CS 340 Home Work # 6 Solution

CLO 2

Total marks: 40

Q 1. (10 Marks) Define the following machine-learning terms in your own words

(a) Training set
(b) Hypothesis
(c) Bias
(d) Variance

a. A set of input–output pair examples, used as input to a machine learning program to create a hypothesis.

b. In machine learning, a hypothesis is a function, learned from the training data and a member of the hypothesis space, that maps inputs to outputs.

c. The amount by which the output of a hypothesis consistently varies from the true answer in a particular direction, regardless of the exact training data.

d. The amount by which the output of a hypothesis randomly varies from the true answer, when trained on slightly different data sets.

Q 2. (10 Marks)

Consider the following data set comprised of three binary input attributes (A_1, A_2, and A_3) and one binary output:

Example	A_1	A_2	A_3	Output y
$\mathbf{x}_1$	1	0	0	0
$\mathbf{x}_2$	1	0	1	0
$\mathbf{x}_3$	0	1	0	0
$\mathbf{x}_4$	1	1	1	1
$\mathbf{x}_5$	1	1	0	1

Use the algorithm in Figure 19.5 (page 660) to learn a decision tree for these data. Show the computations made to determine the attribute to split at each node.

Note that to compute each split, we need to compute $Remainder(A_i)$ for each attribute A_i, and select the attribute the provides the minimal remaining information, since the existing information prior to the split is the same for all attributes we may choose to split on.

Computations for first split: remainders for $A1$, $A2$, and $A3$ are

$$(4/5)(-2/4\log(2/4) - 2/4\log(2/4)) + (1/5)(-0 - 1/1\log(1/1)) = 0.800$$
$$(3/5)(-2/3\log(2/3) - 1/3\log(1/3)) + (2/5)(-0 - 2/2\log(2/2)) \approx 0.551$$
$$(2/5)(-1/2\log(1/2) - 1/2\log(1/2)) + (3/5)(-1/3\log(1/3) - 2/3\log(2/3)) \approx 0.951$$

Choose A_2 for first split since it minimizes the remaining information needed to classify all examples. Note that all examples with $A_2 = 0$, are correctly classified as $B = 0$. So we only need to consider the three remaining examples (x_3, x_4, x_5) for which $A_2 = 1$.

After splitting on A_2, we compute the remaining information for the other two attributes on the three remaining examples (x_3, x_4, x_5) that have $A_2 = 1$. The remainders for $A1$ and $A3$ are

$$(2/3)(-2/2\log(2/2) - 0) + (1/3)(-0 - 1/1\log(1/1)) = 0$$
$$(1/3)(-1/1\log(1/1) - 0) + (2/3)(-1/2\log(1/2) - 1/2\log(1/2)) \approx 0.667.$$

So, we select attribute A1 to split on, which correctly classifies all remaining examples.

Q3. (10 Marks) Suppose you train a classifier and test it on a held-out validation set. It gets 30% classification accuracy on the training set and 30% classification accuracy on the validation set.

(a) From what problem is your model most likely suffering: underfitting or overfitting?

(b) What could reasonably be expected to improve your classifier's performance on the validation set: adding new features or removing some features? Justify your answer.

(c) What could reasonably be expected to improve your classifier's performance on the validation set: collecting more training data or throwing out some training data? Justify your answer.

a. Underfitting.

b. Adding new features. Under the current feature representation, we are unable to accurately model the training data for the purpose of the classification task we're interested in. The classifier may be able to deduce more information about the connections between data points and their classes from additional features, allowing it to better model the data for the classification task. For example, a linear perceptron could not accurately model two classes separated by a circle in a 2-dimensional feature space, but by using quadratic features in a kernel perceptron, we can find a perfect separating hyperplane.

c. Collecting more training data. More training data can't hurt. However, given that training and hold-out validation data sets already achieve the same performance, it may be that the underlying problem is not a lack of training data.

Q4. (10 Marks)

Construct a support vector machine that computes the XOR function. Use values of +1 and −1 (instead of 1 and 0) for both inputs and outputs, so that an example looks like $([-1, 1], 1)$ or $([-1, -1], -1)$. Map the input $[x_1, x_2]$ into a space consisting of x_1 and $x_1\,x_2$. Draw the four input points in this space, and the maximal margin separator. What is the margin? Now draw the separating line back in the original Euclidean input space.

The examples map from $[x_1, x_2]$ to $[x_1, x_1\,x_2]$ coordinates as follows:
$[-1, -1]$ (negative) maps to $[-1, +1]$
$[-1, +1]$ (positive) maps to $[-1, -1]$
$[+1, -1]$ (positive) maps to $[+1, -1]$
$[+1, +1]$ (negative) maps to $[+1, +1]$
Thus, the positive examples have $x_1x_2 = -1$ and the negative examples have $x_1x_2 = +1$. The maximum margin separator is the line $x_1x_2 = 0$, with a margin of 1. The separator corresponds to the $x_1 = 0$ *and* $x_2 = 0$ axes in the original space—this can be thought of as the limit of a hyperbolic separator with two branches.