

Welcome!

First, what is cosmology?

⇒ Study of our Universe on its largest scales

↳ Don't care about individual stars, galaxies, planets etc.

→ Concerned about the large-scale, **statistical distribution** of stuff.

**Analogy:** a public health official cares about what % of a population has caught the 'flu, and not whether an individual is sick or not.

If we want to study our Universe on the largest scales, we should have a sense for how big it is!

What sorts of units do we use to describe distances in cosmology?

Light years :  $1 \text{ ly} \equiv$  distance that light travels in a year  
= (1 year) ( $c$ )  
=  $9.46 \times 10^{15} \text{ m}$ .

More common

Parsecs :  $1 \text{ parsec} \approx 3.26 \text{ ly}$

Anyone know how a parsec is defined?

Typical distances in cosmology: Mpc or Gpc

Between stars in a galaxy :  $\sim \text{pc}$   
" galaxies :  $\sim \text{Mpc}$

Show distance scale slides

This is both humbling and awe-inspiring!

So the universe is definitely big, but ...

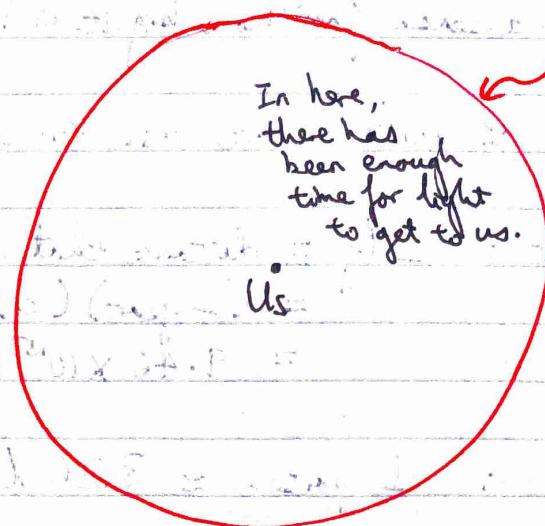
**Discussion:** Is the universe infinite in extent?

**Question:**

We don't know! To check this, we would have to be able to look infinitely far away. All we can say is that the universe is either infinite or at least much, much larger than what we can see.

→ Theoretically, our simplest theory models favour an infinite universe.

Even if "the" universe is infinite, our Observable Universe is finite. Why? Because light travels at a finite speed.



In here,  
there has  
been enough  
time for light  
to get to us.

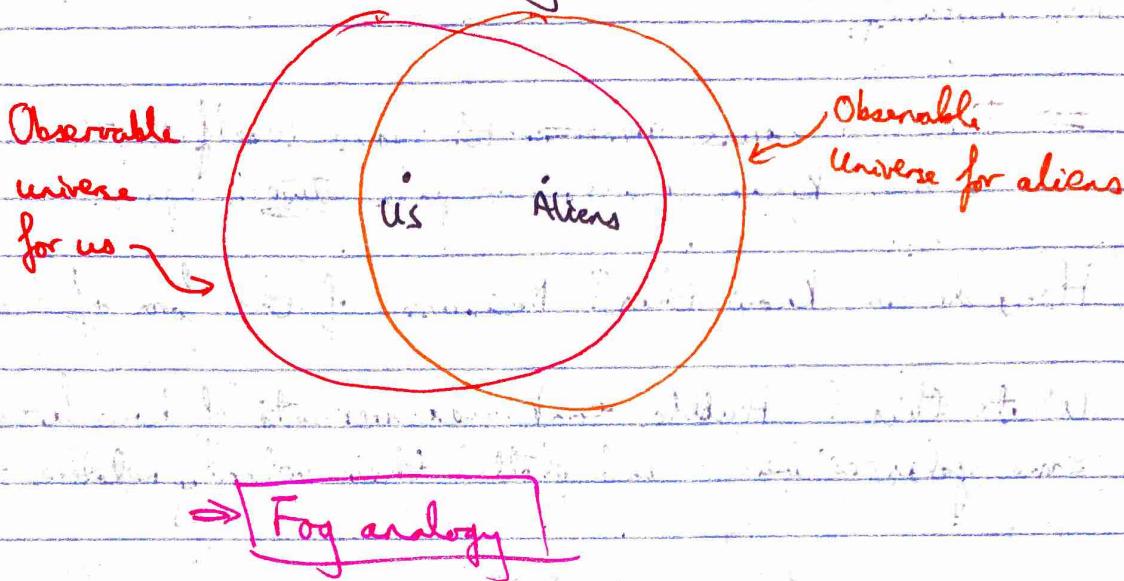
Boundary of our observable  
universe

Out here, ~~or there~~  
things are so far away  
from us that light  
hasn't had enough time  
to travel to us in the  
entire age of the universe.

**Discussion:** Does this mean we are at the center of  
**Question:** the universe?

No! We are at the center of the universe that we can observe

but we are not at the center of "the" universe. Some alien civilization a Gpc away would see a different observable universe



So how big is this observable universe of ours?

If the age ~~is~~ of our Universe is  $\approx 13.6$  Gyr, one might say that the radius is

$$r = ct.$$

This turns out to give about the right number, but it's the ~~the~~ wrong calculation.

Why? It doesn't take into account the fact that the universe is expanding.

We'll get into some of the mathematics of the expansion in a second, but for now we'll first think about two conceptual questions:

① Can something that's infinite expand?

② What, if anything, is the universe "expanding into"? } Discuss!

The answer to the first question is "yes", and the answer to the second question is that it doesn't need to ~~expand~~ expand into anything.

⇒ This is because ~~water~~ it's space itself that's expanding, so we're kinda creating new space between space.

How do we know this? Because of the form of Hubble's Law.

What's this? Hubble took measurements of how far away some galaxies were, and plotted their velocity relative to us.

⇒ Show Hubble plot // Notice anything wrong with the plot?  
Units!

There are two very important observations here:

- ① Except for our closest neighbour, all the galaxies seem to be running away from us!
- ② The ~~velocity~~ <sup>velocities</sup> with which the galaxies are running away (their "recessional velocities") ~~are~~ are proportional to their distance from us:

$$v \propto d$$

We can understand both of these facts if we realize that it's space itself that's stretching.

⇒ Rubber band demo

For ① we can either ~~assume~~ assume that we are smelly or that space itself is stretching. There's space between us and other galaxies, so that as space expands, things move away from us.

For ②, if there is double the space between me and a certain galaxy, since every bit of space is stretching, double the space  $\Rightarrow$  double the recessional velocity.

We can write

$$V = H_0 d$$

$H_0$  is called the Hubble constant / Hubble parameter.

What are its units?  $\frac{1}{t}$

A rough ballpark value is  $H_0 \approx 2.3 \times 10^{-18} \text{ s}^{-1}$

For astronomical purposes, it is more convenient to write

$$H_0 \approx \frac{70 \text{ km/s}}{\text{Mpc}}$$

Why is this handy? Remember that galaxies are typically  $\sim \text{Mpc}$  apart. So this is saying "for every extra Mpc away a galaxy is, it's moving away from me  $\rightarrow$  an extra 70 km/s faster".

Notice also that I've only been giving you rough answers. That's because  $H_0$  is hard to measure, and the community currently has two sort of inconsistent values:

Supernovae:  $H_0 = 73.24 \pm 1.74 \frac{\text{km/s}}{\text{Mpc}}$

CMB:  $H_0 = 67.8 \pm 0.9 \frac{\text{km/s}}{\text{Mpc}}$

Nobody knows what's going on! Maybe something is wrong with one of these observations (or both). Or maybe this is a hint of new physics that we don't yet understand.

Because historically  $H_0$  has been hard to determine, we sometimes write it as

$$H_0 = h \frac{100 \text{ km/s}}{\text{Mpc}}$$

where  $h \approx 0.7$ . The idea is that we can leave everything in terms of  $h$ , and if you want more concrete numbers, you can plug in the numbers that you find the most appropriate.

Now here's something interesting. Let's suppose that since the beginning of time (whatever that means) all the galaxies have been coasting away from each other at a constant speed.

Then  $d = vt \Rightarrow$  If  $v = H_0 d$ , we have  $v = H_0 t$

$$\Rightarrow t_H = \frac{1}{H_0}$$

This is the Hubble time, and in this silly model of our Universe it is the age of our Universe.

If you plug in the numbers, you get  $t_H \sim 14$  Gyr. Which is almost right!

This is mostly just an accident. In the universe, galaxies are not coasting away from each other at a constant rate (and actually, that was a bit of an inconsistent assumption in the "derivation" above).

In R life; the Hubble parameter is a function of time.  
We write

$$H_0 \equiv H(t = \text{today})$$

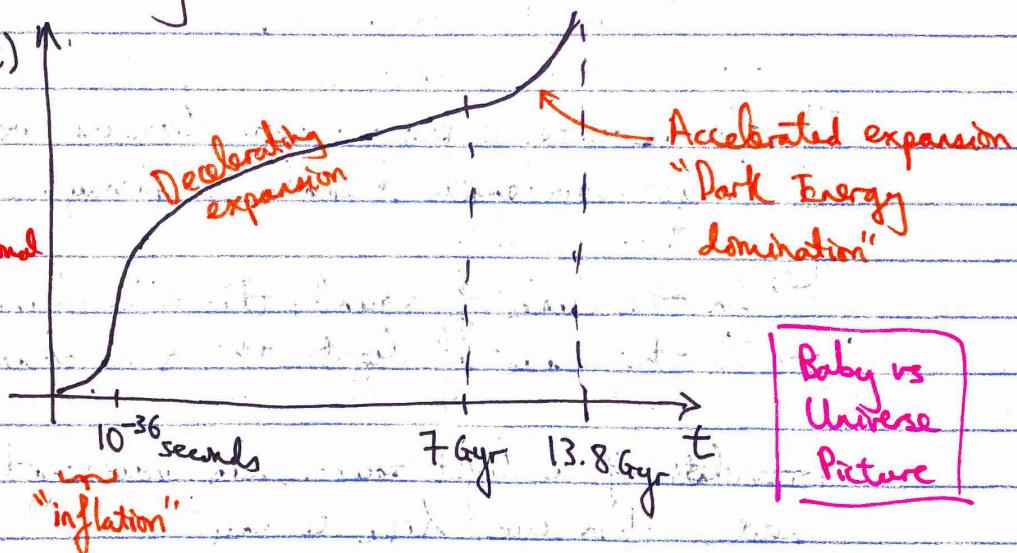
In cosmology, "0" subscript means today.

The age of our Universe is  $t_0 = 13.8 \text{ Gyr}$ . This is very close to  $1/H_0 = t_H$  because if we look at the expansion of our Universe, it looks something like this:

Do not confuse  
with acceleration!

"Scale factor"

Imagine a  
quantity proportional  
to the average  
distance between  
galaxies



These accelerations and decelerations cancel each other out fairly well in the calculation of the age of our Universe, and because of this,  $t_H \approx t_0$ .

What we've seen so far is that the Hubble Law lets us understand how our Universe expands and also allows us to calculate the age of our Universe.

But it's also super helpful as a tool.

**Discussion :** Which components of an astronomical object's position / Question and velocity are easy / hard to measure?

Turns out it's reasonably easy to get velocity along the line of sight if we take advantage of the Doppler Effect

The Doppler effect occurs when we receive waves coming from a moving source

### Bicycle movie clip

If the source is moving away from us, we observe it with a longer wavelength / smaller frequency.

- In the case of sound, this is a change in pitch.
- In the case of light, this is a change in colour.

~~For what~~ If a light source has wavelength  $\lambda_{\text{rest}}$  when it is at rest, then we like to say

$$\frac{\lambda_{\text{obs}}}{\lambda_{\text{rest}}} = 1 + z \quad \text{where } z \text{ is called the redshift.}$$

For low velocities of the source  $z \approx \frac{v}{c}$   $v > 0$  if moving away.

This is very important because it means that if I know what wavelength an astronomical source is emitting at, and I compare it to  $\lambda_{\text{obs}}$  using this formula, I can figure out the velocity of the source.

→ But only the component along the line of sight!

Note also that because  $c = \lambda v$ , we can also write

$$\frac{v_{\text{rest}}}{v_{\text{obs}}} = 1+z$$

In general, in astronomy the Doppler shift can be used to help us figure out the velocities of objects. Which is great. But in cosmology this is an even more powerful result.

↳ If I look @ far away objects, their velocity is dominated by the recession velocity due to the expansion of our Universe. Which means we can use the Doppler effect to find  $v$ , and then use Hubble's Law to find out how far away something is!

Three

~~Two~~ caveats:

- ① In general it's not precise enough to neglect the fact that the Hubble parameter changes with time. Here's the general formula (which I won't prove, since it requires Einstein's Theory of General Relativity)

$$D_c = \frac{c}{H_0} \int_0^z \frac{dy}{\sqrt{\Omega_m(1+y)^3 + \Omega_\Lambda}}$$

where  $\Omega_m \approx 0.3$ , and  $\Omega_\Lambda \approx 0.7$

("30% of our Universe's energy is matter; 70% of it is in "vacuum energy / dark energy").

- ② In general relativity there turns out to be no unique way to define distance. I've given you the formula for the "comoving line of sight distance". This is the measure

of distance that accounts for the fact that the universe expands as the light is travelling towards you, so by the time you receive it, it's even farther away than expected.

- ③ This whole scheme relies on us knowing the original emission wavelength of the light! How do we know this? We need to know something about the system we're studying. In our case, in 21cm cosmology, we use the fact that neutral hydrogen atoms emit radio waves with rest wavelengths of 21cm.

⇒ **21cm line slides**

Now, the redshift of an object not only gives us its distance from us, but it also tells us something about time.

⇒ **Night Sky is a Time Machine slides**

Because looking farther away is the same as looking back in time, we can also use redshift to measure time.

~~Another~~ One way to think about the Doppler shift is in terms of velocities, as we've discussed. But we can also think of it as the universe's expansion stretching out waves.

So if ~~the~~  $a(t_{\text{em}})$  is the size of the universe when a wave was emitted,

$$a(t_{\text{em}}) \propto \lambda_{\text{rest}}$$

$$\text{and } a(t_0) \propto \lambda_{\text{obs}}$$

$$\Rightarrow 1+z = \frac{a(t_0)}{a(t_{\text{em}})} = \frac{1}{a}$$

by convention, set  $a(t_0) = 1$

So at  $z=2$ , the object is so far away that our Universe was  $\frac{1}{3}$  of its current size when the light from this object started travelling towards us.

| $z$ | time before now     | $D_c$              | $-a$           |
|-----|---------------------|--------------------|----------------|
| 0   | 0 yr                | 0 Mpc              | 1              |
| 1   | <del>7.86</del> Gyr | <del>3.4</del> Gpc | $\frac{1}{2}$  |
| 3   | <del>8.90</del> Gyr | <del>6.5</del> Gpc | $\frac{1}{4}$  |
| 6   | <del>9.22</del> Gyr | <del>8.4</del> Gpc | $\frac{1}{7}$  |
| 15  | <del>13.4</del> Gyr | <del>10</del> Gpc  | $\frac{1}{16}$ |
| 27  | <del>13.6</del> Gyr | <del>11</del> Gpc  | $\frac{1}{28}$ |

HERE:  
 $50 \pm 230$  H $_{\text{0}}$   
 $\Rightarrow z \approx 27 \pm 53$   
 You might help determine these bounds!

So redshift measures time, but in a fairly non-linear way.

$$t_{\text{lookback}} = \frac{1}{H_0} \int_0^z \frac{dy}{(1+ty) \sqrt{\Omega_m(1+ty)^3 + \Omega_\Lambda}}$$

$\Rightarrow$  CMB, structure formation, EoR slides