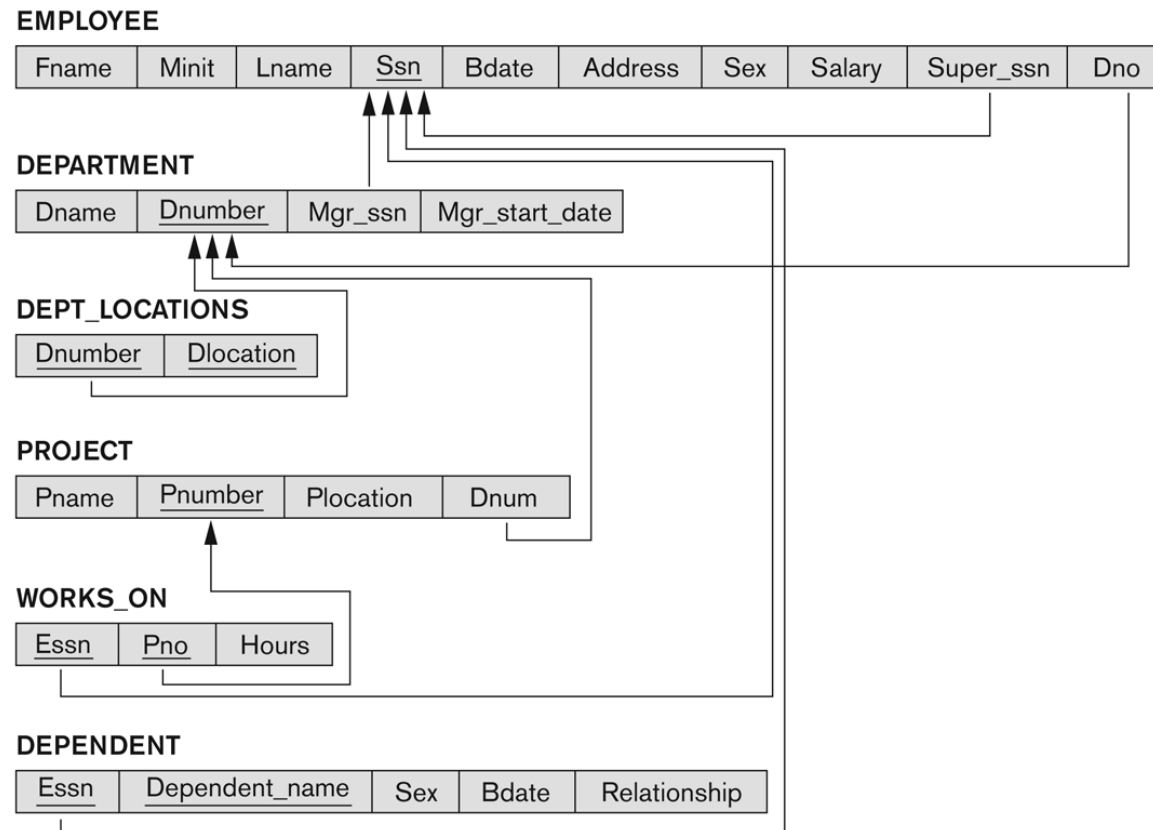
March 2021

4CCS1DBS – Database Systems

Week 8 – Relational Algebra 2

King's College London

Database Schema for COMPANY

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno

DEPARTMENT

Dname	Dnumber	Mgr_ssn	Mgr_start_date

DEPT_LOCATIONS

Dnumber	Dlocation

PROJECT

Pname	Pnumber	Plocation	Dnum

WORKS_ON

Essn	Pno	Hours

DEPENDENT

Essn	Dependent_name	Sex	Bdate	Relationship

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

DEPARTMENT

Dname	Dnumber	Mgr_ssn	Mgr_start_date
Research	5	333445555	1988-05-22
Administration	4	987654321	1995-01-01
Headquarters	1	888665555	1981-06-19

DEPT_LOCATIONS

Dnumber	Dlocation
1	Houston
4	Stafford
5	Bellaire
5	Sugarland
5	Houston

WORKS_ON

Essn	Pno	Hours
123456789	1	32.5
123456789	2	7.5
666884444	3	40.0
453453453	1	20.0
453453453	2	20.0
333445555	2	10.0
333445555	3	10.0
333445555	10	10.0
333445555	20	10.0
999887777	30	30.0
999887777	10	10.0
987987987	10	35.0
987987987	30	5.0
987654321	30	20.0
987654321	20	15.0
888665555	20	NULL

PROJECT

Pname	Pnumber	Plocation	Dnum
ProductX	1	Bellaire	5
ProductY	2	Sugarland	5
ProductZ	3	Houston	5
Computerization	10	Stafford	4
Reorganization	20	Houston	1
Newbenefits	30	Stafford	4

DEPENDENT

Essn	Dependent_name	Sex	Bdate	Relationship
333445555	Alice	F	1986-04-05	Daughter
333445555	Theodore	M	1983-10-25	Son
333445555	Joy	F	1958-05-03	Spouse
987654321	Abner	M	1942-02-28	Spouse
123456789	Michael	M	1988-01-04	Son
123456789	Alice	F	1988-12-30	Daughter
123456789	Elizabeth	F	1967-05-05	Spouse

Relational Algebra Overview

- Relational Algebra consists of several groups of operations
  - Unary Relational Operations
    - SELECT (symbol: σ (sigma))
    - PROJECT (symbol: π (pi))
    - RENAME (symbol: ρ (rho))
  - Relational Algebra Operations From Set Theory
    - UNION ($\cup$), INTERSECTION ($\cap$), DIFFERENCE (or MINUS, –)
    - CARTESIAN PRODUCT (**x**)
  - Binary Relational Operations
    - JOIN (several variations of JOIN exist)
    - DIVISION
  - Additional Relational Operations
    - OUTER JOINS, OUTER UNION
    - AGGREGATE FUNCTIONS (These compute summary of information: for example, SUM, COUNT, AVG, MIN, MAX)

Review: Relational Algebra Expressions

Q1) How many tuples are returned from the following query?
Q2) How many attributes are returned?
Q3) What is the name of the relation?

TEMP ← σ $_{DNO=5}$ (EMPLOYEE)

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

Review: Relational Algebra Expressions

Q1) How many tuples are returned from the following query?
Q2) How many attributes are returned?
Q3) What is the name of the relation?

$$\text{TEMP} \leftarrow \sigma_{DNO=5} (\text{EMPLOYEE})$$

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

Answers:

TEMP

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston,TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston,TX	M	40000	888665555	5
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble,TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

Review: Relational Algebra Expressions

TEMP ← σ $_{DNO=5}$ (EMPLOYEE)

TEMP

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston,TX	M	30000	333445555	5
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Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble,TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

What about the following?

R(First_name, Last_name, Salary) ← π $_{FNAME, LNAME, SALARY}$ (TEMP)

Review: Relational Algebra Expressions

TEMP ← $\sigma_{DNO=5}$ (EMPLOYEE)

TEMP

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston,TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston,TX	M	40000	888665555	5
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble,TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

What about the following?

R(First_name, Last_name, Salary) ← $\pi_{FNAME, LNAME, SALARY}$ (TEMP)

Answer: R

First_name	Last_name	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

Review: Relational Algebra Expressions

R(First_name, Last_name, Salary) ← $\pi_{\text{FNAME, LNAME, SALARY}}$ (TEMP)

R

First_name	Last_name	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

What is another way of writing this?

Review: Relational Algebra Expressions

R(First_name, Last_name, Salary) ← π FNAME, LNAME, SALARY (TEMP)

R

First_name	Last_name	Salary
John	Smith	30000
Franklin	Wong	40000
Ramesh	Narayan	38000
Joyce	English	25000

What is another way of writing this?

ρR(First_name, Last_name, Salary) (π FNAME, LNAME, SALARY (TEMP))

Review: Boolean Expressions AND / OR

DEPENDENT

Essn	Dependent_name	Sex	Bdate	Relationship
333445555	Alice	F	1986-04-05	Daughter
333445555	Theodore	M	1983-10-25	Son
333445555	Joy	F	1958-05-03	Spouse
987654321	Abner	M	1942-02-28	Spouse
123456789	Michael	M	1988-01-04	Son
123456789	Alice	F	1988-12-30	Daughter
123456789	Elizabeth	F	1967-05-05	Spouse

$\sigma_{\text{Relationship="Spouse" AND Sex="M"}}(\text{DEPENDENT})$

$\sigma_{\text{Relationship="Spouse" } \wedge \text{ Sex="M"}}(\text{DEPENDENT})$

Logical AND: ∧

Review: Boolean Expressions AND / OR

DEPENDENT

Essn	Dependent_name	Sex	Bdate	Relationship
333445555	Alice	F	1986-04-05	Daughter
333445555	Theodore	M	1983-10-25	Son
333445555	Joy	F	1958-05-03	Spouse
987654321	Abner	M	1942-02-28	Spouse
123456789	Michael	M	1988-01-04	Son
123456789	Alice	F	1988-12-30	Daughter
123456789	Elizabeth	F	1967-05-05	Spouse

σ Relationship="Spouse" AND Sex="M" (DEPENDENT)

σ Relationship="Spouse" $\wedge$ Sex="M" (DEPENDENT)

Logical AND: $\wedge$

Review: Boolean Expressions AND / OR

DEPENDENT

Essn	Dependent_name	Sex	Bdate	Relationship
333445555	Alice	F	1986-04-05	Daughter
333445555	Theodore	M	1983-10-25	Son
333445555	Joy	F	1958-05-03	Spouse
987654321	Abner	M	1942-02-28	Spouse
123456789	Michael	M	1988-01-04	Son
123456789	Alice	F	1988-12-30	Daughter
123456789	Elizabeth	F	1967-05-05	Spouse

$\sigma_{\text{Relationship="Spouse" OR Sex="M"}}(\text{DEPENDENT})$

$\sigma_{\text{Relationship="Spouse" } \vee \text{ Sex="M"}}(\text{DEPENDENT})$

Logical OR: ∨

Review: Boolean Expressions AND / OR

DEPENDENT

Essn	Dependent_name	Sex	Bdate	Relationship
333445555	Alice	F	1986-04-05	Daughter
333445555	Theodore	M	1983-10-25	Son
333445555	Joy	F	1958-05-03	Spouse
987654321	Abner	M	1942-02-28	Spouse
123456789	Michael	M	1988-01-04	Son
123456789	Alice	F	1988-12-30	Daughter
123456789	Elizabeth	F	1967-05-05	Spouse

$\sigma_{\text{Relationship="Spouse" OR Sex="M"}}(\text{DEPENDENT})$

$\sigma_{\text{Relationship="Spouse" V Sex="M"}}(\text{DEPENDENT})$

Logical OR: V

CROSS PRODUCT with PROJECT and SELECT

- **Example**: Names of female employees and their dependents.

  - Using CROSS PRODUCT (X) with PROJECT (π) and SELECT (σ)
  - **First:** Use a SELECT (σ) to get Female Employees

    - FEMALE_EMPS $\leftarrow \sigma_{SEX='F'}$(EMPLOYEE)

FEMALE_EMPS

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
Alicia	J	Zelaya	999887777	1968-07-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

  - Use Project (π) to get the Name and SSN

    - EMPNAMES $\leftarrow \pi_{FNAME, LNAME, SSN}$ (FEMALE_EMPS)

EMPNAMES

Fname	Lname	Ssn
Alicia	Zelaya	999887777
Jennifer	Wallace	987654321
Joyce	English	453453453

CROSS PRODUCT with PROJECT and SELECT

- **Example**: Names of female employees and their dependents.

  - **Second:** Use Cross Product (X) on Female Employees and Dependents

- EMP_DEPENDENTS ← EMPNAMES x DEPENDENT

EMPNAMES

Fname	Lname	Ssn
Alicia	Zelaya	999887777
Jennifer	Wallace	987654321
Joyce	English	453453453

DEPENDENT

Essn	Dependent_name	Sex	Bdate	Relationship
333445555	Alice	F	1986-04-05	Daughter
333445555	Theodore	M	1983-10-25	Son
333445555	Joy	F	1958-05-03	Spouse
987654321	Abner	M	1942-02-28	Spouse
123456789	Michael	M	1988-01-04	Son
123456789	Alice	F	1988-12-30	Daughter
123456789	Elizabeth	F	1967-05-05	Spouse

How many tuples are returned from the above query?

CROSS PRODUCT with PROJECT and SELECT

- **Example**: Names of female employees and their dependents.

  - **Second:** Use Cross Product (X) on Female Employees and Dependents

- EMP_DEPENDENTS ← EMPNAMES x DEPENDENT

EMPNAMES

Fname	Lname	Ssn
Alicia	Zelaya	999887777
Jennifer	Wallace	987654321
Joyce	English	453453453

DEPENDENT

Essn	Dependent_name	Sex	Bdate	Relationship
333445555	Alice	F	1986-04-05	Daughter
333445555	Theodore	M	1983-10-25	Son
333445555	Joy	F	1958-05-03	Spouse
987654321	Abner	M	1942-02-28	Spouse
123456789	Michael	M	1988-01-04	Son
123456789	Alice	F	1988-12-30	Daughter
123456789	Elizabeth	F	1967-05-05	Spouse

How many tuples are returned from the above query?

Answer: 21
– For each tuple in **EMPNAMES (3)** cross with each tuple in **DEPENDENT (7)**
- **3x7=21**

CROSS PRODUCT with PROJECT and SELECT

EMPNAMES

Fname	Lname	Ssn

DEPENDENT

Essn	Dependent_name	Sex	Bdate	Relationship

EMP_DEPENDENTS

Fname	Lname	Ssn	Essn	Dependent_name	Sex	Bdate	. . .
Alicia	Zelaya	999887777	333445555	Alice	F	1986-04-05	. . .
Alicia	Zelaya	999887777	333445555	Theodore	M	1983-10-25	. . .
Alicia	Zelaya	999887777	333445555	Joy	F	1958-05-03	. . .
Alicia	Zelaya	999887777	987654321	Abner	M	1942-02-28	. . .
Alicia	Zelaya	999887777	123456789	Michael	M	1988-01-04	. . .
Alicia	Zelaya	999887777	123456789	Alice	F	1988-12-30	. . .
Alicia	Zelaya	999887777	123456789	Elizabeth	F	1967-05-05	. . .
Jennifer	Wallace	987654321	333445555	Alice	F	1986-04-05	. . .
Jennifer	Wallace	987654321	333445555	Theodore	M	1983-10-25	. . .
Jennifer	Wallace	987654321	333445555	Joy	F	1958-05-03	. . .
Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28	. . .
Jennifer	Wallace	987654321	123456789	Michael	M	1988-01-04	. . .
Jennifer	Wallace	987654321	123456789	Alice	F	1988-12-30	. . .
Jennifer	Wallace	987654321	123456789	Elizabeth	F	1967-05-05	. . .
Joyce	English	453453453	333445555	Alice	F	1986-04-05	. . .
Joyce	English	453453453	333445555	Theodore	M	1983-10-25	. . .
Joyce	English	453453453	333445555	Joy	F	1958-05-03	. . .
Joyce	English	453453453	987654321	Abner	M	1942-02-28	. . .
Joyce	English	453453453	123456789	Michael	M	1988-01-04	. . .
Joyce	English	453453453	123456789	Alice	F	1988-12-30	. . .
Joyce	English	453453453	123456789	Elizabeth	F	1967-05-05	. . .

CROSS PRODUCT with PROJECT and SELECT

- **Example**: Names of female employees and their dependents.
- **Third:** Use Select (σ) to get the *actual* Employee and Dependent pairs
- ACTUAL_DEPENDENTS $\leftarrow \sigma_{\text{SSN=ESSN}}$(EMP_DEPENDENTS)

EMP_DEPENDENTS

Fname	Lname	Ssn	Essn	Dependent_name	Sex	Bdate	. . .
Alicia	Zelaya	999887777	333445555	Alice	F	1986-04-05	. . .
Alicia	Zelaya	999887777	333445555	Theodore	M	1983-10-25	. . .
Alicia	Zelaya	999887777	333445555	Joy	F	1958-05-03	. . .
Alicia	Zelaya	999887777	987654321	Abner	M	1942-02-28	. . .
Alicia	Zelaya	999887777	123456789	Michael	M	1988-01-04	. . .
Alicia	Zelaya	999887777	123456789	Alice	F	1988-12-30	. . .
Alicia	Zelaya	999887777	123456789	Elizabeth	F	1967-05-05	. . .
Jennifer	Wallace	987654321	333445555	Alice	F	1986-04-05	. . .
Jennifer	Wallace	987654321	333445555	Theodore	M	1983-10-25	. . .
Jennifer	Wallace	987654321	333445555	Joy	F	1958-05-03	. . .
Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28	. . .
Jennifer	Wallace	987654321	123456789	Michael	M	1988-01-04	. . .
Jennifer	Wallace	987654321	123456789	Alice	F	1988-12-30	. . .
Jennifer	Wallace	987654321	123456789	Elizabeth	F	1967-05-05	. . .
Joyce	English	453453453	333445555	Alice	F	1986-04-05	. . .
Joyce	English	453453453	333445555	Theodore	M	1983-10-25	. . .
Joyce	English	453453453	333445555	Joy	F	1958-05-03	. . .
Joyce	English	453453453	987654321	Abner	M	1942-02-28	. . .
Joyce	English	453453453	123456789	Michael	M	1988-01-04	. . .
Joyce	English	453453453	123456789	Alice	F	1988-12-30	. . .
Joyce	English	453453453	123456789	Elizabeth	F	1967-05-05	. . .

CROSS PRODUCT with PROJECT and SELECT

■ **Example**: Names of female employees and their dependents.

■ **Third:** Use Select (σ) to get the *actual* Employee and Dependent pairs

■ ACTUAL_DEPENDENTS $\leftarrow \sigma_{SSN=ESSN}$(EMP_DEPENDENTS)

EMP_DEPENDENTS

Fname	Lname	Ssn	Essn	Dependent_name	Sex	Bdate	. . .
Alicia	Zelaya	999887777	333445555	Alice	F	1986-04-05	. . .
Alicia	Zelaya	999887777	333445555	Theodore	M	1983-10-25	. . .
Alicia	Zelaya	999887777	333445555	Joy	F	1958-05-03	. . .
Alicia	Zelaya	999887777	987654321	Abner	M	1942-02-28	. . .
Alicia	Zelaya	999887777	123456789	Michael	M	1988-01-04	. . .
Alicia	Zelaya	999887777	123456789	Alice	F	1988-12-30	. . .
Alicia	Zelaya	999887777	123456789	Elizabeth	F	1967-05-05	. . .
Jennifer	Wallace	987654321	333445555	Alice	F	1986-04-05	. . .
Jennifer	Wallace	987654321	333445555	Theodore	M	1983-10-25	. . .
Jennifer	Wallace	987654321	333445555	Joy	F	1958-05-03	. . .
Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28	. . .
Jennifer	Wallace	987654321	123456789	Michael	M	1988-01-04	. . .
Jennifer	Wallace	987654321	123456789	Alice	F	1988-12-30	. . .
Jennifer	Wallace	987654321	123456789	Elizabeth	F	1967-05-05	. . .
Joyce	English	453453453	333445555	Alice	F	1986-04-05	. . .
Joyce	English	453453453	333445555	Theodore	M	1983-10-25	. . .
Joyce	English	453453453	333445555	Joy	F	1958-05-03	. . .
Joyce	English	453453453	987654321	Abner	M	1942-02-28	. . .
Joyce	English	453453453	123456789	Michael	M	1988-01-04	. . .
Joyce	English	453453453	123456789	Alice	F	1988-12-30	. . .
Joyce	English	453453453	123456789	Elizabeth	F	1967-05-05	. . .

CROSS PRODUCT with PROJECT and SELECT

- **Example**: Names of female employees and their dependents.

  - **Third:** Use Select (σ) to get the *actual* Employee and Dependent pairs

  - ACTUAL_DEPENDENTS $\leftarrow \sigma_{SSN=ESSN}$(EMP_DEPENDENTS)

ACTUAL_DEPENDENTS

Fname	Lname	Ssn	Essn	Dependent_name	Sex	Bdate	. . .
Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28	. . .

CROSS PRODUCT with PROJECT and SELECT

- **Example**: Names of female employees and their dependents.

  - **Fourth:** Use Project (π) to limit the attributes to just the names in the Result

  - RESULT $\leftarrow \pi_{\text{FNAME, LNAME, DEPENDENT\_NAME}}$ (ACTUAL_DEPS)

ACTUAL_DEPENDENTS

Fname	Lname	Ssn	Essn	Dependent_name	Sex	Bdate	...
Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28	...

CROSS PRODUCT with PROJECT and SELECT

- **Example**: Names of female employees and their dependents.

  - **Fourth:** Use Project (π) to limit the attributes to just the names in the Result

  - RESULT $\leftarrow \pi_{\text{FNAME, LNAME, DEPENDENT\_NAME}}$ (ACTUAL_DEPS)

ACTUAL_DEPENDENTS

Fname	Lname	Ssn	Essn	Dependent_name	Sex	Bdate	...
Jennifer	Wallace	987654321	987654321	Abner	M	1942-02-28	...

RESULT

Fname	Lname	Dependent_name
Jennifer	Wallace	Abner

CROSS PRODUCT with PROJECT and SELECT

- **Example**: Names of female employees and their dependents.

  - **Summary:**
    - **First:** Use a SELECT (σ) to get Female Employees
    - **Second:** Use Cross Product (X) on Female Employees and Dependents
    - **Third:** Use Select (σ) to get the *actual* Employee and Dependent pairs
    - **Fourth:** Use Project (π) to limit the attributes to just the names in the Result

Binary Relational Operations: JOIN

- JOIN Operation (denoted by ⋈)
  - The sequence of CARTESIAN PRODUCT followed by SELECT is used quite commonly to identify and select related tuples from two relations
  - A special operation, called JOIN combines this sequence into a single operation

  - This operation is very important for any relational database with more than a single relation, because it allows us to *combine related tuples* from various relations

  - The general form of a join operation on two relations R(A1, A2, . . ., An) and S(B1, B2, . . ., Bm) is:

$$R \bowtie_{<join\ condition>} S$$

  - where R and S can be any relations that result from general *relational algebra expressions*.

Binary Relational Operations: JOIN (cont.)

- **Example**: Suppose that we want to retrieve the name of the manager of each department.

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

DEPARTMENT

Dname	Dnumber	Mgr_ssn	Mgr_start_date
Research	5	333445555	1988-05-22
Administration	4	987654321	1995-01-01
Headquarters	1	888665555	1981-06-19

- **What is a valid RA statement for the above?**

Binary Relational Operations: JOIN (cont.)

- Example: Suppose that we want to retrieve the name of the manager of each department.

  - DEPT_MGR ← DEPARTMENT ⋈ $_{Mgr\_ssn=Ssn}$ EMPLOYEE

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

DEPARTMENT

Dname	Dnumber	Mgr_ssn	Mgr_start_date
Research	5	333445555	1988-05-22
Administration	4	987654321	1995-01-01
Headquarters	1	888665555	1981-06-19

How many TUPLES will result?

Binary Relational Operations: JOIN (cont.)

- Example: Suppose that we want to retrieve the name of the manager of each department.

  - DEPT_MGR ← DEPARTMENT ⋈ $_{Mgr\_ssn=Ssn}$ EMPLOYEE

    - MGRSSN=SSN is the join condition
      - Combines each department record with the employee who manages the department
      - The join condition can also be specified as DEPARTMENT.MGRSSN= EMPLOYEE.SSN

DEPT_MGR

Dname	Dnumber	Mgr_ssn	· · ·	Fname	Minit	Lname	Ssn	· · ·
Research	5	333445555	· · ·	Franklin	T	Wong	333445555	· · ·
Administration	4	987654321	· · ·	Jennifer	S	Wallace	987654321	· · ·
Headquarters	1	888665555	· · ·	James	E	Borg	888665555	· · ·

How many *attributes*?

Some Properties of JOIN

- Consider the following JOIN operation:

  - R(A1, A2, . . ., An) ⋈ S(B1, B2, . . ., Bm)

    R.Ai=S.Bj

  - Result is a relation Q with degree **n + m attributes**:

    - Q(A1, A2, . . ., An, B1, B2, . . ., Bm), in that order.

  - The resulting relation state has one tuple for each combination of tuples—r from R and s from S, but *only if they satisfy the join condition* r[Ai]=s[Bj]

  - Hence, if R has n_R tuples, and S has n_S tuples, then the join result will generally have *less than* $n_R * n_S$ **tuples**.

  - Only related tuples (based on the join condition) will appear in the result.

THETA JOIN

- The general case of JOIN operation is called a **Theta-join**:

$$R \bowtie_{theta} S$$

- The join condition is called *theta*
- *Theta* can be any general boolean expression on the attributes of R and S; for example:
  - R.Ai<S.Bj AND (R.Ak=S.Bl OR R.Ap<S.Bq)
- Most join conditions involve one or more equality conditions joined through AND, for example:
  - R.Ai=S.Bj AND R.Ak=S.Bl AND R.Ap=S.Bq

Binary Relational Operations: EQUIJOIN

- **EQUIJOIN** Operation
- The most common use of join involves join conditions with *equality comparisons* only
- Such a join, where <u>only the equality comparison operator</u> is used is called an **EQUIJOIN**.
  - In the result of an **EQUIJOIN** we always have one or more pairs of attributes (whose names need not be identical) that have identical values in every tuple.
  - Previous Example is an **EQUIJOIN**:

DEPT_MGR ← DEPARTMENT ⋈ $_{MGRSSN=SSN}$ EMPLOYEE

DEPT_MGR

Dname	Dnumber	Mgr_ssn	· · ·	Fname	Minit	Lname	Ssn	· · ·
Research	5	333445555	· · ·	Franklin	T	Wong	333445555	· · ·
Administration	4	987654321	· · ·	Jennifer	S	Wallace	987654321	· · ·
Headquarters	1	888665555	· · ·	James	E	Borg	888665555	· · ·

Binary Relational Operations: NATURAL JOIN Operation

- **NATURAL JOIN** Operation (*)

R * S

DEPT_MGR

Dname	Dnumber	Mgr_ssn	· · ·	Fname	Minit	Lname	Ssn	· · ·
Research	5	333445555	· · ·	Franklin	T	Wong	333445555	· · ·
Administration	4	987654321	· · ·	Jennifer	S	Wallace	987654321	· · ·
Headquarters	1	888665555	· · ·	James	E	Borg	888665555	· · ·

- Another variation of JOIN called NATURAL JOIN
  — was created to get rid of the second (superfluous) attribute in an EQUIJOIN condition.
  - because one of each pair of attributes with identical values is superfluous

Binary Relational Operations NATURAL JOIN (contd.)

- To apply a natural join on the DNUMBER attributes of DEPARTMENT and DEPT_LOCATIONS, it is sufficient to write:
  - DEPT_LOCS ← DEPARTMENT * DEPT_LOCATIONS

- Only attribute with the same name is DNUMBER
- An implicit join condition is created based on this attribute:
  DEPARTMENT.DNUMBER=DEPT_LOCATIONS.DNUMBER

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno

DEPARTMENT

Dname	Dnumber	Mgr_ssn	Mgr_start_date

DEPT_LOCATIONS
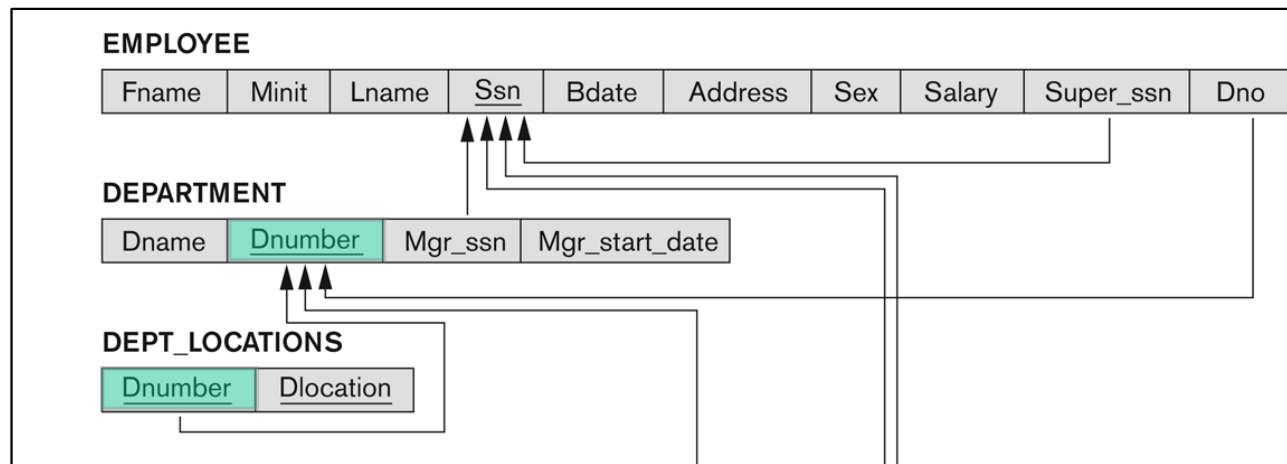
Dnumber	Dlocation

EXERCISE

- **Example**: To apply a natural join on the DNUMBER attributes of DEPARTMENT and DEPT_LOCATIONS, it is sufficient to write:
  - DEPT_LOCS ← DEPARTMENT * DEPT_LOCATIONS

- Only attribute with the same name is DNUMBER
- An implicit join condition is created based on this attribute:
  DEPARTMENT.DNUMBER=DEPT_LOCATIONS.DNUMBER

Dname	Dnumber	Mgr_ssn	Mgr_start_date	Location
Headquarters	1	888665555	1981-06-19	Houston
Administration	4	987654321	1995-01-01	Stafford
Research	5	333445555	1988-05-22	Bellaire
Research	5	333445555	1988-05-22	Sugarland
Research	5	333445555	1988-05-22	Houston

What would be the result with JOIN or EQUIJOIN?

EXERCISE

DEPT_LOCS ← DEPARTMENT * DEPT_LOCATIONS

Dname	Dnumber	Mgr_ssn	Mgr_start_date	Location
Headquarters	1	888665555	1981-06-19	Houston
Administration	4	987654321	1995-01-01	Stafford
Research	5	333445555	1988-05-22	Bellaire
Research	5	333445555	1988-05-22	Sugarland
Research	5	333445555	1988-05-22	Houston

What would be the result with JOIN or EQUIJOIN?

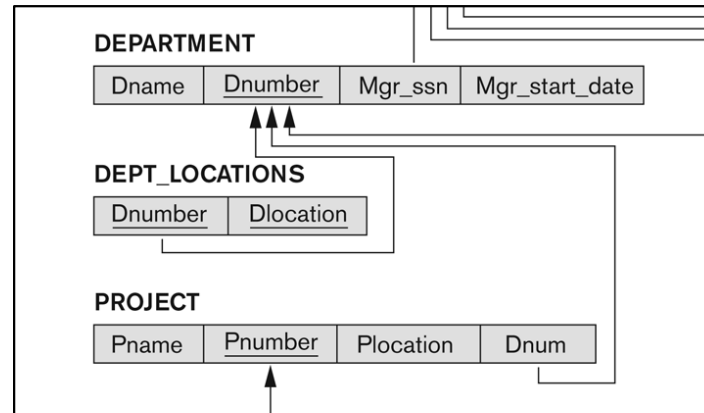| DNAME	DEPARTMENT.DNUMBER	MGRSSN	MGRSTARTDATE	DEPT_LOCATIONS.DNUMBER	DLOCATION
Headquarters	1	888665555	1981-06-19	1	Houston
Administration	4	987654321	1995-01-01	4	Stafford
Research	5	333445555	1988-05-22	5	Bellaire
Research	5	333445555	1988-05-22	5	Houston
Research	5	333445555	1988-05-22	5	Sugarland

Binary Relational Operations NATURAL JOIN (contd.)

- In a general example:
  Q ← R(A,B,C,D) * S(C,D,E)

  - The implicit join condition includes *each pair* of attributes with the same name, joined with AND :
    - R.C=S.C AND R.D=S.D

  - Result keeps only one attribute of each such pair:
    - Q(A,B,C,D,E)

Example of NATURAL JOIN

Need to rename DEPARTMENT attributes:

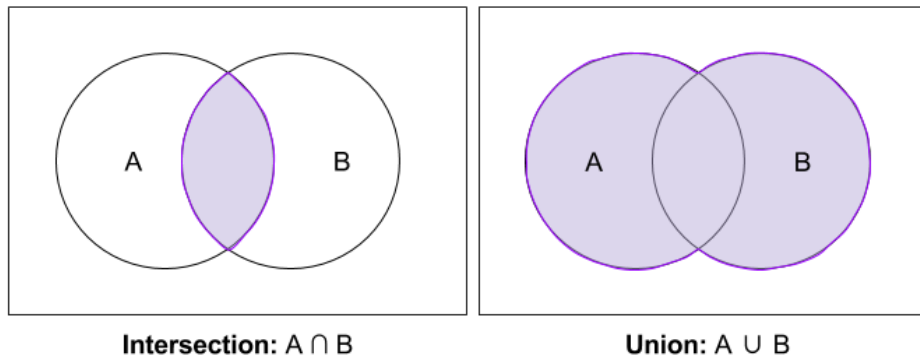DEPT ← ρ(DNAME, DNUM, MGR_SSN, MGRSTART_DATE) (DEPARTMENT)

DEPARTMENT

Dname	Dnumber	Mgr_ssn	Mgr_start_date

DEPT_LOCATIONS

Dnumber	Dlocation

PROJECT

Pname	Pnumber	Plocation	Dnum

PROJ_DEPT ← PROJECT*DEPARTMENT

PROJ_DEPT

Pname	Pnumber	Plocation	Dnum	Dname	Mgr_ssn	Mgr_start_date
ProductX	1	Bellaire	5	Research	333445555	1988-05-22
ProductY	2	Sugarland	5	Research	333445555	1988-05-22
ProductZ	3	Houston	5	Research	333445555	1988-05-22
Computerization	10	Stafford	4	Administration	987654321	1995-01-01
Reorganization	20	Houston	1	Headquarters	888665555	1981-06-19
Newbenefits	30	Stafford	4	Administration	987654321	1995-01-01

Complete Set of Relational Operations

- The set of operations including SELECT σ, PROJECT π , UNION $\cup$, DIFFERENCE $-$, RENAME ρ, and CARTESIAN PRODUCT X is called a *complete set* because any other relational algebra expression can be expressed by a combination of these five operations.

- For example:
  - $A \cap B = (A \cup B) - ((A - B) \cup (B - A))$

Intersection: A ∩ B   Union: A ∪ B

  - $R \bowtie_{<join\ condition>} S = \sigma_{<join\ condition>} (R\ X\ S)$

Binary Relational Operations: DIVISION

- DIVISION Operation
  - The division operation is applied to **two** relations
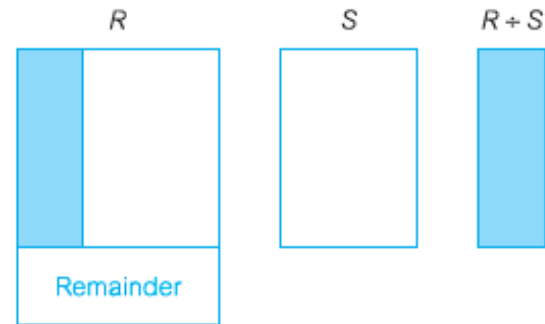
- $R(Z) \div S(X) = T(Y)$
  - where X subset of Z.
  - Let $Y = Z - X$ (and hence $Z = X \cup Y$); that is, let Y be the set of attributes of R that are not attributes of S.

- For a tuple t to appear in the result T of the DIVISION, the values in t must appear in R in combination with *every* tuple in S.

- DIVISION is like having a *dynamic* SELECT operation

Example of DIVISION

R

A	B
a1	b1
a2	b1
a3	b1
a4	b1
a1	b2
a3	b2
a2	b3
a3	b3
a4	b3
a1	b4
a2	b4
a3	b4

S

A
a1
a2
a3

T

B
b1
b4

- For a tuple t to appear in the result T of the DIVISION, the values in t must appear in R in combination with *every* tuple in S.

$R(Z) \div S(X) = T(Y)$

$\{A, B\}$ $\{A\}$ $\{A, B\} - \{A\} = \{B\}$ *attributes*

Example of DIVISION

R

A	B
a1	b1
a2	b1
a3	b1
a4	b1
a1	b2
a3	b2
a2	b3
a3	b3
a4	b3
a1	b4
a2	b4
a3	b4

S

A
a1
a2
a3

T

B
b1
b4

- For a tuple t to appear in the result T of the DIVISION, the values in t must appear in R in combination with *every* tuple in S.

$$R(Z) \div S(X) = T(Y)$$

$$\{A, B\} \quad \{A\} \quad \{A, B\} - \{A\} = \{B\} \quad \textit{attributes}$$

Challenge - write **DIVISION** in terms of the universal set of relational algebra operators (see book for answer)?

Example: Using DIVISION

- **Example**: Find the names of employees who work on _all_ the projects that 'John Smith' works on.

Example: Using DIVISION

- **Example**: Find the names of employees who work on _all_ the projects that 'John Smith' works on.

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

Example

- **Example**: Find the names of employees who work on _all_ the projects that 'John Smith' works on.

  - First, retrieve 'John Smith' the employee:
    - SMITH ← σ $_{FNAME='John' \; AND \; LNAME='Smith'}$(EMPLOYEE)

      SMITH

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5

  - Next retrieve the list of project numbers that 'John Smith' works on:
    - SMITH_PNOS ← π $_{PNO}$ (WORKS_ON ⋈ $_{ESSN=SSN}$ SMITH)

Example

- **Example**: Find the names of employees who work on *all* the projects that 'John Smith' works on.
  - SMITH ← σ $_{FNAME='John'\ AND\ LNAME='Smith'}$(EMPLOYEE)
  - SMITH_PNOS← π $_{PNO}$ (WORKS_ON ⋈ $_{ESSN=SSN}$ SMITH)

SMITH

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5

WORKS_ON

Essn	Pno	Hours
123456789	1	32.5
123456789	2	7.5
666884444	3	40.0
453453453	1	20.0
453453453	2	20.0
333445555	2	10.0
333445555	3	10.0
333445555	10	10.0
333445555	20	10.0
999887777	30	30.0
999887777	10	10.0
987987987	10	35.0
987987987	30	5.0
987654321	30	20.0
987654321	20	15.0
888665555	20	NULL

SMITH_PNOS

- number of attributes?

- number of tuples?

Example

- **Example**: Find the names of employees who work on _all_ the projects that 'John Smith' works on.
  - SMITH ← σ $_{FNAME='John' \ AND \ LNAME='Smith'}$(EMPLOYEE)
  - SMITH_PNOS← π $_{PNO}$ (WORKS_ON ⋈ $_{ESSN=SSN}$ SMITH)

SMITH

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5

SMITH_PNOS

Pno
1
2

WORKS_ON

Essn	Pno	Hours
123456789	1	32.5
123456789	2	7.5
666884444	3	40.0
453453453	1	20.0
453453453	2	20.0
333445555	2	10.0
333445555	3	10.0
333445555	10	10.0
333445555	20	10.0
999887777	30	30.0
999887777	10	10.0
987987987	10	35.0
987987987	30	5.0
987654321	30	20.0
987654321	20	15.0
888665555	20	NULL

Example (contd)

- Next create a relation that includes attributes <PNO, ESSN> (SSN and PNOs for everybody):

WORKS_ON

Essn	Pno	Hours
123456789	1	32.5
123456789	2	7.5
666884444	3	40.0
453453453	1	20.0
453453453	2	20.0
333445555	2	10.0
333445555	3	10.0
333445555	10	10.0
333445555	20	10.0
999887777	30	30.0
999887777	10	10.0
987987987	10	35.0
987987987	30	5.0
987654321	30	20.0
987654321	20	15.0
888665555	20	NULL

Example (contd)

- Next create a relation that includes attributes <PNO, ESSN>
  (SSN and PNOs for everybody):
  - SSN_PNOS ←
    $\pi_{ESSN, PNO}$ (WORKS_ON)

SSN_PNOS

Essn	Pno
123456789	1
123456789	2
666884444	3
453453453	1
453453453	2
333445555	2
333445555	3
333445555	10
333445555	20
999887777	30
999887777	10
987987987	10
987987987	30
987654321	30
987654321	20
888665555	20

WORKS_ON

Essn	Pno	Hours
123456789	1	32.5
123456789	2	7.5
666884444	3	40.0
453453453	1	20.0
453453453	2	20.0
333445555	2	10.0
333445555	3	10.0
333445555	10	10.0
333445555	20	10.0
999887777	30	30.0
999887777	10	10.0
987987987	10	35.0
987987987	30	5.0
987654321	30	20.0
987654321	20	15.0
888665555	20	NULL

Example (contd)

- Given we have the following:
  - SMITH_PNOS← π_{PNO} (WORKS_ON $\bowtie_{ESSN=SSN}$ SMITH)
  - SSN_PNOS← $\pi_{ESSN, PNO}$ (WORKS_ON)
- Finally apply the division operation to the two relations, which will produce the desired outcome:

WORKS_ON

Essn	Pno	Hours
123456789	1	32.5
123456789	2	7.5
666884444	3	40.0
453453453	1	20.0
453453453	2	20.0
333445555	2	10.0
333445555	3	10.0
333445555	10	10.0
333445555	20	10.0
999887777	30	30.0
999887777	10	10.0
987987987	10	35.0
987987987	30	5.0
987654321	30	20.0
987654321	20	15.0
888665555	20	NULL

SSN_PNOS

Essn	Pno
123456789	1
123456789	2
666884444	3
453453453	1
453453453	2
333445555	2
333445555	3
333445555	10
333445555	20
999887777	30
999887777	10
987987987	10
987987987	30
987654321	30
987654321	20
888665555	20

Example (contd)

- SMITH_PNOS← π_{PNO} (WORKS_ON ⋈ $_{ESSN=SSN}$ SMITH)
- SSN_PNOS← $\pi_{ESSN, PNO}$ (WORKS_ON)
- **SSNS(SSN)←SSN_PNOS ÷ SMITH_PNOS**

WORKS_ON

Essn	Pno	Hours
123456789	1	32.5
123456789	2	7.5
666884444	3	40.0
453453453	1	20.0
453453453	2	20.0
333445555	2	10.0
333445555	3	10.0
333445555	10	10.0
333445555	20	10.0
999887777	30	30.0
999887777	10	10.0
987987987	10	35.0
987987987	30	5.0
987654321	30	20.0
987654321	20	15.0
888665555	20	NULL

SSN_PNOS

Essn	Pno
123456789	1
123456789	2
666884444	3
453453453	1
453453453	2
333445555	2
333445555	3
333445555	10
333445555	20
999887777	30
999887777	10
987987987	10
987987987	30
987654321	30
987654321	20
888665555	20

SMITH_PNOS

Pno
1
2

SSNS

Ssn
123456789
453453453

Example (contd)

- JOIN and PROJECT the resulting SSNS with the Employees to get the result:
- RESULT ← $\pi_{FNAME, LNAME}$ (SSNS*EMPLOYEE)

SSNS

Ssn
123456789
453453453

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

What kind of JOIN is this? Number of Attributes?

Example (contd)

- JOIN and PROJECT the resulting SSNS with the Employees to get the result:
- RESULT ← $\pi_{FNAME, LNAME}$ (SSNS*EMPLOYEE)

SSNS

Ssn
123456789
453453453

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

SSNS*EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

Complete Example

SMITH ← σ $_{FNAMe='John'\ AND\ LNAME='Smith'}$ (EMPLOYEE)

SMITH_PNOS← π $_{PNO}$ (WORKS_ON $\bowtie_{ESSN=SSN}$ SMITH)

SSN_PNOS← π $_{ESSN,\ PNO}$ (WORKS_ON)

SSNS(SSN)←SSN_PNOS ÷ SMITH_PNOS

RESULT ← π $_{FNAME,\ LNAME}$ (SSNS*EMPLOYEE)

SSN_PNOS

Essn	Pno
123456789	1
123456789	2
666884444	3
453453453	1
453453453	2
333445555	2
333445555	3
333445555	10
333445555	20
999887777	30
999887777	10
987987987	10
987987987	30
987654321	30
987654321	20
888665555	20

SMITH_PNOS

Pno
1
2

SSNS

Ssn
123456789
453453453

SSNS*EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5

RESULT

Fname	Lname
John	Smith
Joyce	English

Recap of Relational Algebra Operations

Operation	Purpose	Notation
SELECT	Selects all tuples that satisfy the selection condition from a relation R.	$\sigma_{<\text{selection condition}>}(R)$
PROJECT	Produces a new relation with only some of the attributes of R, and removes duplicate tuples.	$\pi_{<\text{attribute list}>}(R)$
THETA JOIN	Produces all combinations of tuples from R_1 and R_2 that satisfy the join condition.	$R_1 \bowtie_{<\text{join condition}>} R_2$
EQUIJOIN	Produces all the combinations of tuples from R_1 and R_2 that satisfy a join condition with only equality comparisons.	$R_1 \bowtie_{<\text{join condition}>} R_2$, OR $R_1 \bowtie_{(<\text{join attributes 1}>),(<\text{join attributes 2}>)} R_2$
NATURAL JOIN	Same as EQUIJOIN except that the join attributes of R_2 are not included in the resulting relation; if the join attributes have the same names, they do not have to be specified at all.	$R_1 *_{<\text{join condition}>} R_2$, OR $R_1 *_{(<\text{join attributes 1}>),(<\text{join attributes 2}>)} R_2$ OR $R_1 * R_2$
UNION	Produces a relation that includes all the tuples in R_1 or R_2 or both R_1 and R_2; R_1 and R_2 must be union compatible.	$R_1 \cup R_2$
INTERSECTION	Produces a relation that includes all the tuples in both R_1 and R_2; R_1 and R_2 must be union compatible.	$R_1 \cap R_2$
DIFFERENCE	Produces a relation that includes all the tuples in R_1 that are not in R_2; R_1 and R_2 must be union compatible.	$R_1 - R_2$
CARTESIAN PRODUCT	Produces a relation that has the attributes of R_1 and R_2 and includes as tuples all possible combinations of tuples from R_1 and R_2.	$R_1 \times R_2$
DIVISION	Produces a relation $R(X)$ that includes all tuples $t[X]$ in $R_1(Z)$ that appear in R_1 in combination with every tuple from $R_2(Y)$, where $Z = X \cup Y$.	$R_1(Z) \div R_2(Y)$

Additional Relational Operations: Aggregate Functions and Grouping

- A type of request that cannot be expressed in the basic relational algebra is to specify mathematical **aggregate functions** on collections of values from the database.
- Examples of such functions include retrieving the *average* or *total* salary of all employees or the *total* number of employee tuples.
  - These functions are used in **simple statistical queries** that **summarise** information from the database tuples.
- Common functions applied to collections of numeric values include
  - SUM, AVERAGE, MAXIMUM, and MINIMUM.
- The COUNT function is used for counting tuples or values.

Aggregate Function Operation

- Use of the Aggregate Functional operation $\mathcal{F}$

  - $\mathcal{F}_{\text{MAX Salary}}$ (EMPLOYEE) retrieves the maximum salary value from the EMPLOYEE relation
  - $\mathcal{F}_{\text{MIN Salary}}$ (EMPLOYEE) retrieves the minimum Salary value from the EMPLOYEE relation
  - $\mathcal{F}_{\text{SUM Salary}}$ (EMPLOYEE) retrieves the sum of the Salary from the EMPLOYEE relation
  - $\mathcal{F}_{\text{COUNT SSN, AVERAGE Salary}}$ (EMPLOYEE) computes the count (number) of employees and their average salary
  - **Note: count just counts the number of rows, without removing duplicates**

  - Parentheses on functions are optional:
    - $\mathcal{F}_{\text{MAX Salary}}$ (EMPLOYEE) is equivalent $\mathcal{F}_{\text{MAX (Salary)}}$(EMPLOYEE)

Using Grouping with Aggregation

- The previous examples all summarised one or more attributes for a set of tuples
  - E.g. Maximum Salary or Count (number of) Ssn
- Grouping can be combined with Aggregate Functions

- **Example**: For each department, retrieve the DNO, COUNT SSN, and AVERAGE SALARY

- A variation of aggregate operation $\mathcal{F}$ allows this:
  - Grouping attribute placed to **left** of symbol
  - Aggregate functions to **right** of symbol
  - DNO $\mathcal{F}$ COUNT SSN, AVERAGE Salary (EMPLOYEE)

- Above operation groups employees by DNO (department number) and computes the count of employees and average salary per department

Examples of Applying Aggregate Functions and Grouping

What is the result of the following operations?

Dno $\mathfrak{I}$ COUNT Ssn, AVERAGE Salary (EMPLOYEE).

Note that the book uses a different F symbol.

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

How many **groups**?

Examples of Applying Aggregate Functions and Grouping

What is the result of the following operations?

Dno $\mathfrak{I}$ COUNT Ssn, AVERAGE Salary (EMPLOYEE).

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

Dno	Count_ssn	Average_salary
5	4	33250
4	3	31000
1	1	55000

Examples of Applying Aggregate Functions and Grouping

What is the result of the following operations?

$$\rho_{R(Dno,\ No\_of\_employees,\ Average\_sal)}\left(_{Dno}\ \Im\ _{COUNT\ Ssn,\ AVERAGE\ Salary}\ (EMPLOYEE)\right).$$

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
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Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

Examples of Applying Aggregate Functions and Grouping

What is the result of the following operations?

$$\rho_{R(Dno,\ No\_of\_employees,\ Average\_sal)} (_{Dno}\mathfrak{I}_{\ COUNT\ Ssn,\ AVERAGE\ Salary} (EMPLOYEE)).$$

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

R

Dno	No_of_employees	Average_sal
5	4	33250
4	3	31000
1	1	55000

Examples of Applying Aggregate Functions and Grouping

What is the result of the following operations?

$\Im$ COUNT Ssn, AVERAGE Salary (EMPLOYEE).

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

Examples of Applying Aggregate Functions and Grouping

What is the result of the following operations?

$\Im$ COUNT Ssn, AVERAGE Salary (EMPLOYEE).

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

Duplicates are not eliminated before counting values.
COUNTing unique values?

Count_ssn	Average_salary
8	35125

Additional Relational Operations (cont.)

- The OUTER JOIN Operation
  - In NATURAL JOIN and EQUIJOIN, tuples without a *matching* (or *related*) tuple are eliminated from the join result
    - Tuples with null in the join attributes are also eliminated
    - This amounts to loss of information.
  - A set of operations, called OUTER joins, can be used when we want to keep all the tuples in R, or all those in S, or all those in both relations in the result of the join, regardless of whether or not they have matching tuples in the other relation.

Additional Relational Operations (cont.)

- LEFT OUTER JOIN: R ⟕ S
    - keeps every tuple in the first or left relation R
    - if no matching tuple is found in S, then attributes of S in join result are filled or "padded" with null values.
- RIGHT OUTER JOIN: R ⟖ S
    - keeps <u>every tuple</u> in the second or right relation S
    - attributes of R are "padded" with null values in unlatching tuples
- FULL OUTER JOIN: R ⟗ S
    - keeps all tuples in both the left and the right relations
    - when no matching tuples are found, padding them with null values as needed.

Example: LEFT JOIN

TEMP_LEFT $\leftarrow$ (EMPLOYEE $⋈$ $_{Ssn=Mgr\_ssn}$ DEPARTMENT)
RESULT $\leftarrow$ π $_{Fname,Minit, Lname, Dname}$ (TEMP_LEFT)

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

DEPARTMENT

Dname	Dnumber	Mgr_ssn	Mgr_start_date
Research	5	333445555	1988-05-22
Administration	4	987654321	1995-01-01
Headquarters	1	888665555	1981-06-19

*TEMP_LEFT: How many **attributes**?*
*How many **tuples**?*

Example: LEFT JOIN

TEMP_LEFT ← (EMPLOYEE ⋈ $_{Ssn=Mgt\_ssn}$ DEPARTMENT)

RESULT ← π $_{Fname,Minit, Lname, Dname}$ (TEMP_LEFT)

TEMP_LEFT

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno	Dname	Dnumber	Mgr_ssn	Mgr_start_date
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5	NULL	NULL	NULL	NULL
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5	Research	5	333445555	1988-05-22
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4	NULL	NULL	NULL	NULL
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4	Administration	4	987654321	1995-01-01
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5	NULL	NULL	NULL	NULL
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5	NULL	NULL	NULL	NULL
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4	NULL	NULL	NULL	NULL
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1	Headquarters	1	888665555	1981-06-19

Example: LEFT JOIN

TEMP_LEFT ← (EMPLOYEE ⋈ $_{Ssn=Mgt\_ssn}$ DEPARTMENT)

RESULT ← $\pi_{Fname,Minit,\ Lname,\ Dname}$ (TEMP_LEFT)

TEMP_LEFT

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno	Dname	Dnumber	Mgr_ssn	Mgr_start_date
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5	NULL	NULL	NULL	NULL
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5	Research	5	333445555	1988-05-22
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4	NULL	NULL	NULL	NULL
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4	Administration	4	987654321	1995-01-01
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5	NULL	NULL	NULL	NULL
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5	NULL	NULL	NULL	NULL
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4	NULL	NULL	NULL	NULL
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1	Headquarters	1	888665555	1981-06-19

RESULT

Fname	Minit	Lname	Dname
John	B	Smith	NULL
Franklin	T	Wong	Research
Alicia	J	Zelaya	NULL
Jennifer	S	Wallace	Administration
Ramesh	K	Narayan	NULL
Joyce	A	English	NULL
Ahmad	V	Jabbar	NULL
James	E	Borg	Headquarters

Example: RIGHT JOIN

TEMP_RIGHT ← (EMPLOYEE ⋈ $_{Ssn=Mgt\_ssn}$ DEPARTMENT)
RESULT ← π $_{Fname,Minit, Lname, Dname}$ (TEMP_RIGHT)

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

DEPARTMENT

Dname	Dnumber	Mgr_ssn	Mgr_start_date
Research	5	333445555	1988-05-22
Administration	4	987654321	1995-01-01
Headquarters	1	888665555	1981-06-19

*TEMP_RIGHT: How many **attributes**?*
*How many **tuples**?*

Example: RIGHT JOIN

TEMP_RIGHT ← (EMPLOYEE ⋈ $_{Ssn=Mgt\_ssn}$ DEPARTMENT)
RESULT ← $\pi_{Fname,Minit, Lname, Dname}$ (TEMP_RIGHT)

RESULT

Fname	Minit	Lname	Dname
Franklin	T	Wong	Research
Jennifer	S	Wallace	Administration
James	E	Borg	Headquarters

All tuples in the RIGHT relations DEPARTMENT are matched

Example: OUTER JOIN

TEMP_OUTER ← (EMPLOYEE ⋈ $_{Ssn=Mgt\_ssn}$ DEPARTMENT)
RESULT ← π $_{Fname,Minit, Lname, Dname}$ (TEMP_OUTER)

EMPLOYEE

Fname	Minit	Lname	Ssn	Bdate	Address	Sex	Salary	Super_ssn	Dno
John	B	Smith	123456789	1965-01-09	731 Fondren, Houston, TX	M	30000	333445555	5
Franklin	T	Wong	333445555	1955-12-08	638 Voss, Houston, TX	M	40000	888665555	5
Alicia	J	Zelaya	999887777	1968-01-19	3321 Castle, Spring, TX	F	25000	987654321	4
Jennifer	S	Wallace	987654321	1941-06-20	291 Berry, Bellaire, TX	F	43000	888665555	4
Ramesh	K	Narayan	666884444	1962-09-15	975 Fire Oak, Humble, TX	M	38000	333445555	5
Joyce	A	English	453453453	1972-07-31	5631 Rice, Houston, TX	F	25000	333445555	5
Ahmad	V	Jabbar	987987987	1969-03-29	980 Dallas, Houston, TX	M	25000	987654321	4
James	E	Borg	888665555	1937-11-10	450 Stone, Houston, TX	M	55000	NULL	1

DEPARTMENT

Dname	Dnumber	Mgr_ssn	Mgr_start_date
Research	5	333445555	1988-05-22
Administration	4	987654321	1995-01-01
Headquarters	1	888665555	1981-06-19

*TEMP_OUTER: How many **attributes**?*
*How many **tuples**?*

Example: OUTER JOIN

TEMP_OUTER ← (EMPLOYEE ⋈ $_{Ssn=Mgt\_ssn}$ DEPARTMENT)
RESULT ← $\pi_{Fname,Minit, Lname, Dname}$ (TEMP_OUTER)

RESULT

Fname	Minit	Lname	Dname
John	B	Smith	NULL
Franklin	T	Wong	Research
Alicia	J	Zelaya	NULL
Jennifer	S	Wallace	Administration
Ramesh	K	Narayan	NULL
Joyce	A	English	NULL
Ahmad	V	Jabbar	NULL
James	E	Borg	Headquarters

Same as LEFT JOIN...

Additional Relational Operations (cont.)

- OUTER UNION Operations
  - The **outer union** operation was developed to take the union of tuples from two relations if the relations are *not type compatible*.
  - This operation will take the union of tuples in two relations R(X, Y) and S(X, Z) that are **partially compatible**, meaning that only some of their attributes, say X, are type compatible.
  - The attributes that are type compatible are represented only once in the result, and those attributes that are not type compatible from either relation are also kept in the result relation T(X, Y, Z).

Additional Relational Operations (cont.)

Employees

Fname	SSN	Dno	Supervisor
John	123456789	5	Franklin
Franklin	333445555	5	James
Alicia	999887777	4	Jennifer
Jennifer	987654321	4	James
Ramesh	666884444	5	Franklin
Joyce	453453453	5	Franklin
Ahmad	987987987	4	Jennifer
James	888665555	1	NULL

Supervisors

Fname	SSN	Dno	Rank
Franklin	333445555	5	Vice President
Jennifer	987654321	4	2
James	888665555	1	1
Lela	222222222	3	3
Martin	333333333	3	3

- Tuples from the two relations are matched based on having the same combination of values of the shared attributes— **Fname, SSN, Dno**.

- If tuples match on these attributes, both Supervisor and Rank will have a value; otherwise, one of these two attributes will be null.

- The result relation EMPLOYEES_OR_SUPERVISORS will have the following attributes : <**Fname, SSN, Dno**, Supervisor, Rank>

Additional Relational Operations (cont.)

- **Example**: Consider an OUTER UNION of these two relations: Employees OUTER UNION Supervisors

Employees

Fname	SSN	Dno	Supervisor
John	123456789	5	Franklin
Franklin	333445555	5	James
Alicia	999887777	4	Jennifer
Jennifer	987654321	4	James
Ramesh	666884444	5	Franklin
Joyce	453453453	5	Franklin
Ahmad	987987987	4	Jennifer
James	888665555	1	NULL

Supervisors

Fname	SSN	Dno	Rank
Franklin	333445555	5	2
Jennifer	987654321	4	2
James	888665555	1	1
Lela	222222222	3	3
Martin	333333333	3	3

Employees_Or_Supervisors

Fname	SSN	Dno	Supervisor	Rank
John	123456789	5	Franklin	NULL
Franklin	333445555	5	James	2
Alicia	999887777	4	Jennifer	NULL
Jennifer	987654321	4	James	2
Ramesh	666884444	5	Franklin	NULL
Joyce	453453453	5	Franklin	NULL
Ahmad	987987987	4	Jennifer	NULL
James	888665555	1	NULL	1
Lela	222222222	3	NULL	3
Martin	333333333	3	NULL	3

Relational Algebra Overview

- Relational Algebra consists of several groups of operations
  - Unary Relational Operations
    - SELECT (symbol: σ (sigma))
    - PROJECT (symbol: π (pi))
    - RENAME (symbol: ρ (rho))
  - Relational Algebra Operations From Set Theory
    - UNION (∪), INTERSECTION (∩), DIFFERENCE (or MINUS, –)
    - CARTESIAN PRODUCT (**x**)
  - Binary Relational Operations
    - JOIN (several variations of JOIN exist)
    - DIVISION
  - Additional Relational Operations
    - OUTER JOINS, OUTER UNION
    - AGGREGATE FUNCTIONS (These compute summary of information: for example, SUM, COUNT, AVG, MIN, MAX)