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Evaluation of $\langle \tilde{V}_2^* \tilde{V}_1 \rangle$ for Flat $P(k)$ at Small Time Separation

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We begin with the definition of the visibility measured by a baseline $\vec{b}(t)$ with primary beam $A(\Omega, \nu, t)$, given the sky intensity $I_\nu(\Omega, \nu, t)$:

$$V(\nu, t) = \int \mathrm{d}\Omega A(\Omega, \nu, t) I_\nu(\Omega, \nu, t) e^{i2\pi\nu\vec{b}\cdot\hat{n}/c}. \tag{1}$$

The delay-transformed visibility is formally obtained by taking the Fourier transform of the visibility along the frequency-axis:

$$\tilde{V}(\tau, t) = \int_{\mathbb{R}} \mathrm{d}\nu V(\nu, t) e^{i2\pi\nu\tau}. \tag{2}$$

In practice, the interferometer output will be filtered, so the delay transform will only be integrated over a finite region, and the electronics may introduce some frequency dependence; we take this into consideration by attaching a function $\phi(\nu)$ under the integral in Eq. 2. As a first approximation, we assume the beam and sky are both frequency-independent. This allows us to rewrite the delay-transformed visibility as:

$$\tilde{V}(\tau, t) \approx \int \mathrm{d}\Omega A(\Omega, t) I(\Omega, t) \int \mathrm{d}\nu \phi(\nu) e^{i2\pi\nu(\tau + \vec{b}\cdot\hat{n}/c)}, \tag{3}$$

where the first integral is over the celestial sphere and the second integral is over the entire real line. Note that the second integral is just the Fourier transform of the filter, evaluated at the delay plus the geometric delay of the baseline—for a broad bandpass filter, the delay transform picks out the visibility at a particular delay mode (or rather a particular strip of sky where $\vec{b}\cdot\hat{n}/c$ is constant). We denote the Fourier transform of the filter as $\tilde{\phi}(\tau)$. We now consider the correlation of two visibilities V_1, V_2 measured by a single baseline, but slightly offset in time. We use subscripts $\{1, 2\}$ to denote at which time the quantity is to be evaluated. A straightforward application of Eq. 3 yields

$$\langle \tilde{V}_2^* \tilde{V}_1 \rangle \approx \left\langle \int \mathrm{d}\Omega \mathrm{d}\Omega' A_2^*(\Omega') I_2^*(\Omega') \tilde{\phi}^* \left(\tau + \frac{\vec{b}_2 \cdot \hat{n}'}{c} \right) A_1(\Omega) I_1(\Omega) \tilde{\phi} \left(\tau + \frac{\vec{b}_1 \cdot \hat{n}}{c} \right) \right\rangle. \tag{4}$$

We now assume that the sky signal is static, white noise. Assuming all other quantities are deterministic, the ensemble average only affects the intensities. Since different locations of the sky are uncorrelated by assumption, we have $\langle I^*(\Omega') I(\Omega) \rangle = I^*(\Omega') I(\Omega) \delta^D(\Omega - \Omega')$, where $\delta^D(x)$ is the Dirac-delta function. Substituting this into Eq. 4 and using the property of the Dirac-delta, we get the following:

$$\langle \tilde{V}_2^* \tilde{V}_1 \rangle \approx \int \mathrm{d}\Omega A_2^*(\Omega) A_1(\Omega) |I(\Omega)|^2 \tilde{\phi}^* \left(\tau + \frac{\vec{b}_2 \cdot \hat{n}}{c} \right) \tilde{\phi} \left(\tau + \frac{\vec{b}_1 \cdot \hat{n}}{c} \right). \tag{5}$$

If we assume the difference in observation times δt is small, then we may expand the Fourier-transformed filter to first-order as

$$\tilde{\phi}(\tau + \vec{b}_2 \cdot \hat{n}/c) \approx \tilde{\phi}(\tau + \vec{b}_1 \cdot \hat{n}/c) + \frac{\partial \tilde{\phi}}{\partial \tau} \delta\tau, \tag{6}$$

where the second term is evaluated at $\tau + \vec{b}_1 \cdot \hat{n}/c$. Now, the change in delay is just due to the change in the baseline, so we have

$$\delta\tau = \frac{1}{c} \frac{\partial \vec{b}_1}{\partial t} \cdot \hat{n} \delta t. \tag{7}$$

Assuming that the beam does not change significantly over δt, then we may rewrite Eq. 5, using Eq. 6 and Eq. 7, as

$$\langle \tilde{V}_2^* \tilde{V}_1 \rangle (\tau, t) \approx \int \mathrm{d}\Omega |A(\Omega)|^2 |I(\Omega)|^2 \left[|\tilde{\phi}|^2 + \tilde{\phi} \frac{\partial \tilde{\phi}^*}{\partial \tau} \frac{\delta t}{c} \frac{\partial \vec{b}}{\partial t} \cdot \hat{n} \right]_{\tau + \vec{b}\cdot\hat{n}/c}. \tag{8}$$

Noting that $\partial_t \vec{b} \approx \vec{\omega}_\oplus \times \vec{b}$, where $\vec{\omega}_\oplus$ is the Earth's angular velocity, we come to our final result:

$$\langle \tilde{V}_2^* \tilde{V}_1 \rangle(\tau, t) \approx \int \mathrm{d}\Omega |A(\Omega)|^2 |I(\Omega)|^2 \left[|\tilde{\phi}|^2 + \tilde{\phi} \frac{\partial \tilde{\phi}^*}{\partial \tau} \frac{\delta t}{c} (\vec{\omega}_\oplus \times \vec{b}) \cdot \hat{n} \right]_{\tau + \vec{b} \cdot \hat{n}/c} . \tag{9}$$

Note that Eq. 9 clearly shows that $\langle \tilde{V}_2^* \tilde{V}_1 \rangle$ is purely real if either δt or $\vec{\omega}_\oplus \times \vec{b}$ vanishes.