Student Name

**CS 480 Spring 2022 Written Assignment #02**

Due: **Thursday, March 3rd, 11:00 PM CST**

Points: **80**

**Instructions:**

1. Use this document template to report your answers. Name the complete document as follows:

LastName\_FirstName\_CS480\_Written02.doc

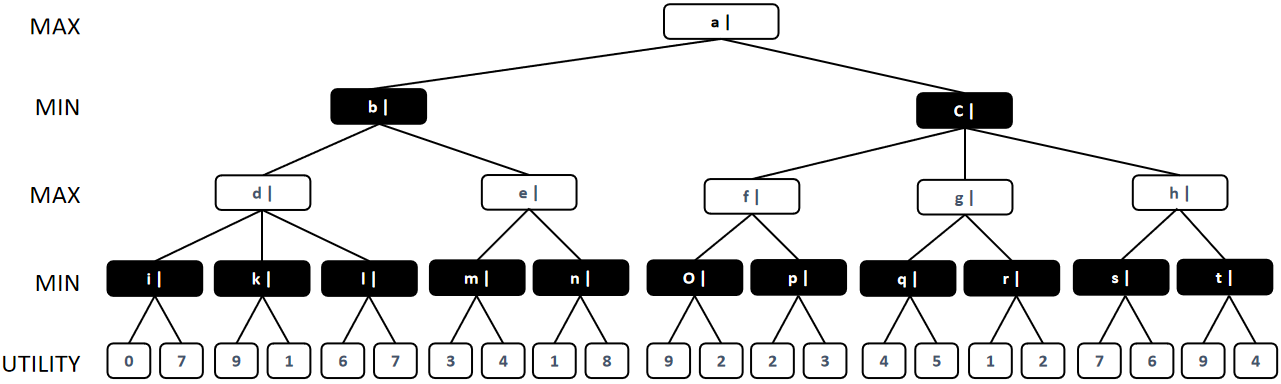
1. Submit the final document to Blackboard Assignments section before the due date. No late submissions will be accepted.

**Objectives:**

1. (20 points) Demonstrate your understanding of MinMax games and a-b pruning algorithm.
2. (60 points) Demonstrate your understanding of propositional logic, its syntax, laws, and inference based on propositional logic.

**Problem 1 [20 pts]:**

Consider the following MinMax game tree



Evaluate MinMax values for all nodes (you can paste in an edited version of this tree below) **[10 pts]**:

|  |
| --- |
| **Your solution:** |
| i=min(0,7)=0  k=min(9,1)=1  l=min(6,7)=6  d=max(I,k, l)=max(0, 1,6)=6  m=min(3,4)=3  n=min(1,8)=1  e=max(m, n)=max(3,1)=3  o=min(9,2)=2  p=min(2,3)=2  f=max(o,p)=max(2,2)=2  q=min(4,5)=4  r=min(1,2)=1  g=max(q,r)=max(4,1)=4  s=min(7,6)=6  t=min(9,4)=4  h=max(s,t)=max(6,4)=6  b=min(d,e)=min(6,3)=3  c=min(f,g, h)=min(2,4,6)=2  a=max(b,c)=max(3,2)=3 |

Now, apply **alpha-beta (a-b) pruning** to prune some of the tree branches. Show (you can paste in an edited version of this tree below) which sections of the tree will be pruned and **justify your answer [10 pts]**:

|  |
| --- |
| **Your solution:** |
|  |

**Problem 2 [20 pts]:**

Use **truth tables** to show that the following sentences are **tautologies [5 pts]**:

1. ¬(p ∧ q) ⇔ ¬p ∨ ¬q **[5 pts]**

Place your truth table here.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| p | q | p ∧ q | ¬(p ∧ q) | ¬p | ¬q | ¬p ∨ ¬q |
| true | true | true | false | false | false | false |
| true | false | false | true | false | true | true |
| false | true | false | true | true | false | true |
| false | false | false | true | true | true | true |

From the truth table above, shows that ¬(p ∧ q) ⇔ ¬p ∨ ¬q is a tautolog

1. p ⇒ q ⇔ ¬q ⇒ ¬p **[5 pts]**

Place your truth table here.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| p | q | p ⇒ q | ¬q | ¬p | ¬q ⇒ ¬p |
| true | true | true | false | false | true |
| true | false | false | true | false | false |
| false | true | true | false | true | true |
| false | false | true | true | true | true |

From the truth table above, shows that p ⇒ q ⇔ ¬q ⇒ ¬p is a tautolog

1. ((p ⇒ q) ∧ (q ⇒ p)) ⇔ (p ⇔ q) **[5 pts]**

Place your truth table here.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| p | q | p ⇒ q | q ⇒ p | (p ⇒ q) ∧ (q ⇒ p) | p ⇔ q |
| true | true | true | true | true | true |
| true | false | false | true | false | false |
| false | true | true | false | false | false |
| false | false | true | true | true | true |

From the truth table above, shows that ((p ⇒ q) ∧ (q ⇒ p)) ⇔ (p ⇔ q) is a tautolog

1. (p ∨ q) ∧ (¬q ∨ r) ⇒ (p ∨ r) **[5 pts]**

Place your truth table here.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| p | q | r | ¬q | p ∨ q | ¬q ∨ r | (p ∨ q) ∧ (¬q ∨ r) | p ∨ r | (p ∨ q) ∧ (¬q ∨ r) ⇒ (p ∨ r) |
| true | true | true | false | true | true | true | true | true |
| true | true | false | false | true | false | false | true | true |
| true | false | true | true | true | true | true | true | true |
| true | false | false | true | true | true | true | true | true |
| false | true | true | false | true | true | true | true | true |
| false | true | false | false | true | false | false | false | true |
| false | false | true | true | false | true | false | true | true |
| false | false | false | true | false | true | false | false | true |

From the truth table above, shows that (p ∨ q) ∧ (¬q ∨ r) ⇒ (p ∨ r) is a tautolog

**Problem 3 [20 pts]:**

Use **deduction** to show (**prove**) that the following sentences are **tautologies**:

1. ¬(p ∧ q) ⇔ ¬p ∨ ¬q **[5 pts]**

|  |  |  |
| --- | --- | --- |
| **Your proof:** | | |
| **Step** | **Resulting sentence** | **Applied law / rule** |
| 1 | ¬ (p ∧ q) ⇔ ¬p ∨ ¬q | De Morgan’s law |
| 2 | ¬p ∨ ¬q⇔ ¬p ∨ ¬q | De Morgan’s law |
| 3 | T |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| Add more rows if necessary | Symbols (copy/paste): T⊥∨∧≡⇔¬⇒∴ | | |

1. p ⇒ q ⇔ ¬q ⇒ ¬p **[5 pts]**

|  |  |  |
| --- | --- | --- |
| **Your proof:** | | |
| **Step** | **Resulting sentence** | **Applied law / rule** |
| 1 | p ⇒ q⇔¬q ⇒ ¬p | Contraposition law |
| 2 | ¬q ⇒ ¬p⇔¬q ⇒ ¬p | Contraposition law |
| 3 | T |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| Add more rows if necessary | Symbols (copy/paste): T⊥∨∧≡⇔¬⇒∴ | | |

1. ((p ⇒ q) ∧ (q ⇒ p)) ⇔ (p ⇔ q) **[5 pts]**

|  |  |  |
| --- | --- | --- |
| **Your proof:** | | |
| **Step** | **Resulting sentence** | **Applied law / rule** |
| 1 | (p ⇒ q) ∧ (q ⇒ p) ⇔ (p ⇔ q) | Equivalence law |
| 2 | (p ⇔ q) ⇔ (p ⇔ q) | Equivalence law |
| 3 | T |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| Add more rows if necessary | Symbols (copy/paste): T⊥∨∧≡⇔¬⇒∴ | | |

1. (p ∨ q) ∧ (¬q ∨ r) ⇒ (p ∨ r) **[5 pts]**

|  |  |  |
| --- | --- | --- |
| **Your proof:** | | |
| **Step** | **Resulting sentence** | **Applied law / rule** |
| 1 | (¬ p⇒ q) ∧ (q⇒ r) ⇒ (p ∨ r) | Implication law |
| 2 | (¬ p⇒ q) ∧ (q⇒ r) ⇒ (¬p⇒ r) | Implication law |
| 3 | (¬ p⇒ r) ⇒ (¬p⇒ r) | Hypothetical Syllogism |
| 4 | ¬ (p ∨ r) ∨(p ∨ r) | Implication law |
| 5 | T |  |
| 6 |  |  |
| Add more rows if necessary | Symbols (copy/paste): T⊥∨∧≡⇔¬⇒∴ | | |

**Problem 4 [20 pts]:**

Convert the following sentences into **conjunctive normal form** (**CNF**):

1. p ⇔ q **[6 pts]**

|  |  |  |
| --- | --- | --- |
| **Your conversion steps:** | | |
| **Step** | **Resulting sentence** | **Applied law / rule** |
| 1 | (p ⇒ q) ∧ (q ⇒ p) | Equivalence law (p ⇒ q) ∧ (q ⇒ p) ⇔ (p ⇔ q) |
| 2 | (¬p∨q) ∧(¬q∨p) | Implication law ¬ p ∨ q ⇔ p ⇒q |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| Add more rows if necessary | Symbols (copy/paste): T⊥∨∧≡⇔¬⇒∴ | | |

1. p ∧ q⇔ p ∨ q **[6 pts]**

|  |  |  |
| --- | --- | --- |
| **Your conversion steps:** | | |
| **Step** | **Resulting sentence** | **Applied law / rule** |
| 1 | (p ∧ q) ⇒(p ∨ q) ∧(p ∨ q) ⇒(p ∧ q) | Equivalence law (p ⇒ q) ∧ (q ⇒ p) ⇔ (p ⇔ q) |
| 2 | (¬ (p ∧ q) ∨(p ∨ q)) ∧(¬(p ∨ q) ∨(p ∧ q)) | Implication law ¬ p ∨ q ⇔ p ⇒q |
| 3 | ((¬ p∨¬ q) ∨(p ∨ q)) ∧((¬ p∧¬ q) ∨(p ∧ q)) | De Morgan’s law ¬ (p ∧ q) ⇔ ¬ q ∨ ¬ |
| 4 | (¬ p∨¬ q ∨p ∨ q) ∧((¬ p∧¬ q) ∨(p ∧ q)) |  |
|  | T ∧((¬ p∧¬ q) ∨(p ∧ q)) | Law of Excluded Middle (Negation Law) |
| 5 | ((¬ p∧¬ q) ∨(p ∧ q)) | Identity law |
| 6 | (¬ p∨(p ∧ q)) ∧(¬ q∨(p ∧ q)) | Distributive law |
|  | (¬ p∨ p) ∧(¬ p∨ q) ∧(¬ q∨p) ∧(¬ q∨ q) | Distributive law |
|  | T∧ (¬ p∨ q) ∧(¬ q∨p) ∧ T | Law of Excluded Middle (Negation Law) |
|  | (¬ p∨ q) ∧(¬ q∨p) | Identity law |
|  |  |  |
|  |  |  |
| Add more rows if necessary | Symbols (copy/paste): T⊥∨∧≡⇔¬⇒∴ | | |

1. p ∧ (p ⇒ q)⇒ q **[8 pts]**

|  |  |  |
| --- | --- | --- |
| **Your conversion steps:** | | |
| **Step** | **Resulting sentence** | **Applied law / rule** |
| 1 | p ∧(¬ p∨ q) ⇒ q | Implication law |
| 2 | p ∧(¬(¬ p∨ q) ∨ q) | Implication law |
| 3 | p ∧(( p∧¬ q) ∨ q) | De Morgan’s law |
| 4 | p ∧( p∨q) ∧(¬ q ∨ q) | Distributive law |
| 5 | p ∧( p∨q) ∧ T | Law of Excluded Middle (Negation Law) |
| 6 | p ∧( p∨q) | Identity law |
| Add more rows if necessary | Symbols (copy/paste): T⊥∨∧≡⇔¬⇒∴ | | |