Reconstruction of transverse and longitudinal impact parameters with Key4HEP

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Abstract

This short letter contains additional information for the τ_{h} reconstruction algorithms found in [1] and used in [2]. Previous work on τ_{h} identification has shown that the finite but non-zero lifetime of the τ lepton provides a handle to improve the discrimination of hadronic τ decays from quark and gluon jets. How these are computed in the context of the afformentioned software package and publication using the Key4HEP [3] format is documented in the following.

The finite distance that a τ lepton travels between its production and decay ($c\tau = 87\ \mu$m) causes the tracks of the $\mathrm{h}^{\pm}$ produced in the τ decay to be displaced from the primary event vertex (PV). The transverse (d_{xy}) and longitudinal (d_{z}) impact parameters quantify this displacement in the directions perpendicular and parallel to the beam axis. The d_{xy} and d_{z} are determined by the point of closest approach (PCA) between the track and the PV:

$$\begin{aligned} d_{\mathrm{xy}} &= \mathrm{sgn}\left((\vec{r}_{\mathrm{PCA}} - \vec{r}_{\mathrm{PV}}) \cdot \vec{p}_{\mathrm{jet}}\right) \sqrt{(x_{\mathrm{PCA}} - x_{\mathrm{PV}})^2 + (y_{\mathrm{PCA}} - y_{\mathrm{PV}})^2} \\ d_{\mathrm{z}} &= \mathrm{sgn}\left((\vec{r}_{\mathrm{PCA}} - \vec{r}_{\mathrm{PV}}) \cdot \vec{p}_{\mathrm{jet}}\right) \left|z_{\mathrm{PCA}} - z_{\mathrm{PV}}\right|, \end{aligned} \tag{1}$$

where sgn refers to the signum function. Concerning the sign of d_{xy} and d_{z}, we follow the convention used in Ref. [4] and use a positive sign in case the momentum vector of the jet that seeds the τ_{h} reconstruction points towards the same hemisphere as the difference between the PV and the PCA, and a negative sign if it points towards the opposite hemisphere. We denote the former by the symbol $\vec{p}_{\mathrm{jet}}$ and the latter by $\vec{r}_{\mathrm{PCA}} - \vec{r}_{\mathrm{PV}}$. The difference between the PV and the PCA represents an estimate of the τ flight direction.

The d_{xy} and d_{z}, and their respective uncertainties, $\sigma_{d\mathrm{xy}}$ and $\sigma_{d\mathrm{z}}$, are not part of the Key4HEP format used in Pandora and thus need to be computed for the work presented in this paper. We compute d_{xy} and d_{z} using the set of standard track parameters (ϕ_0, λ, d_0, z_0, Ω) used in Key4HEP. The parameters d_0 and z_0 denote the distances between the track and the reference point in the x-y plane and in z-direction, respectively; ϕ_0 denotes the angle between the tangent to the track and the x-direction; $\lambda = \cot\theta$ denotes the slope of the track in the r-z plane; and Ω the inverse of the track's radius of curvature. Details on the definition of these parameters can be found in Ref. [5]. The trajectory of charged particles propagating through the solenoidal magnetic field within the detector has the form of a helix. Computing the PCA of the helix with respect to the PV is a non-trivial task, because the relevant equations are non-linear. We simplify this task by taking advantage of the fact that the distances that τ leptons travel between their production and decay are typically small compared to the expected radius of curvature of the tracks produced in the τ decay, which allows us to use a linear approximation of the helix equations.

To obtain the expressions for the linear approximation, we start with the parametrisation of the helix given in Ref. [6]:

$$\vec{r}_{\mathrm{trk}}(s) = \begin{pmatrix} x_c + 1/\Omega\,\sin(\phi_0 + s\,\Omega) \\ y_c - 1/\Omega\,\cos(\phi_0 + s\,\Omega) \\ z_c + s\,\lambda \end{pmatrix}\,, \tag{2}$$

where $\vec{r}_{\mathrm{trk}}(s)$ denotes a point on the helix and s the travel distance of the charged particle in the transverse plane. To obtain a linear approximation of this equation we perform a Taylor expansion with respect to the travel distance s at the point $s = 0$ and keep the constant term and the term linear in s. This yields:

$$\vec{r}_{\mathrm{trk}}(s) = \begin{pmatrix} s\,\cos(\phi_0) + x_c + 1/\Omega\,\sin(\phi_0) \\ s\,\sin(\phi_0) + y_c - 1/\Omega\,\cos(\phi_0) \\ z_c + s\,\lambda \end{pmatrix}\,. \tag{3}$$

The parametrisations in Eqs. (2) and (3) use the centre, $\vec{r}_c = (x_c,\, y_c,\, z_c)$, of the helix trajectory in the x-y plane as the reference point. The track parameters $(\phi_0,\, \lambda,\, d_0,\, z_0,\, \Omega)$ are available for different reference points $\vec{r}_{\mathrm{ref}} = (x_{\mathrm{ref}},\, y_{\mathrm{ref}},\, z_{\mathrm{ref}})$ in Key4HEP. Among the available $\vec{r}_{\mathrm{ref}}$, we choose the one that we expect to be closest to the PV: the reference point coinciding with the nominal interaction point at the centre of the detector, $\vec{r}_{\mathrm{ref}} = (0,\, 0,\, 0)$. We follow the convention of Ref. [5] and change the parametrisation such that it refers to a general reference point $\vec{r}_{\mathrm{ref}} = (x_{\mathrm{ref}},\, y_{\mathrm{ref}},\, z_{\mathrm{ref}})$. By definition of the track parameters ϕ_0, d_0, and z_0 the position of the point on the helix at the travel distance $s = 0$ is given by:

$$\vec{r}_{\mathrm{trk}}(s=0) = \begin{pmatrix} x_{\mathrm{ref}} + \cos(\frac{\pi}{2} - \phi_0)\,d_0 \\ y_{\mathrm{ref}} - \sin(\frac{\pi}{2} - \phi_0)\,d_0 \\ z_{\mathrm{ref}} + z_0 \end{pmatrix} \tag{4}$$

By comparing Eqs. (3) and (4), we find that the linear parametrisation with respect to the reference point $\vec{r}_{\mathrm{ref}}$ is given by the expression:

$$\vec{r}_{\mathrm{trk}}(s) = \begin{pmatrix} s\,\cos(\phi_0) + x_{\mathrm{ref}} + \cos(\frac{\pi}{2} - \phi_0)\,d_0 \\ s\,\sin(\phi_0) + y_{\mathrm{ref}} - \sin(\frac{\pi}{2} - \phi_0)\,d_0 \\ s\,\lambda + z_{\mathrm{ref}} + z_0\,. \end{pmatrix} \tag{5}$$

We rewrite Eq. (5) in the form:

$$\vec{r}_{\mathrm{trk}}(s) = \vec{r}_a + (\vec{r}_b - \vec{r}_a)\;s\,, \tag{6}$$

with:

$$\vec{r}_a = \begin{pmatrix} x_{\mathrm{ref}} + \cos(\frac{\pi}{2} - \phi_0)\,d_0 \\ y_{\mathrm{ref}} - \sin(\frac{\pi}{2} - \phi_0)\,d_0 \\ z_{\mathrm{ref}} + z_0 \end{pmatrix} \quad \text{and} \quad \vec{r}_b - \vec{r}_a = \begin{pmatrix} \cos(\phi_0) \\ \sin(\phi_0) \\ \lambda \end{pmatrix}\,. \tag{7}$$

This is the equation of a straight line in three dimensions.

Using this linear approximation, the PCA between the track and the PV is given by the following expression for the travel distance s [7]:

$$s_{\mathrm{PCA}} = -\frac{(\vec{r}_a - \vec{r}_{\mathrm{PV}}) \cdot (\vec{r}_b - \vec{r}_a)}{|\vec{r}_b - \vec{r}_a|^2}\,. \tag{8}$$

The above expression holds in three dimensions. We restrict the computation of the PCA between the track and the PV to the x-y plane. The corresponding expression for the travel distance s in two dimensions is:

$$s_{\mathrm{PCA}} = -\frac{(x_a - x_{\mathrm{PV}}) \cdot (x_b - x_a) + (y_a - y_{\mathrm{PV}}) \cdot (y_b - y_a)}{(x_b - x_a)^2 + (y_b - y_a)^2}\,. \tag{9}$$

Inserting Eq. (9) into Eq. (6), we obtain the location of the PCA: $\vec{r}_{\mathrm{PCA}} = \vec{r}_{\mathrm{trk}}(s_{\mathrm{PCA}})$, which allows us to compute the transverse and longitudinal impact parameters by means of Eq. (1). The uncertainties $\sigma_{d\mathrm{xy}}$ and $\sigma_{d\mathrm{z}}$ are computed by standard error propagation [8], assuming the uncertainties on the five track parameters (ϕ_0, λ, d_0, z_0, Ω) to be uncorrelated.

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References

[1] T. Lange, S. Nandan, J. Pata, L. Tani, C. Veelken, HEP-KBFI/ml-tau-reco: Submission (Jul. 2023). doi:10.5281/zenodo.8113344.
URL https://doi.org/10.5281/zenodo.8113344

[2] T. Lange, S. Nandan, J. Pata, L. Tani, C. Veelken, Particle-flow based tau identification at future e^+e^- colliders (7 2023). arXiv:2307.07747.

[3] G. Ganis, C. Helsens, V. Völkl, Key4hep, a framework for future HEP experiments and its use in FCC, Eur. Phys. J. Plus 137 (2022) 149. arXiv:2111.09874, doi:10.1140/epjp/s13360-021-02213-1.

[4] CMS Collaboration, Status of b-tagging and vertexing tools for 2011 data analysis, CMS physics analysis summary (2011).
URL http://cds.cern.ch/record/1395489

[5] T. Kramer, Track parameters in LCIO, Linear collider note (2021).
URL https://bib-pubdb1.desy.de/record/81214/files/LC-DET-2006-004[1].pdf

[6] T. Kuhr, Rekonstruktion von V0s mit dem H1-Silizium-Detektor, Master's thesis, Hamburg U. (1998).

[7] E. W. Weisstein, Point-line distance–3-dimensional. - from mathworld–a wolfram web resource., https://mathworld.wolfram.com/Point-LineDistance3-Dimensional.html (2023).

[8] G. Cowan, Statistical data analysis, Clarendon Press, 1998.