External memory merge-sort

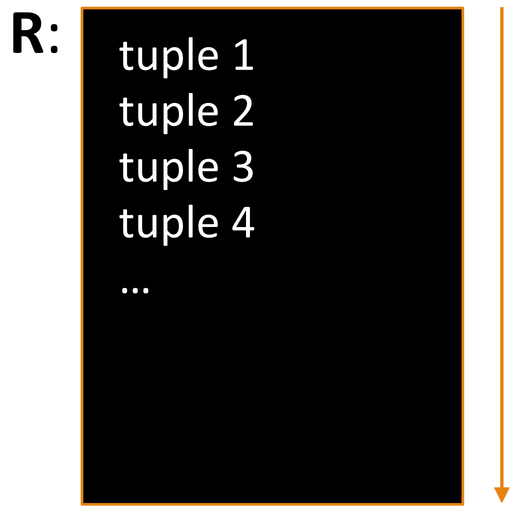
THIS ONE IS NOT REQUIRED FOR THE EXAM!

Overview over this video

This video (which is not required for the exam), we will cover external memory merge sort

Reminder: Computing $\sigma_{condition}(R)$

Basic procedure:



```
for each tuple t in R:
    if t satisfies condition:
        output t
```

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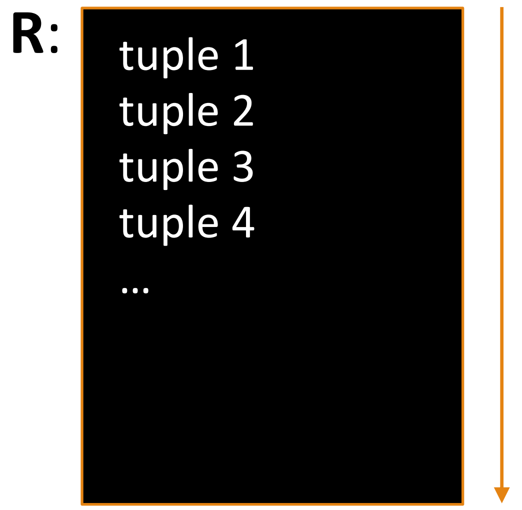

Needs to read the entire relation

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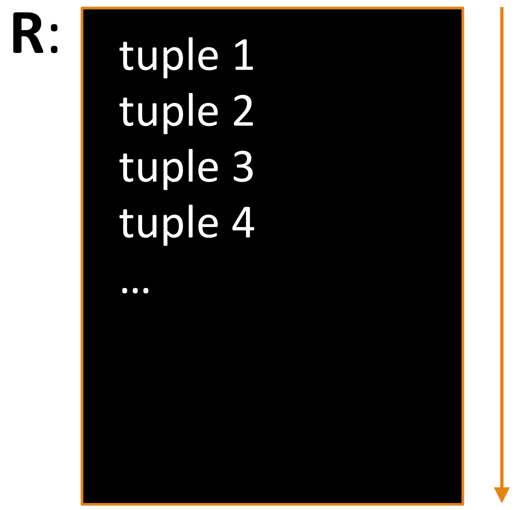

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Can this be done faster?

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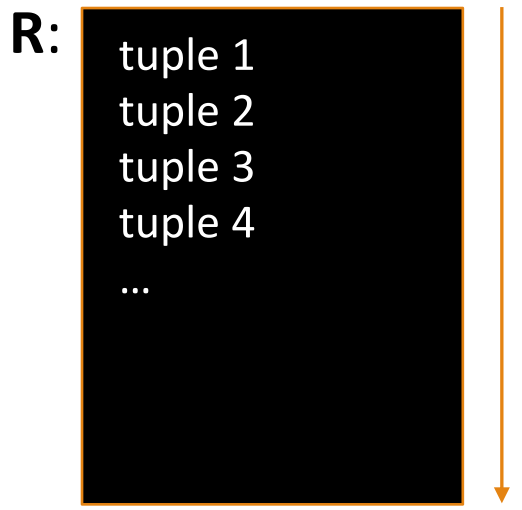

```
for each tuple t in R:
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Needs to read the entire relation

Can this be done faster?

Yes, sometimes!

Example

$\sigma_{\text{programme='G401'}}(\text{Students})$

Students

id	name	programme
…	…	….
1234	Anna	G401
2345	Ben	G701
3456	Chloe	G401
4567	Dave	G401
…	…	…

Example

$\sigma_{\text{programme='G401'}}(\text{Students})$

Students

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…	…	….
1234	Anna	G401
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…	…	…

Selection can be performed faster if we know **where to find the rows for 'G401'**

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Two solutions: **sorting** & **index**

Example

$\sigma_{\text{programme}=\text{'G401'}}(\text{Students})$

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Two solutions: **sorting** & **index**

This video

Faster Joins With Sorting

(from video "Faster Joins With Sorting")

Sort Join Algorithm:

Compute R $\bowtie_{A=B}$ S:

1. Sort **R** on **A**
2. Sort **S** on **B**
3. Merge the sorted **R** and **S**

Running time: $O(|R| \times \log_2 |R|)$

Running time: $O(|S| \times \log_2 |S|)$

Running time: $O(\text{size of output})$

Having a run time depending on the size of output is called output sensitive

Typical running time: $O(|\mathbf{R}|\log_2|\mathbf{R}| + |\mathbf{S}|\log_2|\mathbf{S}|)$

- If not "too many" values in **A** occur multiple times
- E.g., this is the case if **A** is a key

Typically much faster than Nested Loop Join

- Same time in the worst case, because output can have size up to $|\mathbf{R}| \times |\mathbf{S}|$

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Want to do that faster!

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Outside databases

Many problems are much easier then the data is sorted and in general, a significant fraction of all computing time is spend on sorting

- Was estimated to be around 25% back in the 60s
  - Found some claims that it is still in the range 25%-50%, but it is hard to verify

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Some problems were you need sorting are “small” in that they fit in main memory

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Some ain’t

Understanding the issue solved

To find one record on hard disk (with very fast "normal" drive of 15,000 rpm):

- 2.00 **milli-seconds** or $2 \cdot 10^6$ **nano-seconds**

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To find one record on hard disk (with very fast “normal” drive of 15,000 rpm):

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SLOW!!!

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slow

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fast

Merge Sort

(Internal memory) merge sort:

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**Divide** input in two parts $P_1$, $P_2$
**Sort** $P_1$ & $P_2$ **recursively**
**While** $P_1$ or $P_2$ is not empty:
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External Merge Sort

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External merge sort:

Number of disk blocks that fit in RAM

Merge

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Merge in more details

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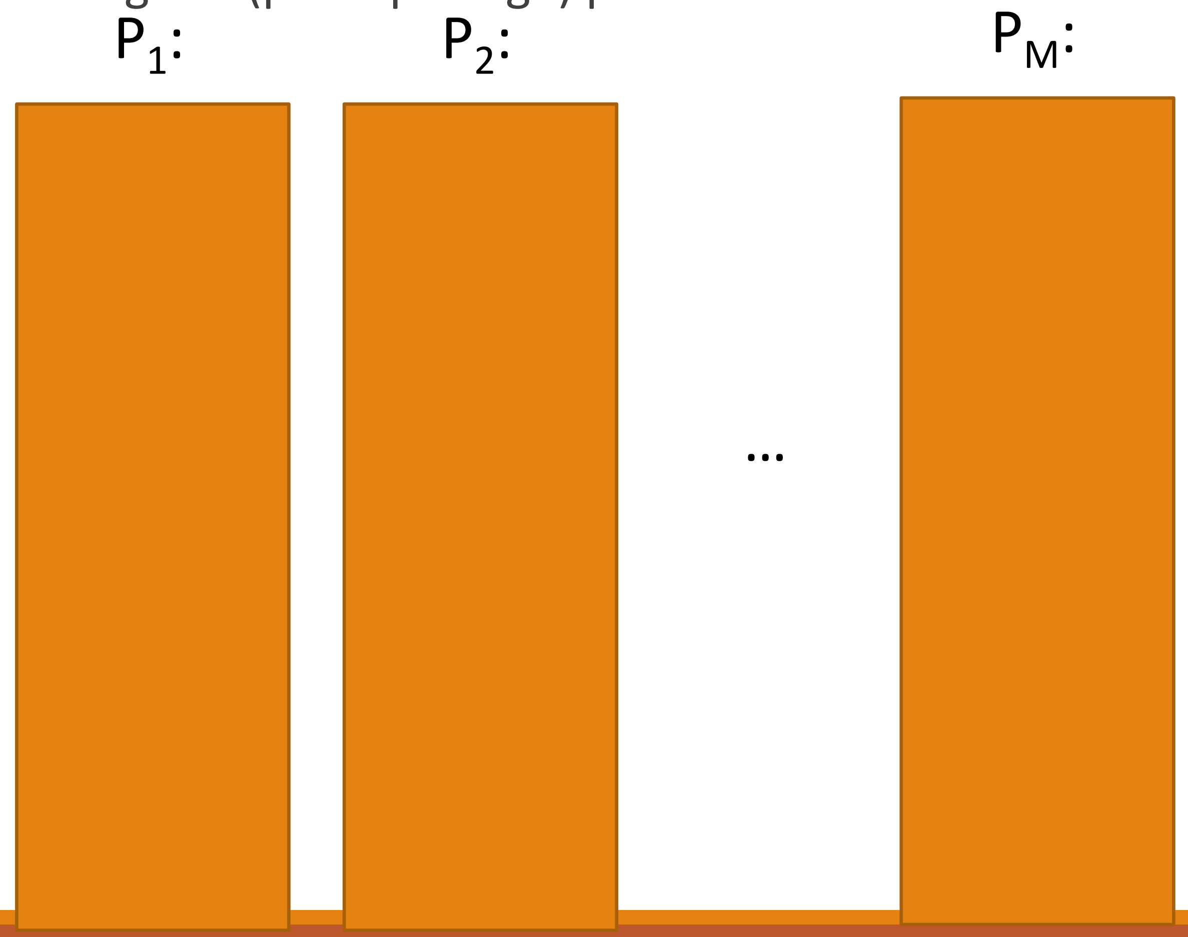

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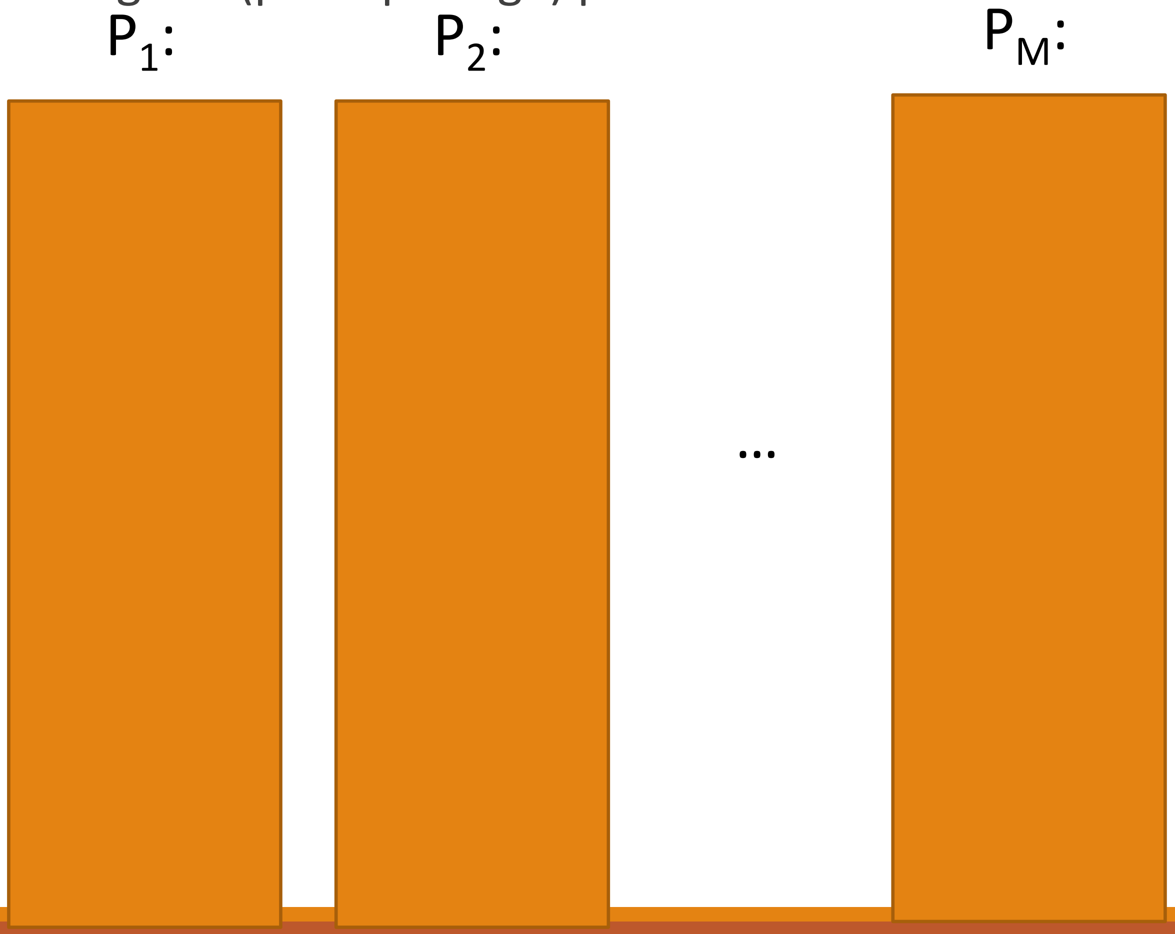
Colors:
Blue in RAM
Orange in disk
White does not exists yet



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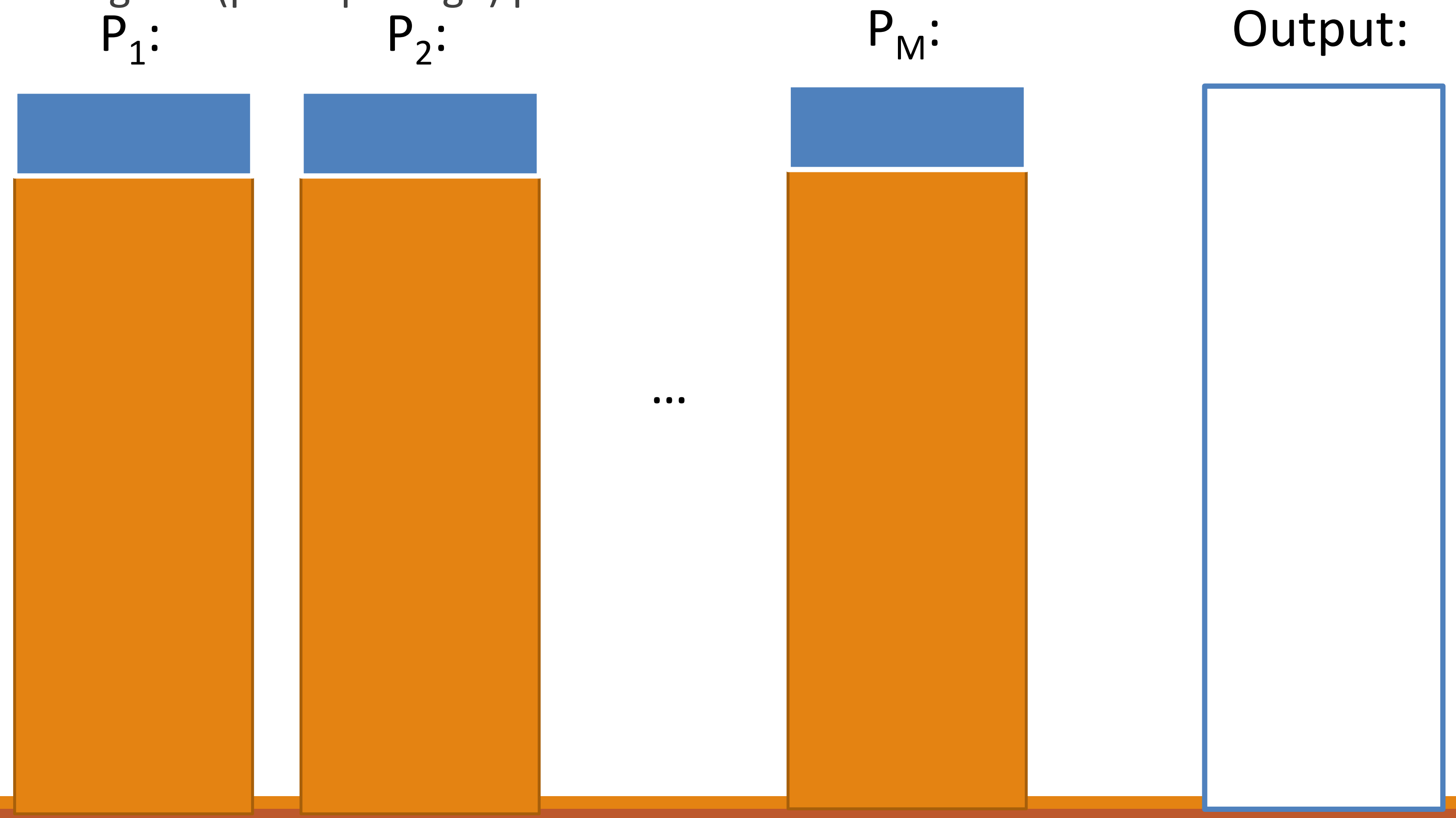

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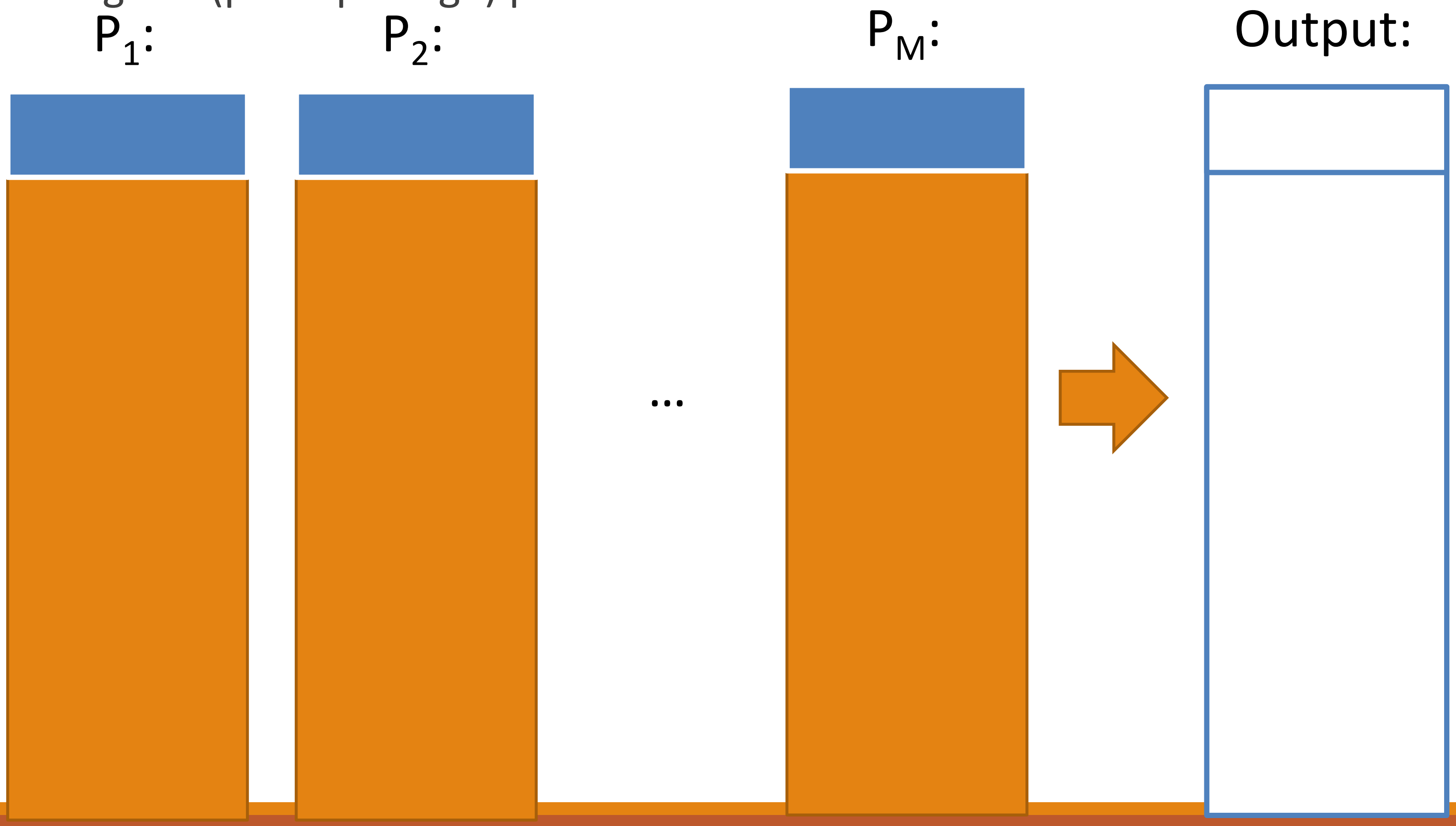

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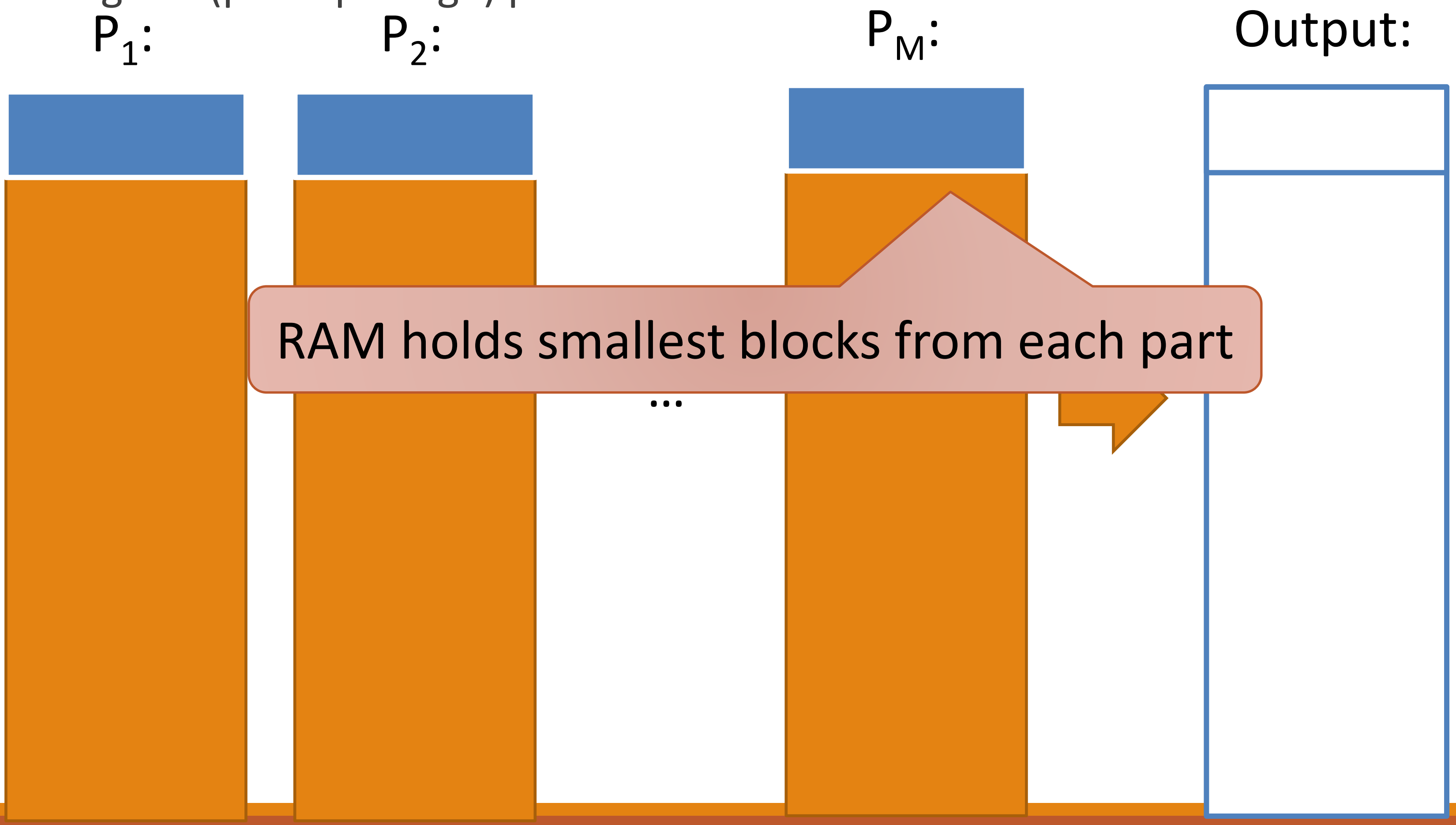

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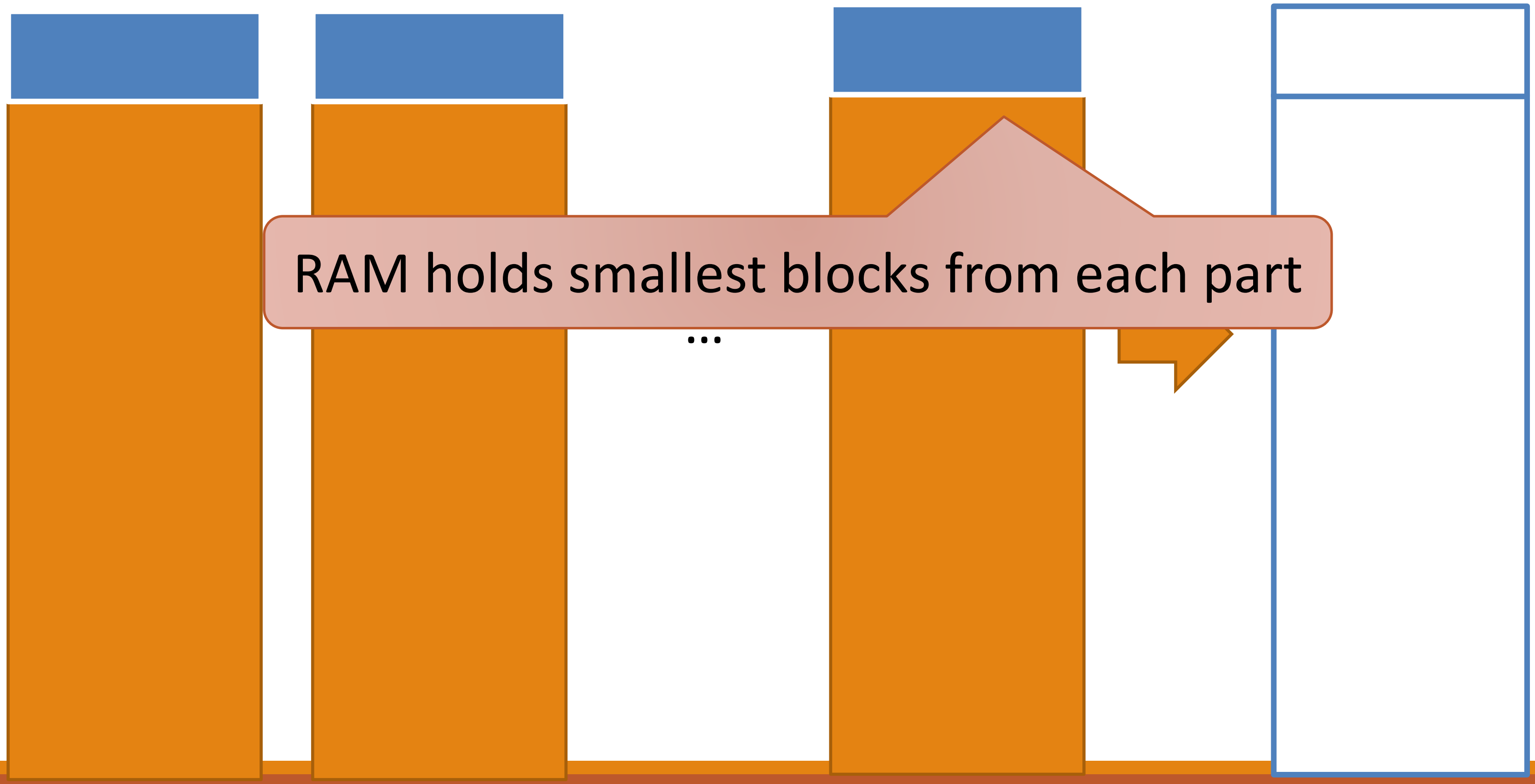

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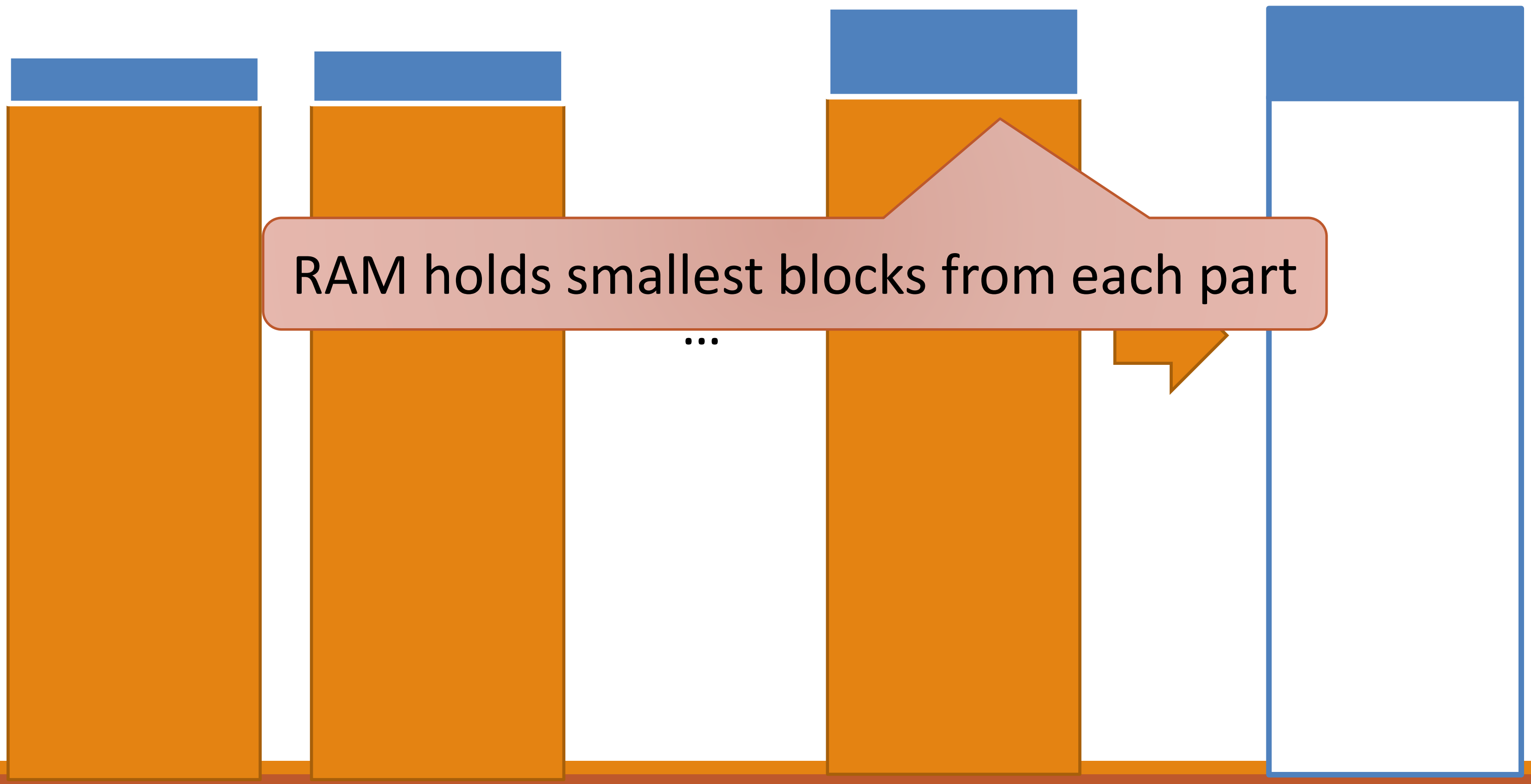

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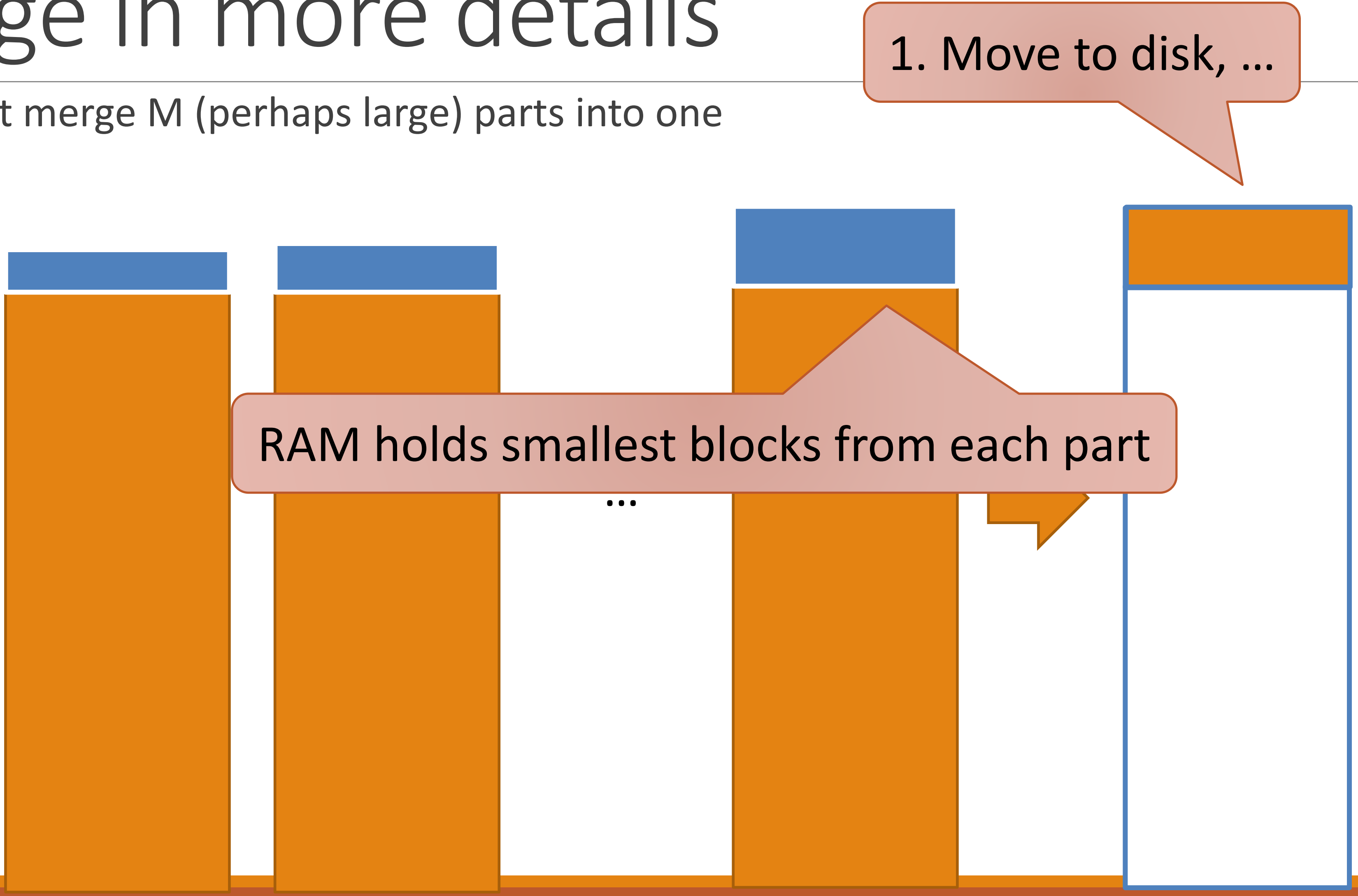

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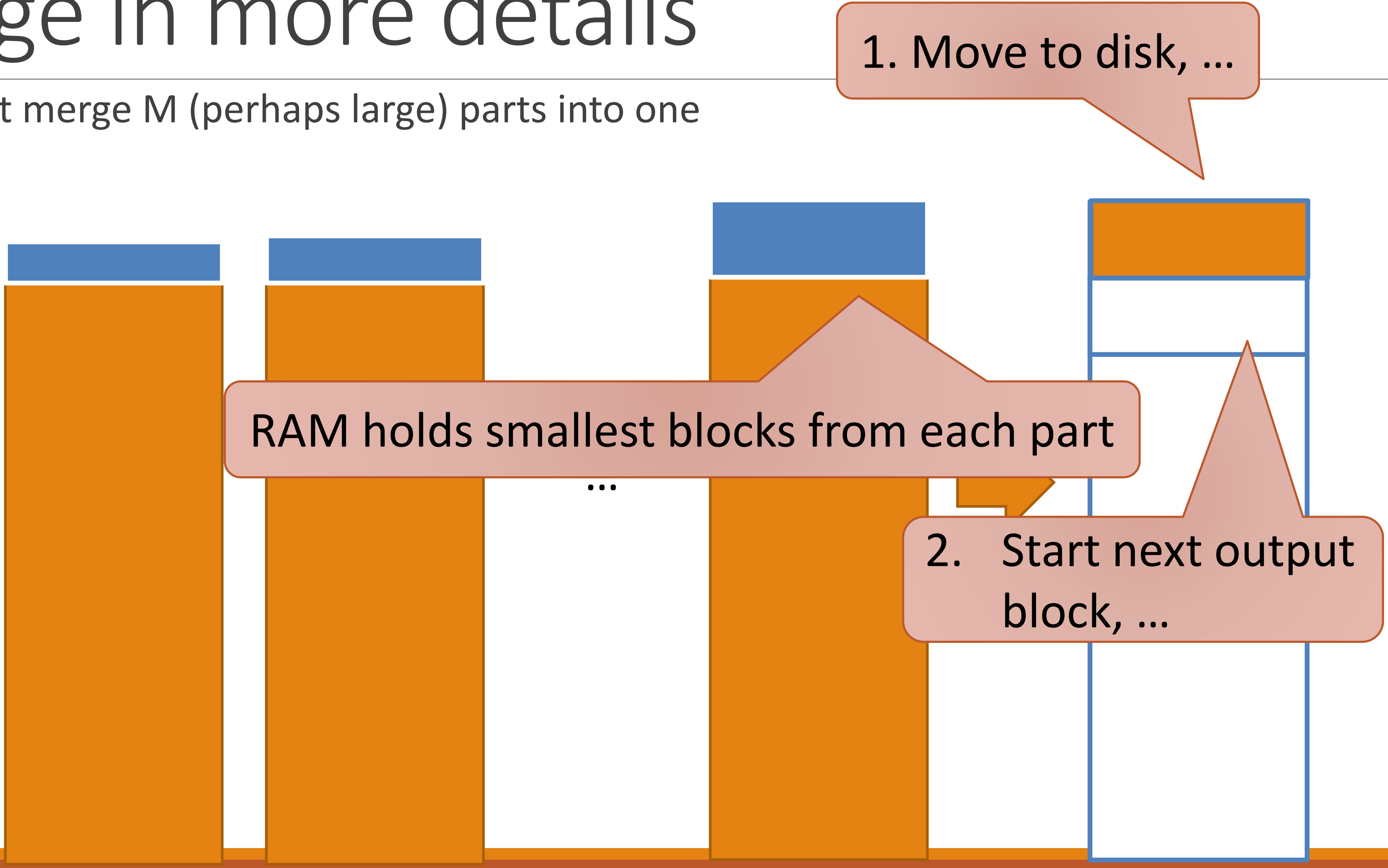
1. Move to disk, …
RAM holds smallest blocks from each part
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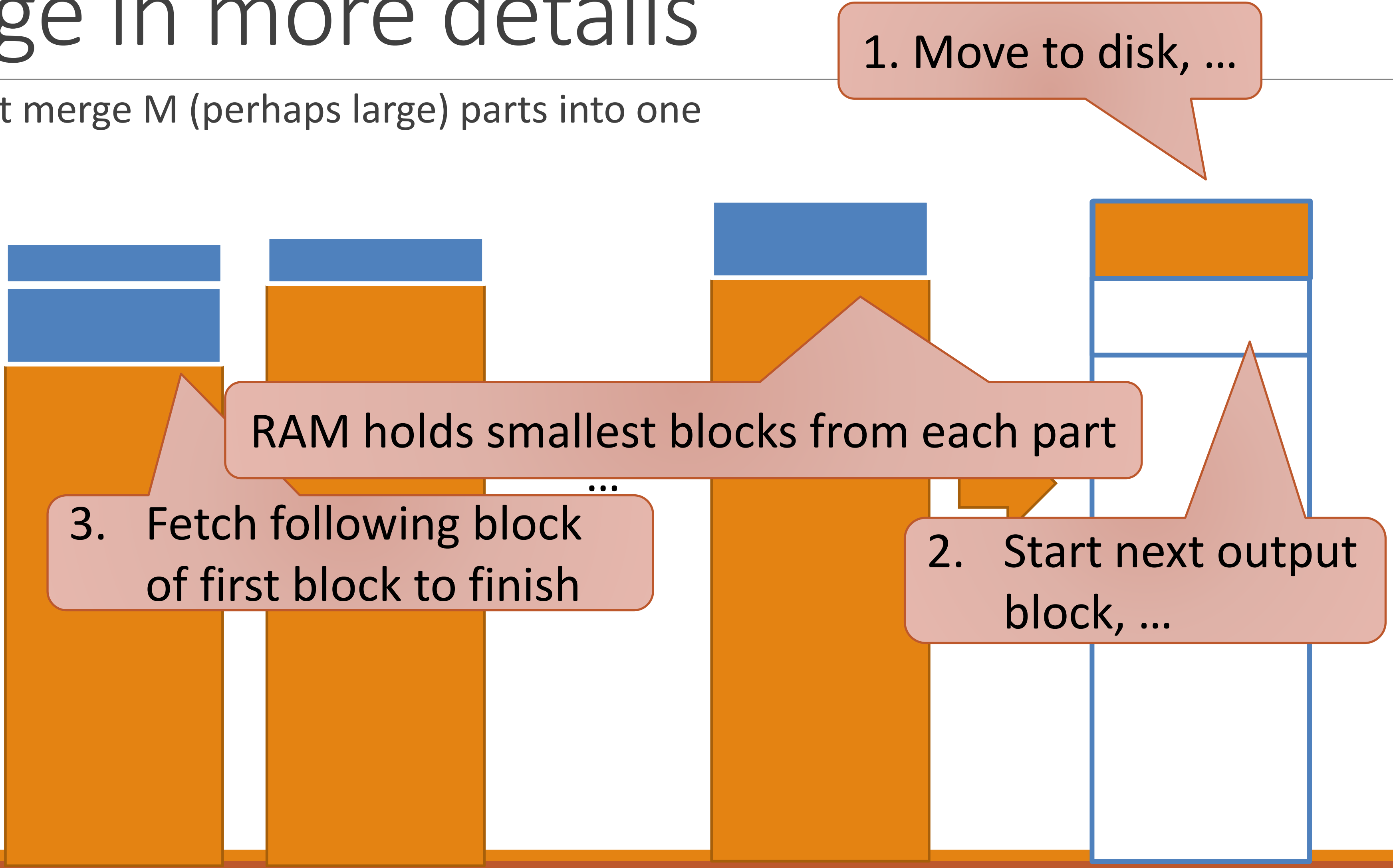
1. Move to disk, …
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3. Fetch following block of first block to finish
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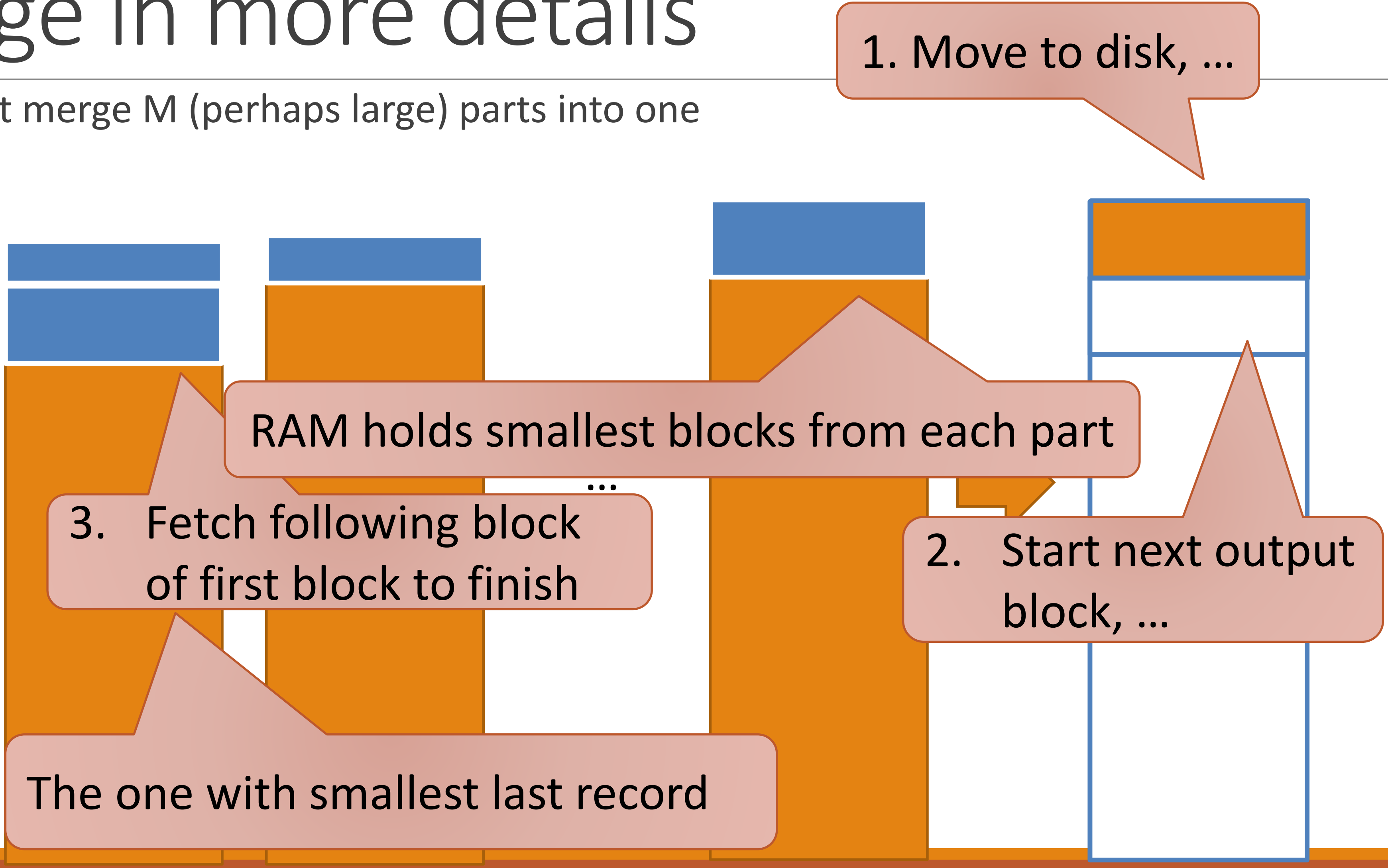
1. Move to disk, …
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…
3. Fetch following block of first block to finish
2. Start next output block, …
The one with smallest last record

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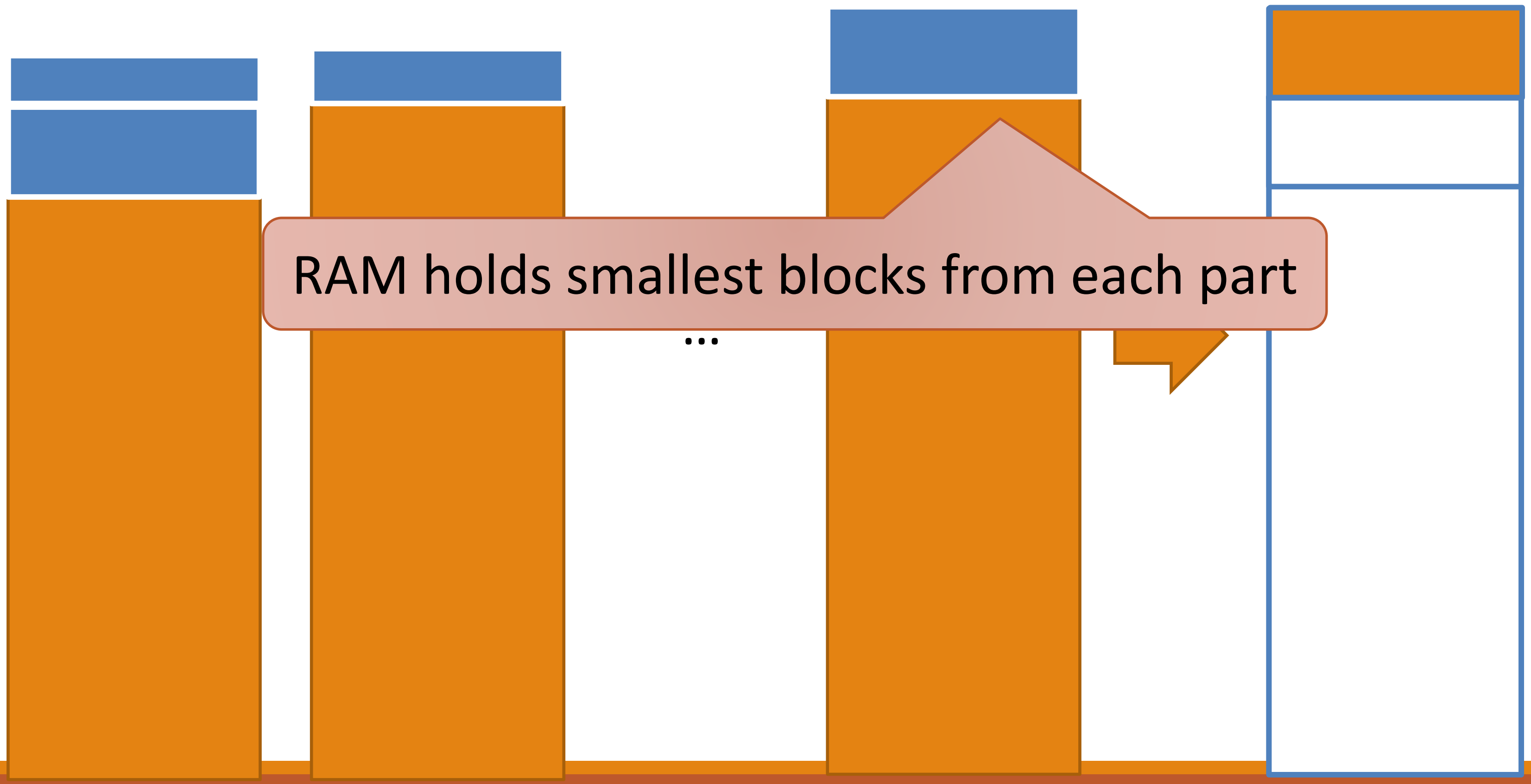

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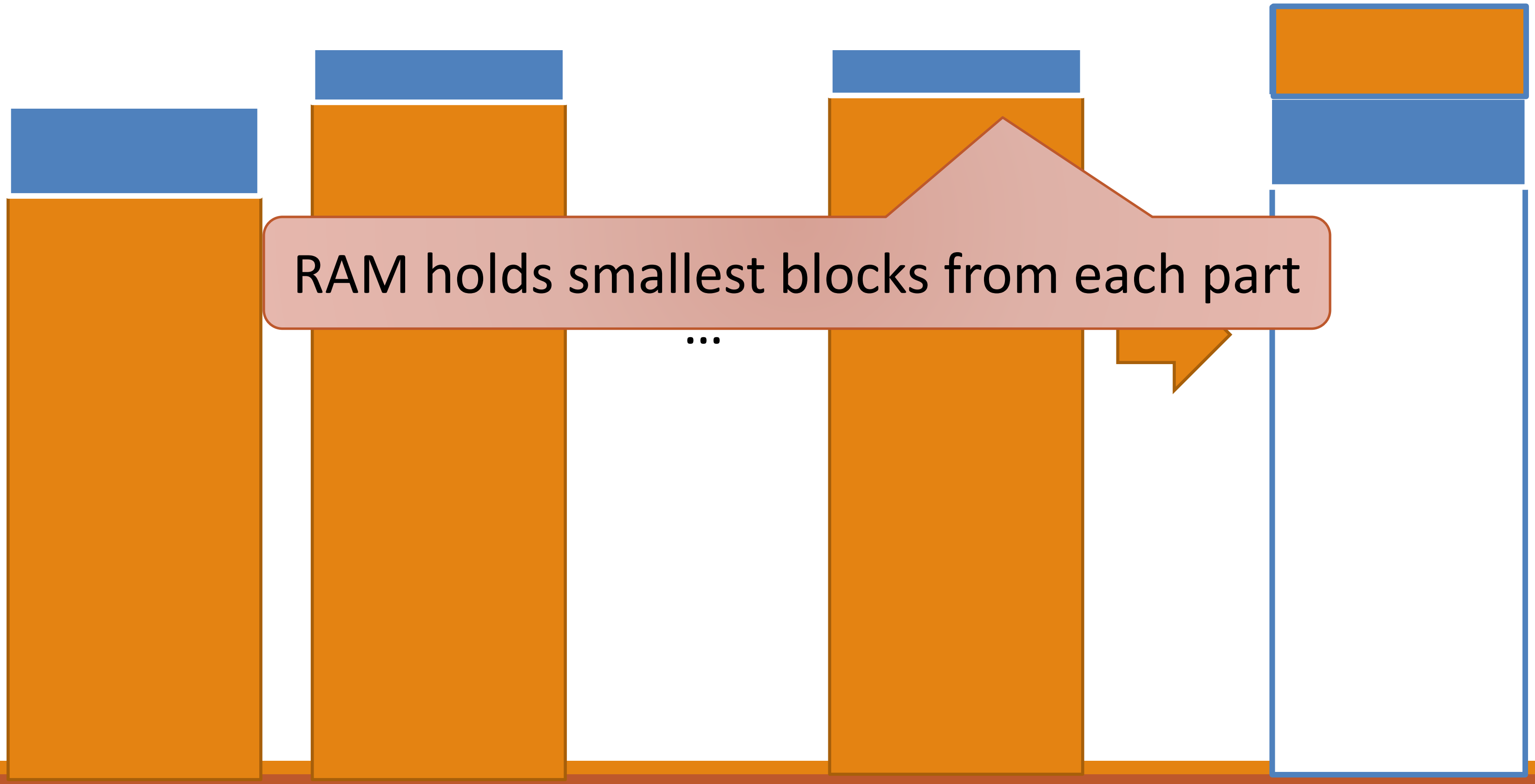

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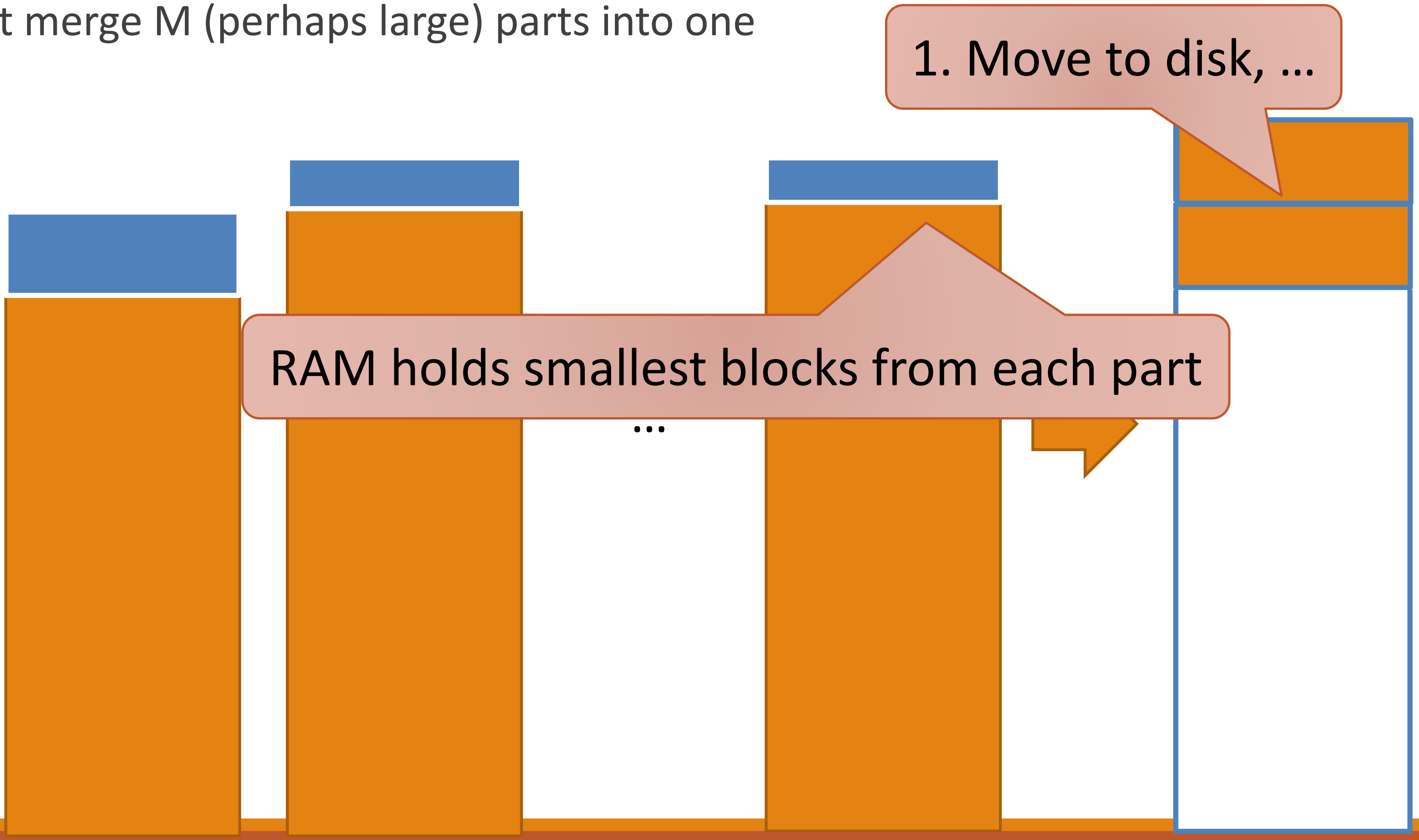

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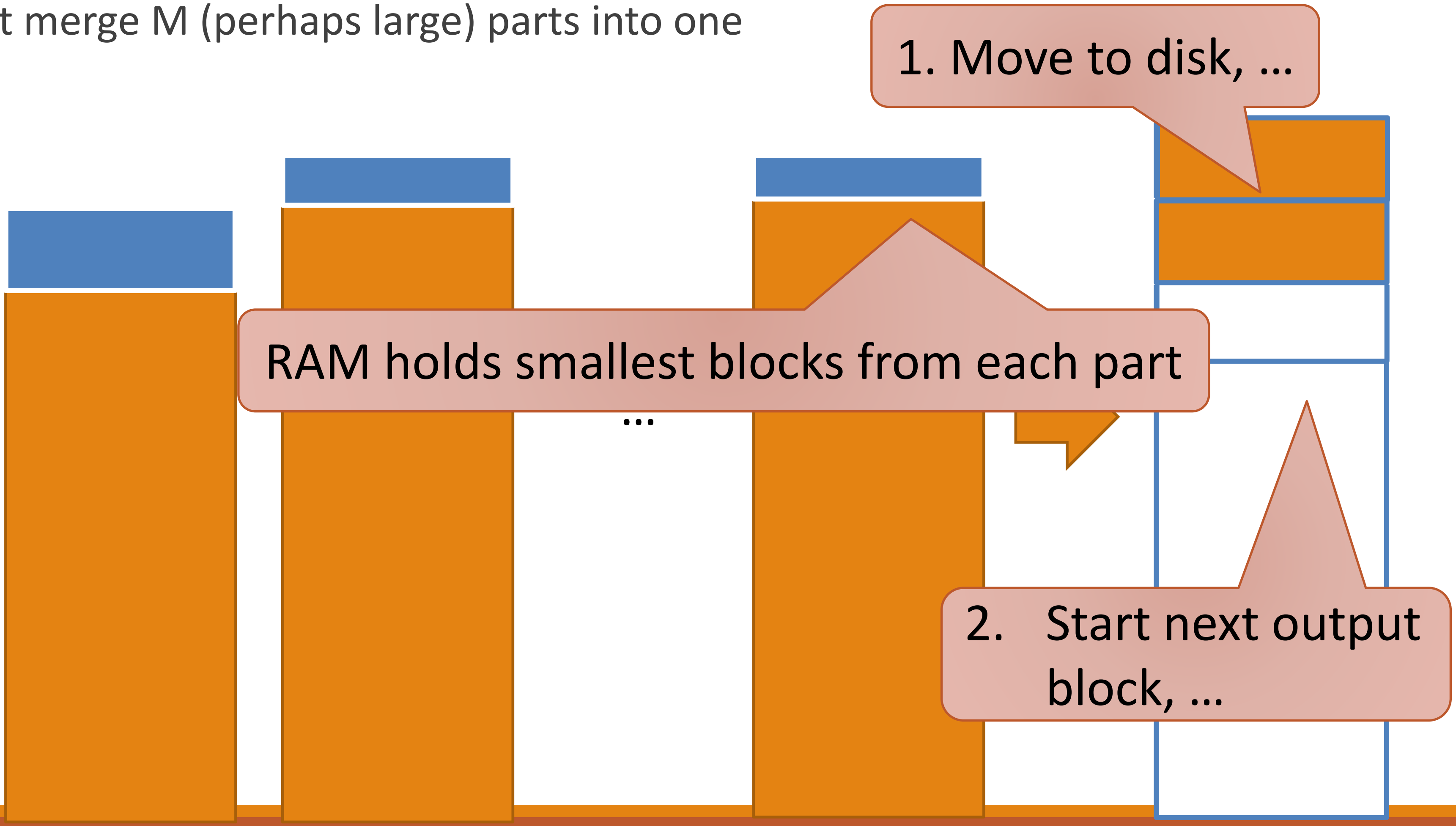
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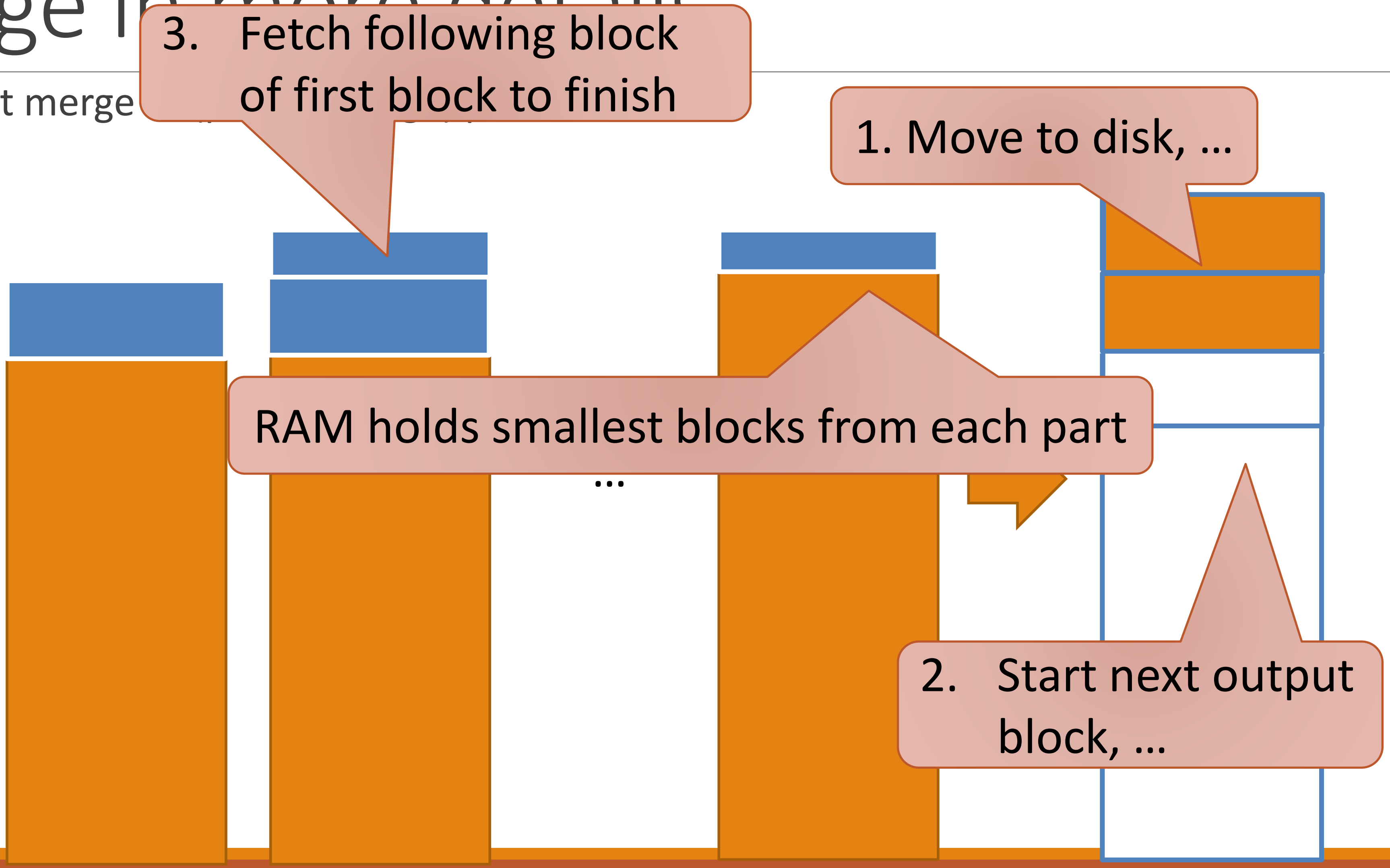

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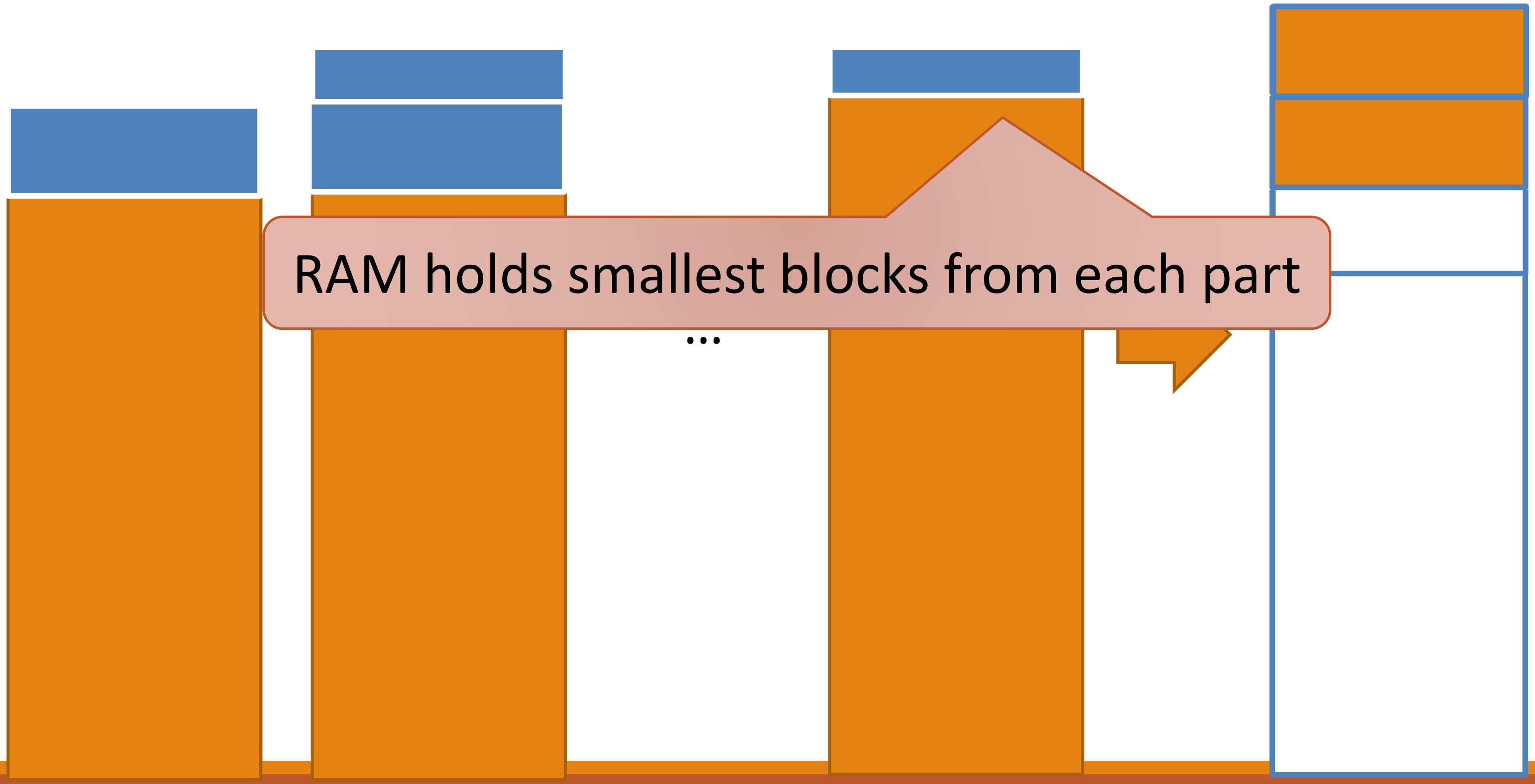

Analysis of External Merge Sort

External merge sort:

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Merge

On each level of recursion we scan through all blocks once = O(N/B) disk operations, where B is block size

Analy ort

We split in M buckets in each level of recursion until reminder is below MB (where it fits in RAM and can be sorted directly): $\log_M(N/(MB))$ levels

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Total disk operations:
$O(N/B \log_M(N/(MB)))$

On each level of recursion we scan through all blocks once = O(N/B) disk operations, where B is block size

Comparison

In practice, since hard disks are slow:

- D)
- D is time for a disk operation

Quicksort (or other internal memory sorting algorithms) uses $O(N/B \log_2(N) \times D)$ time!

Common values for D, B and M:

- D was around $3.1 \cdot 10^4$ **nano-seconds** on SSDs and $2 \cdot 10^6$ **nano-seconds** on fast normal hard disks
- B is typically around 512-4096 bytes
- M is the size of your RAM divided by B, so say 8-32 GB RAM / 512-4096 bytes: $2.0 \cdot 10^6$ to $6.5 \cdot 10^7$

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Assuming we are lucky about pivot elements

Summary

External memory sorting is very much faster on inputs that are much larger than main memory