# 표준모형의 이해

## Ch2

- Lagrangian

$L(q_s, \dot{q}_s) = T(\dot{q}_s) - V(q_s)$

- $\delta S = 0$

$S = S(t_1, t_2, q_s(t), \dot{q}_s(t))$

$\frac{\partial L}{\partial q_s} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_s}\right) = 0$   $\delta S = 0$

$S(t_1, t_2, q_s(t), \dot{q}_s(t)) = \int_{t_1}^{t_2} L(q_s(t), \dot{q}_s(t))\, dt$

$\therefore\ 0 = \delta S = \int_{t_1}^{t_2} \left( \frac{\partial L}{\partial q_s} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_s(t)}\right)\right) \delta q_s(t)\, dt + \left[ \frac{\partial L}{\partial \dot{q}_s} \delta q_s \right]_{t_1}^{t_2}$

- Fourvector

$x^\mu\ (t, x, y, z)$ ⇒ 4가 여러 개 좌표를 합치는 $q_s(t)$에 대응한다.

⇒ $L = L(\phi_s, \partial_\mu \phi_s)$

↳ $L = \int \mathcal{L}\, d^3x$

$S = S(t_1, t_2, \phi_s, \partial^\mu \phi_s) = \int_{t_1}^{t_2} L(\phi_s, \partial^\mu \phi_s) = \int_{t_1}^{t_2}\int_V dx\, \mathcal{L}(\phi_s, \partial^\mu \phi_s) = \int_1^2 d^4x\, \mathcal{L}(\phi_s, \partial^\mu \phi_s)$

$0 = \delta S = \int_1^2 d^4x \left( \frac{\partial \mathcal{L}}{\partial \phi_s}\delta\phi_s + \frac{\partial \mathcal{L}}{\partial(\partial^\mu \phi_s)} \delta(\partial^\mu \phi_s)\right)$

$0 = \int_1^2 d^4x \left( \frac{\partial \mathcal{L}}{\partial \phi_s} - \partial^\mu \frac{\partial \mathcal{L}}{\partial(\partial^\mu \phi_s)} \right)\delta\phi_s + \left[ \partial^\mu \frac{\partial \mathcal{L}}{\partial(\partial^\mu \phi_s)} \delta \phi_s \right]_1^2$   $\therefore\ \frac{\partial \mathcal{L}}{\partial \phi_s} - \partial^\mu \frac{\partial \mathcal{L}}{\partial(\partial^\mu \phi_s)} = 0$

## Ch3 : 여러 종류의 입자들

- **fermi-dirac particle (fermion)**

$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$ ⇒ $\sigma_1 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_2 \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_3 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ (pauli matrix)

$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$

$\gamma^1 = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}\begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_1 \end{pmatrix} = \begin{pmatrix} 0 & \sigma_1 \\ -\sigma_1 & 0 \end{pmatrix}$

$\gamma^2 = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}\begin{pmatrix} \sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix} = \begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix}$

$\gamma^3 = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}\begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix} = \begin{pmatrix} 0 & \sigma_3 \\ -\sigma_3 & 0 \end{pmatrix}$

Weyl, chiral (gamma matrix)

- $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ → $g^{\mu\nu} = g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$ ($\mu\nu$ = 0,1,2,3)
- $\gamma^5\gamma^\mu + \gamma^\mu\gamma^5 = \{\gamma^5, \gamma^\mu\} = 0$
- $\gamma^0\gamma^0 = I$, $(\gamma^k)^2 = -I$ (k=1,2,3)
- $\gamma^{\mu\dagger} = \gamma^0\gamma^\mu\gamma^0$

- $x^\mu = (t, x, y, z)$, $x_\mu = (t, -x, -y, -z)$

$\partial^\mu = \left(\frac{\partial}{\partial t}, -\vec{\nabla}\right)$, $\partial_\mu = \left(\frac{\partial}{\partial t}, \vec{\nabla}\right)$

$\partial^\mu = g^{\mu\nu}\partial_\nu$

⇒ dirac eq.

- $(i\gamma^\mu\partial_\mu - m)\psi = 0$ , $\bar{\psi} = \psi^\dagger\gamma^0$

$\dagger$: $\psi^\dagger(-i\partial_\mu\gamma^{\mu\dagger} - m) = 0$

$= \psi^\dagger(-i\partial_\mu\gamma^0\gamma^\mu\gamma^0 - m) = 0$

$= \psi^\dagger\gamma^0(-i\partial_\mu\gamma^\mu\gamma^0 - \gamma^0 m) = 0$

$= \bar{\psi}(-i\partial_\mu\gamma^\mu - m) = 0$

$\Rightarrow \bar{\psi}(i\partial_\mu\gamma^\mu + m) = 0$

$$\boxed{\begin{aligned} L &= i\bar{\psi}\partial_\mu\gamma^\mu\psi - m\bar{\psi}\psi \\ &= \bar{\psi}(i\partial_\mu\gamma^\mu - m)\psi \end{aligned}}$$

- $\phi_2 = \psi$: $\frac{\partial \mathcal{L}}{\partial \psi} - \partial^\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu\psi)} = 0$ → $-m\bar{\psi} - \partial^\mu(i\bar{\psi}\gamma^\mu) = 0$

- $\phi_2 = \bar{\psi}$: $\frac{\partial \mathcal{L}}{\partial \bar{\psi}} - \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \bar{\psi})}\right) = 0$ → $-m\psi + i\partial_\mu\gamma^\mu\psi = 0$ ⇒ $(i\partial_\mu\gamma^\mu - m)\psi = 0$

$\therefore\ \mathcal{L} = i\bar{\psi}\partial_\mu\gamma^\mu\psi - m\bar{\psi}\psi$ 일 때 Field Lagrangian으로 적합.

$*$ SU(2)

$\bar{\chi} = \begin{pmatrix} \psi \\ \phi \end{pmatrix}$, $\chi^\dagger = (\psi^\dagger\ \phi^\dagger)$

$\bar{\chi} = \chi^\dagger\gamma^0 = (\bar{\psi}, \bar{\phi})$

↳ gauge particle (boson)

- photon (maxwell eq.)

$\vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t}$, $\vec{B} = \nabla\times\vec{A}$

$\nabla\cdot\vec{E} = \rho$, $\nabla\times\vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{J}$, $\nabla\cdot\vec{B} = 0$, $\nabla\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

⇒ $\boxed{F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu}$

$A^\mu \Rightarrow (\phi, A_x, A_y, A_z)$, $A_\mu = (\phi, -\vec{A})$

$(\phi, \vec{A})$

$J^\mu \Rightarrow (\rho, \vec{J})$

⇒ $0 = \partial_\lambda F_{\mu\nu} + \partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu}$ ($\lambda, \mu, \nu$ = 1,2,3), $J^\nu = \partial_\mu F^{\mu\nu}$ ($\mu\nu$ = 0,1,2,3)

maxwell eq.

$\boxed{\begin{aligned} 0 &= \varepsilon^{\mu\nu\lambda\rho}\partial_\lambda F_{\mu\nu} \\ J_\nu &= \partial^\mu F_{\mu\nu} \end{aligned}}$

- Lagrangian

$\boxed{\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - J^\mu A_\mu}$

$F_{\mu\nu} = F^{\rho\sigma}g_{\rho\mu}g_{\sigma\nu} = F^{\mu\nu}$

$\therefore$ 질량 있는 입자 ($\text{gauge particle} = B_\mu$)

$\mathcal{L} = -\frac{1}{4}B^{\mu\nu}B_{\mu\nu} + \frac{1}{2}m^2 B^\nu B_\nu - J^\mu B_\mu$

$\partial_\mu B^{\mu\nu} + m^2 B^\nu = J^\nu$

- SU(2) → 3 gauge particle

$\vec{W}_\mu = (W_\mu^1, W_\mu^2, W_\mu^3)$ ⇒ 3개 벡터 성분을 가짐.

$*$ no mass, no J

$\mathcal{L} = -\frac{1}{4}(W^{1\mu\nu}W^1_{\mu\nu} + W^{2\mu\nu}W^2_{\mu\nu} + W^{3\mu\nu}W^3_{\mu\nu}) = -\frac{1}{4}\sum_{a=1}^{3} W^{a\mu\nu}W^a_{\mu\nu} = -\frac{1}{4}\vec{W}^{\mu\nu}\cdot\vec{W}_{\mu\nu}$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$*$ no mass, J (o)

$\mathcal{L} = -\frac{1}{4}\vec{W}^{\mu\nu}\vec{W}_{\mu\nu} - \vec{J}^\mu\vec{W}_\mu$

- mass, J (o)

$\mathcal{L} = -\frac{1}{4}\vec{W}^{\mu\nu}\vec{W}_{\mu\nu} + \frac{1}{2}\vec{W}^\mu M^2 \vec{W}_\mu - \vec{J}^\mu\vec{W}_\mu$ → $0 = \frac{\partial \mathcal{L}}{\partial W^3_\nu} - \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu W^3_\nu)}\right)$

$\frac{\partial \mathcal{L}}{\partial W^3_\nu} = -\partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu W^3_\nu)}\right)$

$\frac{\partial \mathcal{L}}{\partial W^3_\nu} = \frac{\partial}{\partial W^3_\nu}\left(\frac{1}{2}\vec{W}^\mu M^2\vec{W}_\mu - \vec{J}^\nu\cdot\vec{W}_\nu\right) + \frac{\partial}{\partial W^3_\nu}\left(-\frac{1}{4}\vec{W}^{\mu\nu}\cdot\vec{W}_{\mu\nu}\right)$

$= m_3^2 W^{3\nu} - J^{3\nu} +$

$\vec{W}^3_{\mu\nu} = \partial_\mu W^3_\nu - \partial_\nu W^3_\mu - g\,\varepsilon\, W^i_\mu W^k_\nu$

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$\therefore\ \partial_\mu \vec{W}^{\mu\nu} + m^2\vec{W}^\nu = \vec{J}^\nu + g(\vec{W}_\mu \times \vec{W}^{\mu\nu})$

↳ scalar particle

⇒ klein gordon eq.

$\boxed{(\partial^\mu\partial_\mu + m^2)\phi = 0}$

$\mathcal{L} = \frac{1}{2}\partial^\mu\phi\,\partial_\mu\phi - \frac{1}{2}m^2\phi^2 = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2$

$\mathcal{L}$

- Dirac particle (fermion) (Dirac eq.)

spinor $\psi$ : $\bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$

$q\begin{pmatrix} u \\ d \end{pmatrix}$ : $\bar{q}(i\gamma^\mu\partial_\mu - m_q)q = \bar{u}(i\gamma^\mu\partial_\mu - m_u)u + \bar{d}(i\gamma^\mu\partial_\mu - m_d)d$

gauge particle.

전자기장 A : $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - j^\mu A_\mu$ $\quad (F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu)$

[illegible] (B) : $-\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - j^\mu B_\mu + \frac{1}{2}m^2 B^\mu B_\mu$

↳ proca eq. ([illegible])

$\vec{W}_\mu\ (\vec{W}_\mu^1, \vec{W}_\mu^2, \vec{W}_\mu^3)$ ([illegible]) : $-\frac{1}{4}\vec{W}^{\mu\nu}\cdot\vec{W}_{\mu\nu} - \vec{j}^\mu\vec{W}_\mu + \frac{1}{2}(W^{1\mu}\, W^{2\mu}\, W^{3\mu})\begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}\begin{pmatrix} W^{1\mu} \\ W^{2\mu} \\ W^{3\mu} \end{pmatrix}$

- ~~[illegible]~~ → [illegible] 이름.
- scalar particle (Higgs)

$\phi$ : $\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m^2\phi^2$

$\phi = \frac{1}{\sqrt{2}}(\varepsilon_1 + i\varepsilon_2)$ : $\partial^\mu\phi^*\partial_\mu\phi - m^2\phi^*\phi$
$= \frac{1}{2}\partial^\mu\varepsilon_1\partial_\mu\varepsilon_1 + \frac{1}{2}\partial^\mu\varepsilon_2\partial_\mu\varepsilon_2 - \frac{1}{2}m^2(\varepsilon_1^2 + \varepsilon_2^2)$

$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$ (doublet) : $(\partial^\mu\phi)^\dagger\partial_\mu\phi - \phi^\dagger M_\phi^2\phi$
$= \frac{1}{2}\partial^\mu\phi_1^*\partial_\mu\phi_1 + \frac{1}{2}\partial^\mu\phi_2^*\partial_\mu\phi_2 - \frac{1}{2}(\phi_1^*\ \phi_2^*)\begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix}\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$

# CH4 gauge symmetry

global $\partial_\mu\chi(x) = 0$ ([illegible] X)
$\rightarrow \Delta A_\mu = 0.$

$\therefore$ global → gauge [illegible] X.

ex
- $\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - j^\mu A_\mu$
에서 $A_\mu'(x) \to A_\mu(x) - \partial_\mu\chi(x)$ 시
$\mathcal{L}' = \mathcal{L} + \partial_\mu(j^\mu\chi)$
↳ [illegible] ⇒ 0.
([illegible] → [illegible] X)
($\partial_\mu j^\mu = 0$ 이면)

## ① global U(1)

(gauge particle X)

$\psi' \to e^{ia\theta}\psi, \quad \phi' \to e^{ia_0\theta}\phi$

| | |
|---|---|
| spinor $\psi$ | $\psi' = e^{ia\theta}\psi$ |
| $q$ | $q' = \begin{pmatrix} e^{ia_u\theta} & 0 \\ 0 & e^{ia_d\theta} \end{pmatrix} q$ |
| scalar $\phi = \frac{1}{\sqrt{2}}(\varepsilon_1 + i\varepsilon_2)$ | $\phi' = e^{ia_0\theta}\phi$ |
| $\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$ | $\phi' = \begin{pmatrix} e^{ia_1\theta} & 0 \\ 0 & e^{ia_2\theta} \end{pmatrix}\phi$ |

## ② local U(1)

$(e^{ia\theta(x)})$

(gauge + particle)

추가항

- Dirac particle.

① spinor $(\psi \to \psi e^{ia\theta(x)})$
$\mathcal{L}' \to \bar{\psi}'(i\gamma^\mu\partial_\mu - m)\psi'$
$= \mathcal{L} + i\partial_\mu\theta(x)[ia\bar{\psi}\gamma^\mu\psi]$

hypercharge 관련?

② quark (2중항)
$\bar{q}' m_q q' = \bar{q} m_q q$
$\left(q' = \begin{pmatrix} e^{ia_u\theta(x)} & 0 \\ 0 & e^{ia_d\theta(x)} \end{pmatrix} q\right)$
$\therefore$
$\mathcal{L}' = \mathcal{L} + i\partial_\mu\theta(x)[ia_u\bar{u}\gamma^\mu u + ia_d\bar{d}\gamma^\mu d]$

⇒ [illegible] 불변, 운동에너지항에 의해 추가항.
(운동에너지항) → $\mathcal{L}$ = 운동에너지 + 질량항?, 그럼 이렇게 되는건가?

- Scalar particle

$\phi' = e^{ia_0\theta(x)}\phi, \quad \partial_\mu\phi' = ia_0\partial_\mu\theta e^{ia_0\theta}\phi + e^{ia_0\theta}\partial_\mu\phi$

③ $\phi = (\varepsilon_1 + i\varepsilon_2)/\sqrt{2}$
$\mathcal{L}' \to \mathcal{L} + ia_0(\phi\partial^\mu\phi^* - \phi^*\partial^\mu\phi)\partial_\mu\theta + (a_0)^2\phi^*\phi\partial^\mu\theta\partial_\mu\theta$

* $A\overleftrightarrow{\partial}B \equiv (\partial A)B - A(\partial B)$
$\phi^*\overleftrightarrow{\partial}\phi \equiv \phi(\partial^\mu\phi^*) - \phi^*\partial^\mu\phi)$

④ 이중항
* $U(\theta) = \begin{pmatrix} e^{ia_1\theta} & 0 \\ 0 & e^{ia_2\theta} \end{pmatrix}$
$U^\dagger(\theta) = U(-\theta) = U^{-1}(\theta)$

$\phi' = U(\theta)\phi, \quad \phi'^\dagger = \phi^\dagger U^\dagger(\theta), \quad \partial_\mu\phi' = \cdots$

- 질량항. $-\phi^\dagger M_\phi^2\phi$ 그대로.
- 운동항. 바뀜 ($\theta^2 \approx 0$)

⇒ $\mathcal{L}' = \mathcal{L} + (\partial^\mu\phi)^\dagger U^\dagger(\partial_\mu U)\phi + \phi^\dagger(\partial^\mu U)^\dagger U\partial_\mu\phi$
$= \mathcal{L} + \frac{1}{2}i\partial_\mu\theta\left(a_1\phi_1^*\overleftrightarrow{\partial}^\mu\phi_1 + a_2\phi_2^*\overleftrightarrow{\partial}^\mu\phi_2\right)$

- gauge particle (for $B_\mu$)

- $B_\mu' = B_\mu - \frac{1}{g_1}\partial_\mu\chi(x) \qquad B_{\mu\nu}' \Rightarrow (\partial_\mu B_\nu' - \partial_\nu B_\mu') = B_{\mu\nu}$

- 질량항 : $\frac{1}{2}m^2 B^\mu B_\mu \Rightarrow \frac{1}{2}m^2 B^\mu B_\mu \underline{- \frac{1}{2}m^2\left(\frac{1}{g_1}B^\mu\partial_\mu\chi + \frac{1}{g_1}B_\mu\partial^\mu\chi - \frac{1}{g_1^2}\partial^\mu\chi\partial_\mu\chi\right)}$
$= 0.\ (\chi, B, g \neq 0)$
$\therefore m = 0.$

추가항, 상쇄 ⇒ gauge 변환에 대해 불변 $e^{ia_0\theta(x)}$ 의 상쇄항.

① Dirac particle. (Dirac 입자 gauge particle 이 상호작용시).

Dirac 추가항 : $i\partial_\mu\theta\,(ia\bar{\psi}\gamma^\mu\psi)$
$+$ gauge 추가항 : $\frac{1}{g_1}j^\mu\partial_\mu\chi$
$\therefore 0$

⇒ $\theta = \chi$ 일 경우.
$\therefore j^\mu = ag_1\bar{\psi}\gamma^\mu\psi$

$\therefore \mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} - ag_1\bar{\psi}\gamma^\mu\psi B_\mu$
$= \bar{\psi}(i\gamma^\mu\underbrace{(\partial_\mu + iag_1B_\mu)}_{D_\mu}-m)\psi - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}$

$D_\mu$ ([illegible], covariant derivative)

a: dirac
$g_1$: gauge

$= \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}$

② Dirac 이중항 q

Dirac 추가항 : $i\partial_\mu\theta\,(ia_u\bar{u}\gamma^\mu u + ia_d\bar{d}\gamma^\mu d)$
gauge 추가항 : $\frac{1}{g_1}j^\mu\partial_\mu\theta$

(1) 데이터상 같은결과?

$\therefore j^\mu = a_u g_1\bar{u}\gamma^\mu u + a_d g_1\bar{d}\gamma^\mu d$

⇒ $\mathcal{L} = \bar{q}(i\gamma^\mu\partial_\mu - m_q)q - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} - (a_u g_1\bar{u}\gamma^\mu u + a_d g_1\bar{d}\gamma^\mu d)B_\mu$

$i(\bar{u}\ \bar{d})i\gamma^\mu\begin{pmatrix} a_u g_1 & 0 \\ 0 & a_d g_1 \end{pmatrix}\begin{pmatrix} u \\ d \end{pmatrix}B_\mu$
$= \bar{q}i\gamma^\mu\begin{pmatrix} ia_u g_1 B_\mu & 0 \\ 0 & ia_d g_1 B_\mu \end{pmatrix}q$

$= \bar{q}\left(i\gamma^\mu\begin{pmatrix} \partial_\mu + ia_u g_1 B_\mu & 0 \\ 0 & \partial_\mu + ia_d g_1 B_\mu \end{pmatrix} - m_q\right)q - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}$

$\therefore D_\mu^u = \partial_\mu + ia_u g_1 B_\mu$
$D_\mu^d = \partial_\mu + ia_d g_1 B_\mu$

⇒ $D_\mu = \partial_\mu + ig_1\begin{pmatrix} a_u & 0 \\ 0 & a_d \end{pmatrix}B_\mu$

$\mathcal{L} = \bar{u}(i\gamma^\mu D_\mu^u - m_u)u + \bar{d}(i\gamma^\mu D_\mu^d - m_d)d - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}.$

③ Scalar particle $\phi$

추가항 : $ia_0(\phi\partial^\mu\phi^* - \phi^*\partial^\mu\phi)\partial_\mu\theta + (a_0)^2\phi^*\phi\partial^\mu\theta\partial_\mu\theta.$

$D_\mu$ : $\partial_\mu + ia_0 g_1 B_\mu$

$\mathcal{L}$ : $(D^\mu\phi)^* D_\mu\phi - m^2\phi^*\phi - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}$

④ Scalar 이중항 $\phi$

$\mathcal{L} = \partial^\mu\phi^\dagger\partial_\mu\phi - \phi^\dagger M_\phi^2\phi$
$= \frac{1}{2}\partial^\mu\phi_1^*\partial_\mu\phi_1 - \frac{1}{2}m_1^2\phi_1^*\phi_1 + \frac{1}{2}\partial^\mu\phi_2^*\partial_\mu\phi_2 - \frac{1}{2}m_2^2\phi_2^*\phi_2,$

$D_\mu = \begin{pmatrix} D_\mu^1 & 0 \\ 0 & D_\mu^2 \end{pmatrix} = \begin{pmatrix} \partial_\mu + ia_1 g_1 B_\mu & 0 \\ 0 & \partial_\mu + ia_2 g_1 B_\mu \end{pmatrix}.$

$\therefore \mathcal{L} = (D^\mu\phi)^\dagger D_\mu\phi - \phi^\dagger M_\phi^2\phi - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}$
$= \frac{1}{2}(D^\mu\phi_1)^* D_\mu^1\phi_1 - \frac{1}{2}m_1^2\phi_1^*\phi_1 + \frac{1}{2}(D^{2\mu}\phi_2)^* D_\mu^2\phi_2 - \frac{1}{2}m_2^2\phi_2^*\phi_2 - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}$

상호작용 : gauge → scalar, → Dirac 과의 상호작용.

$\therefore$ $\theta(x)$ 가 임의의 함수로 미분항에 의해 추가항이 발생하므로, gauge 에서 상쇄항으로, dirac scalar 장의 추가항이 대해 대칭성이 유지된다.

## ③ global SU(2) gauge symmetry.

SU(2) ⇒ 3가지 axial
doublet → SU(2)공간에서 두가지로 변환가능.
triplet → 〃 3가지로 〃

* $\tau_i$ → 아이소스핀 표현 (iso spin $\frac{1}{2}$)
⇒ 2×2로 표현.
singlet → 고려X.

- 발생자 $\tau_1$ $\tau_2$ $\tau_3$ 연관된 개개 → 3개 변수 $\theta_1, \theta_2, \theta_3$ : 변환의 강도.

$[\tau_i, \tau_j] = i\epsilon_{ijk}\tau_k \qquad \tau_i = \frac{1}{2}\sigma_i$

– SU(2)의 일반 원소 $U(\theta) = e^{i\tau_1\theta_1 + i\tau_2\theta_2 + i\tau_3\theta_3}$
$= e^{i\vec{\tau}\cdot\vec{\theta}}$

U(1) → 위상변화 (한가지만 변환) 성분 한개

SU(2) → 3차원 공간에서 변환(2가지 변환) ⇒ 2가지 (doublet) 3가지 변환으로 변환가능 (triplet).

- 가능한 SU(2) globally ⇒ dirac doublet, scalar doublet.

  - Dirac doublet.

    $q' = U(\vec{\theta})q = e^{i\vec{\tau}\cdot\vec{\theta}}\begin{pmatrix} u \\ d \end{pmatrix}$

    → Lagrangian 운동에너지 변화X, 질량항 $m_q = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}$ ($m_u \neq m_d$) ⇒ 대칭성 만들기 위해
    ① $m_u = m_d$
    ② $m_u, m_d = 0$ ⇒ by chiral 대칭

  - Scalar doublet.

    $\phi' = \frac{1}{\sqrt{2}}e^{i\vec{\tau}\cdot\vec{\theta}}\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$

    → 동일, $M_\phi = \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix}$ ⇒ (chiral 대칭) 불가 ⇒ scalar particle은 관계X

    $m_1, m_2 = 0$

    ∴ $m_1 = m_2$. ( scalar potential에 의해 질량항 발생 )

## ④ local SU(2) gauge symmetry.

$\partial_\mu$ 추가, $W_\mu^1\ W_\mu^2\ W_\mu^3$에 의해 정의됨.

gauge

운동에너지항이 유지되려면 → 이 항이 추가되어야 함.

- $W_\mu'^i = W_\mu^i - \frac{1}{g_2}\partial_\mu\theta_i - \epsilon^{ijk}\theta_j W_\mu^k$
  $- \vec{\theta} \times \vec{W_\mu}$

  $\vec{\tau}\cdot\vec{W_\mu}' = U(\vec{\tau}\cdot\vec{W_\mu})U^{-1} - \frac{i}{g_2}U\partial_\mu U^{-1}$
  ⇒ 아이소스핀 회전 (pure gauge 항.)
  $= \vec{\tau}\cdot\vec{W_\mu} - \vec{\tau}\cdot(\vec{\theta}\times\vec{W_\mu}) \qquad -\frac{1}{g_2}\vec{\tau}\cdot\partial_\mu\vec{\theta}$
  $= \vec{\tau}\cdot(\vec{W_\mu} - \frac{1}{g_2}\partial_\mu\vec{\theta} - (\vec{\theta}\times\vec{W_\mu}))$
  $W_\mu'$

- $\vec{W}_{\mu\nu} = \partial_\mu\vec{W_\nu} - \partial_\nu\vec{W_\mu} - g_2\vec{W_\mu}\times\vec{W_\nu}$ 이면은

  $\vec{W}_{\mu\nu}' = \vec{W}_{\mu\nu} - \vec{\theta}\times\vec{W}_{\mu\nu}$

  ↳ gauge. $\mathcal{L}$ 운동에너지항
  → $\vec{W}_{\mu\nu}'\vec{W}'^{\mu\nu} = \vec{W}_{\mu\nu}\vec{W}^{\mu\nu}$

질량항: gauge 변환시 불변X ( $M^2 = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}$ )
→ $W_\mu = 0$. (질량 0)

* Dirac local SU(2)
  $q' \to e^{i\vec{\tau}\cdot\vec{\theta}}q$ , $\mathcal{L} = \bar{q}(i\gamma^\mu\partial_\mu - m_q)q$
  $\mathcal{L}' \to$ 질량항이 0이되어야 ; $D_\mu = \partial_\mu + ig_2\vec{\tau}\cdot\vec{W_\mu}$

  ∴ $\mathcal{L} = \bar{q}(i\gamma^\mu D_\mu)q - \frac{1}{4}\vec{W}^{\mu\nu}\vec{W}_{\mu\nu}$

* Scalar local SU(2)

  $D_\mu = \partial_\mu + ig_2\vec{\tau}\cdot\vec{W_\mu} \qquad$ ∴ $\mathcal{L} = (D^\mu\phi)^\dagger D_\mu\phi - \frac{1}{4}\vec{W}^{\mu\nu}\vec{W}_{\mu\nu}$

U(1)

| 입자 | 변환식 | 라그랑지안 밀도 |
|---|---|---|
| $\psi$ | $\psi' = e^{ia\theta}\psi$ | $\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$<br>$D_\mu = \partial_\mu + iag_1B_\mu$ |
| $q = \begin{pmatrix} u \\ d \end{pmatrix}$ | $q' = e^{ia_q\theta}q$ | $\mathcal{L} = \bar{q}(i\gamma^\mu D_\mu - m_q)q$<br>$D_\mu = \partial_\mu + ia_qg_1B_\mu$ |
| $B_\mu$ | $B_\mu' = B_\mu - \frac{1}{g_1}\partial_\mu\theta$ | $\mathcal{L} = -\frac{1}{4}B^{\mu\nu}B_{\mu\nu}$ |
| $\vec{W}_\mu = (W_\mu^1, W_\mu^2, W_\mu^3)$ | 변환하지 않음 | |
| $\phi = \frac{1}{\sqrt{2}}(\xi_1 + i\xi_2)$ | $\phi' = e^{ia_0\theta}\phi$ | $\mathcal{L} = (D^\mu\phi)^* D_\mu\phi - m^2\phi^*\phi$<br>$D_\mu = \partial_\mu + ia_0g_1B_\mu$ |
| $\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$ | $\phi' = e^{ia_\phi\theta}\phi$ | $\mathcal{L} = (D^\mu\phi)^\dagger D_\mu\phi - \phi^\dagger M_\phi^2\phi$<br>$D_\mu = \partial_\mu + ia_\phi g_1B_\mu$ |

SU(2)

| 입자 | 변환식 | 라그랑지안 밀도 |
|---|---|---|
| $\psi$ | 변환하지 않음 | |
| $q = \begin{pmatrix} u \\ d \end{pmatrix}$ | $q' = e^{i\vec{\tau}\cdot\vec{\theta}}q$ | $\mathcal{L} = \bar{q}(i\gamma^\mu D_\mu)q$<br>$D_\mu = \partial_\mu + ig_2\vec{\tau}\cdot\vec{W}_\mu$ |
| $B_\mu$ | 변환하지 않음 | |
| $\vec{W}_\mu = (W_\mu^1, W_\mu^2, W_\mu^3)$ | $\vec{W}_\mu' = \vec{W}_\mu - \frac{1}{g_2}\partial_\mu\vec{\theta} - \vec{\theta}\times\vec{W}_\mu$ | $-\frac{1}{4}\vec{W}^{\mu\nu}\cdot\vec{W}_{\mu\nu}$ |
| $\phi = \frac{1}{\sqrt{2}}(\xi_1 + i\xi_2)$ | 변환하지 않음 | |
| $\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$ | $\phi' = e^{i\vec{\tau}\cdot\vec{\theta}}\phi$ | $\mathcal{L} = (D^\mu\phi)^\dagger D_\mu\phi$<br>$D_\mu = \partial_\mu + ig_2\vec{\tau}\cdot\vec{W}_\mu$ |

## ⑤ chiral U(1) symmetry

↳ by dirac spinor 4개 성분. (for dirac 입자),

- $P_L = \frac{1}{2}(1-\gamma^5)$, $P_R = \frac{1}{2}(1+\gamma^5)$ ↳ 왜?
- $P_L\psi = \psi_L$
  $P_R\psi = \psi_R$. $\qquad \psi_L + \psi_R = \psi$.
- $\bar{\psi}_L = (\psi_L)^\dagger\gamma^0$ (앞에서 나온 걸까 왜?)
  $= (P_L\psi)^\dagger\gamma^0$
  $= \psi^\dagger P_L^\dagger\gamma^0 = \psi^\dagger\frac{1}{2}(1-\gamma^5)\gamma^0$
  $= \psi^\dagger\frac{1}{2}\gamma^0(1+\gamma^5)$
  $= \bar{\psi}\frac{1}{2}(1+\gamma^5)$
  $= \bar{\psi}P_R$
  $\qquad \bar{\psi}_R = \bar{\psi}P_L$.

* $P_L + P_R = 1$
  $P_L \cdot P_R = P_R \cdot P_L = 0$
  $P_L^2 = P_L$ , $P_R^2 = P_R$.

* 성질.
  ① $\bar{\psi} = \bar{\psi}_L + \bar{\psi}_R$
  ② $\bar{\psi}_L\psi_L = \bar{\psi}P_RP_L\psi = 0$.
  ③ $\bar{\psi}_R\psi_R = \bar{\psi}P_L \cdot P_R\psi = 0$.
  ④ $\bar{\psi}\psi = (\bar{\psi}_L + \bar{\psi}_R)(\psi_L + \psi_R)$
  $= 0 + 0 + \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L$.
  ⑤ $\bar{\psi}_L\gamma^\mu\psi_R = \bar{\psi}_R\gamma^\mu\psi_L = 0$
  ⑥ $\bar{\psi}\gamma^\mu\psi = (\bar{\psi}_L + \bar{\psi}_R)\gamma^\mu(\psi_L + \psi_R)$
  $= \bar{\psi}_L\gamma^\mu\psi_L + \bar{\psi}_R\gamma^\mu\psi_R$.

⇒ $\mathcal{L}_{Dirac} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$
$= (\bar{\psi}_L + \bar{\psi}_R)(i\gamma^\mu\partial_\mu - m)(\psi_L + \psi_R)$
$= i\bar{\psi}_L\partial_\mu\gamma^\mu\psi_L + i\bar{\psi}_R\partial_\mu\gamma^\mu\psi_R - m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$

↳ $U_A(1)$ gauge symmetry. ( U(1) → $e^{i\alpha}$ 아님! )

$\psi' = e^{i\alpha\gamma^5}\psi$
$\bar{\psi}' = \bar{\psi}e^{i\alpha\gamma^5}$
⇒ $\mathcal{L}' = \bar{\psi}'(i\gamma^\mu\partial_\mu - m)\psi'$
$= \bar{\psi}(i\gamma^\mu\partial_\mu - me^{2i\alpha\gamma^5})\psi$ ?

$m = 0$ 이어야 $U_A(1)$에서 대칭성.

$\psi_L' = e^{i\alpha P_L} \cdot P_L\psi$ , $\psi_R' = \psi_R$.
$= (P_R + P_Le^{i\alpha})P_L\psi$
$= e^{i\alpha}P_L\psi = e^{i\alpha}\psi_L$
$\bar{\psi}_L' = \bar{\psi}_Le^{-i\alpha}$ , $\bar{\psi}_R' = \bar{\psi}_R$.
$\mathcal{L}' = \mathcal{L}$.

⇒ U(1) + $U_A$(1): $U_L(1)$ $e^{i\alpha\frac{1}{2}(1-\gamma^5)} = e^{i\alpha P_L}$ 변환 ≈ $P_R + P_Le^{i\alpha}$
$U_R(1)$ $e^{i\alpha\frac{1}{2}(1+\gamma^5)} = e^{i\alpha P_R}$ 변환. ≈ $P_L + P_Re^{i\alpha}$

$e^{i\alpha P_R} = e^{i\alpha\frac{1}{2}(1+\gamma^5)}$
$= 1 + i\alpha P_R + \frac{1}{2!}(i\alpha P_R)^2 \cdots$
$= P_L + P_Re^{i\alpha}$
$= P_L + P_Re^{i\alpha}$
$\mathcal{L}' = \mathcal{L}$.

$U(1)$ 불변 X
$U_L(1) \times U_R(1)$ 불변 가능.

$U_L(1) \times U_R(1)$
U(1) chiral symmetry.
(질량이 없는 경우).

∴ Dirac particle 이 질량이 없는경우 chirality가 대칭성을 가짐.

* Chirality는 half integer spin에만 적용됨.
* dirac particle은 spinor에 의해 변화가 결정되고
  gauge, scalar particle은 성분수가 변화 방향 결정됨.

## ⑥ chiral SU(2) symmetry.

(Dirac) ⇒ $m = 0$ 인 경우.

$q_L = P_Lq = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$
$q_R = P_Rq = \begin{pmatrix} u_R \\ d_R \end{pmatrix}$
$q = q_L + q_R$

$\bar{q}_L = q_L^\dagger\gamma^0 = (\bar{u}_L, \bar{d}_L)$
$\bar{q}_R = q_R^\dagger\gamma^0 = (\bar{u}_R, \bar{d}_R)$
$\bar{q} = \bar{q}_L + \bar{q}_R$

* $SU_L(2)$ 변환.
  $e^{i\vec{\tau}\cdot\vec{\theta}P_L} \sim P_R + e^{i\vec{\tau}\cdot\vec{\theta}}P_L$

  $q_L' = e^{i\vec{\tau}\cdot\vec{\theta}}q_L \qquad \bar{q}_L' = \bar{q}_Le^{-i\vec{\tau}\cdot\vec{\theta}}$
  $q_R' = q_R \qquad \bar{q}_R' = \bar{q}_R$

  $\mathcal{L}' = i\bar{q}_L'\gamma^\mu\partial_\mu q_L' + i\bar{q}_R'\gamma^\mu\partial_\mu q_R'$
  $= \mathcal{L}$.

* $SU_R(2)$ 변환.
  $e^{i\vec{\tau}\cdot\vec{\theta}P_R} \sim P_L + e^{i\vec{\tau}\cdot\vec{\theta}}P_R$.

  $q_L' = q_L \qquad \bar{q}_L' = \bar{q}_L$
  $q_R' = e^{i\vec{\tau}\cdot\vec{\theta}}q_R \qquad \bar{q}_R' = \bar{q}_R \cdot e^{-i\vec{\tau}\cdot\vec{\theta}}$

  $\mathcal{L}' = i\bar{q}_L'\gamma^\mu\partial_\mu q_L' + i\bar{q}_R'\gamma^\mu\partial_\mu q_R'$.
  $= \mathcal{L}$.

∴ chiral symmetry를 위해 $m = 0$,
$\mathcal{L}$ 파트의 변화는 주는 관계X.

# ch5. Higgs mechanism

In low Energy → symmetry breaks spontaneously.

진공 → scalar 장의 기대값이 0이 아닌 값.

$\phi = \frac{1}{\sqrt{2}}(\xi_1 + i\xi_2)$, $\mathcal{L} = \partial^\mu \phi^* \partial_\mu \phi - m^2 \phi^* \phi$

$T - V$ → potential.

짝수 차항만 가능
⇒ 4차까지 확장
$V(|\phi|^2) = \alpha|\phi|^2 + \beta|\phi|^4$ 로 제한.

$$\mathcal{L} \Rightarrow \partial^\mu \phi^* \partial_\mu \phi - (\mu^2|\phi|^2 + \lambda|\phi|^4)$$

potential $-\infty$ X) ⇒ $\lambda > 0$, $\mu^2$ $>0 \rightarrow m = \mu$; $=0 \rightarrow m = 0$; $<0 \rightarrow$ 정의X.

$\lambda = 0$ $\mu^2 > 0$

⇒ $T = 0$ 일때 (최소)
$\partial^\mu \phi^* \partial_\mu \phi = 0$
$\Rightarrow \partial_\mu \phi(x) = 0$
∴ $\phi(x) = C$
↳ "공간 어디에서나 일정한 값"

(potential 최소)
$V = V(\phi(x))$
$= V(\phi)$
potential 이 최소 ⇒ $V$ 극값
$0 = \frac{\partial}{\partial \phi^*}[\mu^2 \phi^* \phi + \lambda(\phi^* \phi)^2]|_\phi$
$= \mu^2 \phi + 2\lambda(\phi^* \phi)\phi$
$= \mu^2 \phi + 2\lambda|\phi|^2 \phi$

$\lambda = 0 \rightarrow \mu^2 > 0$

$\lambda > 0 \rightarrow \mu^2 > 0 \Rightarrow \phi = 0$

$\mu^2 = 0 \Rightarrow \phi = 0$

$\mu^2 < 0 \Rightarrow \phi = 0$
or
$|\phi|^2 = -\frac{\mu^2}{2\lambda} = \frac{1}{2}v^2$ (기댓값), ∴ $\phi = 0$ 보다 $|\phi| = \frac{v}{\sqrt{2}}$ 일때 최소의 potential 을 가짐.

## ① Global U(1)

∴ $\langle\phi\rangle = \frac{v}{\sqrt{2}}$ * $e^{i\theta}\langle\phi\rangle$ (global U(1) 변환) ⇒ 무수히 많은 $\langle\phi\rangle$ 가능.

(섭동이론)
↓
진공에서
섭동 전개
$\phi = 0$ 에서 X.
진공이
$\phi = \langle\phi\rangle$ 에서.

$\phi = \eta + \langle\phi\rangle$

$\mathcal{L} = \partial^\mu \phi^* \partial_\mu \phi - (\mu^2|\phi|^2 + \lambda|\phi|^4)$
$= \partial^\mu \eta^* \partial_\mu \eta - (\mu^2|\eta + \langle\phi\rangle|^2 + \lambda|\eta + \langle\phi\rangle|^4)$

* $\phi' = \eta' + \langle\phi\rangle'$
$= e^{i\theta}(\eta + \langle\phi\rangle)$
$(\frac{v}{\sqrt{2}}, 0)$ 으로 선택.
$\neq e^{i\theta}\eta + \langle\phi\rangle$ ⇒ $\eta = \frac{1}{\sqrt{2}}(\chi_1 + i\chi_2)$ 로 설정시, $\eta + \langle\phi\rangle = \frac{1}{\sqrt{2}}(\chi_1 + v + i\chi_2)$
$e^{i\theta}$ 사라짐 (진공 $\phi$ 선택)
⇒ U(1) sym 깨짐.

($\lambda v^2 \chi_1 = \frac{1}{2} m \chi_1$)

적용시.
$\mathcal{L} = \frac{1}{2}\partial^\mu \chi_1 \partial_\mu \chi_1 + \frac{1}{2}\partial^\mu \chi_2 \partial_\mu \chi_2 + \frac{1}{2}\lambda v^2[(\chi_1 + v)^2 + \chi_2^2] - \frac{1}{4}\lambda[(\chi_1 + v)^2 + \chi_2^2]^2$
$= \frac{1}{2}\partial^\mu \chi_1 \partial_\mu \chi_1 + \frac{1}{2}\partial^\mu \chi_2 \partial_\mu \chi_2 - \lambda v^2 \chi_1^2 \cdots$
$\chi_2^2$ 항 없음X. → $m_{\chi_2} = 0$
$m_{\chi_1} = \sqrt{2\lambda v^2}$ ?

↳ 진공 선택 → U(1) sym X
∴ 질량 가능, U(1) 깨짐.

$\xi_1 \xi_2$ ($\phi$) 일경우
$\mathcal{L} = \frac{1}{2}\partial^\mu \xi_1 \partial_\mu \xi_1 + \frac{1}{2}\partial^\mu \xi_2 \partial_\mu \xi_2 + \frac{1}{2}\lambda v^2(\xi_1^2 + \xi_2^2) - \frac{1}{4}\lambda(\xi_1^2 + \xi_2^2)^2$
$\mu^2 < 0$, $\lambda > 0$ 에서 질량이 정의X.
→ 따라서 $\xi_1, \xi_2$ 질량 $m = 0$.

∴ $\phi = \frac{1}{\sqrt{2}}(\xi_1 + i\xi_2)$ 에서 $m_{\xi_1}, m_{\xi_2} = 0$ U(1) global
↓
U(1) X

$\mu^2 < 0$, $\lambda > 0$ 인 경우
$\phi = \langle\phi\rangle + \eta$
$= \frac{1}{\sqrt{2}}(v + \chi_1 + i\chi_2)$
⇒ $m_{\chi_1} = \sqrt{2\lambda v^2}$
$m_{\chi_2} = 0$

## ② Global SU(2)×U(1)

$\mathcal{L} = (\partial^\mu \phi)^\dagger \partial_\mu \phi - \phi^\dagger \mu_\phi^2 \phi$
↳ $\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2$

⇒ $\mu^2 < 0$, $\lambda > 0$ 인 경우

$\phi^\dagger \phi = -\frac{\mu^2}{2\lambda} = \frac{1}{2}v^2$

$V(\phi^\dagger \phi) = \lambda(\phi^\dagger \phi - \frac{1}{2}v^2)^2$ ⇒ 최솟값 0으로 잡음. 대칭유지 ($m = 0$),

$\phi = \eta + \langle\phi\rangle$ 정의.

$\eta = \frac{1}{\sqrt{2}}\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \chi_3 + i\chi_4 \\ \chi_1 + i\chi_2 \end{pmatrix}$

↳ 진공에너지 정하기 전 SU(2), U(1) symmetry.
$(\langle\phi\rangle' = \langle\phi\rangle \cdot e^{i\vec{\sigma}\cdot\vec{\theta}})$, $(\phi = e^{i a_\phi \theta}(\langle\phi\rangle + \eta))$
hypercharge.

$(e^{i\vec{\sigma}\cdot\vec{\theta}}\langle\phi\rangle)^\dagger e^{i\vec{\sigma}\cdot\vec{\theta}}\langle\phi\rangle = \langle\phi\rangle^\dagger\langle\phi\rangle =$
$(e^{i a_\phi \theta}\langle\phi\rangle)^\dagger e^{i a_\phi \theta}\langle\phi\rangle = \langle\phi\rangle^\dagger\langle\phi\rangle = \frac{1}{2}v^2$
⇒ SU(2), U(1) 대칭성 유지.
($m = 0$),

⊛ $\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$ → 대칭성 변환수 4개 ($\theta, \theta_1, \theta_2, \theta_3$) ? 왜 4개?

진공 특정 (최소 V)로 정할시 변환불가 ⇒ 변환이 다른상태.
→ SU(2)×U(1) 깨짐.

$\mathcal{L} = (\partial^\mu \eta)^\dagger \partial_\mu \eta - \lambda\left((\eta + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix})^\dagger(\eta + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}) - \frac{1}{2}v^2\right)^2.$

* 결국 다 깨지는 것은.
$\begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix}$ SU(2)×U(1) 의 특정 U(1)은 살아남음. ($\theta\theta_3$ 의 조합으로).
↳ $e^{i\alpha}$ (U(1)×SU(2) 변환).
$\begin{pmatrix} e^{i\alpha} & 0 \\ 0 & 1 \end{pmatrix} = e^{i a_\phi \theta} \cdot e^{i\frac{\sigma_3 \theta_3}{2}} = \begin{pmatrix} e^{i a_\phi \theta} & 0 \\ 0 & e^{i a_\phi \theta} \end{pmatrix}\begin{pmatrix} e^{i\frac{\theta_3}{2}} & 0 \\ 0 & e^{-i\frac{\theta_3}{2}} \end{pmatrix}$
∴ $a_\phi \theta + \frac{\theta_3}{2} = \alpha$
$a_\phi \theta - \frac{\theta_3}{2} = 0$ 인 경우.

$(\phi = \langle\phi\rangle + \eta = \frac{1}{\sqrt{2}}\begin{pmatrix} \eta_1 \\ \eta_2 + v \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \chi_3 + i\chi_4 \\ \chi_1 + i\chi_2 \end{pmatrix})$

$= \frac{1}{2}(\partial^\mu \chi_1 \partial_\mu \chi_1 + \partial^\mu \chi_2 \partial_\mu \chi_2 + \partial^\mu \chi_3 \partial_\mu \chi_3 + \partial^\mu \chi_4 \partial_\mu \chi_4)$
$-\lambda\left(\frac{1}{2}\begin{pmatrix} \chi_3 + i\chi_4 \\ \chi_1 + v + i\chi_2 \end{pmatrix}^\dagger\begin{pmatrix} \chi_3 + i\chi_4 \\ \chi_1 + v + i\chi_2 \end{pmatrix} - \frac{1}{2}v^2\right)^2 = \frac{1}{2}\sum_{a=1}^{4}\partial^\mu \chi_a \partial_\mu \chi_a - \frac{1}{4}\lambda\left((\chi_1 + v)^2 + \chi_2^2 + \chi_3^2 + \chi_4^2 - v^2\right)^2.$

=

$\chi_1^2$ 항의 계수만 있음. ⇒ $\chi_1$ 의 질량만 있음.

$-\lambda v^2 \chi_1^2$
$-\frac{1}{2}m^2\chi_1^2$

∴ $m_{\chi_1} = \sqrt{2\lambda v^2}$
U(1) 보존.

$\chi_2, \chi_3, \chi_4$ ⇒ gold-boe particle.

## ③ local U(1) sym. breaking

review for local U(1)

$\mathcal{L} = (D^\mu \phi)^* D_\mu \phi - V(\phi^*\phi) - \frac{1}{4} B^{\mu\nu} B_{\mu\nu}$

$( D_\mu = \partial_\mu + i a_\phi g_1 B_\mu \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu )$

$V(\phi^*\phi) = \lambda (\phi^*\phi - \frac{1}{2} v^2)^2 \quad (m=0) \rightarrow$ ?? $\begin{cases} \chi_1 = 0 \\ \chi_2 = \sqrt{\ } \end{cases}$ 아닌가?

- $\phi' = e^{i a_\phi \theta} \phi, \quad B_\mu' = B_\mu - \frac{\partial_\mu \theta}{g_1}$ 변환에 대하여 V는 대칭성을 갖는가.

여기서 $\phi' = \langle \phi \rangle + \eta$ 를 생각하면 $\langle \phi \rangle$를 정하지 않아도 국소 U(1)이 깨지지 않음.
global U(1) 변환
$\Rightarrow$ gauge 장의 [illegible]

$\hookrightarrow$ [illegible]

$\phi \rightarrow$ 실함수로 고정 (복소 $\rightarrow$ 실함수)
$\longrightarrow$ 대칭성 깨짐
(실함수로 선택해서 [illegible]). $\longrightarrow$ 실수 자유도 하나 제거

$\mathcal{L} = (D^\mu \xi)^* D_\mu \xi - \lambda (\xi^2 - \frac{1}{2} v^2)^2 - \frac{1}{4} B^{\mu\nu} B_{\mu\nu}$

$\langle \xi \rangle = \frac{1}{\sqrt{2}} v$

$\xi = \eta + \langle \xi \rangle = \frac{1}{\sqrt{2}} (h + v)$
$\hookrightarrow \frac{1}{\sqrt{2}} h$

$\Rightarrow \mathcal{L} = \frac{1}{2} \partial^\mu h \partial_\mu h + a_\phi^2 g_1^2 B^\mu B_\mu h^2 + \underbrace{\frac{1}{2} a_\phi^2 g_1^2 v^2}_{m^2} B^\mu B_\mu - \frac{\lambda}{4} (h^2 + 2vh)^2 - \frac{1}{4} B^{\mu\nu} B_{\mu\nu}$

$\cdots \quad m_h = \sqrt{2\lambda v^2}$

$m = a_\phi g_1 v$

$\Rightarrow$ 게이지 보존 $m=0$ 이 $U(1)_{global}$ 에서 나왔기에 여기선 X.

* 복소수 $\rightarrow$ 실수 (자유도 1↓)
$\Rightarrow$ gauge 질량 (자유도 1↑) ) 자유도 유지

* spin 1 vector particle은
질량(O) $\rightarrow$ 편광 3
(X) $\rightarrow$ 〃 2

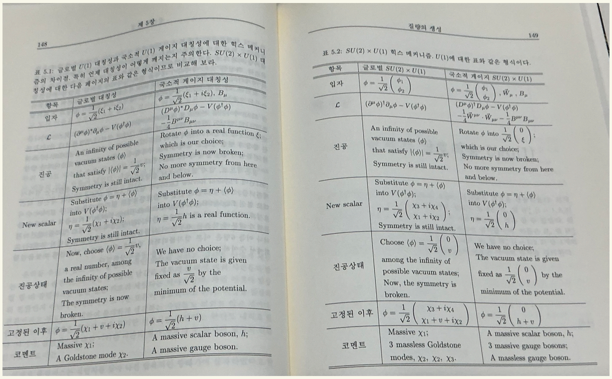



표 5.1: 글로벌 U(1) 대칭성과 국소적 U(1) 게이지 대칭성에 대한 힉스 메커니즘의 차이점. 특히 언제 대칭성이 어떻게 깨지는지 주의한다. $SU(2) \times U(1)$ 대칭성에 대한 다음 페이지의 표와 같은 형식이므로 비교해 보라.

| 항목 | 글로벌 대칭성 | 국소적 게이지 대칭성 |
|---|---|---|
| 입자 | $\phi = \frac{1}{\sqrt{2}}(\xi_1 + i\xi_2)$ | $\phi = \frac{1}{\sqrt{2}}(\xi_1 + i\xi_2)$, $B_\mu$ |
| $\mathcal{L}$ | $(\partial^\mu\phi)^*\partial_\mu\phi - V(\phi^\dagger\phi)$ | $(D^\mu\phi)^* D_\mu\phi - V(\phi^\dagger\phi)$ $-\frac{1}{4}B^{\mu\nu}B_{\mu\nu}$ |
| 진공 | An infinity of possible vacuum states $(\phi)$ that satisfy $\lvert\langle\phi\rangle\rvert = \frac{1}{\sqrt{2}}v$; Symmetry is still intact. | Rotate $\phi$ into a real function $\xi_1$ which is our choice; Symmetry is now broken; No more symmetry from here and below. |
| New scalar | Substitute $\phi = \eta + \langle\phi\rangle$ into $V(\phi^\dagger\phi)$; $\eta = \frac{1}{\sqrt{2}}(\chi_1 + i\chi_2)$; Symmetry is still intact. | Substitute $\phi = \eta + \langle\phi\rangle$ into $V(\phi^\dagger\phi)$; $\eta = \frac{1}{\sqrt{2}}h$ is a real function. |
| 진공상태 | Now, choose $\langle\phi\rangle = \frac{1}{\sqrt{2}}v$, a real number, among the infinity of possible vacuum states; The symmetry is now broken. | We have no choice; The vacuum state is given fixed as $\frac{v}{\sqrt{2}}$ by the minimum of the potential. |
| 고정된 이후 | $\phi = \frac{1}{\sqrt{2}}(\chi_1 + v + i\chi_2)$ | $\phi = \frac{1}{\sqrt{2}}(h + v)$ |
| 코멘트 | Massive $\chi_1$; A Goldstone mode $\chi_2$. | A massive scalar boson, $h$; A massive gauge boson. |



표 5.2: $SU(2) \times U(1)$ 힉스 메커니즘. $U(1)$에 대한 표와 같은 형식이다.

| 항목 | 글로벌 $SU(2) \times U(1)$ | 국소적 게이지 $SU(2) \times U(1)$ |
|---|---|---|
| 입자 | $\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}$ | $\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}$, $\vec{W}_\mu$, $B_\mu$ |
| $\mathcal{L}$ | $(\partial^\mu\phi)^\dagger\partial_\mu\phi - V(\phi^\dagger\phi)$ | $(D^\mu\phi)^\dagger D_\mu\phi - V(\phi^\dagger\phi)$ $-\frac{1}{4}\vec{W}^{\mu\nu}\cdot\vec{W}_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}$ |
| 진공 | An infinity of possible vacuum states $(\phi)$ that satisfy $\lvert\langle\phi\rangle\rvert = \frac{1}{\sqrt{2}}v$; Symmetry is still intact. | Rotate $\phi$ into $\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ \xi\end{pmatrix}$, which is our choice; Symmetry is now broken; No more symmetry from here and below. |
| New scalar | Substitute $\phi = \eta + \langle\phi\rangle$ into $V(\phi^\dagger\phi)$; $\eta = \frac{1}{\sqrt{2}}\begin{pmatrix}\chi_3 + i\chi_4\\ \chi_1 + i\chi_2\end{pmatrix}$; Symmetry is still intact. | Substitute $\phi = \eta + \langle\phi\rangle$ into $V(\phi^\dagger\phi)$; $\eta = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ h\end{pmatrix}$ |
| 진공상태 | Choose $\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}$ among the infinity of possible vacuum states; Now, the symmetry is broken. | We have no choice; The vacuum state is given fixed as $\frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}$ by the minimum of the potential. |
| 고정된 이후 | $\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\chi_3 + i\chi_4\\ \chi_1 + v + i\chi_2\end{pmatrix}$ | $\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ h + v\end{pmatrix}$ |
| 코멘트 | Massive $\chi_1$; 3 massless Goldstone modes, $\chi_2$, $\chi_3$, $\chi_4$. | A massive scalar boson, $h$; 3 massive gauge bosons; A massless gauge boson. |

## ④ local SU(2)×U(1) sym breaking.

$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \xi_3 + i\xi_4 \\ \xi_1 + i\xi_2 \end{pmatrix}$

$V(\phi^\dagger\phi) = \lambda (\phi^\dagger\phi - \frac{1}{2} v^2)^2$
$\hookrightarrow$ SU(2)×U(1) 대칭성 가짐.

SU(2)×U(1) Lagrangian

$\mathcal{L} = (D^\mu\phi)^\dagger D_\mu\phi - \lambda(\phi^\dagger\phi - \frac{1}{2}v^2)^2 - \frac{1}{4}\vec{W}^{\mu\nu}\cdot\vec{W}_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}$

$D_\mu = \partial_\mu + i a_\phi g_1 B_\mu + i g_2 \vec{\tau} \cdot \vec{W}_\mu$

$\vec{W}_{\mu\nu} = \partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu - g_2 \vec{W}_\mu \times \vec{W}_\nu, \quad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$ 일때, gauge 장 때문에 $\phi = \langle\phi\rangle + \eta$ 불가.

$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \xi_3 + i\xi_4 \\ \xi_1 + i\xi_2 \end{pmatrix} \Rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \xi \end{pmatrix} = \begin{pmatrix} 0 \\ \chi \end{pmatrix}$

$(\theta_3, \theta_1, \theta_2)$ 세 각 변환. 실함수 고정. (대칭성 깨짐)

$\phi = \begin{pmatrix} 0 \\ \chi \end{pmatrix} = \eta + \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h + v \end{pmatrix}$

$\mathcal{L} = \left( \frac{1}{\sqrt{2}} D^\mu \begin{pmatrix} 0 \\ h+v \end{pmatrix} \right)^\dagger \frac{1}{\sqrt{2}} D_\mu \begin{pmatrix} 0 \\ h+v \end{pmatrix} - \frac{\lambda}{4} \left( (h+v)^2 - v^2 \right)^2 - \frac{1}{4} \vec{W}^{\mu\nu} \cdot \vec{W}_{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu}$

$\rightarrow$ ① $h$ 입자 질량 가짐
② gauge 입자 질량 가짐.
③ SU(2)×U(1) 깨졌지만 U(1) 대칭성 남음.

① $\lambda v^2 h^2 + \lambda v h^3 + \frac{1}{4}\lambda h^4$
$m_h = \sqrt{2\lambda v^2}$

② $D_\mu \phi = \left( \partial_\mu + i a_\phi g_1 B_\mu + i g_2 \vec{\tau} \cdot \vec{W}_\mu \right) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h+v \end{pmatrix}$

$= \frac{1}{\sqrt{2}} \partial_\mu h + \frac{1}{\sqrt{2}} (i a_\phi g_1 B_\mu + i g_2 \vec{\tau} \cdot \vec{W}_\mu) \begin{pmatrix} 0 \\ h \end{pmatrix}$

$+ \underbrace{\frac{1}{\sqrt{2}} (i a_\phi g_1 B_\mu + i g_2 \vec{\tau} \cdot \vec{W}_\mu) \begin{pmatrix} 0 \\ v \end{pmatrix}}$

$\hookrightarrow \frac{1}{\sqrt{2}} \left( i a_\phi g_1 \begin{pmatrix} B_\mu & 0 \\ 0 & B_\mu \end{pmatrix} + i g_2 \begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{pmatrix} \right) \begin{pmatrix} 0 \\ v \end{pmatrix}$

$= \frac{iv}{\sqrt{2}} \begin{pmatrix} g_2 (W_\mu^1 - iW_\mu^2) \\ a_\phi g_1 B_\mu - g_2 W_\mu^3 \end{pmatrix}$

$(D_\mu \phi)^\dagger D_\mu \phi \sim \frac{v^2}{2} g_2^2 (W^{1\mu} W_\mu^1 + W^{2\mu} W_\mu^2)$
$+ \frac{v^2}{2} (a_\phi g_1 B^\mu - g_2 W^{3\mu})(a_\phi g_1 B_\mu - g_2 W_\mu^3)$

$\therefore W_\mu^1, W_\mu^2, a_\phi g_1 B^\mu - g_2 W_\mu^3$ 가 질량을 가짐.

4개의 보존 중 3개가 질량을 가지고,
나머지 하나는 대칭성 유지 $\rightarrow$ U(1).

(Goldstone boson 이 각각 흡수되어 gauge particle이 질량이 됨).

|  |  |  |
|---|---|---|
| Q. | Y. | T |
| $U(1)_{em}$ | $U(1)_Y$ | SU(2) |
|  |  | $e^{i\vec{\tau}\cdot\vec{\theta}}$ |

## Ch6. $U_{EM}(1)$

$W_\mu^+ = \frac{1}{\sqrt{2}} (W_\mu^1 - iW_\mu^2) \qquad Z_\mu = \frac{1}{\sqrt{g_1^2 + g_2^2}} (g_2 W_\mu^3 - g_1 B_\mu)$

$W_\mu^- = \frac{1}{\sqrt{2}} (W_\mu^1 + iW_\mu^2) \qquad A_\mu = \frac{1}{\sqrt{g_1^2 + g_2^2}} (g_1 W_\mu^3 + g_2 B_\mu)$

$W_\mu^1 = \frac{1}{\sqrt{2}} (W_\mu^+ + W_\mu^-) \qquad W_\mu^3 = \frac{1}{\sqrt{g_1^2 + g_2^2}} (g_2 Z_\mu + g_1 A_\mu)$

$W_\mu^2 = \frac{1}{\sqrt{2}} i (W_\mu^+ - W_\mu^-) \qquad B_\mu = \frac{1}{\sqrt{g_1^2 + g_2^2}} (-g_1 Z_\mu + g_2 A_\mu)$

* $g_1$ for U(1)
$g_2$ for SU(2) $\Rightarrow \tan\theta_W = \frac{g_1}{g_2}$
(weak mixing angle)
(weinberg angle)

$e = g_1 \cos\theta_W$
$= g_2 \sin\theta_W$

$g_2 W_\mu^3 = g_2 Z_\mu \cos\theta_W + eA_\mu$
$g_1 B_\mu = -g_1 Z_\mu \sin\theta_W + eA_\mu$ $\Rightarrow \cdots$ $W_\mu^3 = Z_\mu \cos\theta_W + A_\mu \sin\theta_W$
$B_\mu = -Z_\mu \sin\theta_W + A_\mu \cos\theta_W$

$A_\mu$ 항이 결합 상수 찾기.

(예시) $D_\mu = \partial_\mu + \frac{i}{2} g_1 Y B_\mu + \frac{i}{2} g_2 (\sigma_1 W_\mu^1 + \sigma_2 W_\mu^2 + \sigma_3 W_\mu^3)$

$= \partial_\mu + \frac{i}{2} (g_1 Y B_\mu + g_2 \sigma_3 W_\mu^3) + \frac{i}{2} (g_2 \sigma_1 W_\mu^1 + g_2 \sigma_2 W_\mu^2)$

$= \cdots \partial_\mu + \frac{i}{2} e \begin{pmatrix} Y+1 & 0 \\ 0 & Y-1 \end{pmatrix} A_\mu - \frac{i}{2} \frac{1}{\sqrt{g_1^2 + g_2^2}} \begin{pmatrix} g_1^2 Y - g_2^2 & \\ & g_1^2 Y + g_2^2 \end{pmatrix} Z_\mu + \frac{i}{\sqrt{2}} g_2 \begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix}$

(예시) $D_\mu = \partial_\mu + \frac{i}{2} g_1 Y B_\mu$

$= \partial_\mu + \frac{i}{2} Y e A_\mu - \frac{i}{2} Y g_1 \sin\theta_W Z_\mu$

- lepton ( using $T_3$ )

$Y \rightarrow B_\mu$ 게이지 장에서 변환

- 왼손잡이 경입자 ($Y = -1$)

$- D_\mu = \partial_\mu + ie \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} A_\mu + i \frac{g_2}{2\cos\theta_W} \begin{pmatrix} 1 & 0 \\ 0 & -\cos 2\theta_W \end{pmatrix} Z_\mu + \frac{i}{\sqrt{2}} g_2 \begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix}$

$- \mathcal{L} = \bar{l}_L i\gamma^\mu D_\mu l_L \rightarrow (\nu, e)$

$= \bar{\nu}_L i\gamma^\mu (\partial_\mu + i \frac{g_2}{2\cos\theta_W} Z_\mu) \nu_L$
$+ \bar{e}_L i\gamma^\mu (\partial_\mu + i(-1) eA_\mu - i \frac{g_2}{2\cos\theta_W} \cos 2\theta_W Z_\mu) e_L$
$+ (\frac{i g_2}{\sqrt{2}}) (\bar{\nu}_L i\gamma^\mu W_\mu^+ e_L + \bar{e}_L i\gamma^\mu W_\mu^- \nu_L)$

neutrino $\Rightarrow A_\mu$ 항 X.

$\frac{Y}{2} + T_3 = Q$

$Y = -1 \quad -\frac{1}{2} \begin{cases} \frac{1}{2} & 0 & \nu_L \\ -\frac{1}{2} & -1 & e_L \end{cases}$

- 오른손잡이 경입자 $e_R$ ($Y = -2$)
(singlet) $\Rightarrow$ [illegible]

$- D_\mu = \partial_\mu - ieA_\mu + i \frac{g_2}{\cos\theta_W} \sin^2\theta_W Z_\mu$

$- \mathcal{L} = \bar{e}_R i\gamma^\mu D_\mu e_R = \bar{e}_R i\gamma^\mu (\partial_\mu + i(-1)eA_\mu + i \frac{g_2}{\cos\theta_W} \sin^2\theta_W Z_\mu) e_R$

* $l_L + e_R$.

전자 $\Rightarrow A_\mu$와 상호작용, 세기 $\rightarrow e$.

$\bar{l}_L i\gamma^\mu D_\mu l_L + \bar{e}_R i\gamma^\mu D_\mu e_R = \bar{\nu}_L i\gamma^\mu \partial_\mu \nu_L + \bar{e} i\gamma^\mu \partial_\mu e - e(-1)(\bar{e}\gamma^\mu e) A_\mu - \frac{g_2}{2\cos\theta_W} (\bar{\nu}_L \gamma^\mu \nu_L + g_{eV} (\bar{e}\gamma^\mu e) -$
$g_{eA} (\bar{e}\gamma^\mu \gamma^5 e)) Z_\mu - (\frac{1}{\sqrt{2}} g_2) ((\bar{\nu}_L \gamma^\mu e_L) W_\mu^+ + (\bar{e}_L \gamma^\mu \nu_L) W_\mu^-)$

$\hookrightarrow$ e가 Z와 상호작용,
L, R 세기 다름.

$\hookrightarrow$ W는 $e_L$만 반응.

$( g_{eV} = T_{e3} - 2Q_e \sin^2\theta_W, \ g_{eA} = -\frac{1}{2} = T_{e3}. )$
$\hookrightarrow -\frac{1}{2} \quad \hookrightarrow -1$

- 쿼크의 공변도함수 $(\nu SU(2)_3)$

- Left hand doublet $q_L$ $(Y=\frac{1}{3})$

$\cdot\ D_\mu = \partial_\mu + \frac{2}{3}e\begin{pmatrix}\frac{2}{3} & 0\\ 0 & -\frac{1}{3}\end{pmatrix}A_\mu - \frac{2}{3}\begin{pmatrix}\frac{1}{3}g_1\sin\theta_W - g_2\cos\theta_W & 0 \\ 0 & \frac{1}{3}g_1\sin\theta_W + g_2\cos\theta_W\end{pmatrix}Z_\mu + \frac{3}{\sqrt{3}}g_2\begin{pmatrix}0 & W_\mu^+\\ W_\mu^- & 0\end{pmatrix}$

$\cdot\ \mathcal{L} = \bar{q}_L\gamma^\mu D_\mu q_L = \bar{u}_L\gamma^\mu(\partial_\mu + \frac{2}{3}(\frac{2}{3})eA_\mu)u_L + \bar{d}_L\gamma^\mu(\partial_\mu + \frac{2}{3}(-\frac{1}{3})eA_\mu)d_L$
$-\frac{2}{3}(\frac{1}{3}g_1\sin\theta_W - g_2\cos\theta_W)\bar{u}_L\gamma^\mu Z_\mu u_L - \frac{2}{3}(\frac{1}{3}g_1\sin\theta_W + g_2\cos\theta_W)\bar{d}_L\gamma^\mu Z_\mu d_L + (\frac{3}{\sqrt{3}}g_2)(\bar{u}_L\gamma^\mu W_\mu^+ d_L + \bar{d}_L\gamma^\mu W_\mu^- u_L)$

- right hand $u_R$ $(Y=\frac{4}{3})$ singlet, $d_R$ $(Y=-\frac{2}{3})$ singlet

$u_R$

$-D_\mu = \partial_\mu + 2\frac{2}{3}eA_\mu - 2\frac{2}{3}g_1\sin\theta_W Z_\mu$

$-\mathcal{L} = \bar{u}_R\gamma^\mu D_\mu u_R = \bar{u}_R\gamma^\mu(\partial_\mu + 2(\frac{2}{3})eA_\mu - 2\frac{2}{3}g_1\sin\theta_W Z_\mu)u_R$

$d_R$

$-D_\mu = \partial_\mu - 2\frac{1}{3}eA_\mu + 2\frac{1}{3}g_1\sin\theta_W Z_\mu$

$-\mathcal{L} = \bar{d}_R\gamma^\mu D_\mu d_R = \bar{d}_R\gamma^\mu(\partial_\mu + 2(-\frac{1}{3})eA_\mu + 2\frac{1}{3}g_1\sin\theta_W Z_\mu)d_R$

$\mathcal{L}_q = \bar{u}\gamma^\mu\partial_\mu u + \bar{d}\gamma^\mu\partial_\mu d - e((\frac{2}{3})\bar{u}\gamma^\mu u + (-\frac{1}{3})\bar{d}\gamma^\mu d)A_\mu$
$-\frac{g_2}{2\cos\theta_W}(\bar{u}\gamma^\mu(g_V - g_A\gamma^5)u + \bar{d}\gamma^\mu(g_V - g_A\gamma^5)d)Z_\mu - \frac{3}{\sqrt{3}}g_2(\bar{u}_L\gamma^\mu W_\mu^+ d_L + \bar{d}_L\gamma^\mu W_\mu^- u_L)$

↳ 계수들이 복잡.. 생략.
계수가 일치하려면 → quark 군의 전하가.. ☹,

- 스칼라 입자의 공변도함수. $(Y=1)$
(doublet 만 사용)*

$-D_\mu = \partial_\mu + \frac{2}{3}e\begin{pmatrix}Y+1 & 0\\ 0 & Y-1\end{pmatrix}A_\mu - \frac{2}{3}\begin{pmatrix}g_1Y\sin\theta_W - g_2\cos\theta_W & 0\\ 0 & g_1Y\sin\theta_W + g_2\cos\theta_W\end{pmatrix}Z_\mu + \frac{3}{\sqrt{3}}g_2\begin{pmatrix}0 & W_\mu^+\\ W_\mu^- & 0\end{pmatrix}$

$= \partial_\mu + 2e\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}A_\mu - \frac{2}{3}\begin{pmatrix}-\sqrt{g_1^2+g_2^2}\cos2\theta_W & 0\\ 0 & \sqrt{g_1^2+g_2^2}\end{pmatrix}Z_\mu + \frac{3}{\sqrt{3}}g_2\begin{pmatrix}0 & W_\mu^+\\ W_\mu^- & 0\end{pmatrix}$

$-D_\mu\phi \quad (\phi = \frac{1}{\sqrt{3}}\begin{pmatrix}0\\ h+v\end{pmatrix})$

$= \frac{1}{\sqrt{3}}\begin{pmatrix}0\\ \partial_\mu h\end{pmatrix} + \frac{2}{3\sqrt{3}}\begin{pmatrix}\sqrt{3}g_2W_\mu^+\\ -\sqrt{g_1^2+g_2^2}Z_\mu\end{pmatrix}(h+v)$

→ 중성 → $A_\mu$와 상호작용X.

$-(D_\mu\phi)^\dagger D_\mu\phi = \frac{1}{4}g_2^2(h+v)^2W_\mu^-W_\mu^+ + \frac{1}{3}(\partial^\mu h\partial_\mu h + \frac{1}{4}(g_1^2+g_2^2)(h+v)^2Z^\mu Z_\mu)$

$m_W = m_{W^\pm} = \frac{1}{2}vg_2$
$m_Z = \frac{1}{3}v\sqrt{g_1^2+g_2^2}$
$m_A = 0.$

- gauge particle 정리.

$-\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}\vec{W}^{\mu\nu}\vec{W}_{\mu\nu} \Rightarrow$ Higgs 3개의 gauge particles.

$B_\mu = A_\mu\cos\theta_W - Z_\mu\sin\theta_W$
$B_{\mu\nu} = \underbrace{(\partial_\mu A_\nu - \partial_\nu A_\mu)}_{F_{\mu\nu}}\cos\theta_W - \underbrace{(\partial_\mu Z_\nu - \partial_\nu Z_\mu)}_{Z_{\mu\nu}}\sin\theta_W$

$W_\mu^3 = A_\mu\sin\theta_W + Z_\mu\cos\theta_W$
$W_{\mu\nu}^3 = \partial_\mu W_\nu^3 - \partial_\nu W_\mu^3 - g_2(W_\mu^1W_\nu^2 - W_\nu^1W_\mu^2)$
$= F_{\mu\nu}\sin\theta_W + Z_{\mu\nu}\cos\theta_W - g_2(W_\mu^1W_\nu^2 - W_\nu^1W_\mu^2)$
$2(W_\mu^-W_\nu^+ - W_\mu^+W_\nu^-)$

$W_{\mu\nu}^1 = \partial_\mu W_\nu^1 - \partial_\nu W_\mu^1 - g_2(W_\mu^2W_\nu^3 - W_\nu^2W_\mu^3) = (\partial\times W^1)_{\mu\nu} - g_2(W^2\times W^3)_{\mu\nu}$
$W_{\mu\nu}^2 = \partial_\mu W_\nu^2 - \partial_\nu W_\mu^2 - g_2(W_\mu^3W_\nu^1 - W_\nu^3W_\mu^1) = (\partial\times W^2)_{\mu\nu} - g_2(W^3\times W^1)_{\mu\nu}$

$(\partial\times W^1)_{\mu\nu} = \frac{1}{\sqrt{3}}(\partial\times W^+)_{\mu\nu} + \frac{1}{\sqrt{3}}(\partial\times W^-)_{\mu\nu}$
$(W^2\times W^3)_{\mu\nu} = -\frac{3}{\sqrt{3}}\sin\theta_W(A\times W^+)_{\mu\nu} + \frac{3}{\sqrt{3}}\sin\theta_W(A\times W^-)_{\mu\nu}$
$-\frac{3}{\sqrt{3}}\cos\theta_W(Z\times(W^+ + W^-))_{\mu\nu}$

$\therefore -\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}\vec{W}^{\mu\nu}\vec{W}_{\mu\nu}$
$= -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}Z^{\mu\nu}Z_{\mu\nu} + 2\frac{1}{4}g_2(F^{\mu\nu}\sin\theta_W + Z^{\mu\nu}\cos\theta_W)(W_\mu^-W_\nu^+ - W_\mu^+W_\nu^-)$
$+ 2\frac{1}{4}g_2(F_{\mu\nu}\sin\theta_W + Z_{\mu\nu}\cos\theta_W)(W^{-\mu}W^{+\nu} - W^{+\mu}W^{-\nu}) + \frac{g_2^2}{4}(W^{-\mu}W^{+\nu} - W^{+\mu}W^{-\nu})(W_\mu^-W_\nu^+ - W_\mu^+W_\nu^-)$
$-\frac{1}{4}(W^{+\mu\nu}W_{\mu\nu}^- + W^{-\mu\nu}W_{\mu\nu}^+)$

$\cdot\ W_{\mu\nu}^1 = \frac{1}{\sqrt{3}}(D\times W^+)_{\mu\nu} + \frac{1}{\sqrt{3}}(D\times W^-)_{\mu\nu}$
$+\frac{3}{\sqrt{3}}g_2\cos\theta_W(Z\times(W^+ - W^-))_{\mu\nu}$

$(D_\mu = \partial_\mu + 3eA_\mu \quad (W^+)$
$D_\mu = \partial_\mu - 3eA_\mu \quad (W^-))$

$\cdot\ W_{\mu\nu}^2 = \frac{1}{\sqrt{3}}(D\times W^+)_{\mu\nu} - \frac{3}{\sqrt{3}}(D\times W^-)_{\mu\nu} - \frac{1}{\sqrt{3}}g_2\cos\theta_W(Z\times(W^+ + W^-))_{\mu\nu}$
$-W_{\mu\nu}^+ = \frac{1}{\sqrt{3}}(W_{\mu\nu}^1 - 2W_{\mu\nu}^2) = (D\times W^+)_{\mu\nu} + \frac{3}{\sqrt{3}}g_2\cos\theta_W(Z\times W^+)_{\mu\nu}$
$-W_{\mu\nu}^- = \frac{1}{\sqrt{3}}(W_{\mu\nu}^1 + 2W_{\mu\nu}^2) = (D\times W^-)_{\mu\nu} - \frac{3}{\sqrt{3}}g_2\cos\theta_W(Z\times W^-)_{\mu\nu}$

$\Rightarrow W_{\mu\nu}^+ = \frac{1}{\sqrt{3}}(W_{\mu\nu}^1 - 2W_{\mu\nu}^2) = (D\times W^+)_{\mu\nu} + \frac{3}{\sqrt{3}}g_2\cos\theta_W(Z\times W^+)_{\mu\nu}$
$W_{\mu\nu}^- = \frac{1}{\sqrt{3}}(W_{\mu\nu}^1 + 2W_{\mu\nu}^2) = (D\times W^-)_{\mu\nu} - \frac{3}{\sqrt{3}}g_2\cos\theta_W(Z\times W^-)_{\mu\nu}$

↳ $W^{+\mu\nu}W_{\mu\nu}^- + W^{-\mu\nu}W_{\mu\nu}^+ = W^{1\mu\nu}W_{\mu\nu}^1 + W^{2\mu\nu}W_{\mu\nu}^2$

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- 국소적 $U_{em}(1)$ 대칭성. $SU(2)_3 + U(1) \rightarrow U(1)_{EM}$
$I_3 + \frac{Y}{2} = Q$

- gauge particle.

$U(1)$: $B_\mu' = B_\mu - \frac{1}{g_1}\partial_\mu\theta$
$W_\mu^{1\prime} = W_\mu^1 - \frac{1}{g_2}\partial_\mu\theta_1 - (\theta_2W_\mu^3 - \theta_3W_\mu^2) = W_\mu^1 + \theta_3W_\mu^2$
$W_\mu^{2\prime} = W_\mu^2 - \frac{1}{g_2}\partial_\mu\theta_2 - (\theta_3W_\mu^1 - \theta_1W_\mu^3) = W_\mu^2 - \theta_3W_\mu^1$
$W_\mu^{3\prime} = W_\mu^3 - \frac{1}{g_2}\partial_\mu\theta_3 - (\theta_1W_\mu^2 - \theta_2W_\mu^1) = W_\mu^3 - \frac{1}{g_2}\partial_\mu\theta_3$

$U(1)_{EM}$ $(\theta_1, \theta_2 = 0,\ \theta_3, \theta$ 일정$)$: $= B_\mu - \frac{1}{g_1}\partial_\mu\theta$

$\rightarrow$ $A_\mu' = A_\mu - \frac{1}{e}\partial_\mu\theta$
$W_\mu^{+\prime} = (1+3\theta)W_\mu^+$
$W_\mu^{-\prime} = (1-3\theta)W_\mu^-$
$Z_\mu' = Z_\mu$

$\rightarrow$ $F_{\mu\nu}' = F_{\mu\nu}$
$Z_{\mu\nu}' = Z_{\mu\nu}$
$W_{\mu\nu}^{+\prime} = (1+3\theta)W_{\mu\nu}^+$
$W_{\mu\nu}^{-\prime} = (1-3\theta)W_{\mu\nu}^-$

$\Rightarrow \mathcal{L} = \theta^2$ 이상항만 불변.

- higgs particle.

중성 ⇒ $A_\mu$와 작용X.
$Z_\mu$
$W_\mu^\pm$ 도 전하 상쇄되어 불변.

$\mathcal{L} = \frac{1}{3}\partial^\mu h\partial_\mu h + \frac{1}{8}(g_2^2((W^{+\mu})^*W_\mu^+ + (W^{-\mu})^*W_\mu^-)(g_1^2+g_2^2)Z^\mu Z_\mu)(h+v)^2$

- dirac particle (lepton, quark)

- quark

$\mathcal{L} = \bar{u}_L\gamma^\mu(\partial_\mu + 2(\frac{2}{3})eA_\mu)u_L + \bar{d}_L\gamma^\mu(\partial_\mu + 2(-\frac{1}{3})eA_\mu)d_L$
$-\frac{2}{3}(\frac{1}{3}g_1\sin\theta_W - g_2\cos\theta_W)(\bar{u}_L\gamma^\mu Z_\mu u_L) - \frac{2}{3}(\frac{1}{3}g_1\sin\theta_W + g_2\cos\theta_W)(\bar{d}_L\gamma^\mu Z_\mu d_L)$
$-\frac{3}{3}g_2\sin\theta_W(\bar{u}_R\gamma^\mu Z_\mu u_R) + \frac{1}{3}g_1\sin\theta_W(\bar{d}_R\gamma^\mu Z_\mu d_R) + \frac{3}{\sqrt{3}}g_2(\bar{u}_L\gamma^\mu W_\mu^+ d_L + \bar{d}_L\gamma^\mu W_\mu^- u_L)$

$\longrightarrow U(1)_{EM}$

$u_R' = e^{2Q_u\theta}u_R$
$\bar{u}_R' = e^{-2Q_u\theta}\bar{u}_R$
$q_L' = \begin{pmatrix}e^{2Q_u\theta}u_L\\ e^{2Q_d\theta}d_L\end{pmatrix}$

$d_R' = e^{2Q_d\theta}d_R$
$\bar{d}_R' = e^{-2Q_d\theta}\bar{d}_R$
$\bar{q}_L' = (e^{-2Q_u\theta}\bar{u}_L,\ e^{-2Q_d\theta}\bar{d}_L)$

⇒ $\theta^2$ 이상항 무시

- lepton

$\mathcal{L} = \bar{\nu}_L\gamma^\mu\partial_\mu\nu_L + \frac{2}{3}\sqrt{g_1^2+g_2^2}(\bar{\nu}_L\gamma^\mu Z_\mu\nu_L)Z_\mu + \bar{e}_L\gamma^\mu(\partial_\mu + 2(-1)eA_\mu)e_L - \frac{2}{3}\sqrt{g_1^2+g_2^2}\cos2\theta_W(\bar{e}_L\gamma^\mu e_L)Z_\mu$
$+ \frac{3}{\sqrt{3}}g_2((\bar{\nu}_L\gamma^\mu e_L)W_\mu^+ + (\bar{e}_L\gamma^\mu\nu_L)W_\mu^-)$

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* 질량이 있는 gauge particle ( $B, W_{123} \Rightarrow A, W^\pm, Z$ 와의 상호작용)

- quark, lepton.

$\frac{1}{2}\ \ I_3 + \frac{Y}{2} = Q\ \ \frac{2}{3}$

$q_L\begin{pmatrix}u_L\\ d_L\end{pmatrix}$ $\mathcal{L} = \bar{q}_L2\gamma^\mu D_\mu q_L = \bar{u}_L2\gamma^\mu(\partial_\mu + 2(\frac{2}{3})eA_\mu)u_L + \bar{d}_L2\gamma^\mu(\partial_\mu + 2(-\frac{1}{3})eA_\mu)d_L - \frac{2}{3}(\frac{1}{3}g_1\sin\theta_W - g_2\cos\theta_W)\bar{u}_L2\gamma^\mu Z_\mu u_L$
$-\frac{2}{3}(\frac{1}{3}g_1\sin\theta_W + g_2\cos\theta_W)\bar{d}_L2\gamma^\mu Z_\mu d_L + (\frac{3}{\sqrt{3}}g_2)(\bar{u}_L2\gamma^\mu W_\mu^+ d_L + \bar{d}_L2\gamma^\mu W_\mu^- u_L)$

$u_R$, $d_R$: $\mathcal{L} = \bar{u}_R2\gamma^\mu D_\mu u_R = \bar{u}_R2\gamma^\mu(\partial_\mu + 2(\frac{2}{3})eA_\mu)u_R - \frac{2\cdot2}{3}g_1\sin\theta_W\bar{u}_R2\gamma^\mu Z_\mu u_R.$
$\bar{d}_R2\gamma^\mu D_\mu d_R = \bar{d}_R2\gamma^\mu(\partial_\mu + 2(-\frac{1}{3})eA_\mu)d_R + \frac{1\cdot2}{3}g_1\sin\theta_W\bar{d}_R2\gamma^\mu Z_\mu d_R.$

$l_L\begin{pmatrix}\nu_L\\ e_L\end{pmatrix}$ $\mathcal{L} = \bar{l}_L2\gamma^\mu D_\mu l_L = \bar{\nu}_L2\gamma^\mu(\partial_\mu + \frac{2}{3}\sqrt{g_1^2+g_2^2}Z_\mu)\nu_L + \bar{e}_L2\gamma^\mu(\partial_\mu + 2(-1)eA_\mu)e_L - \bar{e}_L2\gamma^\mu(\frac{2}{3}\sqrt{g_1^2+g_2^2}\cos2\theta_W Z_\mu)e_L + (\frac{3}{\sqrt{3}}g_2)(\bar{\nu}_L2\gamma^\mu W_\mu^+ e_L + \bar{e}_L2\gamma^\mu W_\mu^- \nu_L)$

$e_R$ $\mathcal{L} = \bar{e}_R2\gamma^\mu D_\mu e_R = \bar{e}_R2\gamma^\mu(\partial_\mu + 2(-1)eA_\mu)e_R + 2g_1\sin\theta_W\bar{e}_R2\gamma^\mu Z_\mu e_R.$

- scalar

$\phi = \frac{1}{\sqrt{3}}\begin{pmatrix}0\\ h+v\end{pmatrix}$ $\mathcal{L} = (D^\mu\phi)^\dagger D_\mu\phi = \frac{1}{3}\partial^\mu h\partial_\mu h + \frac{1}{8}(g_2^2((W^{+\mu})^*W_\mu^+ + (W^{-\mu})^*W_\mu^-) + (g_1^2+g_2^2)Z^\mu Z_\mu)(h+v)^2$

- gauge particle.

$B_\mu$, $W_{\mu\nu}^3$: $\mathcal{L} = -\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}\vec{W}^{\mu\nu}\vec{W}_{\mu\nu}^3 = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}Z^{\mu\nu}Z_{\mu\nu} + \frac{3}{2}g_2(F^{\mu\nu}\sin\theta_W + Z^{\mu\nu}\cos\theta_W)\cdot(W_\mu^-W_\nu^+ - W_\mu^+W_\nu^-) + \frac{g_2^2}{4}(W^{-\mu}W^{+\nu} - W^{+\mu}W^{-\nu})(W_\mu^-W_\nu^+ - W_\mu^+W_\nu^-)$

$W_\mu^1$, $W_\mu^2$: $\mathcal{L} = -\frac{1}{4}\vec{W}^{\mu\nu}_{1,2}\vec{W}_{\mu\nu}^{1,2} = -\frac{1}{4}(W^{+\mu\nu}W_{\mu\nu}^- + W^{-\mu\nu}W_{\mu\nu}^+)$

# Ch 7. 표준모형.

(여러 세대 추가시 세대간 혼합)

- $SU(2) \times U(1)$
  ↳ left hand 만이 이중항 구성.
  dirac particle right hand 성분은 모두 $SU(1)$, $SU(2)$ 변환에 대해 동일.
  scalar, gauge도 $\gamma$ 없기 때문에 상관X.

- 입자들의 $SU(2) \times U(1)$ 변환.

$$l_L' = e^{i(\frac{Y_l}{2}\theta + \vec{\tau}\cdot\vec{\theta})} l_L$$
$$e_R' = e^{i\frac{Y_e}{2}\theta} e_R$$
$$q_L' = e^{i(\frac{Y_q}{2}\theta + \vec{\tau}\cdot\vec{\theta})} q_L$$
$$u_R' = e^{i\frac{Y_u}{2}\theta} u_R$$
$$d_R' = e^{i\frac{Y_d}{2}\theta} d_R$$

$$B_\mu' = B_\mu - \frac{1}{g_1}\partial_\mu\theta$$
$$\vec{W}_\mu' = \vec{W}_\mu - \frac{1}{g_2}\partial_\mu\vec{\theta} - \vec{\theta}\times\vec{W}_\mu$$
$$\phi' = e^{i(\frac{Y_\phi}{2}\theta + \vec{\tau}\cdot\vec{\theta})}\phi$$

$$a_q = \frac{Y_q}{2}$$
→ 전하량에서 결정됨.

$SU(2)_L \times U(1)_Y$ 대칭시. (질량X)

$$\mathcal{L} = \bar{l}_L i\gamma^\mu(\partial_\mu - i\frac{1}{2}g_1 B_\mu + i g_2\vec{\tau}\cdot\vec{W}_\mu) l_L + \bar{e}_R i\gamma^\mu(\partial_\mu - i g_1 B_\mu) e_R + \bar{q}_L i\gamma^\mu(\partial_\mu + i\frac{1}{6}g_1 B_\mu + i g_2\vec{\tau}\cdot\vec{W}_\mu) q_L$$
$$+ \bar{u}_R i\gamma^\mu(\partial_\mu + \frac{2}{3} i g_1 B_\mu) u_R + \bar{d}_R i\gamma^\mu(\partial_\mu - i\frac{1}{3}g_1 B_\mu) d_R + [(\partial_\mu + i\frac{1}{2}g_1 B_\mu + i g_2\vec{\tau}\cdot\vec{W}_\mu)\phi]^\dagger($$
$$\partial_\mu + \frac{i}{2}g_1 B_\mu + i g_2\vec{\tau}\cdot\vec{W}_\mu)\phi - \lambda(\phi^\dagger\phi - \frac{v^2}{2})^2 - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}\vec{W}^{\mu\nu}\vec{W}_{\mu\nu}$$

- mass

- scalar.