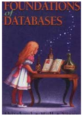
Week 5 Workshop



Alice: Your model reduces the most interesting information to something flat and boring.

Vittorio: You're right, and this causes a lot of problems.

Sergio: Designing the schema for a complex application is tough, and it is easy to make mistakes when updating a database.

Riccardo: Also, the system knows so little about the data that it is hard to obtain good performance.

Alice: Are you telling me that the model is bad?

Vittorio: No, wait, we are going to fix it!

(Foundations of Databases, S. Abiteboul, R. Hull, V. Vianu, Addison-Wesley, 1995)

Housekeeping

1. **Assignment 1 (SQL)** (due 11:59pm, 3 Sep 2021)

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    - Pay attention to which attributes you need to list, whether you need to order the tuples, syntax issues,etc. (Partial marks may be awarded)
    - **Do not wait until the last minute to check/submit your solution.** (Refer to the instructions in the assignment specification.)

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2. **More consultation hours in Week 6**
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Update Anomalies

- **What could happen to insert, delete and update operations?**

ENROLMENT					
Name	StudentID	DoB	CourseNo	Semester	Unit
Tom	123456	25/01/1989	COMP2400	2010 S2	6
Tom	123456	25/01/1989	COMP8740	2011 S2	12
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Australian National University

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  - **Insertion anomalies**: If inserting a new course COMP3000, then ...

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  - **Deletion anomalies**: If deleting the enrolled course COMP2400 of Fran, then ...

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  - **Insertion anomalies**: If inserting a new course COMP3000, then ... (i.e., cannot insert NULL values into Course because of the entity integrity constraint).
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  - **Insertion anomalies**: If inserting a new course COMP3000, then ... (i.e., cannot insert NULL values into Course because of the entity integrity constraint).
  - **Deletion anomalies**: If deleting the enrolled course COMP2400 of Fran, then ... the personal information of Fran, such as DoB, will be lost as well.
  - **Modification anomalies**: If changing the DoB of Michael, then ...

Update Anomalies

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- **Insertion anomalies**: If inserting a new course COMP3000, then ... (i.e., cannot insert NULL values into Course because of the entity integrity constraint).
- **Deletion anomalies**: If deleting the enrolled course COMP2400 of Fran, then ... the personal information of Fran, such as DoB, will be lost as well.
- **Modification anomalies**: If changing the DoB of Michael, then ... update every tuple that records the DoB of this student.

Update Anomalies?

Enrolment					
Name	StudentID	DoB	CourseNo	Semester	Unit
Tom	123456	25/01/1988	COMP2400	2010 S2	6
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⇓

Student		
Name	StudentID	DoB
Tom	123456	25/01/1988
Michael	123458	21/04/1985
Fran	123457	11/09/1987

Course	
CourseNo	Unit
COMP2400	6
COMP8740	12

Enrol		
StudentID	CourseNo	Semester
123456	COMP2400	2010 S2
123456	COMP8740	2011 S2
123458	COMP2400	2009 S2
123458	COMP8740	2011 S2
123457	COMP2400	2009 S2

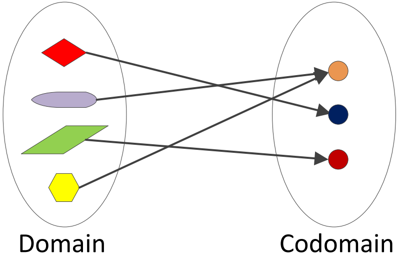
Why Functional Dependencies?

FDs tell us "relationship between and among attributes"!

- FDs are developed to define the **goodness** and **badness** of (relational) database design in a formal way.
  - **Top down**: start with a relation schema and FDs, and produce smaller relation schemas in certain normal form (called *normalisation*).
  - **Bottom up**: start with attributes and FDs, and produce relation schemas (*not popular in practice*).

What is "Functional" about Functional Dependencies?

- The notion of functional dependency is very close to the notion of function.
- A (total) **function** $f : X \rightarrow Y$ describes a relationship between two sets X and Y such that each element of X is mapped to a unique element of Y.



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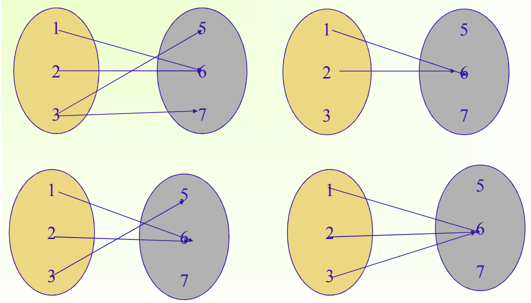
- **Exercise:** *which of them represent a function?*



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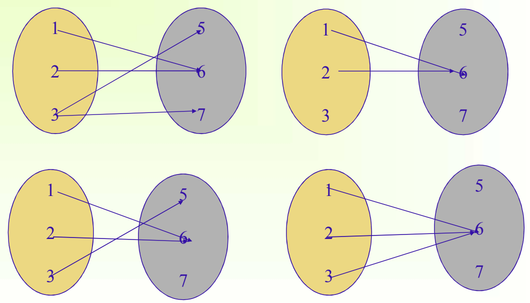
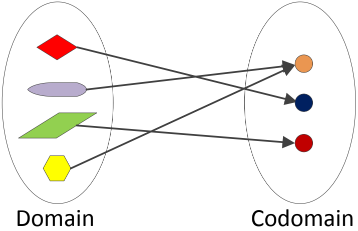


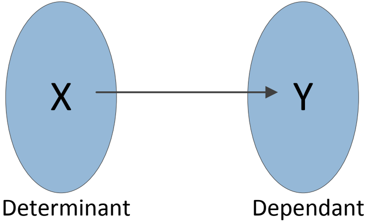
Answer: **The ones at the bottom.**

Functions vs Functional Dependencies

Function



Functional dependency



Functions vs Functional Dependencies

$$f(x) = x^2$$

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x	$f(x)$
1	1
2	4
3	9
4	16
5	25
6	36
...	...

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$$X \rightarrow f(x)$$

Formal Definition

- Let R be a relation schema.
  - A **FD** on R is an expression $X \rightarrow Y$ with attribute sets $X, Y \subseteq R$.
  - A relation $r(R)$ **satisfies $X \rightarrow Y$ on R** if, for any two tuples $t_1, t_2 \in r(R)$, whenever the tuples t_1 and t_2 coincide on values of X, they also coincide on values of Y.

$$\begin{array}{c} t_1[X] = t_2[X] \\ \Downarrow \\ t_1[Y] = t_2[Y] \end{array}$$

- A FD is **trivial** if it can *always* be satisfied, e.g.,
  - $\{A, B\} \rightarrow \{A\}$
  - $\{A, B, C\} \rightarrow \{A, B, C\}$
- **Syntactical convention**: (1) Instead of $\{A, B, C\}$, we may use ABC. (2) $A, B, \ldots$ for individual attributes and $X, Y, \ldots$ for sets of attributes.

Exercise - Functional Dependencies

- A functional dependency specifies a constraint on the relation schema that must hold **at all times**.
- Consider the following relation with attributes $\{$A,B,C,D,E$\}$. Do they satisfy the given FDs?

$r(R)$				
A	**B**	**C**	**D**	**E**
1	2	3	4	5
1	2	2	2	2
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3. $E \rightarrow ABCD$

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1. $ABC \rightarrow AB$ Yes.
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3. $E \rightarrow ABCD$ Yes.

How to Identify FDs in General?

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  Can be provided in the form of discussion with application users and/or data requirement specifications.

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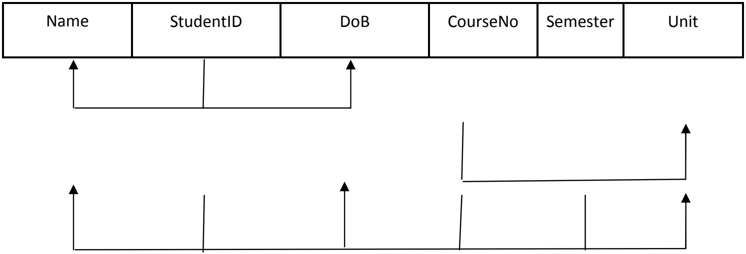
- In real-life applications, we often use the following approaches:

  (1) **Analyse data requirements**
  Can be provided in the form of discussion with application users and/or data requirement specifications.

  (2) **Analyse sample data**
  Useful when application users are unavailable for consultation and/or the document is incomplete.

(1) Analyse Data Requirements and FD Diagram



- StudentID $\rightarrow$ Name, DoB;
- CourseNo $\rightarrow$ Unit;
- StudentID, CourseNo, Semester $\rightarrow$ Name, DoB, Unit.

(2) Analyse Sample Data

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- We may have:
  - $\{$StudentID$\} \rightarrow \{$Name, DoB$\}$;

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  - $\{$StudentID$\} \rightarrow \{$Name, DoB$\}$;
  - $\{$StudentID, Name$\} \rightarrow \{$DoB$\}$;
  - $\{$Name$\} \rightarrow \{$StudentID$\}$ $\times$;
  - ......

(2) Analyse Sample Data

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- We may have:
  - $\{\text{StudentID}\} \rightarrow \{\text{Name, DoB}\}$;
  - $\{\text{StudentID, Name}\} \rightarrow \{\text{DoB}\}$;
  - $\{\text{Name}\} \rightarrow \{\text{StudentID}\}$ $\times$;
  - ......

Limitations:
(1) Sample data needs to be a true representation of **all possible values** in the database.
(2) Do we need all FDs?

Inference?

- To design a good database, we need to consider **all possible FDs**.

  Example:

  If $\{StudentID\} \rightarrow \{ProjectNo\}$ and $\{ProjectNo\} \rightarrow \{Supervisor\}$, we can infer $\{StudentID\} \rightarrow \{Supervisor\}$.

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Example:

If $\{StudentID\} \rightarrow \{ProjectNo\}$ and $\{ProjectNo\} \rightarrow \{Supervisor\}$, we can infer $\{StudentID\} \rightarrow \{Supervisor\}$.

If each student works on one project and each project has one supervisor, then each student must have one project supervisor.

Inference?

- To design a good database, we need to consider **all possible FDs**.

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  If $\{StudentID\} \rightarrow \{ProjectNo\}$ and $\{ProjectNo\} \rightarrow \{Supervisor\}$, we can infer $\{StudentID\} \rightarrow \{Supervisor\}$.

  If each student works on one project and each project has one supervisor, then each student must have one project supervisor.

- **Can we systematically infer all possible FDs?**

Armstrong's Inference Rules (Slides 16-25 will not to be assessed)

- The **Armstrong's inference rules** consist of the following three rules:
  - **Reflexive rule**: $XY \rightarrow Y$
  - **Augmentation rule**: $\{X \rightarrow Y\} \models XZ \rightarrow YZ$
  - **Transitive rule**: $\{X \rightarrow Y, Y \rightarrow Z\} \models X \rightarrow Z$
- We use the notation $\Sigma \models X \rightarrow Y$ to denote that $X \rightarrow Y$ is **inferred** from the set Σ of functional dependencies.

Rule 1 – Reflexive Rule

- $XY \rightarrow Y$.

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Name	StudentID	DoB	CourseNo	Semester	Unit
Tom	123456	25/01/1988	COMP2400	2010 S2	6
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- **Example**:
  {StudentID, CourseNo, Semester} → {CourseNo, Semester}, where
  - X={StudentID};
  - Y={CourseNo, Semester}.

Rule 2 – Augmentation Rule

- $\{X \rightarrow Y\} \models XZ \rightarrow YZ$.

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Name	StudentID	DoB	CourseNo	Semester	Unit
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- **Example**:
$\{\{\text{CourseNo}\} \rightarrow \{\text{Unit}\}\} \models \{\text{CourseNo}, \text{Semester}\} \rightarrow \{\text{Unit}, \text{Semester}\}$,
where
    - X={CourseNo};
    - Y={Unit};
    - Z={Semester}.

Rule 3 – Transitive Rule

- $\{X \rightarrow Y, Y \rightarrow Z\} \models X \rightarrow Z$.

Enrolment					
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- **Example**: {StudentID, CourseNo} $\rightarrow$ {CourseNo}, {CourseNo} $\rightarrow$ {Unit} $\models$ {StudentID, CourseNo} $\rightarrow$ {Unit}, where
  - X={StudentID, CourseNo};
  - Y={CourseNo};
  - Z={Unit}.

Other Derived Rules

- From Armstrong's axioms (i.e., reflexive, augmentation, transitive rules), we can derive the following rules:

Other Derived Rules

- From Armstrong's axioms (i.e., reflexive, augmentation, transitive rules), we can derive the following rules:
  - **Union rule**: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$
  - **Example:** If StudentID $\rightarrow$ Name and StudentID $\rightarrow$ DoB hold, then we have StudentID $\rightarrow$ Name, DoB, where
    - X=StudentID;
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Other Derived Rules

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  - **Decomposition rule**: If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$
  - **Example:** If StudentID $\rightarrow$ Name, DoB holds, then we have StudentID $\rightarrow$ Name and StudentID $\rightarrow$ DoB, where
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Example on Armstrong's Inference Rules

- If each student works on one project and each project has one supervisor, does each student have one project supervisor?

$$\begin{array}{l}\{\{\text{StudentID}\}\rightarrow\{\text{ProjectNo}\},\\ \{\text{ProjectNo}\}\rightarrow\{\text{Supervisor}\}\}\end{array} \quad \models \quad \{\text{StudentID}\}\rightarrow\{\text{Supervisor}\}$$

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- This can be proven by using the Transitive rule:

$$\{X \rightarrow Y, Y \rightarrow Z\} \models X \rightarrow Z$$

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X	Y
0	2
1	2

Armstrong's Inference Rules

- **Two questions:**

[1] William Ward Armstrong: Dependency Structures of Data Base Relationships, page 580-583. IFIP Congress, 1974.

Armstrong's Inference Rules

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    $\rightsquigarrow$ **completeness** (you can prove anything that is right)
- **Theorem** (W. W. Armstrong, 1974[1])
  - The Armstrong's inference rules are both **sound** and **complete**.

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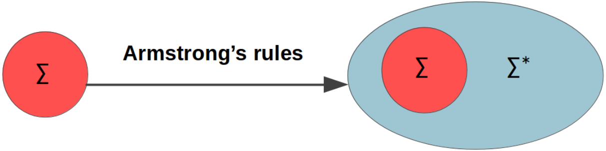

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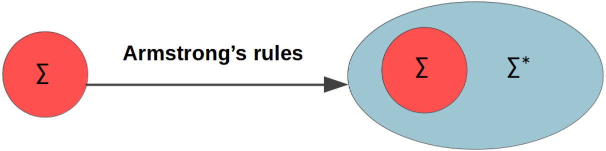

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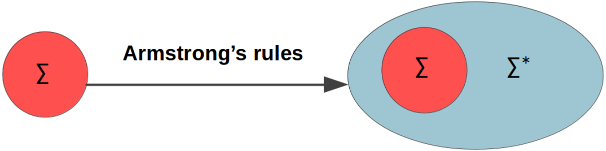

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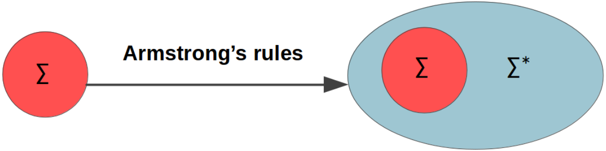

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  Example: Consider a relation schema $R = \{A, B, C, D, E\}$ and a set of FDs $\Sigma = \{AB \to CD, B \to E, DE \to A\}$. How can we use the Armstrong rules to show that $DB \to A \in \Sigma^*$?

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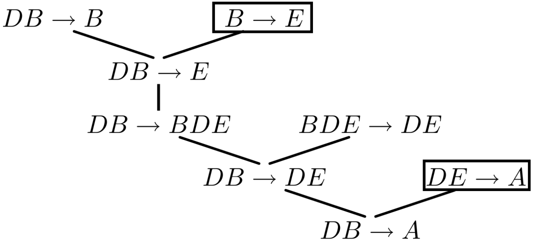

- **How can we derive the proof more efficiently?**

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- Let Σ be a set of FDs. Check whether or not $\Sigma \models X \rightarrow W$ holds?

[2] See Algorithm 15.1 on Page 538 in [Elmasri & Navathe, 7th edition] or Algorithm 1 on Page 555 in [Elmasri & Navathe, 6th edition]

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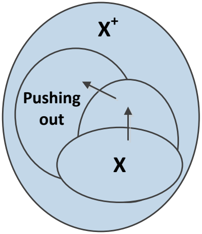
  2. $\Sigma \models X \rightarrow W$ holds iff $W \subseteq X^+$.
- **Algorithm**[2]
  - $X^+ := X$;
  - repeat until no more change on X^+
    - for each $Y \rightarrow Z \in \Sigma$ with $Y \subseteq X^+$, add all the attributes in Z to X^+, i.e., replace X^+ by $X^+ \cup Z$.



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- Consider a relation schema $R = \{A, B, C, D, E, F\}$, a set of FDs $\Sigma = \{AC \rightarrow B, B \rightarrow CD, C \rightarrow E, AF \rightarrow B\}$ on R.
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2. Then we check that $DE \subseteq (AC)^+$. Hence $\Sigma \models AC \rightarrow DE$.

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  2. Then we check that $DE \subseteq (AC)^+$. Hence $\Sigma \models AC \rightarrow DE$.

- **Can you quickly tell whether or not $\Sigma \models AC \rightarrow EF$ holds?**

  $\Sigma \not\models AC \rightarrow EF$ does not hold because $EF \not\subseteq (AC)^+$

Exercise – Implied Functional Dependencies

- Consider a relation schema $R = \{A, B, C, D, E\}$ and a set of functional dependencies $\Sigma = \{A \rightarrow C, B \rightarrow C, CD \rightarrow E\}$ on R.
- Decide whether or not
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  1. $\Sigma \models AD \rightarrow CE$ holds
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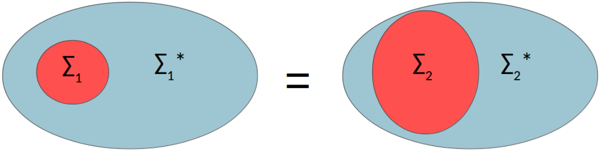
- We build the closure for the set of attributes and check:
  1. $(AD)^+ = (ACD)^+ = (ACDE)^+ = ACDE$ and $CE \subseteq (AD)^+$, hence $\Sigma \models AD \rightarrow CE$.
  2. $(BD)^+ = (BCD)^+ = (BCDE)^+ = BCDE$ and $AC \not\subseteq (BD)^+$, hence $\Sigma \not\models BD \rightarrow AC$.

Equivalence of Functional Dependencies

- Σ_1 and Σ_2 are **equivalent** if $\Sigma_1^* = \Sigma_2^*$.



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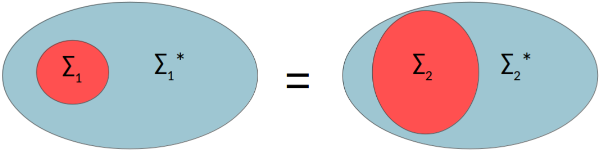

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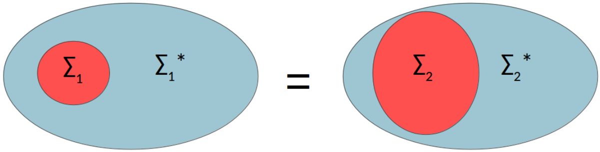

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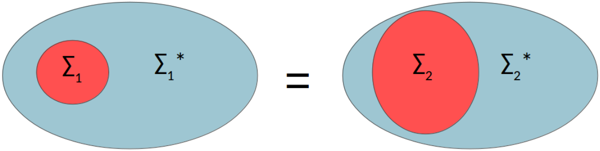

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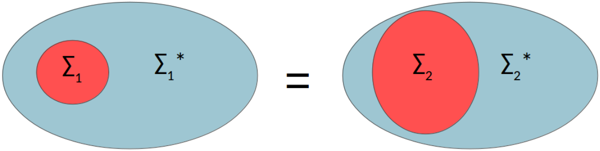

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- If $\Sigma_1 \models \Sigma_2$ and $\Sigma_2 \models \Sigma_1$, are Σ_1 and Σ_2 equivalent? Yes.
- **Questions:** Can we find the **minimal** one among equivalent sets of FDs?

Minimal Cover – The Hard Part!

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- Let Σ be a set of FDs. A **minimal cover** Σ_m of Σ is a set of FDs such that
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  3. **Determinant:** each FD has as few attributes on the left hand side as possible, i.e., for each FD $X \rightarrow A$ in Σ_m, check each attribute B of X to see if we can replace $X \rightarrow A$ with $(X - B) \rightarrow A$ in Σ_m;

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  4. Remove a FD from Σ_m if it is redundant.

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     - $\Sigma = \{B \rightarrow A, D \rightarrow A, \mathbf{AB} \rightarrow \mathbf{D}\}$, $\Sigma_1 = \{B \rightarrow A, D \rightarrow A, \mathbf{A} \rightarrow \mathbf{D}\}$

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     - check whether $\Sigma^* = \Sigma_1^*$?

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       If $\Sigma \not\models \mathbf{A \rightarrow D}$, then $\Sigma^* \neq \Sigma_1^*$.
     - $\Sigma \not\models \mathbf{A \rightarrow D}$ because $D \not\subseteq (A)^+$.

     No. $AB \rightarrow D$ cannot be replaced by $A \rightarrow D$.

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       If $\Sigma \models \mathbf{B} \rightarrow \mathbf{D}$, then $\Sigma \models \Sigma_2$ and $\Sigma_2 \models \Sigma$, indicating $\Sigma^* = \Sigma_2^*$.
       If $\Sigma \not\models \mathbf{B} \rightarrow \mathbf{D}$, then $\Sigma^* \neq \Sigma_2^*$
     - $\Sigma \models \mathbf{B} \rightarrow \mathbf{D}$ because $D \subseteq (B)^+$.

     Yes. $AB \rightarrow D$ can be replaced by $B \rightarrow D$.

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     - check whether $B \rightarrow A$ is redundant?

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     - check whether $B \rightarrow A$ is redundant?
     - $B \rightarrow A$ is redundant because $\{D \rightarrow A, B \rightarrow D\} \models B \rightarrow A$;

  Therefore, the minimal cover of Σ is $\{D \rightarrow A, B \rightarrow D\}$.

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- **Theorem:**

> The minimal cover of a set of functional dependencies Σ always exists but is not necessarily unique.

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Σ has two different minimal covers:

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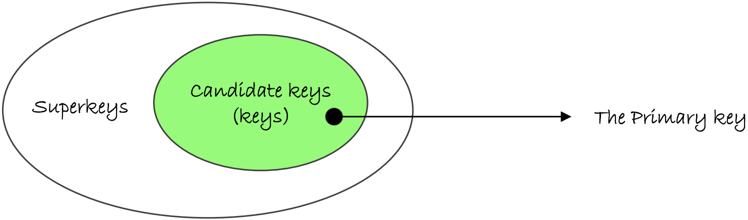
- The algorithm in the previous slide can find one, but not all minimal covers of a set of functional dependencies Σ.

Finding Keys

- Given a set Σ of FDs on a relation *R*, the question is:

How can we find all the (candidate) keys of *R*?



Finding Keys

- **Fact**: A key K of R always defines a FD $K \rightarrow R$.

[3] It extends Algorithm 15.2(a) in [Elmasri & Navathe, 7th edition, pp. 542], or Algorithm 2(a) or in Algorithm 2(a) in [Elmasri & Navathe, 6th edition pp. 558] to finding all keys of R

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  - if $X^+ = R$, then X is a superkey.
  - if no proper subset Y of X with $Y^+ = R$, then X is a key.
- A **prime attribute** is an attribute occurring in a key, and a **non-prime attribute** is an attribute that is not a prime attribute.

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Exercises - Keys and Minimal Cover

- Consider RENTAL={CustID, CustName, PropertyNo, DateStart, Owner} and the following set Σ of FDs:
  - $\{\text{CustID}\} \rightarrow \{\text{CustName}\}$
  - $\{\text{PropertyNo, StartDate}\} \rightarrow \{\text{CustID}\}$
  - $\{\text{PropertyNo, CustID}\} \rightarrow \{\text{StartDate}\}$
  - $\{\text{CustID, StartDate}\} \rightarrow \{\text{PropertyNo}\}$
  - $\{\text{Owner}\} \rightarrow \{\text{PropertyNo}\}$
- **Questions:**
  1. What are the keys of RENTAL?
  2. What is a minimal cover of Σ?

Exercises - Keys and Minimal Cover

- Consider RENTAL={CustID, CustName, PropertyNo, DateStart, Owner} and its FDs in the abbreviated form as
  - R={C, N, P, D, O}, and
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- **Solution**: Check $(X)^+$ for every subset of {C, N, P, D, O}.
  - *O* never appears in the dependent of any FD, *O* must be part of each key.

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- **Solution**: Check $(X)^+$ for every subset of {C, N, P, D, O}.
  - O never appears in the dependent of any FD, O must be part of each key.
  - $(O)^+ = OP$
  - $(CO)^+ = CPNDO$, $(DO)^+ = CPNDO \dots$
  - Thus, {CustID, Owner} and {Owner, DateStart} are the keys.

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- **Solution**:
  1. start from Σ;
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  Therefore, Σ is a minimal cover itself.

Accommodation Database

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK $\{$hotelNo$\}$
  - ROOM(roomNo, hotelNo, type, price) with PK $\{$roomNo, hotelNo$\}$
  - GUEST(guestNo, guestName, guestAddress) with PK $\{$guestNo$\}$
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK $\{?\}$

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  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}

- We have some requirements on BOOKING:

R1 A booking can be made for one day only.
R2 A guest can make several bookings in a hotel for different days.
R3 A guest cannot make two or more bookings in the same hotel for the same day.
R4 A guest can make two or more bookings in different hotels for the same day.
R5 A room in any hotel can only be booked by one guest on the same date, i.e., no *double-booking*.

How to Identify FDs?

- Consider the following:
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- Which functional dependency does the following requirement imply?

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- Which functional dependency does the following requirement imply?

R1 A booking can be made for one day only.

$\hookrightarrow$ $\{$guestNo, hotelNo, roomNo$\}$ $\rightarrow$ $\{$date$\}$?

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}

- Which functional dependency does the following requirement imply?

R1 A booking can be made for one day only.

$\hookrightarrow$ {guestNo, hotelNo, roomNo} $\rightarrow$ {date}? **No**

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK $\{$hotelNo$\}$
  - ROOM(roomNo, hotelNo, type, price) with PK $\{$roomNo, hotelNo$\}$
  - GUEST(guestNo, guestName, guestAddress) with PK $\{$guestNo$\}$
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK $\{?\}$

- Which functional dependency does the following requirement imply?

R1 A booking can be made for one day only.

$\hookrightarrow$ $\{$guestNo, hotelNo, roomNo$\}$ $\rightarrow$ $\{$date$\}$? **No**

guestNo	hotelNo	roomNo	Date
001	H1	R101	28/08/2020
001	H1	R101	29/08/2020

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}
- Which functional dependency does the following requirement imply?

R2 A guest can make several bookings in a hotel for different days.

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}
- Which functional dependency does the following requirement imply?

  R2 A guest can make several bookings in a hotel for different days.

  None

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}

- Which functional dependency does the following requirement imply?

  R3 A guest cannot make two or more bookings in the same hotel for the same day.

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}

- Which functional dependency does the following requirement imply?

  R3 A guest cannot make two or more bookings in the same hotel for the same day.

  $\hookrightarrow$ {guestNo, hotelNo, date} $\rightarrow$ {roomNo}?

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}

- Which functional dependency does the following requirement imply?

R3 A guest cannot make two or more bookings in the same hotel for the same day.

$\hookrightarrow$ {guestNo, hotelNo, date} $\rightarrow$ {roomNo}? **Yes**

guestNo	hotelNo	roomNo	Date
001	H1	R101	29/08/2020
001	H1	R102 ×	29/08/2020

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}
- Which functional dependency does the following requirement imply?

  R4 A guest can make two or more bookings in different hotels for the same day.

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}
- Which functional dependency does the following requirement imply?

  R4 A guest can make two or more bookings in different hotels for the same day.

  None

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}

- Which functional dependency does the following requirement imply?

  R5 A room in any hotel can only be booked by one guest on the same date, i.e., no *double-booking*.

  $\hookrightarrow$ {hotelNo, date, roomNo} $\rightarrow$ {guestNo}

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}

- Which functional dependency does the following requirement imply?

  R5 A room in any hotel can only be booked by one guest on the same date, i.e., no *double-booking*.

  $\hookrightarrow$ {hotelNo, date, roomNo} $\rightarrow$ {guestNo} **Yes**

guestNo	hotelNo	roomNo	Date
001	H1	R101	29/08/2020
002 ×	H1	R101	29/08/2020

How to Find Candidate Keys?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}

- FDs on BOOKING
  - {guestNo, hotelNo, date} $\rightarrow$ {roomNo} by **R3**
  - {hotelNo, date, roomNo} $\rightarrow$ {guestNo} by **R5**

How to Find Candidate Keys?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}
- FDs on BOOKING
  - {guestNo, hotelNo, date} $\rightarrow$ {roomNo} by **R3**
  - {hotelNo, date, roomNo} $\rightarrow$ {guestNo} by **R5**
- Candidate keys on BOOKING
  - {guestNo, hotelNo, date}
  - {hotelNo, date, roomNo}

How to Identify FDs?

- Consider BOOKING(guestNo, hotelNo, date, roomNo) and the following changes:

  R1 A booking can be made for one day only.
  R2 A guest can make several bookings in a hotel for different days.
  ~~**R3** A guest cannot make two or more bookings in the same hotel for the same day.~~
  ~~**R4** A guest can make two or more bookings in different hotels for the same day.~~
  R5 A room in any hotel can only be booked by one guest on the same date, i.e., no *double-booking*.
  R6 A guest is not allowed to make more than one booking for the same day even in the different hotels.

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}
- Which functional dependency does the following requirement imply?

  R6 A guest is not allowed to make more than one booking for the same day even in the different hotels.

How to Identify FDs?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}
- Which functional dependency does the following requirement imply?

R6 A guest is not allowed to make more than one booking for the same day even in the different hotels.

$\hookrightarrow$ {guestNo, date} $\rightarrow$ {hotelNo, roomNo}

How to Find Candidate Keys?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}

- FDs on BOOKING
  - {hotelNo, date, roomNo} $\rightarrow$ {guestNo} by **R5**
  - {guestNo, date} $\rightarrow$ {hotelNo, roomNo} by **R6**

How to Find Candidate Keys?

- Consider the following:
  - HOTEL(hotelNo, hotelName, city) with PK {hotelNo}
  - ROOM(roomNo, hotelNo, type, price) with PK {roomNo, hotelNo}
  - GUEST(guestNo, guestName, guestAddress) with PK {guestNo}
  - BOOKING(guestNo, hotelNo, date, roomNo) with PK {?}
- FDs on BOOKING
  - {hotelNo, date, roomNo} $\rightarrow$ {guestNo} by **R5**
  - {guestNo, date} $\rightarrow$ {hotelNo, roomNo} by **R6**
- Candidate keys on BOOKING
  - {hotelNo, date, roomNo}
  - {guestNo, date}

Australian National University

(credit cookie) Kurt Gödel and Incompleteness Theorem

Kurt Gödel (1906-1978)

Armstrong's Inference Rules

- **Two questions:**
  - Are all the FDs inferred using the Armstrong's inference rules correct?
    $\rightsquigarrow$ **soundness (you cannot prove anything that is wrong)**
  - Can we use the Armstrong's inference rules to infer all possible FDs?
    $\rightsquigarrow$ **completeness (you can prove anything that is right)**
- **Theorem** (W. W. Armstrong)
  - The Armstrong's inference rules are both **sound** and **complete**.

Hilbert's program (1920s)

- **Formulation of mathematics**: formalize all true mathematical statements
- **Completeness**: all true mathematical statements can be proved
- **Consistency**: no contradiction can be obtained in the formalism
- **Decidability**: decide the truth or falsity of any mathematical statement.

Hilbert's program (1920s)

- **Formulation of mathematics**: formalize all true mathematical statements
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- **Consistency**: no contradiction can be obtained in the formalism
- **Decidability**: decide the truth or falsity of any mathematical statement.

David Hilbert (1862-1943)

We must know. We will know.

Kurt Gödel and Incompleteness Theorem

Kurt Gödel
(1906-1978)

- **Theorem** (Kurt Gödel, 1931)
  For any computable axiomatic system that is powerful enough to describe the arithmetic of the natural numbers, **there will always be at least one true but unprovable statement**.

Kurt Gödel and Gödel Prize

Kurt Gödel
(1906-1978)

John von Neumann
(1903-1957)

- Kurt Gödel's achievement in modern logic is singular and monumental – indeed it is more than a monument, it is a landmark which will remain visible far in space and time. — **John von Neumann**

Kurt Gödel and Gödel Prize

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(1903-1957)

- Kurt Gödel's achievement in modern logic is singular and monumental – indeed it is more than a monument, it is a landmark which will remain visible far in space and time. — **John von Neumann**
- The **Gödel prize** became an annual prize for outstanding papers in the area of theoretical computer science since 1993.