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Model Insight

The Barra Japan Equity Model (JPE4)

Empirical Notes

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1. Introduction

1.1. Model Highlights

This document provides empirical results and analysis for the new Barra Japan Equity Model (JPE4). These notes include extensive information on the structure, the performance, and the explanatory power of the factors. Furthermore, these notes also include backtesting results and a thorough side-by-side comparison of the forecasting accuracy of the JPE4 Model and the JPE3 Model, its predecessor.

The JPE4 Model leverages the same methodologies used for the Barra US Equity Model (USE4). These details may be found in the companion document by Menchero, Orr, and Wang (2011).

Briefly, the main advances are:

- Enhanced Style factors reflecting the latest research on Systematic Equity Strategies
- Full daily updates of the model including daily exposures
- New daily-horizon model to cater to investors with short investment investment-horizons
- An innovative Optimization Bias Adjustment designed to improve the factor risk forecasts of optimized portfolios by reducing the effects of sampling error on the factor covariance matrix
- A Volatility Regime Adjustment designed to calibrate factor volatilities and specific risk forecasts to current market levels in daily-horizon model
- The introduction of a country factor to separate the pure industry effect from the overall market, and provide timelier correlation forecasts
- A new specific risk model based on daily asset-level specific returns
- A Bayesian adjustment technique to reduce specific risk biases due to sampling error
- A uniform responsiveness for factor and specific components, providing greater stability in sources of portfolio risk

The JPE4 Model is offered in short-term (JPE4S), long-term (JPE4L) and daily (JPE4D) versions. The three versions have identical factor exposures and factor returns, but differ in their factor covariance matrices and specific risk forecasts. The JPE4S Model is designed to be more responsive and provide more accurate forecasts at a monthly prediction horizon. The JPE4L model is designed for longer-term investors willing to trade some degree of accuracy for greater stability in risk forecasts. The JPE4D model provides investors of all horizons with a tactical, one-day risk forecast.

2. Methodology Highlights

2.1. Systematic Equity Strategies

The Japan Barra Equity Model implements the concept of Systematic Equity Strategies (SES), introduced in Bayraktar (2013a). We understand them as rule-based investment approaches, which are underpinned by empirical evidence in literature. Whereas traditionally Barra models only included two Systematic Equity Strategies, namely Value and Momentum, the JPE4 model expands this coverage to Management, Quality and Sentiment strategies that also are commonly applied by practitioners.

SES factors allow investors to measure their exposure to popular but potentially crowded strategies. Furthermore, asset managers are able to attribute realized returns to SES factors and as a consequence obtain a more meaningful insight into the risk and return drivers of their proprietary strategy.

The empirical analysis confirmed our intuition that the inclusion of these factors into a risk model can lead to more accurate risk forecasts and enhanced performance, particularly for portfolios which are based on a systematic approach. Apart from this benefit, SES factors can also potentially enhance portfolio construction, even if the followed approach, such as index tracking, is not directly related to the new set of SES factors.

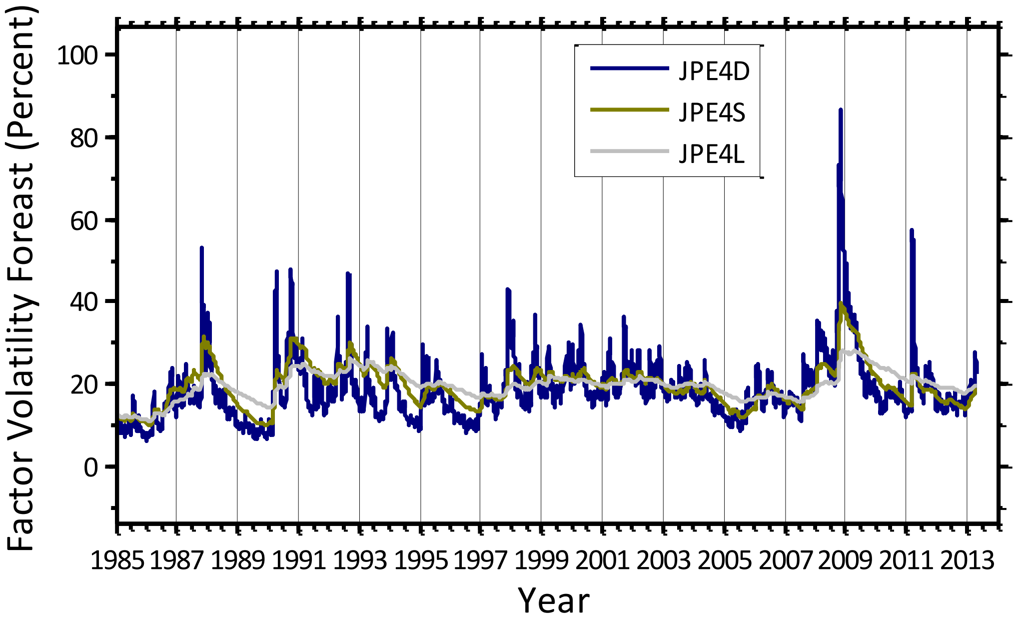
For a detailed test case using empirical results, please refer to Bayraktar (2012b).

2.2. Daily Horizon

The new Barra Japan Equity Daily Model daily version is designed to be more responsive and provide accurate forecasts at a daily prediction horizon. This is achieved by the selection of shorter half-lives in the factor covariance matrix and specific risk estimators, by the removal of Newey-West serial correlation correction, and by an adjusted setting for the Bayesian shrinkage intensity in the specific risk model. The factor structure and the rest of model parameters are the same as in JPE4S.

The JPE4D model has 42-day volatility and four-day Volatility Regime Adjustment half-lives (for more about the VRA, see Appendix E). The combination of these half-lives makes the model more responsive to the volatility changes in recent history. The following graph shows a JPE4D country factor volatility forecast using daily and monthly models.

Figure 2.1: Country Factor Volatility Forecast: JPE4D, JPE4S, and JPE4L.



From **Figure 2.1**, one can clearly see that the JPE4D volatility forecast is significantly higher than the forecast given by the JPE4S model immediately after each of the significant market shocks. As volatility subsides in the time periods following the market events, the JPE4D forecast also goes down more quickly than JPE4S. Some may perceive this behavior as the forecast being too "jumpy." However, as is shown by the empirical results below, the VRA-adjusted volatility forecast of the JPE4D model is in fact more accurate than JPE4S on a daily horizon.

To ensure consistent responsiveness in specific risk of JPE4D, we choose the volatility and VRA half-lives that match those used for the factor covariance forecast. The specific risk Bayesian shrinkage intensity parameter is also different for the daily model and is calibrated to be smaller than the parameter used in the monthly model.

For detailed account of parameter selection methodology, please refer to Menchero (2012).

2.3. Optimization Bias Adjustment

One significant bias of risk models is the tendency to underpredict the risk of optimized portfolios, as demonstrated empirically by Muller (1993). More recently, Bender et al (2009) derived an analytic result for the magnitude of the bias, showing that the underforecasting becomes increasingly severe as the number of factors grows relative to the number of time periods used to estimate the factor covariance matrix. The basic source of this bias is estimation error. Namely, spurious correlations may cause certain stocks to appear as good hedges in-sample, while these hedges fail to perform as effectively out-of-sample.

An important innovation is the identification of portfolios that capture these biases and to devise a procedure for correcting these biases directly within the factor covariance matrix. As shown by Menchero, Wang, and Orr (2011), the *eigenfactors* of the sample covariance matrix are systematically biased. More specifically, the sample covariance matrix tends to underpredict the risk of low-volatility eigenfactors, while overpredicting the risk of high-volatility eigenfactors. Eigenfactors represent portfolios of the original pure factors; they, however, are special in the sense that they are mutually uncorrelated. Also note that the number of eigenfactors equals the number of pure factors within the model.

In Optimization Bias Adjustment, we estimate the biases of the eigenfactors via Monte Carlo simulation, then adjust the predicted eigenvalues of the eigenfactors to correct for these biases (see Appendix D). This procedure helps improve factor risk forecasts of optimized portfolios. Lee et al (2011) demonstrate the effectiveness of Optimization Bias Adjustment by backtesting active portfolios. Furthermore, it has the benefit of building the corrections directly into the factor covariance matrix, while fully preserving the meaning and intuition of the pure factors.

2.4. Volatility Regime Adjustment

Another major source of risk model bias is due to the fact that volatilities are not stable over time, a characteristic known as *non-stationarity*. Since risk models must look backward to make predictions about the future, they exhibit a tendency to underpredict risk in times of rising volatility, and to overpredict risk in times of falling volatility.

Another important innovation in the JPE4 Model is the introduction of a Volatility Regime Adjustment for estimating factor volatilities (see Appendix E). As described in Menchero, Wang, and Orr (2011), the Volatility Regime Adjustment relies on the notion of a cross-sectional bias statistic, which may be interpreted as an *instantaneous* measure of risk model bias for that particular day. By taking a weighted average of this quantity over a suitable interval, the non-stationarity bias can be significantly reduced.

Just as factor volatilities are not stable across time, the same holds for specific risk. In the JPE4 Model, we estimate the adjustment by computing the cross-sectional bias statistic for the specific returns.

We find that in Japan equity market Volatility-Regime Adjustment is effective on daily-horizon frequency and apply it in the factor variance and specific risk of JPE4D.

2.5. Country Factor

Traditionally, single country models (e.g., JPE3) have included industry and style factors, but no Country factor. An important improvement with the JPE4 Model is to explicitly include the Country factor, which is analogous to the World factor in the Barra Global Equity Model (first introduced in GEM2), as described by Menchero, Morozov, and Shepard (2008, 2010).

One significant benefit of the Country factor is the insight and intuition that it affords to investors. For instance, as discussed in Menchero, Orr, and Wang (2011), the Country factor portfolio can be cleanly interpreted as the cap-weighted country portfolio. Furthermore, the Country factor disentangles the pure industry effect from the overall market effect, thus providing a cleaner interpretation of the industry factors.

Without the Country factor, industry factors represent portfolios that are 100 percent net long the particular industry, with zero net weight in every other industry. With the Country factor, by contrast, industry factors represent *dollar-neutral* portfolios that are 100 percent long the industry and 100 percent short the Country factor; that is, industry performance is measured net of the market.

Dollar-neutral industry factor portfolios are important from an attribution perspective. For instance, suppose that a portfolio manager is overweight an industry that *underperforms* the market, but which nonetheless has a *positive* return. Clearly, overweighting an underperforming industry *detracts* from performance. If the industry factors are represented by net-long portfolios, however, an attribution analysis would spuriously show that overweighting the underperforming industry contributed *positively* to performance. This non-intuitive result is resolved by introducing the Country factor, which makes the industry factor portfolios dollar-neutral and thereby produces the intuitive result that overweighting an underperforming industry detracts from performance. Including the Country factor also resolves other problematic issues in risk attribution, as described by Davis and Menchero (2011).

Another benefit of the Country factor pertains to improvements in risk forecasting. Intuitively and empirically, we know that industries tend to become more highly correlated in times of financial crisis. As shown in Menchero, Orr, and Wang (2011), the Country factor is able to capture these changes in industry correlation in a timelier fashion. The underlying mechanism for this effect is that net-long industry portfolios have common exposure to the Country factor, and when the volatility of the Country factor rises during times of market stress, it explains the increased correlations for the industries.

2.6. Specific Risk Model with Bayesian Shrinkage

The JPE4 specific risk model builds upon methodological advances introduced with the European Equity Model (EUE3), as described by Briner, Smith, and Ward (2009). The EUE3 model utilizes daily observations to provide timely estimates of specific risk directly from the time series of specific returns. A significant benefit of this approach is that specific risk is estimated individually for every stock, thus reflecting the idiosyncratic nature of this risk source.

A potential shortcoming of the pure time-series approach is that specific volatilities may not fully persist out-of-sample. In fact, as shown in Menchero, Orr, and Wang (2011), there is a tendency for time-series volatility forecasts to overpredict the specific risk of high-volatility stocks, and underpredict the risk of low-volatility stocks.

To reduce these biases, we apply a Bayesian shrinkage technique. We segment stocks into deciles based on their market capitalization. Within each size bucket, we compute the mean and standard deviation of the specific risk forecasts. We then pull or "shrink" the volatility forecast to the mean within the size decile, with the shrinkage intensity increasing with the number of standard deviations away from the mean.

Appendix E provides further technical details.

3. Factor Structure Overview

3.1. Estimation Universe

The JPE4 model utilizes a rules-based approach in determining the estimation universe, which is the set of securities used to estimate the model. The rules are designed to maintain sufficient market capitalization coverage by selecting the most liquid stocks present in the market. The coverage rule adds the stocks to the estimation universe when they become part of the top 95 percent of the market capitalization, and drops them when they become too small. Similarly, the liquidity criteria screens for the stocks that had sufficient trading activity in the recent history, and excludes them when liquidity drops. Special provisions are made to add new significant IPOs, when the liquidity criteria cannot be satisfied due to limited history. To avoid overly-restrictive filtering during the early history (prior to May 29,1992) we leverage the JPE3 estimation universe as a base.

3.2. Industry Factors

Industries are important variables for explaining the sources of equity return co-movement. JPE3 employs an industry structure that includes 41 industries, each containing one or several sub-industries. There are 97 sub-industries in total. The mapping from sub-industries to industries can be found in the JPE3 Research Notes.

JPE3 assigns multiple industry exposures to stocks as follows. Sales of a firm are provided by our data provider along sub-industries. We first aggregate them to the level of industries. We then regress market capitalizations of the estimation universe stocks against their Sales from different industries to determine the slope coefficients, or “industry betas,” which represent the price-to-sales ratios of the industries. The industry exposures are then given by the fraction of market capitalization explained by each industry. Detailed procedure can be found in the JPE3 Research Notes.

JPE4 keeps this industry structure, except for allowing two new industries, namely, Internet and Real Estate Investment Trusts (REITs) to separate from parent industries when they have reached sufficient depth of coverage. Internet separates from Media and enters the model on December 30, 1999. REITs separates from Real Estate and enters the model on June 30, 2004. Similar to JPE3, JPE4 also assigns multiple industry exposures to stocks using the methodology described above.

For a small number of stocks (about 2 percent of the coverage universe on April 30, 2013), we do not have their sub-industry-level Sales information from our data provider. In this case, we rely on their GICS codes to assign a single industry to them.

In **Table 3.1** we report the average weights of the industries from the sample period of January 1, 1985 to April 30, 2013, and also the end-of-period weights. In **Table 3.2**, we report the mapping from GICS codes to JPE4 industry factors used to assign single industries to stocks missing sub-industry-level Sales information. In **Table 3.3** we report the largest firm within each industry at the end of the sample period. In **Table 3.4** we report average industry betas; also reported in Table 3.4 is the weight of stocks with multiple industry exposures at the end of the sample period. It is clear that multiple-industry exposures are well represented in JPE4.

Table 3.1: JPE4 Industry Factors. Weights were determined within the JPE4 estimation universe using total market capitalization. Averages were computed over the sample period from January 1, 1985 to April 30, 2013.

Sector	JPE4 Industry Factor Name	Average Weight	30-Apr-2013 Weight
Basic Materials	Chemicals	3.07	2.77
	Paper and Pulp	0.61	0.25
	Metals	2.71	1.59
	Glass	0.79	0.53
	Energy	1.12	1.07
Industrial Machinery	Industrial Parts	1.64	1.82
	Factory Equipment	1.58	1.61
	Precision Equipment	1.56	2.41
	Plant Machinery and Engineering	0.77	0.83
	Special Vehicles	1.49	2.03
Automobiles	Auto and Parts	7.86	12.75
Electronics	Electronic Parts	3.80	3.02
	Semiconductor and Equipment	1.22	0.91
	Computer and Telecommunication Equipment	1.94	0.86
	Office and Home Electronics	3.87	2.04
	Miscellaneous Electric Goods	1.61	1.74
Consumer Goods & Services	Drugs	3.99	5.42
	Food and Beverage	3.06	4.50
	Household and Cosmetics	0.92	1.32
	Fiber and Apparel	0.84	0.41
	Miscellaneous Consumer Products	1.23	1.33
	Games	0.92	1.04
	Consumer Services	1.32	1.87
Information Media	Broadcasting Media	2.42	0.61
	Telecommunication	3.64	5.01
	Internet	0.67	2.21
Retail	Department Stores, GMS and Super Markets	3.06	3.00
	Specialty Retailers	1.63	3.01
Business Services	Trading Houses	2.27	2.82
	Business Outsourcing Services	1.29	1.42
	Software System	1.34	1.27
Logistics	Passenger Transportation Services	2.49	2.40
	Goods and Materials Logistics	1.29	1.10
Shelters	Infrastructure Construction	1.37	0.71
	Housing and Building	2.38	1.55
	Real Estate	2.76	4.58

Financials	Banks	10.98	4.78
	Regional Banks	3.47	2.47
	Consumer and Business Loans	1.74	3.07
	Securities	2.73	2.19
	Insurance	1.86	1.95
	REITs	0.29	1.64
Utilities	Electricity and Gas Utilities	4.32	2.12

Table 3.2: Mapping of GICS codes to JPE4 industry factors. This is used to assign single industries to stocks missing sub-industry-level Sales information.

JPE4 Industry Factor Name	GICS Codes
Chemicals	151010
Paper and Pulp	15103020, 151050
Metals	151040
Energy	10
Factory Equipment	20106020
Precision Equipment	35101010, 351030
Special Vehicles	20106010
Auto and Parts	2510
Electronic Parts	452030
Semiconductor and Equipment	452050, 4530
Computer and Telecommunication Equipment	452010, 452020
Office and Home Electronics	25201010, 25201040, 452040
Miscellaneous Electric Goods	201010, 201040, 201050
Drugs	35101020, 3520
Food and Beverage	30101020, 3020
Household and Cosmetics	3030
Fiber and Apparel	252030
Miscellaneous Consumer Products	15103010, 25201020, 25201050, 252020
Consumer Services	25301010, 25301020, 25301030, 253020, 25401030
	35102015, 35102020, 35102030
Broadcasting Media	25401020, 25401025, 25401040
Telecommunication	50
Internet	451010
Department Stores, GMS and Super Markets	255010, 255030, 30101030, 30101040
Specialty Retailers	25301040, 255020, 255040, 30101010
Trading Houses	201070, 35102010
Business Outsourcing Services	2020, 25401010
Software System	451020, 451030
Passenger Transportation Services	203020, 20304010, 20305010
Goods and Materials Logistics	203010, 203030, 20304020, 20305020, 20305030
Infrastructure Construction	151020, 201030

Housing and Building	201020, 25201030
Real Estate	40401020, 404030
Banks	40101010
Regional Banks	40101015
Consumer and Business Loans	401020, 402010, 402020
Securities	402030
Insurance	4030
REITs	40401010, 404020
Electricity and Gas Utilities	55

Table 3.3: Largest stock within each industry at the end of the sample period (30-Apr-2013). Market capitalizations are reported in billions of US dollars.

JPE3 Industry Factor Name	Largest Stock (30-Apr-2013)	Market Cap ($bln)
Chemicals	SHIN-ETSU CHEMICAL	29.1
Paper and Pulp	OJI HOLDINGS CORP	3.8
Metals	NIPPON STEEL & SUMITOMO METAL CORP	25.3
Glass	ASAHI GLASS	9.3
Energy	INPEX	17.6
Industrial Parts	SMC	14.4
Factory Equipment	FANUC CORP	36.1
Precision Equipment	KEYENCE	19.3
Plant Machinery and Engineering	JGC	7.7
Special Vehicles	KOMATSU	26.8
Auto and Parts	TOYOTA MOTOR	199.6
Electronic Parts	KYOCERA	19.5
Semiconductor and Equipment	TOKYO ELECTRON	9.3
Computer and Telecommunication Equipment	NEC	6.8
Office and Home Electronics	CANON	47.9
Miscellaneous Electric Goods	HITACHI	30.1
Drugs	TAKEDA PHARMACEUTICAL	43.4
Food and Beverage	JAPAN TOBACCO	75.7
Household and Cosmetics	KAO	18.2
Fiber and Apparel	TORAY INDUSTRIES	11.5
Miscellaneous Consumer Products	NIKON	8.7
Games	NINTENDO	15.7
Consumer Services	ORIENTAL LAND	14.7
Broadcasting Media	JUPITER TELECOMMUNICATIONS	8.8
Telecommunication	NTT DOCOMO	72.1
Internet	YAHOO JAPAN	29.1
Department Stores, GMS and Super Markets	SEVEN & I HOLDINGS	34.0

Specialty Retailers	FAST RETAILING	38.9
Trading Houses	MITSUBISHI	29.7
Business Outsourcing Services	SECOM	13.0
Software System	NTT DATA	8.9
Passenger Transportation Services	EAST JAPAN RAILWAY	33.4
Goods and Materials Logistics	YAMATO HOLDINGS	9.0
Infrastructure Construction	KINDEN	1.9
Housing and Building	DAIWA HOUSE IND.	13.6
Real Estate	MITSUBISHI ESTATE	45.2
Banks	MITSUBISHI UFJ FINANCIAL GROUP	96.4
Regional Banks	THE BANK OF YOKOHAMA	8.2
Consumer and Business Loans	ORIX	16.9
Securities	NOMURA HOLDINGS	31.1
Insurance	TOKIO MARINE HOLDINGS	24.4
REITs	NIPPON BUILDING FUND	9.9
Electricity and Gas Utilities	TOKYO GAS	14.7

Table 3.4: Industry betas. The final column reports the weight of stocks with multiple industry exposures as of 30-Apr-2013.

JPE4 Industry Factor Name	Average Sales Beta	30-Apr-2013 Multi-Ind Weight
Chemicals	0.80	72.46
Paper and Pulp	0.80	11.90
Metals	0.76	81.67
Glass	1.25	79.51
Energy	0.28	41.31
Industrial Parts	0.70	70.95
Factory Equipment	1.03	63.13
Precision Equipment	1.25	60.87
Plant Machinery and Engineering	1.02	89.35
Special Vehicles	1.24	85.92
Auto and Parts	1.00	73.41
Electronic Parts	0.82	61.48
Semiconductor and Equipment	0.83	81.06
Computer and Telecommunication Equipment	0.25	82.74
Office and Home Electronics	0.74	94.84
Miscellaneous Electric Goods	0.72	89.72
Drugs	0.52	13.89
Food and Beverage	0.48	70.73
Household and Cosmetics	0.67	46.07

Fiber and Apparel	0.79	62.36
Miscellaneous Consumer Products	1.12	71.30
Games	2.24	15.74
Consumer Services	0.85	54.31
Broadcasting Media	1.13	50.07
Telecommunication	1.32	46.83
Internet	2.32	43.66
Department Stores, GMS and Super Markets	0.63	58.12
Specialty Retailers	0.56	19.98
Trading Houses	0.36	0.06
Business Outsourcing Services	0.51	46.98
Software System	0.63	32.23
Passenger Transportation Services	2.45	90.92
Goods and Materials Logistics	0.60	62.24
Infrastructure Construction	0.40	57.56
Housing and Building	0.37	82.91
Real Estate	2.53	79.10
Banks	1.20	87.43
Regional Banks	1.41	59.47
Consumer and Business Loans	0.63	60.85
Securities	15.18	34.65
Insurance	0.70	13.46
REITs	0.94	2.35
Electricity and Gas Utilities	2.37	47.24

3.3. Style Factors

Investment style represents another major source of systematic risk for equity portfolios. Style factors are constructed from financially intuitive stock attributes called *descriptors*, which serve as effective predictors of equity return covariance. Appendix A summarizes the descriptor definitions for each style factor. The descriptor weights are retained as proprietary.

In order to facilitate comparison across style factors– except NK225 – the factors are standardized to have a cap-weighted mean of 0 and a regression-weighted standard deviation of 1. The cap-weighted estimation universe, therefore, is *style neutral*. For more details, see Menchero, Orr, and Wang (2011).

A summary list of all style factors are as follows:

- The *Beta* factor captures market risk that cannot be explained by the Country factor. We compute Beta by time-series regression of stock returns against the cap-weighted estimation universe.
- The *Earnings Yield* factor describes return differences based on a company's earnings relative to its price. Earnings Yield is considered by many investors to be a strong value signal. The descriptors in this factor are the analyst-predicted 12-month forward earnings-to-price ratio and earnings-before-interests-taxes-to-enterprise-value.

- The *Financial Leverage* factor captures return differences between high-leverage and low-leverage stocks. The descriptors within this style factor include market leverage, book leverage, and debt-to-assets ratio.
- The *Foreign Sensitivity* factor measures the risk due to revenues and economic events outside Japan. This factor includes FX rate sensitivity and foreign-sales-to-total-sales ratios.
- The *Growth* factor differentiates stocks based on their prospects for sales or earnings growth. This factor contains forward-looking descriptors in the form of long/short-term analyst predicted earnings growth as well as historical descriptors for sales and earnings growth over the trailing five years.
- The *Industry Momentum* factor differentiates stocks based on both their performance over the trailing six months and the industry performance over the same time. This factor and the *Short-Term Reversal* factor, respectively, are orthogonalized with the rest of the JPE4 factors.
- The *Liquidity* factor describes return differences due to relative trading activity. The descriptors are based on the fraction of total shares outstanding that trade over a recent window.
- The *Long-Term Reversal* factor captures how stocks under/over-perform the market over the long history.
- The *Macro Sensitivity* factor reflects the extent to which macro economy affects asset prices. This factor includes interest rate sensitivity, oil sensitivity, gold sensitivity and US market sensitivity.
- The *Management* factor differentiates stocks based on their management strength. Poorly managed companies will often increase their shares outstanding as they raise capital to pursue questionable acquisitions or investments; better managed companies will often reduce shares outstanding as they return excess capital to shareholders through share repurchase programs.
- The *Membership in NK225 Indicator* factor identifies the common behavior of stocks in the NK255 index.
- The *Momentum* factor differentiates stocks based on their performance over the trailing 12 months. When computing Momentum exposures, we exclude the last month of returns (21 trading days) to avoid the effects of short-term reversal.
- The *Non-linear Size* factor describes the non-linearity in payoff to the Size factor across the market-cap spectrum. This factor roughly captures the risk of a “barbell portfolio” that is long mid-cap stocks and short small-cap and large-cap stocks.
- The *Quality* factor explains return differences between high quality and low quality stocks. Companies with a large accrual component to their earnings are more likely to disappoint investors in the future than are companies with strong cash earnings. The descriptors in this factor are accrual and cash-earnings-to-earnings.
- The *Residual Volatility* factor captures the volatility of the residual return in the times series regression of the *Beta* factor.
- The *Sentiment* factor measures how bullish/bearish (or excessively optimistic/pessimistic) the market participants are in reaction to the change of analyst forecasts.
- The *Short-Term Reversal* factor captures how stocks under/over-perform the market over the recent past as they are expected to over/under-perform in the near future. This factor and the *Industry Momentum* factor, respectively, are orthogonalized with the rest of the JPE4 factors.

- The *Size* factor represents the strongest source of equity return covariance, and captures return differences between large-cap stocks and small-cap stocks. We measure Size by the log of market capitalization.
- The *Value* factor captures the extent to which a company's ongoing business is priced inexpensively in the marketplace. The descriptors in this factor are book-to-price, yield, and sales-to-price.

3.4. Performance of Factors

It is helpful to consider the performance of individual factors. In the following figures, we report cumulative returns to the JPE4 factors. The Country factor return essentially represents the excess return (i.e., above the risk-free rate) of the cap-weighted country portfolio. Style factor returns represent the returns of pure factor portfolios that have exposure only to the style in question. In other words, they have net zero weight in every industry, and have zero exposure to every other style factor. Industry factor returns represent the performance of the pure industry relative to the overall market, net of all style effects. In other words, the pure industry factor portfolio is dollar neutral and has zero exposure to every style.

Figure 3.1: Cumulative returns of the USE4 Country factor and the USE4 estimation universe.

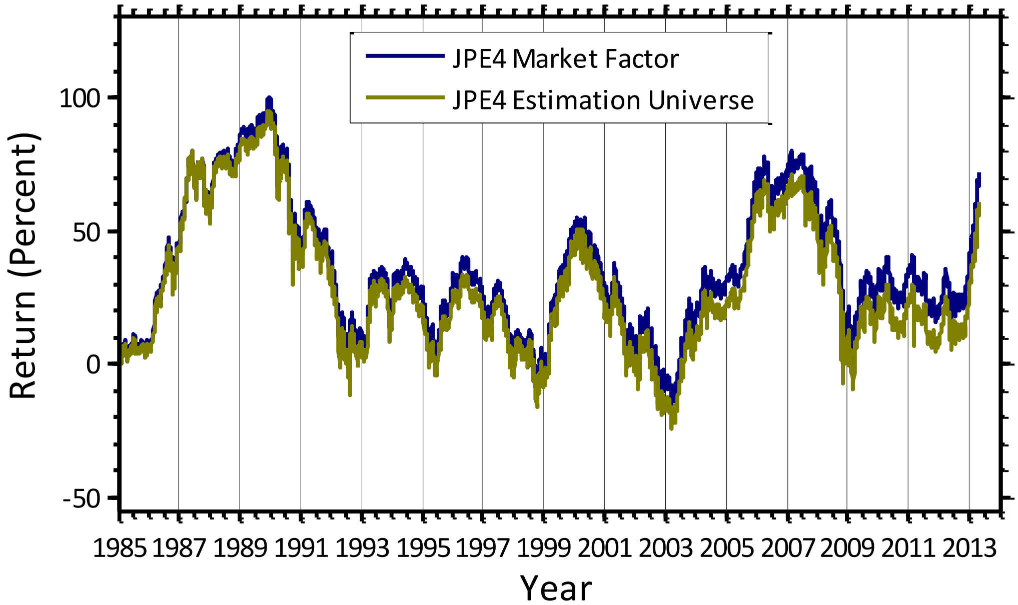


Figure 3.2: Cumulative returns of JPE4 Chemicals factor and Paper and Pulp factor.

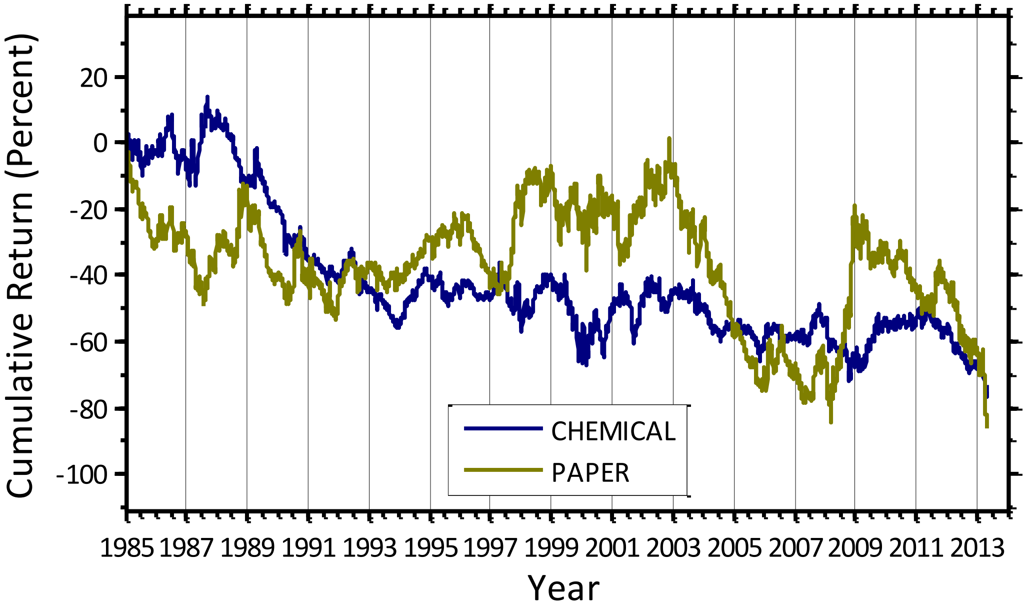


Figure 3.3: Cumulative returns of JPE4 Metals factor, Glass factor, and Energy factor.

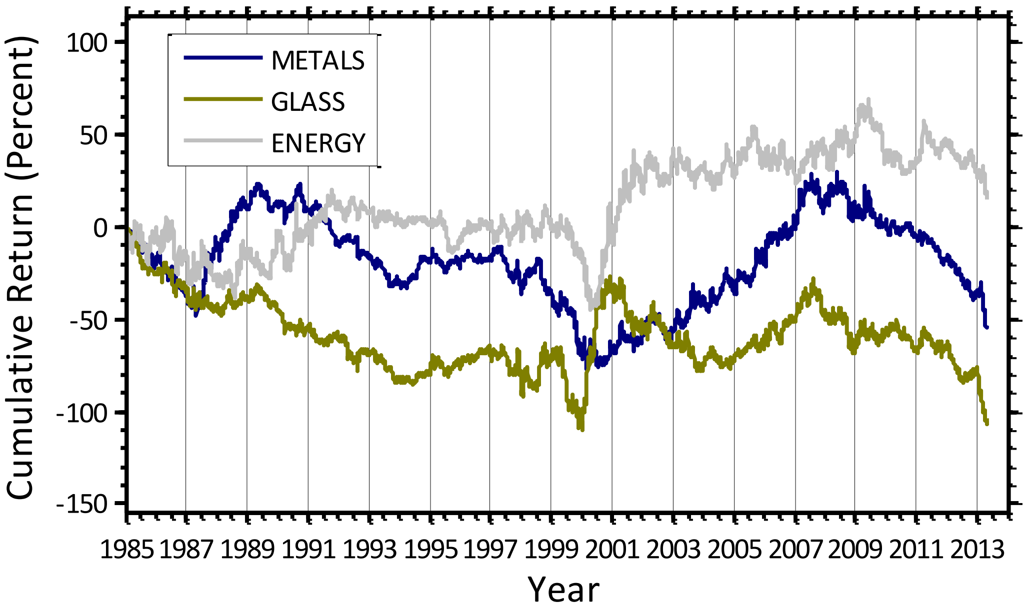


Figure 3.4: Cumulative returns of JPE4 Industrial Parts factor, Factory Equipment factor, and Precision Equipment factor.

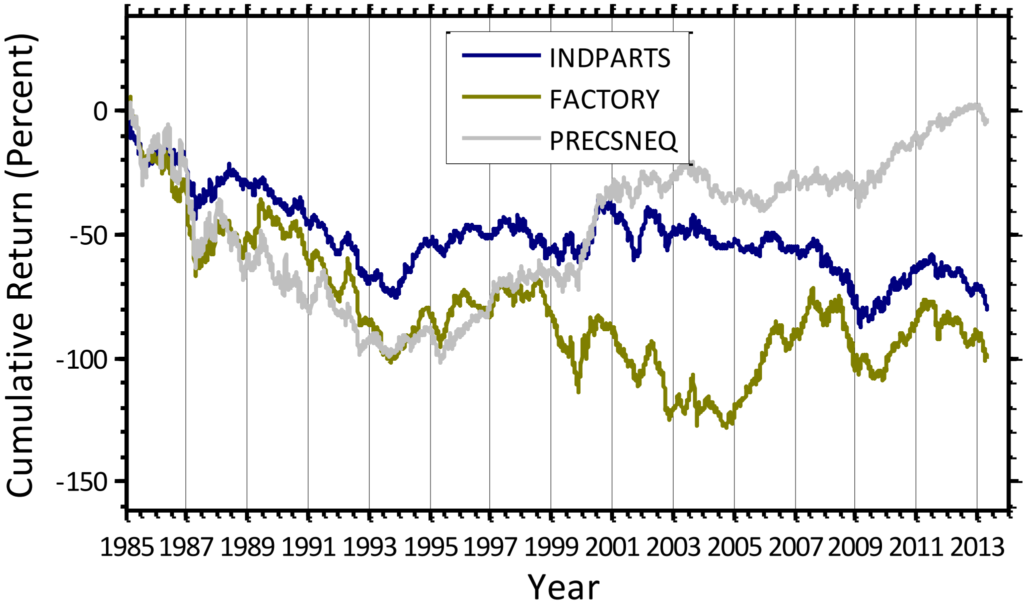


Figure 3.5: Cumulative returns of JPE4 Plant Machinery and Engineering factor, Special Vehicles factor, and Auto and Parts factor.

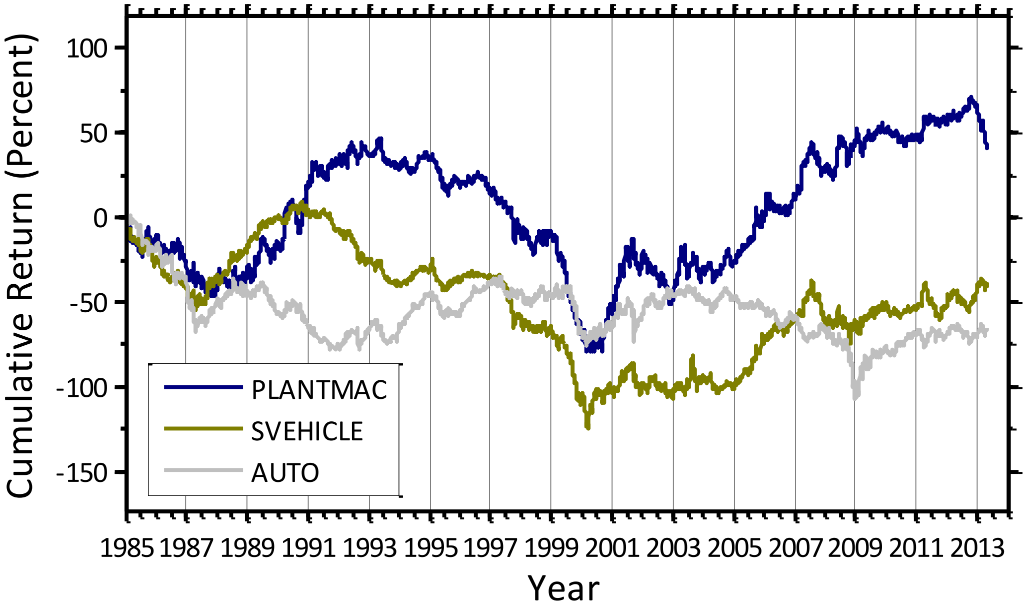


Figure 3.6: Cumulative returns of JPE4 Electronic Parts factor, Semiconductor and Equipment factor, and Computer and Telecommunication Equipment factor.

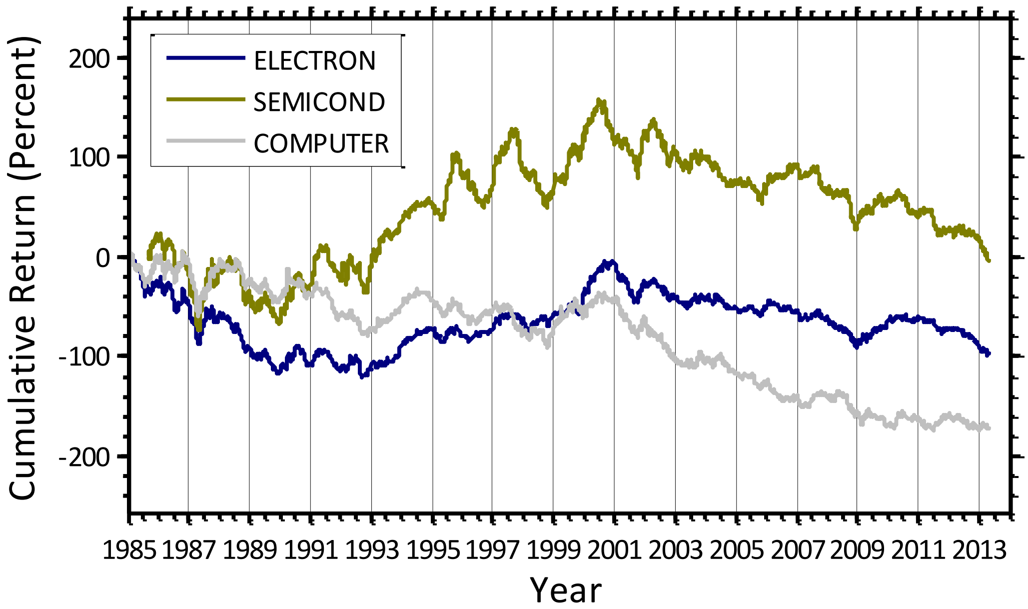


Figure 3.7: Cumulative returns of JPE4 Office and Home Electronics factor and Miscellaneous Electric Goods factor.

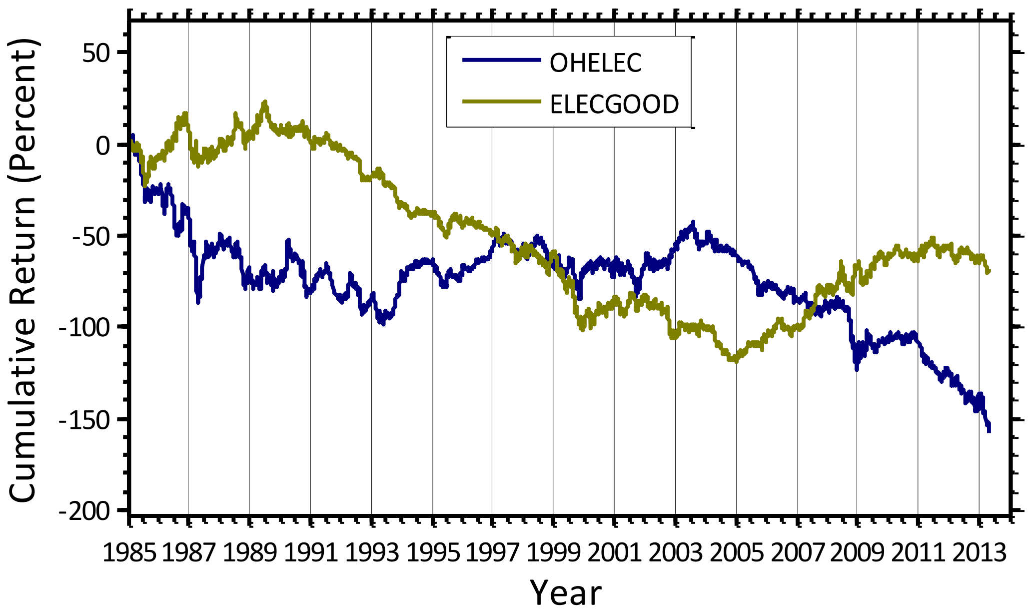


Figure 3.8: Cumulative returns of JPE4 Drugs factor, Food and Beverage factor, and Household and Cosmetics factor.

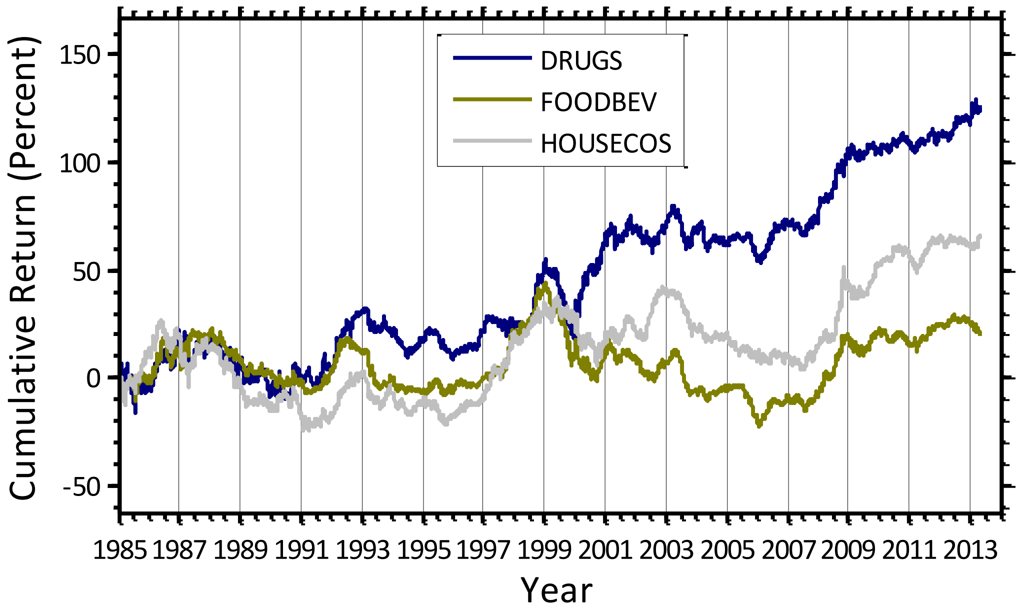


Figure 3.9: Cumulative returns of JPE4 Fiber and Apparel factor and Miscellaneous Consumer Products factor.

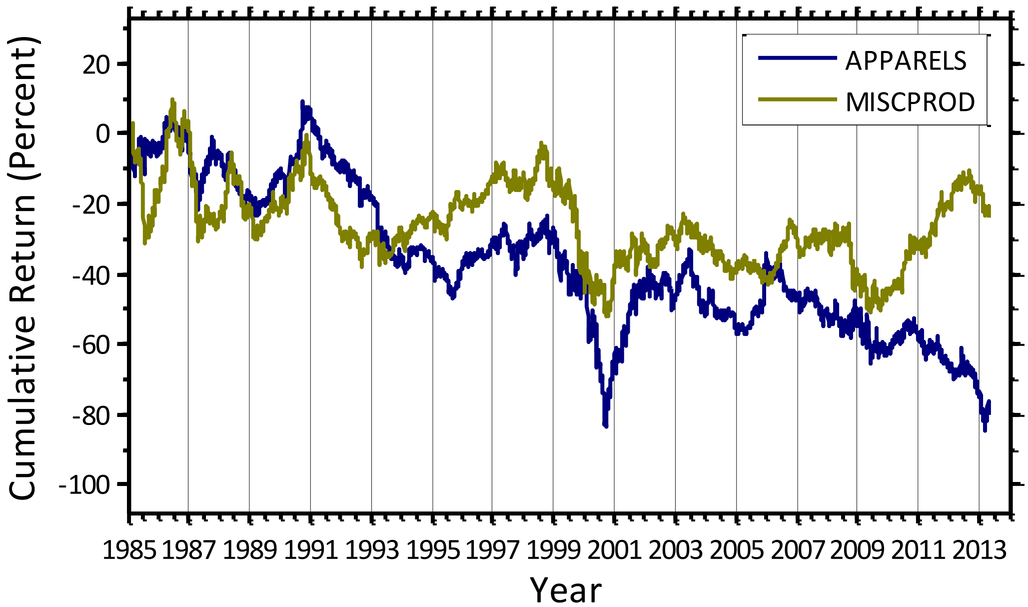


Figure 3.10: Cumulative returns of JPE4 Games factor and Consumer Services factor.

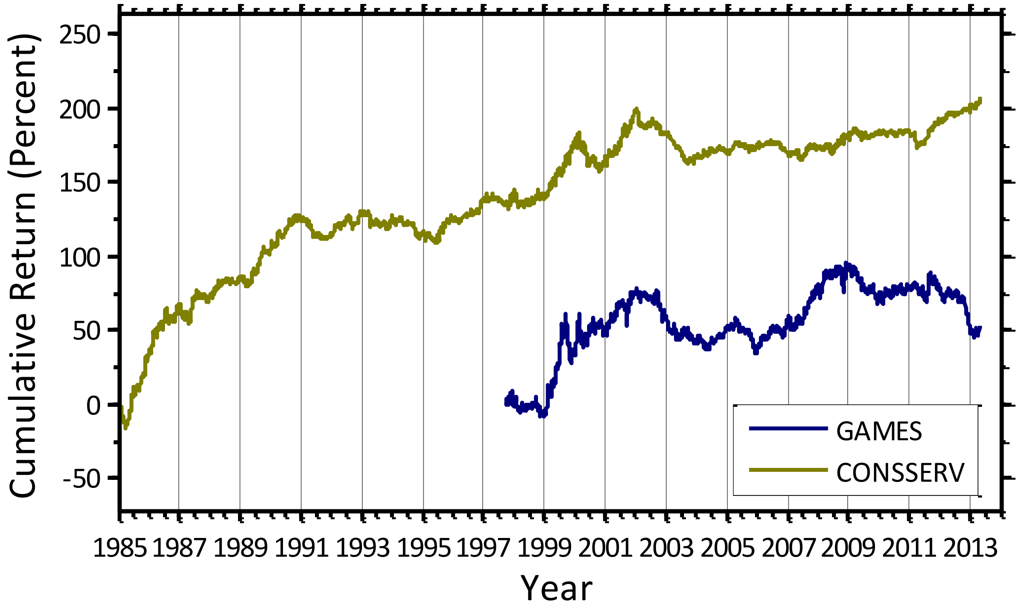


Figure 3.11: Cumulative returns of JPE4 Broadcasting Media factor, Telecommunication factor, and Internet factor.

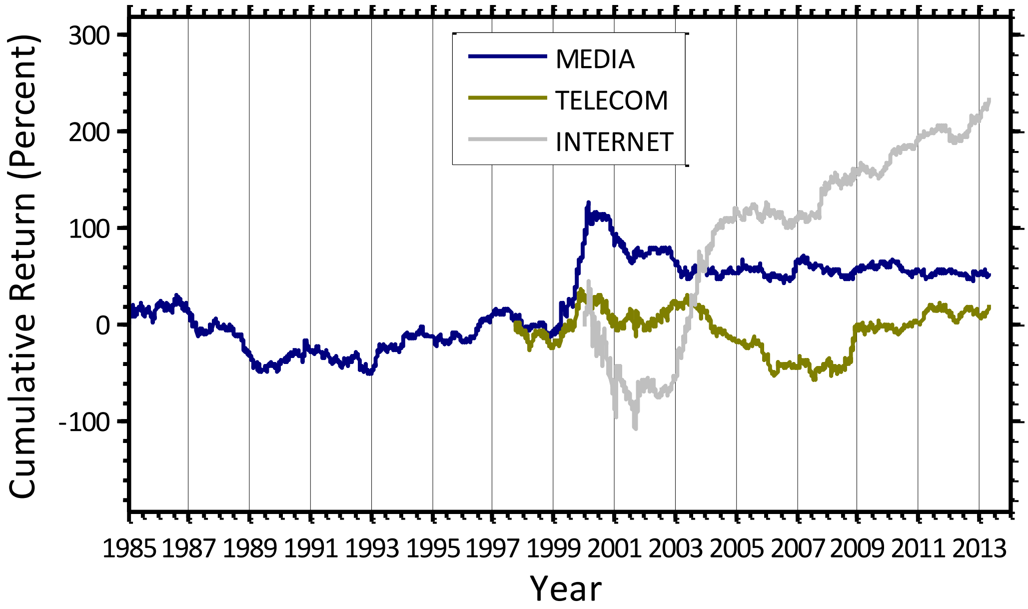


Figure 3.12: Cumulative returns of JPE4 Department Stores GMS and Super Markets factor and Specialty Retailers factor.

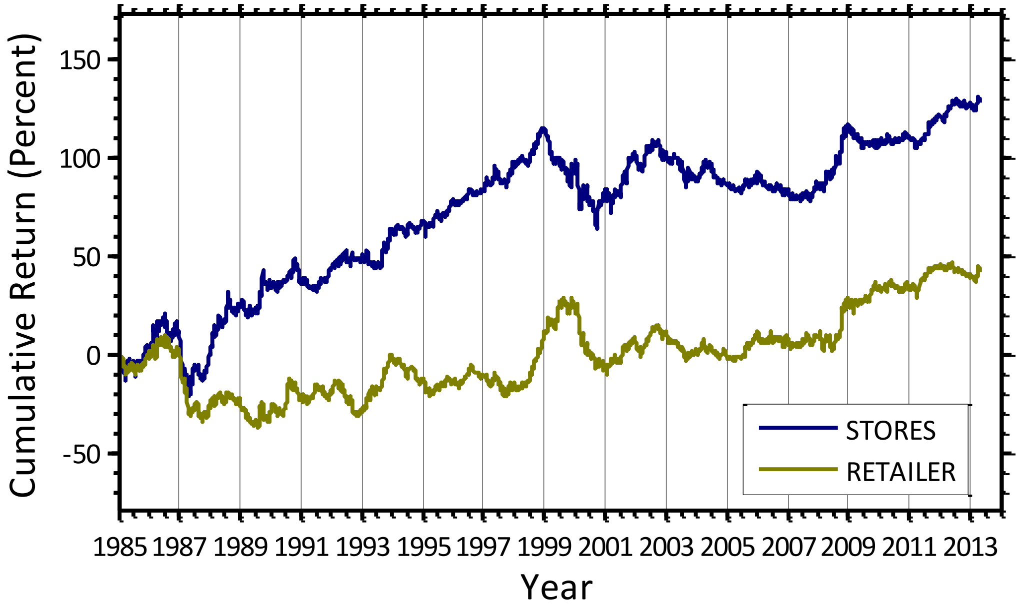


Figure 3.13: Cumulative returns of JPE4 Trading Houses factor, Business Outsourcing Services factor, and Software System factor.

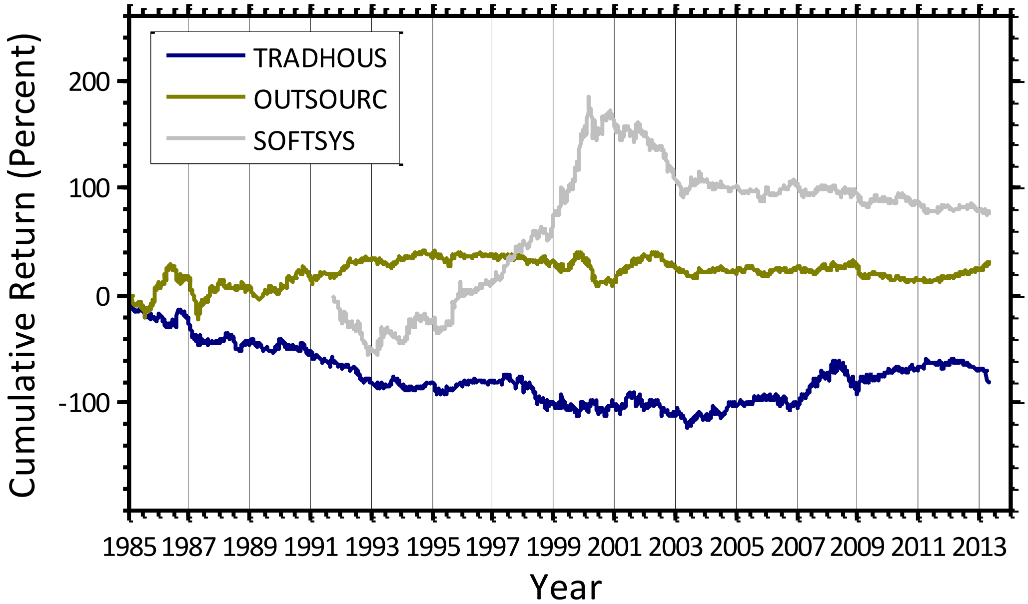


Figure 3.14: Cumulative returns of JPE4 Passenger Transportation Services factor and Goods and Materials Logistics factor.

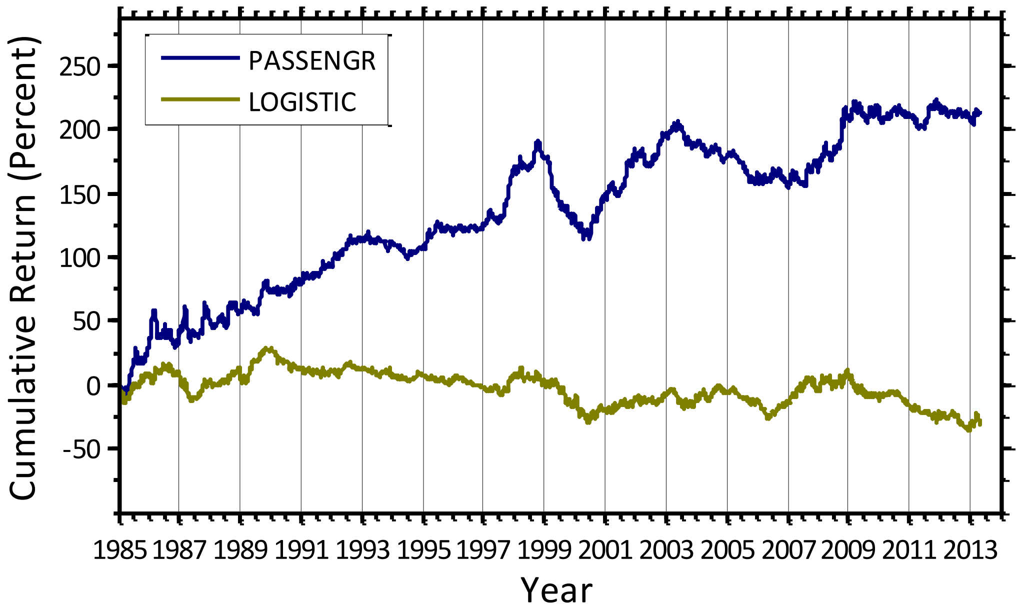


Figure 3.15: Cumulative returns of JPE4 Infrastructure Construction factor, Housing and Building factor, and Real Estate factor.

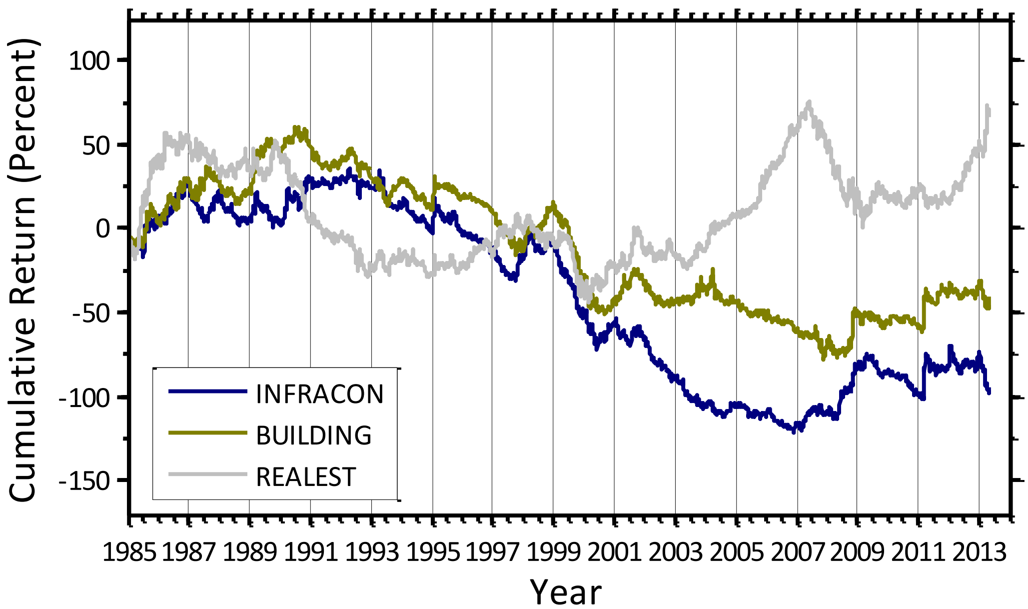


Figure 3.16: Cumulative returns of JPE4 Banks factor, Regional Banks factor, and Consumer and Business Loans factor.

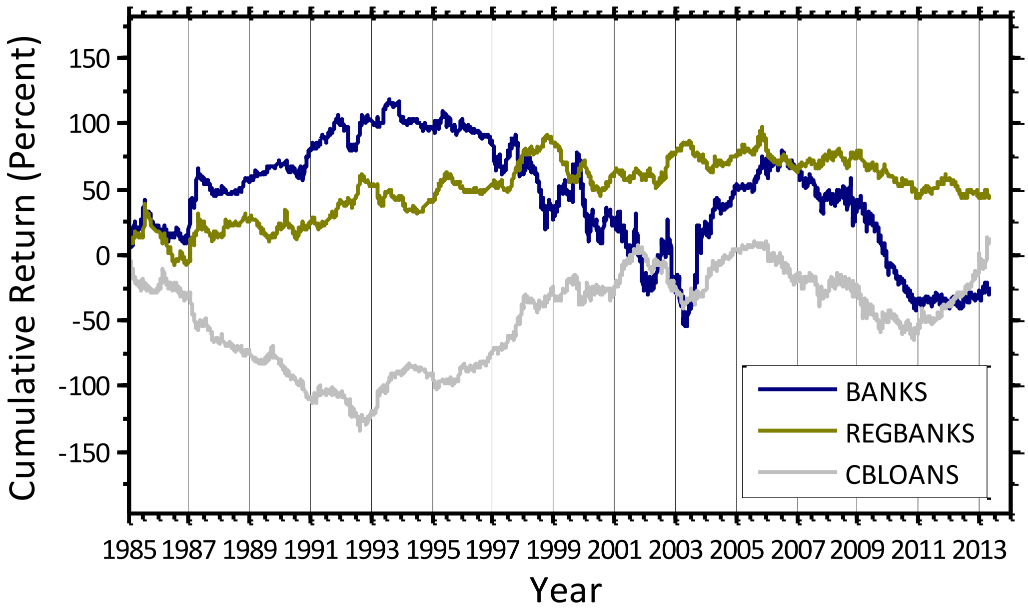


Figure 3.17: Cumulative returns of JPE4 Securities factor and Insurance factor.

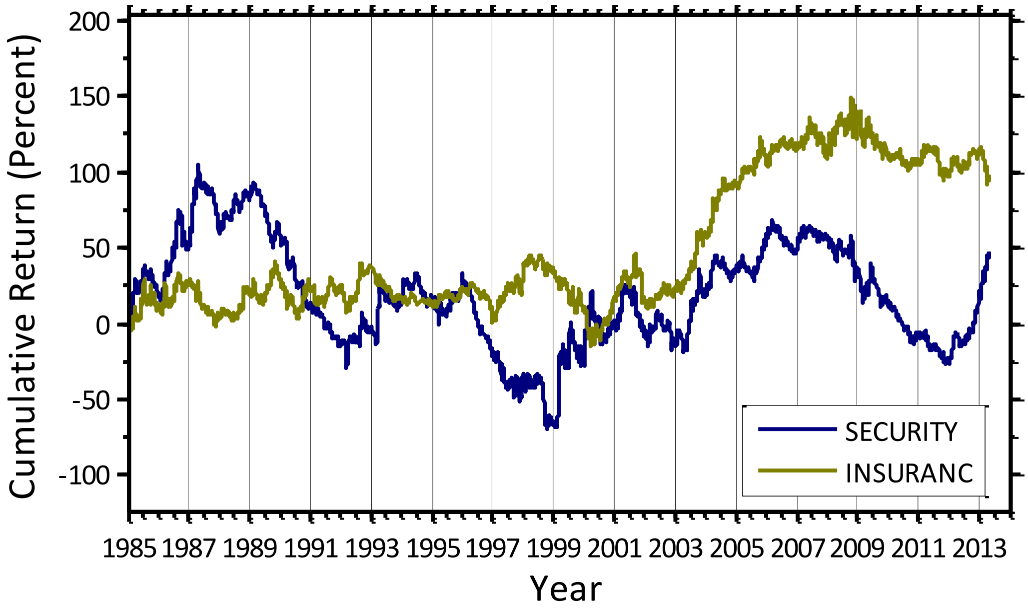


Figure 3.18: Cumulative returns of JPE4 REITs factor and Electricity and Gas Utilities factor.

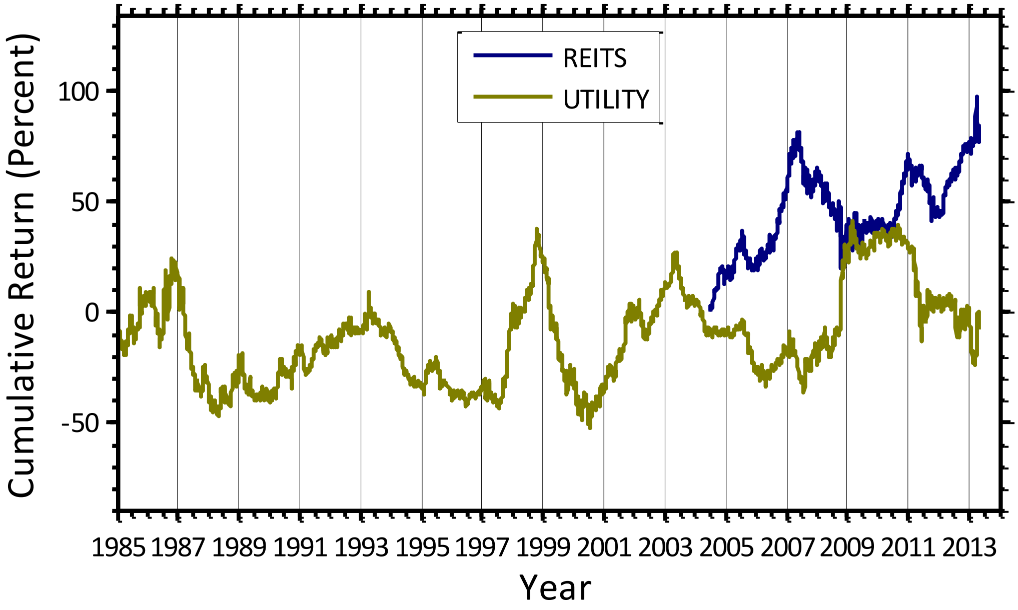


Figure 3.19: Cumulative returns of JPE4 Size factor, NonLinear Size factor, and Membership in NK225 Indicator factor.

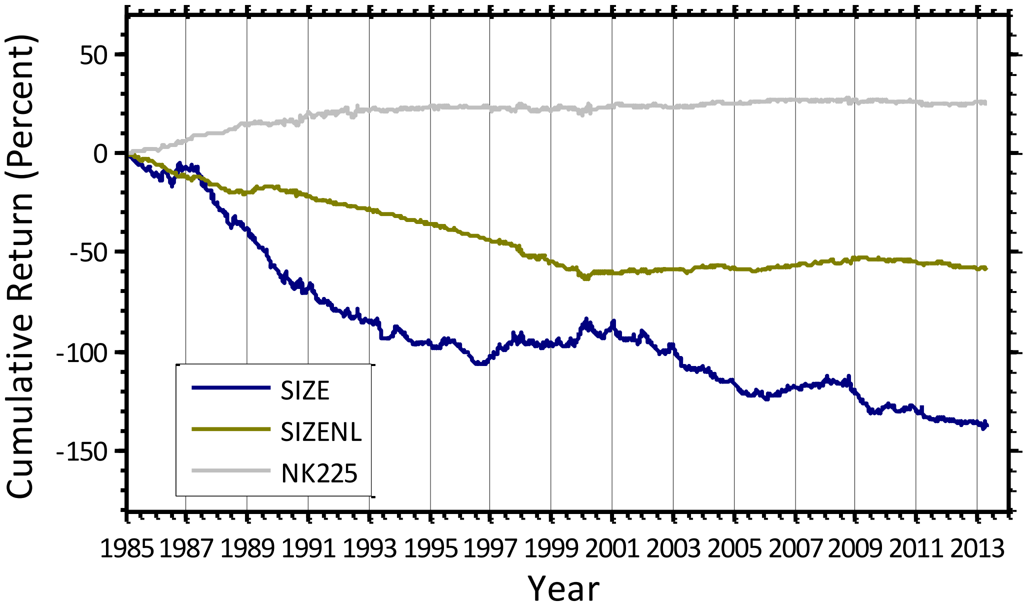


Figure 3.20: Cumulative returns of JPE4 Beta factor and Residual Volatility factor.

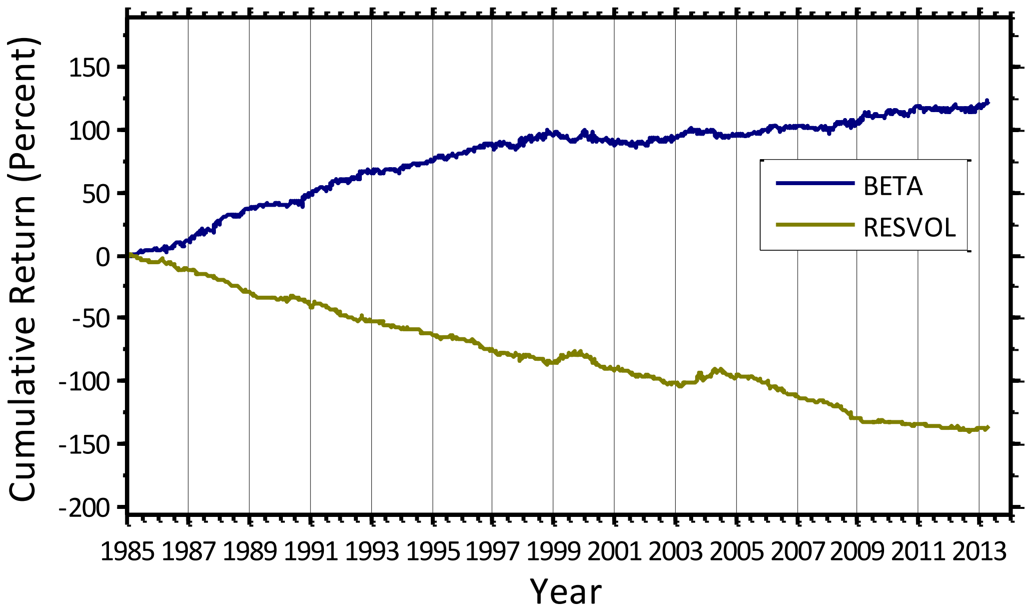


Figure 3.21: Cumulative returns of JPE4 Growth factor, Financial Leverage, and Liquidity factor.

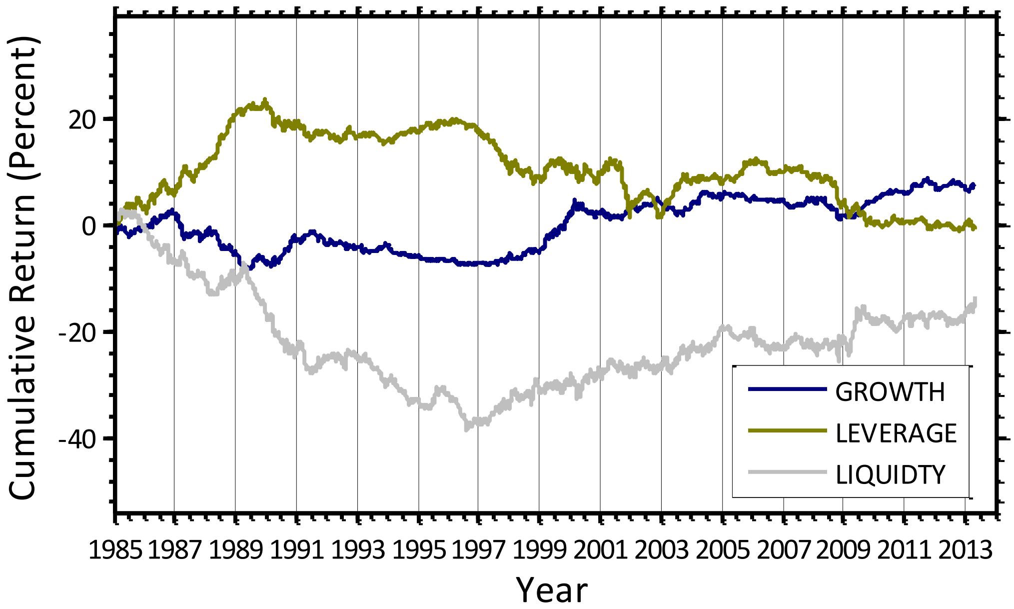


Figure 3.22: Cumulative returns of JPE4 Earnings Yield factor and Value factor.

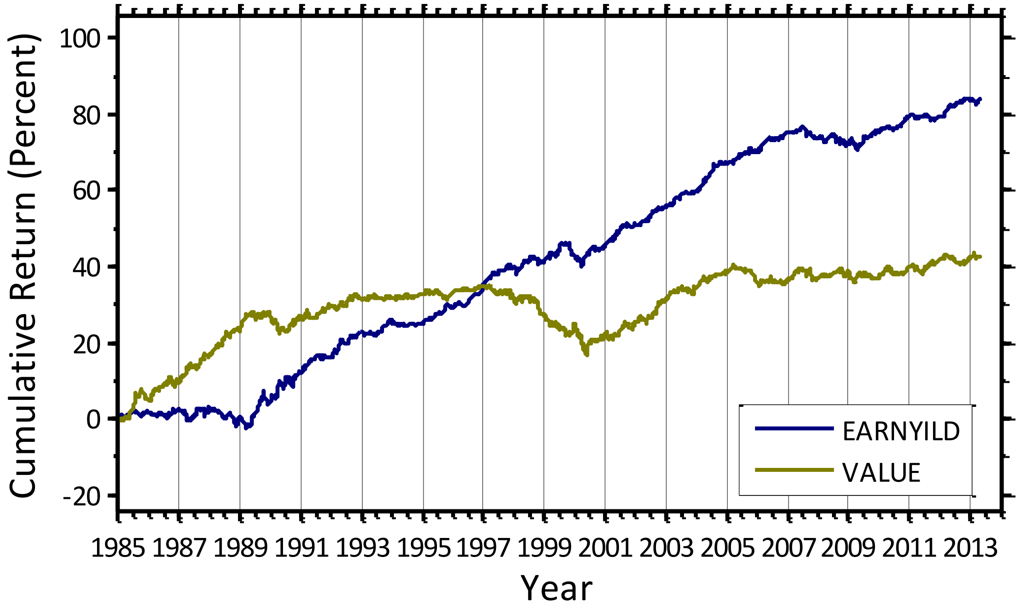


Figure 3.23: Cumulative returns of JPE4 Quality factor, Sentiment factor, and Industry Momentum factor.

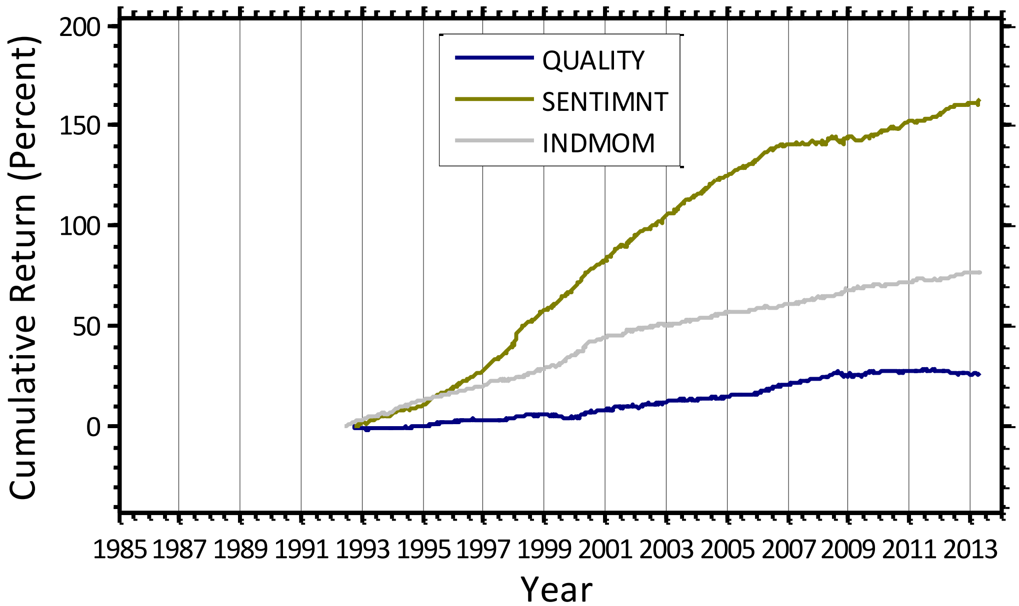


Figure 3.24: Cumulative returns of JPE4 Short-Term Reversal factor.

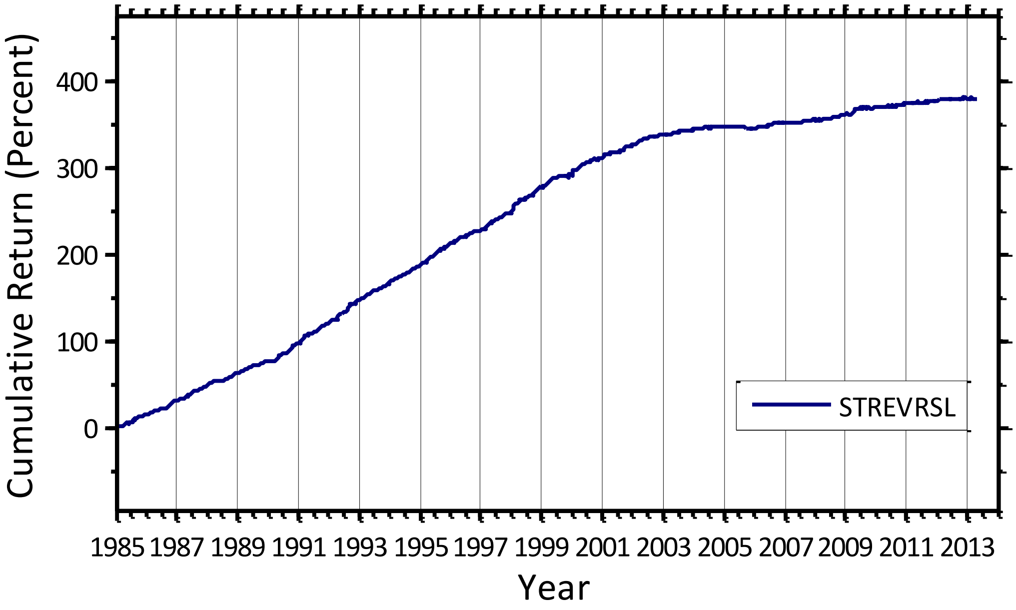


Figure 3.25: Cumulative returns of JPE4 Long-Term Reversal factor, Momentum factor, and Management factor.

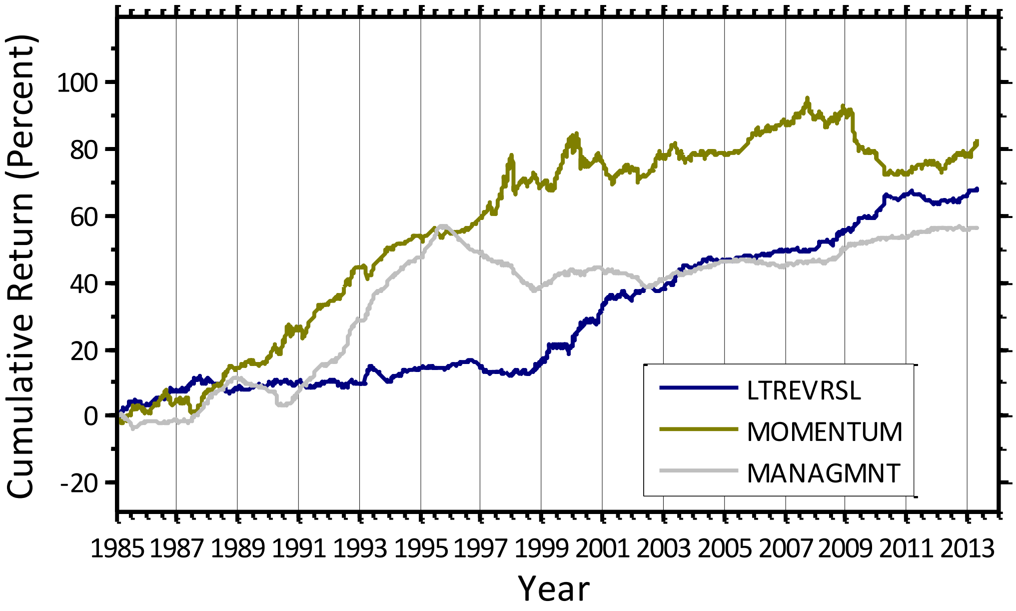
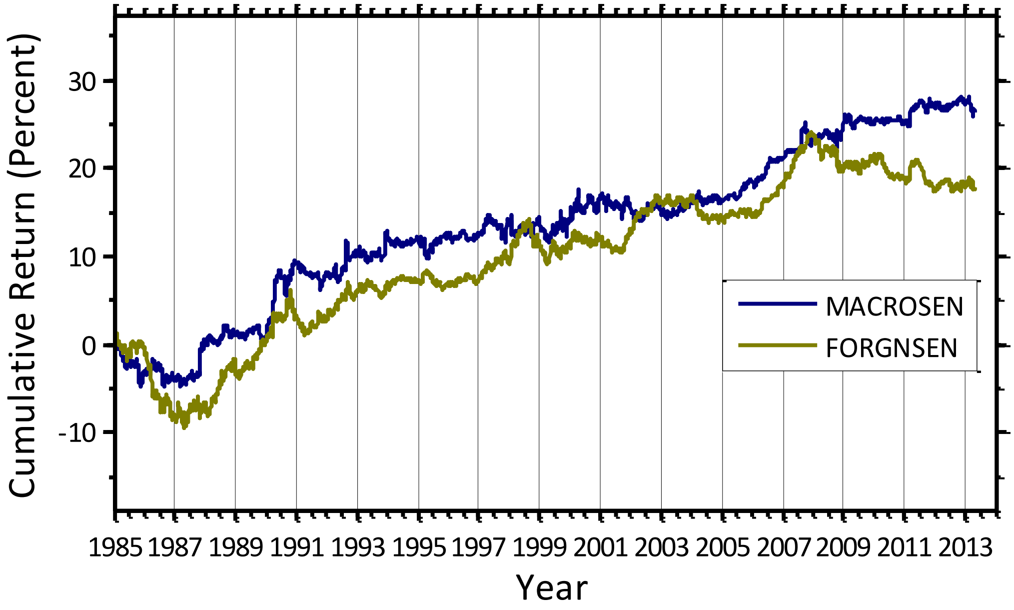


Figure 3.26: Cumulative returns of JPE4 Macro Sensitivity factor and Foreign Sensitivity factor.



4. Model Characteristics and Properties

4.1. Country and Industry Factors

One requirement of a high-quality factor structure is that the factor returns be statistically significant. This helps prevent weak or noisy factors from finding their way into the model. We measure statistical significance by the *t*-statistic of the factor return. Assuming normality, absolute *t*-statistics greater than 2 are considered significant at the 95-percent confidence level. In other words, if the factor had no explanatory power (i.e., was pure noise), then by chance we would observe $|t|>2$ about 5 percent of the time.

In **Table 4.1** we report mean absolute *t*-statistics for the JPE4 Country factor and industry factors, as well as the percentage of observations with $|t|>2$. We also report the returns, volatilities, Sharpe Ratio and the correlations of the daily factor returns with the estimation universe.

Table 4.1: Industry factor summary statistics computed using daily cross-sectional regressions. The sample period is from 01-Jan-1985 to 30-Apr-2013 (6971 days)

Factor Name	Average Absolute *t*-stat	Percent Observ. |*t*|>2	Annual. Factor Return	Annual. Factor Volatility	Factor Sharpe Ratio	Correl. with ESTU
Chemicals	1.42	25.63	-2.65	9.26	-0.29	0.03
Paper and Pulp	1.19	17.59	-3.00	14.75	-0.20	-0.06
Metals	1.75	32.94	-1.91	13.44	-0.14	-0.01
Glass	1.17	16.35	-3.81	14.80	-0.26	-0.01
Energy	1.64	31.09	0.61	18.60	0.03	-0.06
Industrial Parts	1.08	14.27	-2.87	9.08	-0.32	0.05
Factory Equipment	1.17	16.50	-3.57	9.75	-0.37	0.00
Precision Equipment	1.10	14.62	-0.16	11.16	-0.01	-0.03
Plant Machinery and Engineering	1.08	13.96	1.48	15.28	0.10	-0.02
Special Vehicles	1.20	17.72	-1.40	11.50	-0.12	-0.01
Auto and Parts	1.71	32.19	-2.39	10.00	-0.24	-0.02
Electronic Parts	1.51	28.00	-3.45	11.29	-0.31	0.01
Semiconductor and Equipment	1.59	29.24	-0.10	19.12	-0.01	0.00
Computer and Telecommunication Equipment	1.28	20.57	-6.26	14.73	-0.43	0.00
Office and Home Electronics	1.42	24.83	-5.70	12.45	-0.46	-0.03
Miscellaneous Electric Goods	1.07	14.04	-2.46	11.44	-0.22	0.01
Drugs	1.71	32.91	4.57	11.48	0.40	-0.10
Food and Beverage	1.39	25.41	0.70	8.26	0.09	-0.13
Household and Cosmetics	0.95	9.96	2.41	10.69	0.23	-0.13
Fiber and Apparel	1.03	11.42	-2.90	10.90	-0.27	-0.08
Miscellaneous Consumer Products	1.01	11.89	-0.86	10.33	-0.08	-0.06
Games	1.57	16.51	3.53	15.67	0.23	-0.04
Consumer Services	1.11	14.92	7.35	10.44	0.70	-0.12
Broadcasting Media	1.30	21.14	1.90	16.50	0.12	-0.03
Telecommunication	1.77	18.96	1.33	19.46	0.07	-0.02
Internet	2.30	21.22	17.80	30.51	0.58	0.04
Department Stores GMS and Super Markets	1.51	27.00	4.60	10.21	0.45	-0.08
Specialty Retailers	1.24	19.78	1.51	8.63	0.18	-0.15
Trading Houses	1.42	24.87	-2.87	12.25	-0.23	0.01
Business Outsourcing Services	1.08	14.35	1.11	10.11	0.11	-0.05
Software System	1.57	21.79	3.58	13.97	0.26	0.02
Passenger Transportation Services	1.38	24.03	7.72	14.66	0.53	-0.12
Goods and Materials Logistics	1.26	19.31	-1.11	10.65	-0.10	-0.02
Infrastructure Construction	1.21	17.39	-3.43	10.62	-0.32	-0.02
Housing and Building	1.36	21.79	-1.55	11.06	-0.14	-0.03
Real Estate	1.72	31.90	2.40	13.83	0.17	0.06
Banks	2.41	46.09	-0.97	21.81	-0.04	0.06

Regional Banks	1.72	33.08	1.62	10.42	0.16	0.00
Consumer and Business Loans	1.64	28.88	0.41	14.62	0.03	0.02
Securities	1.83	34.73	1.59	18.40	0.09	0.17
Insurance	1.73	32.62	3.55	17.86	0.20	0.08
REITs	2.36	13.94	8.93	19.62	0.45	-0.09
Electricity and Gas Utilities	1.80	33.25	-0.27	15.41	-0.02	-0.10
Average	1.46	22.76		13.60		

4.2. Style Factors

In **Table 4.2**, we report summary statistics for the JPE4 style factors, during the sample period. The sample is broken up into two equal sub-periods.

Also reported in Table 4.2 is the factor stability coefficient, described in Menchero, Orr and Wang (2011). In short, this coefficient is computed as the cross-sectional correlation of factor exposures from one day to the next.

Table 4.2 also reports the Variance Inflation Factor (VIF). As explained in Menchero, Orr and Wang (2011), VIF measures the degree of collinearity among the factors. Excessive collinearity can lead to increased estimation error in the factor returns and non-intuitive correlations among factors.

Table 4.2: Style factor summary statistics computed using daily cross-sectional regressions. The entire sample period is divided into two sub-periods.

Subperiod 1: 01-Jan-1985 to 28-Feb-1999 (3,490 days).

Factor Name	Average Absolute *t*-stat	Percent Observ. |*t*|>2	Annual. Factor Return	Annual. Factor Volatility	Factor Sharpe Ratio	Correl. with ESTU	Factor Stability Coeff	Variance Inflation Factor
Market	10.12	84.93	0.25	18.56	0.01	1.00		
Beta	3.11	55.87	6.95	6.48	1.07	0.78	0.998	2.97
Earnings Yield	1.20	17.54	3.11	2.04	1.53	-0.19	0.999	2.17
Financial Leverage	1.20	18.54	0.66	2.11	0.31	0.14	1.000	2.35
Foreign Sensitivity	1.14	16.48	0.74	2.26	0.33	-0.02	0.999	2.63
Growth	1.03	12.21	-0.29	1.71	-0.17	-0.09	0.999	2.10
Industry Momentum	1.02	6.13	4.56	1.00	4.58	0.00	0.957	1.01
Liquidity	1.80	37.19	-2.25	3.03	-0.74	0.29	0.996	2.21
Long-Term Reversal	1.16	16.93	1.26	1.93	0.65	0.08	0.996	2.13
Macro Sensitivity	1.33	22.38	0.92	1.91	0.48	0.07	0.987	1.35
Management	1.10	14.15	2.86	1.56	1.83	0.04	0.979	1.54
Membership in NK225 Indicator	1.33	22.35	1.65	2.88	0.57	0.11	1.000	
Momentum	1.43	25.24	5.06	3.05	1.66	-0.18	0.993	2.89
NonLinear Size	1.44	26.10	-4.01	1.92	-2.09	0.07	0.999	1.33
Quality	0.92	3.55	0.97	1.21	0.80	0.02	0.999	1.65
Residual Volatility	1.71	33.32	-6.01	3.10	-1.94	0.12	0.999	2.73

Sentiment	1.19	8.14	9.42	1.33	7.09	0.11	0.982	1.32
Short-Term Reversal	2.32	48.65	20.25	2.55	7.95	0.25	0.887	1.00
Size	2.16	44.73	-6.97	4.66	-1.50	0.27	1.000	5.35
Value	1.18	16.88	1.80	2.22	0.81	0.11	0.999	2.63
Average	1.89	26.57	2.05	3.28	1.16	0.15	0.99	2.19

Subperiod 2: 28-Feb-1999 to 30-Apr-2013 (3,481 days).

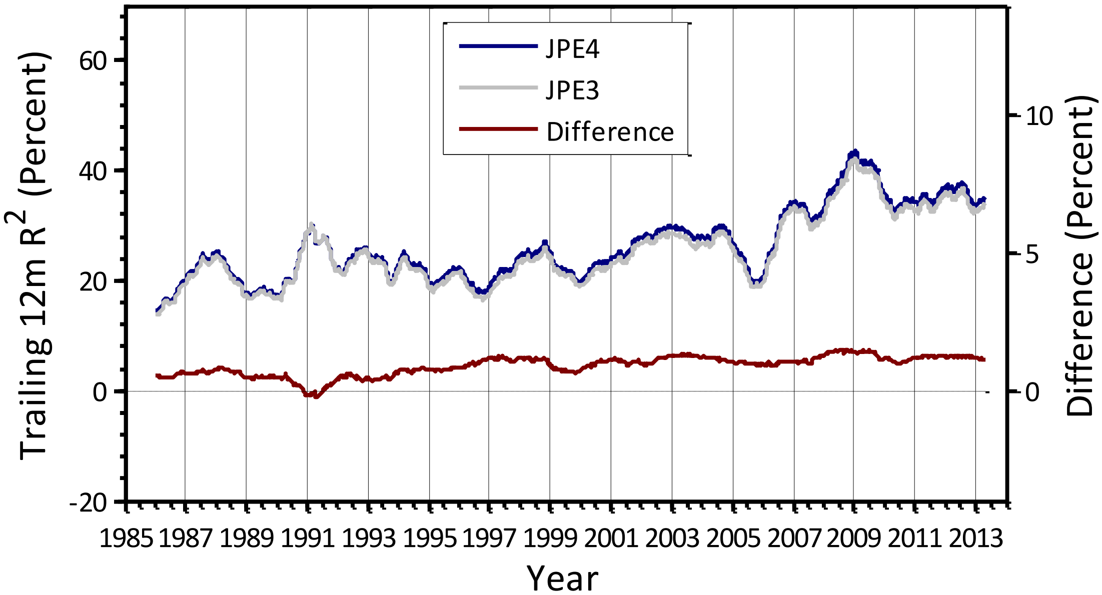
Factor Name	Average Absolute *t*-stat	Percent Observ. |*t*|>2	Annual. Factor Return	Annual. Factor Volatility	Factor Sharpe Ratio	Correl. with ESTU	Factor Stability Coeff	Variance Inflation Factor
Market	14.41	89.51	4.91	21.87	0.22	1.00		
Beta	3.45	62.94	1.81	7.12	0.25	0.80	0.999	4.36
Earnings Yield	1.33	22.38	2.95	1.74	1.69	-0.13	0.998	1.69
Financial Leverage	1.41	25.25	-0.72	2.28	-0.32	0.03	1.000	2.27
Foreign Sensitivity	1.25	19.33	0.53	1.91	0.28	0.09	0.999	2.15
Growth	1.20	18.50	0.84	1.59	0.53	0.00	0.998	1.56
Industry Momentum	1.23	19.74	3.42	1.29	2.66	-0.04	0.964	1.01
Liquidity	2.09	43.90	1.29	3.29	0.39	0.31	0.999	2.52
Long-Term Reversal	1.30	21.63	3.67	2.06	1.78	0.03	0.997	2.01
Macro Sensitivity	1.42	24.48	1.01	2.00	0.50	0.16	0.989	1.54
Management	1.05	12.90	1.17	1.35	0.87	0.04	0.986	1.55
Membership in NK225 Indicator	1.11	15.14	0.18	2.39	0.08	0.03	1.000	
Momentum	2.08	43.75	0.91	3.32	0.27	0.01	0.995	2.17
NonLinear Size	1.66	33.35	-0.15	2.03	-0.07	0.04	0.999	1.37
Quality	1.10	15.66	1.48	1.61	0.92	0.07	0.999	2.13
Residual Volatility	2.11	43.26	-3.81	3.53	-1.08	0.12	0.999	2.75
Sentiment	1.43	26.52	7.51	1.75	4.30	0.14	0.984	1.43
Short-Term Reversal	2.29	47.37	7.22	2.45	2.95	0.14	0.916	1.00
Size	2.48	50.19	-3.01	4.65	-0.65	0.10	1.000	5.15
Value	1.32	22.03	1.26	2.11	0.60	0.02	0.999	2.69
Average	2.29	32.89	1.62	3.52	0.81	0.15	0.99	2.19

4.3. Explanatory Power

The explanatory power of the factors, as measured by adjusted R-squared, is a key metric of model quality. The value of adjusted R-squared, however, can be significantly impacted by the regression weighting scheme, the estimation universe, and the time period under consideration. Caution must be exercised, therefore, when comparing adjusted R-squared values across different models. Nevertheless, if each of these variables is carefully controlled, then a meaningful apples-to-apples comparison between models is possible.

In **Figure 4.1**, we report the trailing 252-day adjusted *R*-squared for the JPE3 and JPE4 models. In order to ensure a fair comparison; the estimation universe and regression weighting scheme (square root of market capitalization) were identical for the two sets of regressions.

Figure 4.1: Trailing 252-day adjusted *R*-squared for JPE4 and JPE3 models (based on the first stage regression).



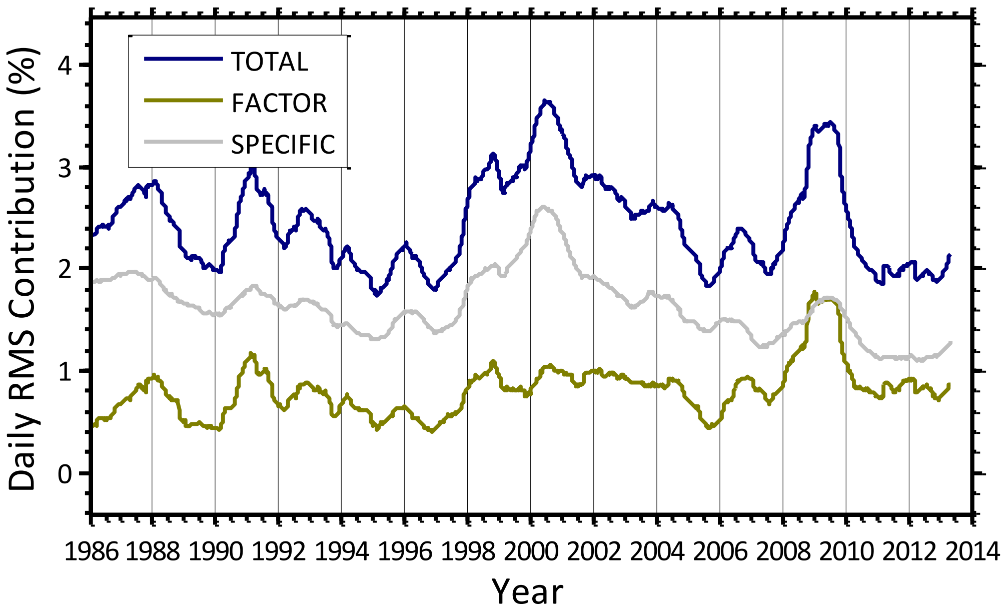
4.4. Cross-Sectional Dispersion

It is informative to study the cross-sectional dispersion of monthly stock returns. As discussed by Menchero and Morozov (2011), dispersion can be measured in one of two ways. The first is by cross-sectional volatility (CSV), which measures the dispersion relative to the *mean* return. The second way is by root mean square (RMS) return, which measures the dispersion relative to *zero* return. The main difference between the two is that the Country factor makes no contribution to CSV, whereas it does contribute to RMS levels.

As discussed by Menchero and Morozov (2011), and shown in Appendix B, the RMS return can be decomposed and attributed to individual factors or groups of factors.

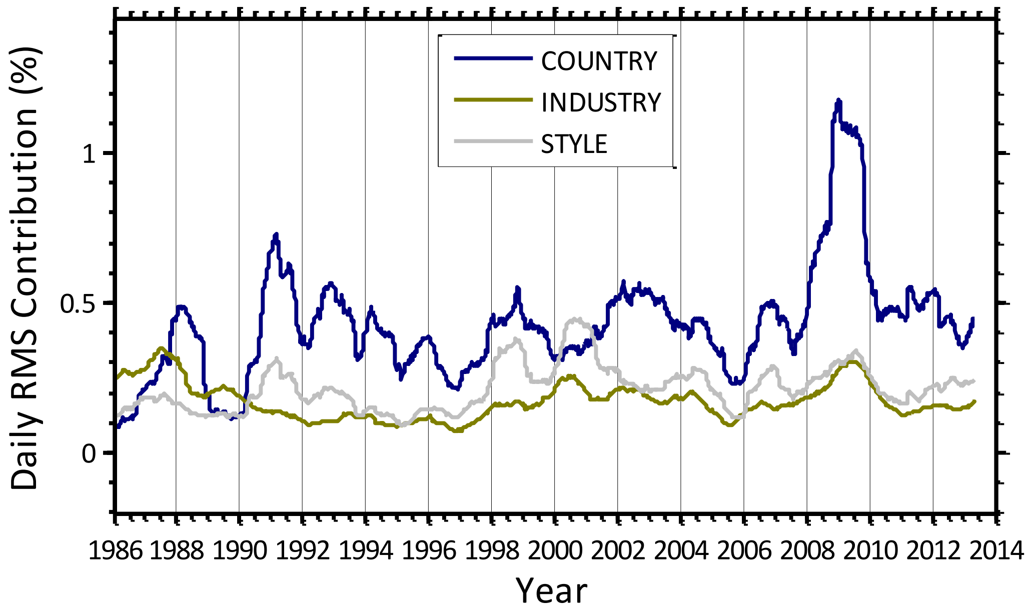
Figure 4.2 shows the net RMS contributions from factors and stock-specific sources with a trailing 252-day total RMS return.

Figure 4.2: Total daily cross-sectional dispersion as measured by root mean square (RMS) return.



In **Figure 4.3**, we further decompose the factor RMS components into the Country factor, industries, and styles.

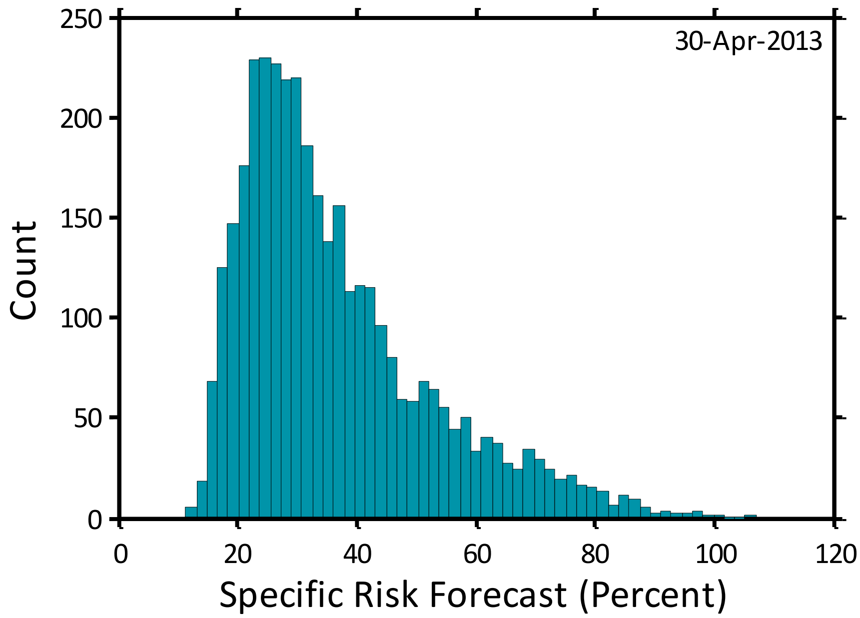
Figure 4.3: Contributions to daily root mean square (RMS) return from Country factor, industries and styles.



4.5. Specific Risk

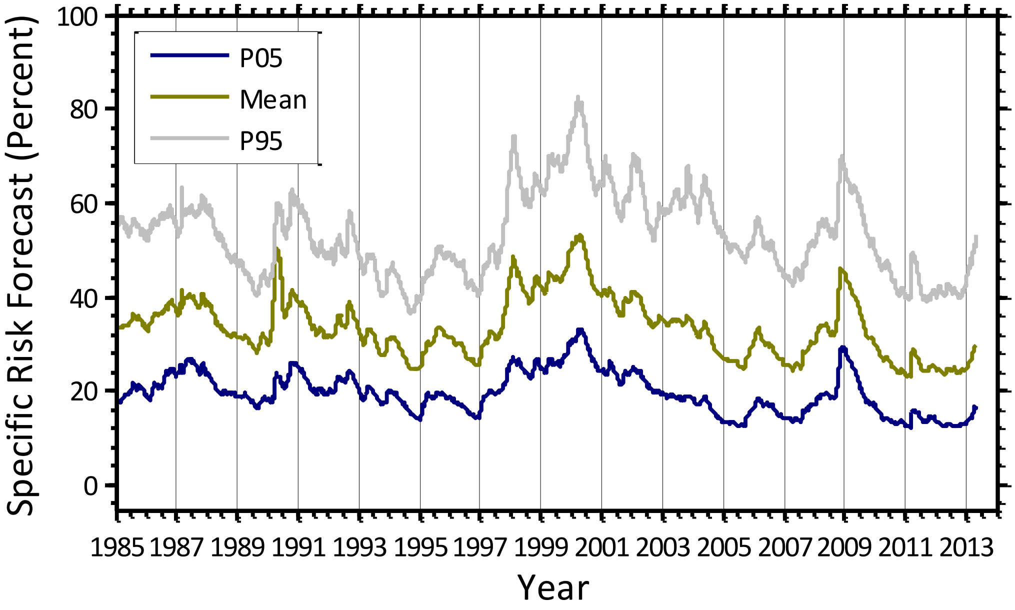
The distribution of specific volatilities is an important characteristic to examine. In **Figure 4.4** we plot the histogram of JPE4S specific risk forecasts for the end-of-period.

Figure 4.4: Histogram of JPE4S specific-risk forecasts.



It is also interesting to study how the distribution of specific risk varied over time. In **Figure 4.5**, we plot the 5-percentile, mean, and 95-percentile values for the specific risk distribution.

Figure 4.5: Specific risk levels versus time for JPE4S.



5. Model Evaluation

5.1 Backtesting

In this section we compare the JPE3 and JPE4 models via backtests, focusing on three kinds of long-only portfolios: minimum risk, benchmark tracking, and active portfolios.

5.1.1 Minimum risk portfolio

In this set of backtests, we construct long-only, monthly rebalanced minimum-risk portfolios. The investment universe is the intersection of JPE3 and JPE4 estimation universe. The backtest period was September 1992 through April 2013. We set a turnover bound of 8 percent, and the maximum asset weight was set to 5 percent.

Figure 5.1 plots the rolling 12-month volatility of the minimum-risk portfolio that was constructed using JPE3S and JPE4S, respectively. The upper subplot is for the portfolio when there was no beta constraint, and the lower subplot is when a minimum forecast beta of 0.5 was forced.

Figure 5.1: Comparison of JPE3S and JPE4S Models for monthly rebalanced minimum volatility portfolio.

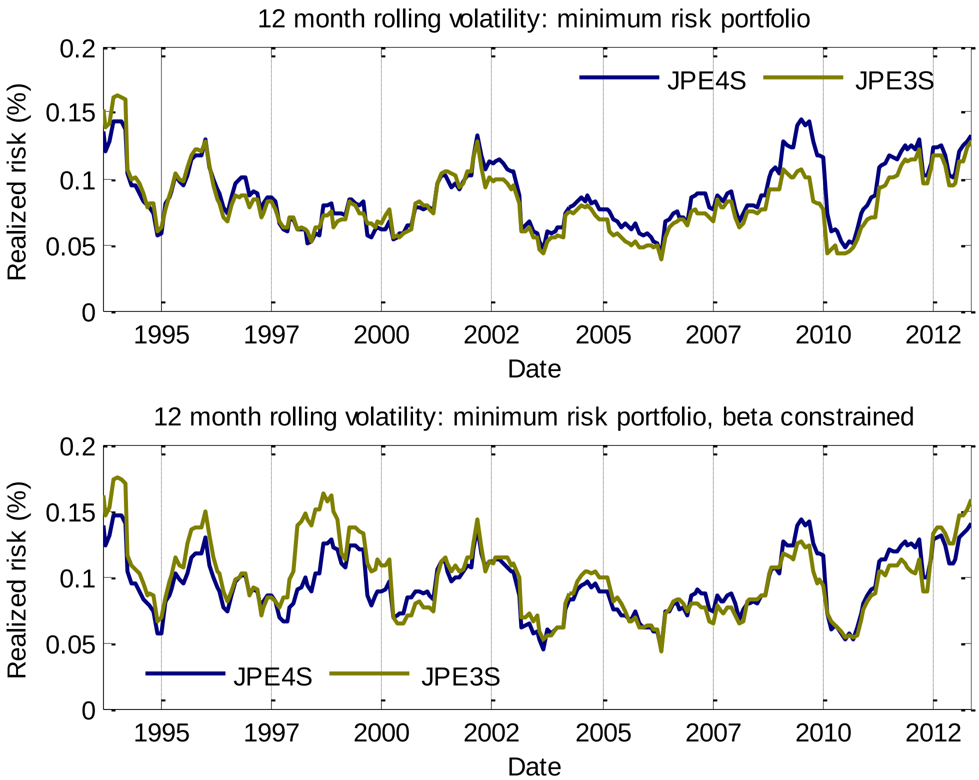
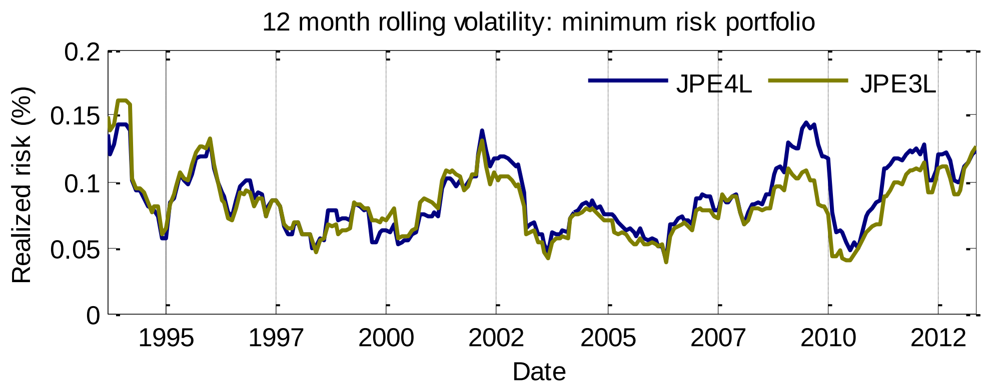


Figure 5.2 is the rolling 12-month volatility of the minimum-risk portfolio that was constructed using JPE3L and JPE4L, respectively. Similar to Figure 5.1, the upper subplot is for the portfolio when there was no beta constraint, and the lower subplot is when a minimum forecast beta of 0.5 was forced.



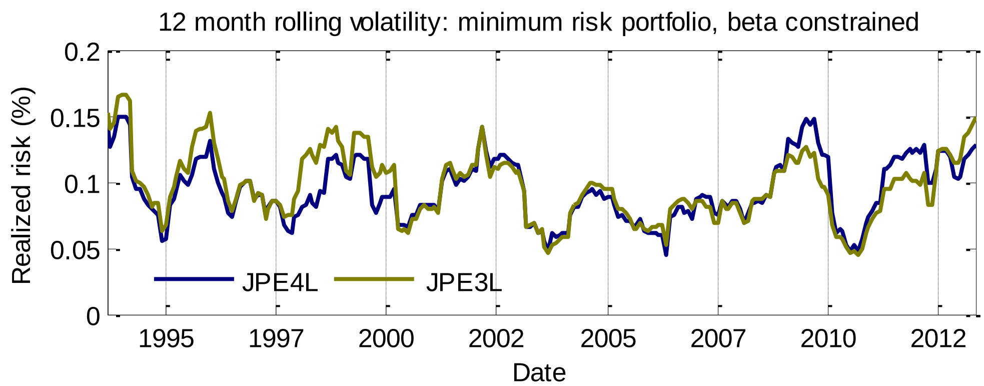


Table 5.1 summarizes the realized volatility, bias statistic, Q statistics and annual turnover over the entire sample period for the portfolio with the constraint of a minimum 0.5 beta.

Table 5.1: Summary of beta-constrained minimum-risk portfolio, entire sample period from September 1992 through Apr 2013.

Model	Realized volatility (%)	Bias	Q	Annual turnover (%)
JPE3S	10.80	0.99	2.60	96
JPE3L	10.48	0.88	2.68	96
JPE4S	10.14	0.84	2.62	96
JPE4L	10.05	0.77	2.55	96

5.1.2 Index-Tracking portfolio

In this comparison, we backtested long-only, monthly rebalanced portfolios that tracked MSCI Japan IMI index. The monthly turnover was set to the maximum of 8 percent and the asset weight was limited to no more than 5 percent. The backtest period was July 1994 through April 2013.

Figure 5.3 shows the rolling 12-month volatility of the index-tracking portfolio that was constructed using JPE3S and JPE4S, respectively. The upper subplot is for the portfolio that had 100 names, and the lower subplot is of 150 names.

Figure 5.3: Comparison of JPE3S and JPE4S Models for monthly rebalanced index-tracking portfolio.

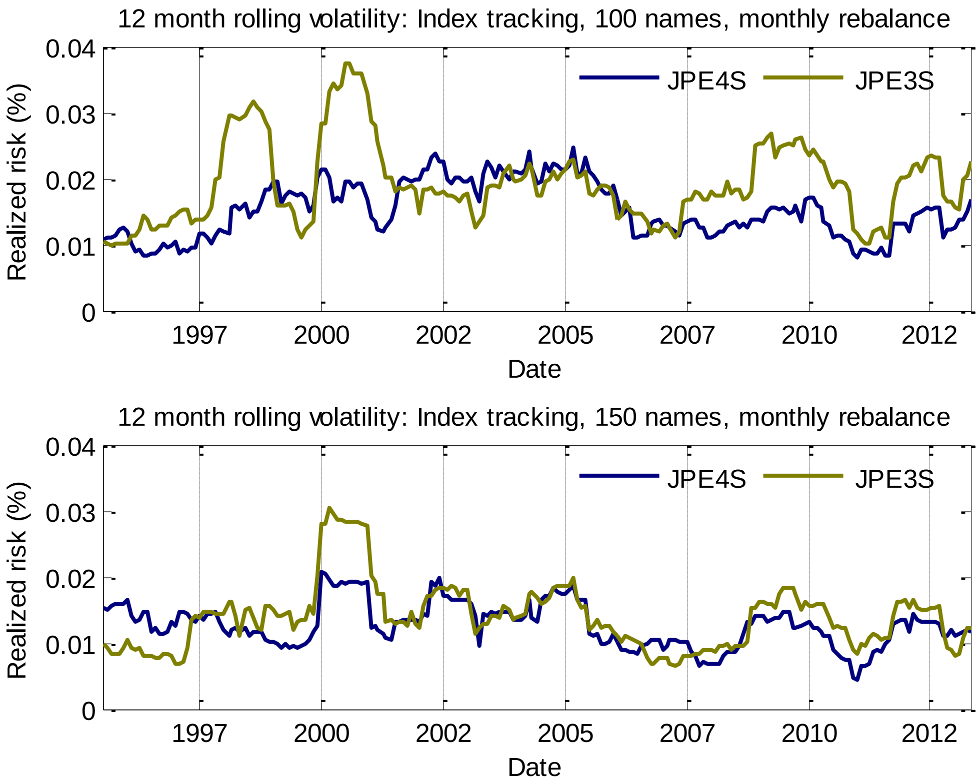


Figure 5.4 shows the rolling 12-month volatility of the index-tracking portfolio that was constructed using JPE3L and JPE4L, respectively. The upper subplot is for the portfolio of 100 names, and the lower subplot is 150 names.

Figure 5.4: Comparison of JPE3L and JPE4L Models for monthly rebalanced index-tracking portfolio.

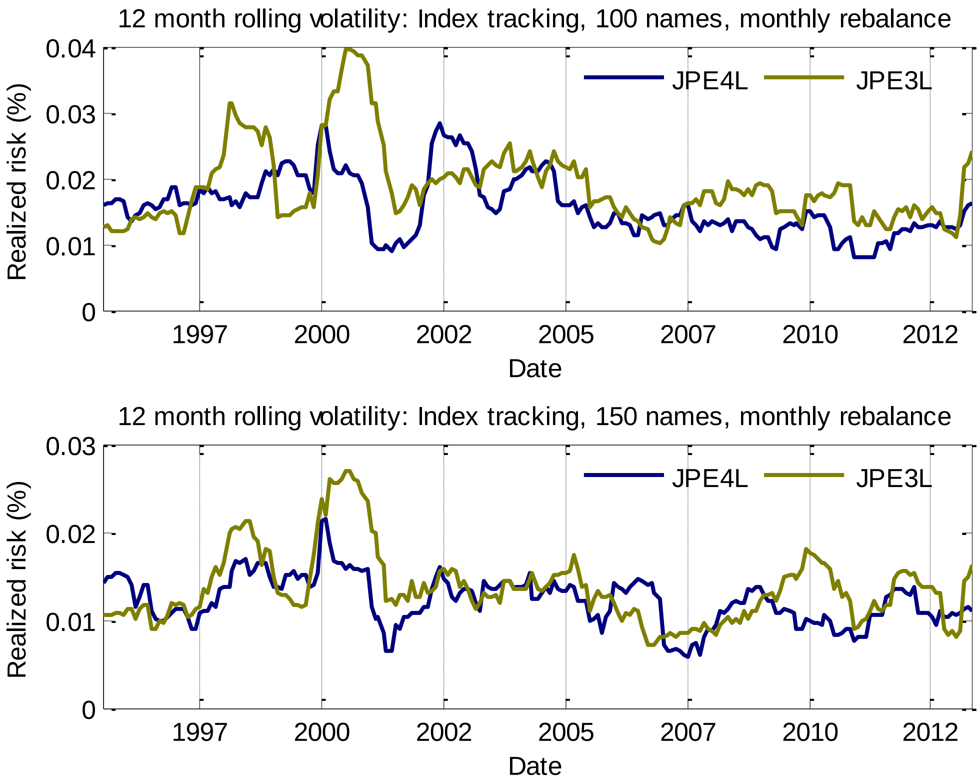


Table 5.2 summarizes the realized tracking error, bias statistic, Q statistics and annual turnover over the entire sample period for the portfolios in **Figure 5.3** and **5.4**.

Table 5.2: Summary of index tracking portfolio, entire sample period from July 1994 through Apr 2013.

Model	# of names	Realized TE (%)	Bias	Q	Annual turnover (%)
JPE3S	100	2.00	1.52	2.70	67
JPE3L	100	1.99	1.49	2.76	69
JPE4S	100	1.62	1.21	2.54	73
JPE4L	100	1.67	1.21	2.47	74
JPE3S	150	1.47	1.55	2.95	62
JPE3L	150	1.43	1.52	2.93	62
JPE4S	150	1.31	1.35	2.77	68
JPE4L	150	1.27	1.32	2.59	69

5.1.3 Active strategy

In this comparison, we first backtested long-only, monthly rebalanced active portfolios. The alpha is the combination of SES factors with a weight of 0.5 on non-orthogonalized Short-Term Reversal and 1 on Earnings Yield, Management, Momentum, Industry Momentum (non-orthogonalized), Long-Term Reversal, Quality, Sentiment, and Value. MSCI Japan IMI index serves as both the benchmark and investment universe. The monthly turnover was set to a limit of 8 percent. The asset weight was constrained to no more than 5 percent. In addition, we alter the risk aversion parameter so as to construct the efficient frontier. The backtest period I was July 1994 through April 2013.

In **Figure 5.5** we plot the realized performance, before transaction costs, of the portfolio that was constructed using JPE3S and JPE4S, respectively. The X axis is the realized annual active risk and the Y axis the realized annual active return.

Figure 5.5: Comparison of JPE3S and JPE4S Models for realized performance of monthly rebalanced active portfolio, before transaction costs.

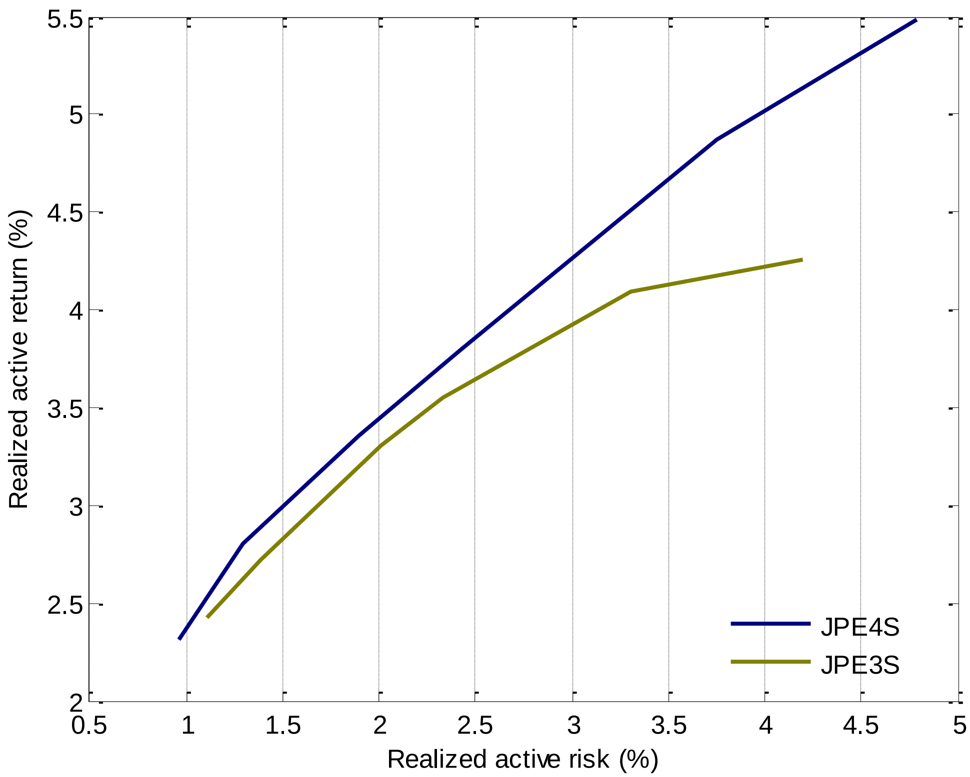


Figure 5.6 is the realized performance of the portfolio (before transaction costs) that is constructed using JPE3L and JPE4L, respectively. The X axis is the realized annual active risk and the Y axis the realized annual active return. Constructing the active portfolio via JPE4S yielded a more efficient performance than via JPE4L.

Figure 5.6: Comparison of JPE3L and JPE4L Models for realized performance of monthly rebalanced active portfolio, before transaction costs.

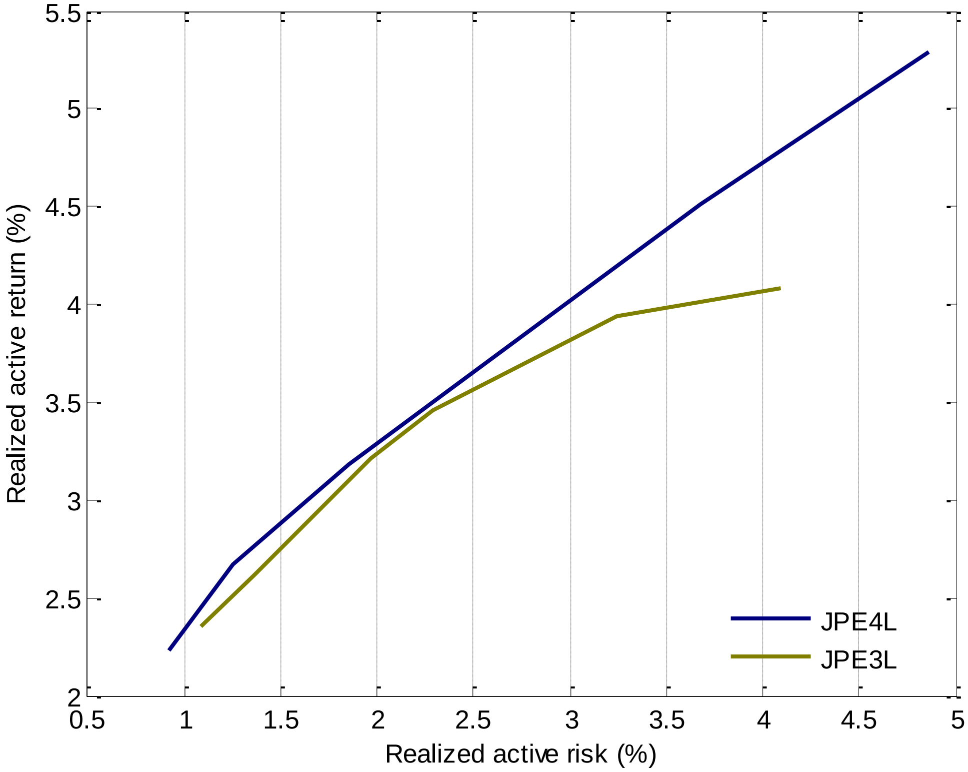
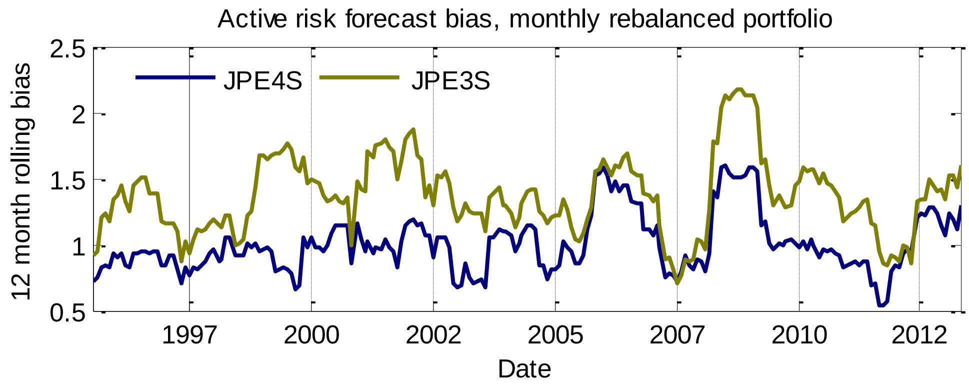


Figure 5.7 compares the 12-month rolling bias of the active portfolios constructed using JPE4S, JPE3S, JPE4L and JPE3L, respectively. We chose appropriate risk aversion parameters so that all these portfolios had an equally realized active risk of 3 percent. As shown, the active risk forecasts from JPE4 models were more accurate than from JPE3 models. There are two reasons for this: 1) JPE4 models rectify forecast bias via optimization bias and volatility regime adjustment, and more importantly, 2) JPE4 is aligned with SES factors, whereas JPE3 is not (See Lee at al 2012 for more details).

Figure 5.7: Comparison of JPE3 and JPE4 Models for risk forecast bias of optimized active portfolio.



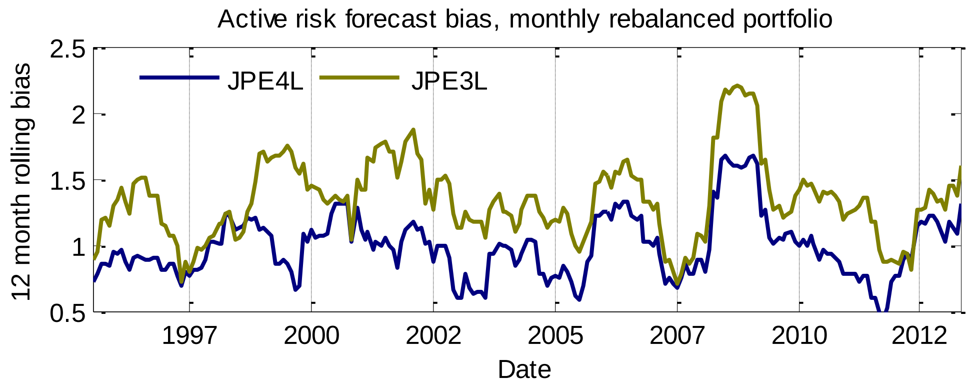


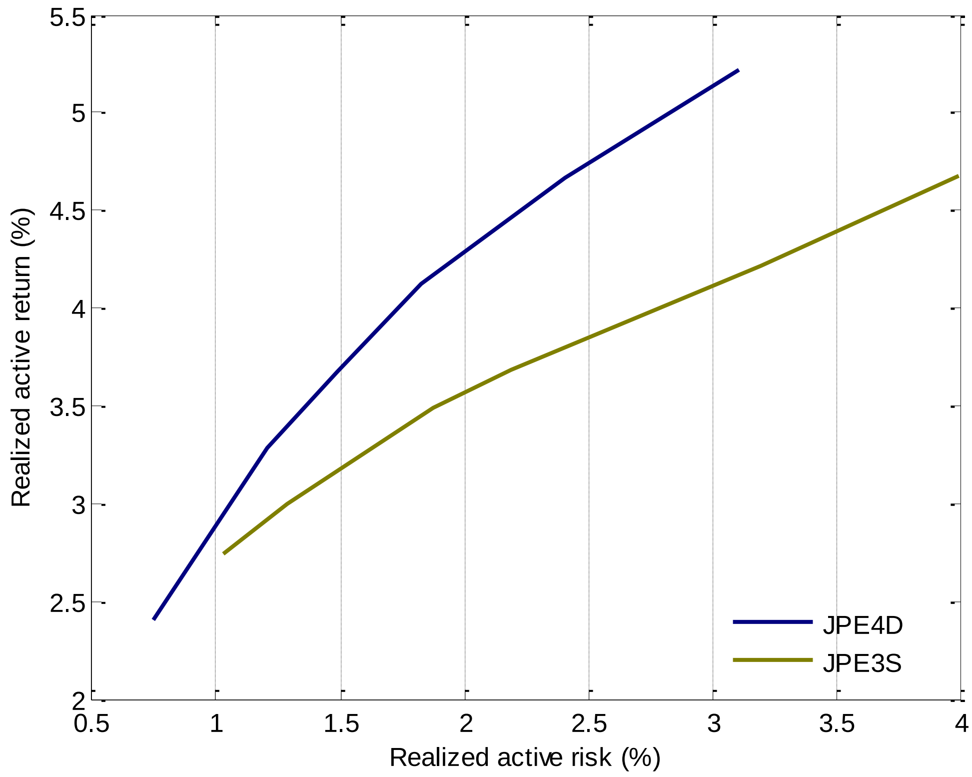
Table 5.3 summarizes the realized annual return, realized annual risk, bias statistic, Q statistics and annual turnover of the sample period for the portfolio in **Figure 5.7**.

Table 5.3: Summary of active portfolio with 3% realized active risk, entire sample period from July 1994 through Apr 2013.

Model	Realized annual active return (%) (before t-cost)	Realized annual active risk (%)	Realized IR (before t-cost)	Bias	Q	Annual turnover (%)
JPE3S	3.92	3	1.30	1.44	2.71	96
JPE3L	3.82	3	1.27	1.41	2.74	96
JPE4S	4.27	3	1.42	1.05	2.22	96
JPE4L	4.00	3	1.33	1.03	2.07	96

For the period of July 1994 through April 2013, we performed a set of backtests with weekly rebalanced long-only active portfolios. The alpha was the equal combination of SES factors: Earnings Yield, Management, Momentum, Industry Momentum (non-orthogonalized), Long-Term Reversal, Quality, Sentiment, Short-Term Reversal (non-orthogonalized) and Value. The MSCI Japan IMI Index served as both the benchmark and investment universe. The weekly turnover was set to the limit of 2 percent. The asset weight was constrained to no more than 5 percent. **Figure 5.8** compares the realized performance (before-transaction-cost) when JPE4D and JPE3S are separately used to construct the active portfolio.

Figure 5.8: Comparison of JPE3S and JPE4D Models for realized performance of weekly rebalanced active portfolio, before transaction costs.



5.2 Forecasting Accuracy

In this section we show side-by-side comparisons of the JPE3 and JPE4 models. Our methodology for evaluating and comparing the accuracy of risk model forecasts was based on the Q-statistic and bias statistic (described in Appendix H). We used the Q-statistic to quantify the differences between models and the bias statistic to build intuition about the periods when a model exhibited under or over forecasts.

Conceptually, the bias statistic is an out-of-sample measure that represents the ratio of realized risk to predicted risk. The ideal bias statistic for perfect risk forecasts should be close to 1. By plotting the mean 252-day rolling-window bias statistic across time for a collection of portfolios in Figures 5.9-5.14, we quickly visualized the magnitude of the average biases and could judge whether they were persistent or regime-dependent.

One potential shortcoming of the bias statistic is that over long windows, we may have sub-periods of overforecasting and underforecasting, yet obtain a bias statistic close to 1 over the entire window. In other words, forecasting errors may cancel out over the long term, even though the risk forecasts may be poor over shorter periods. For a portfolio manager who may be adversely affected by short period of poor performance, it is small consolation knowing that a risk forecast is good *on average*. For this reason, we focus on the mean Q-statistic. The Q-statistic provides a measure of the forecast error and grows with the error size. The mean Q-statistic is not prone to the "error cancellation" and is minimized by having the right forecast for every portfolio for every time period. This gives us a tool to measure the improvements between models on the same set of portfolios. The better model will have lower mean Q-statistic. The details on Q-statistic can be found in Appendix H and Patton (2011).

The following tables present horizon summary mean bias statistic and Q-statistic numbers for the test cases presented in Figures 5.9 to 5.14 for the entire sample period (Jan 1, 1985 to Apr 30, 2013).

Table 5.4: Summary of daily horizon mean bias statistics and Q-statistics for JPE3S and JPE4D Models, entire sample period (January 01, 1985 to Apr 30, 2013).

Figures	Mean B JPE4D	Q JPE4D	Mean B JPE3S	Q JPE3S	Q Diff (bp)	Portfolio Type
5.9	1.03	2.4765	1.05	2.6367	1602	Pure Factors
5.10	1.03	2.3974	1.00	2.5075	1101	Random Active
5.11	1.03	2.5026	1.07	2.7027	2001	Factor Tilts Long
5.12	1.03	2.4502	1.03	2.6076	1574	Factor Tilts Active
5.13	1.00	2.4219	1.13	2.5632	1413	Optimized Styles
5.14	0.99	2.5305	1.21	2.8017	2712	Specific Risk
Average	1.0175	2.4632	1.0810	2.6366	1734	

Table 5.5: Summary of monthly horizon mean bias statistics and Q-statistics for JPE3S and JPE4S Models, entire sample period (January 01, 1985 to Apr 30, 2013).

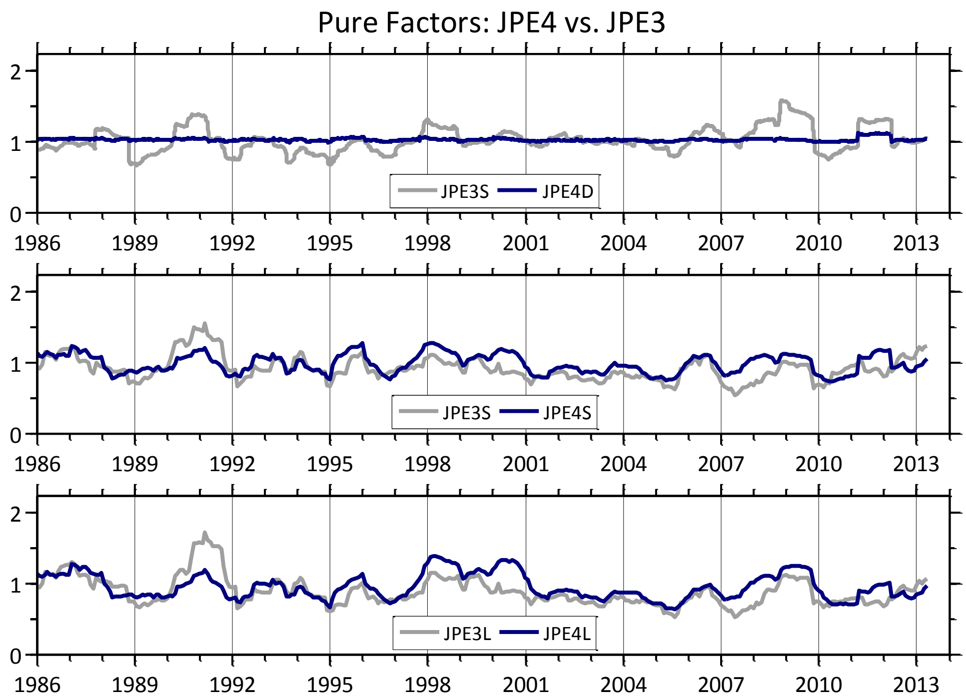
Figures	Mean B JPE4S	Q JPE4S	Mean B JPE3S	Q JPE3S	Q Diff (bp)	Portfolio Type
5.9	1.05	2.4996	1.05	2.5010	14	Pure Factors
5.10	1.01	2.4567	0.98	2.4464	-103	Random Active
5.11	1.06	2.4822	1.09	2.4823	1	Factor Tilts Long
5.12	1.09	2.4799	1.10	2.4807	8	Factor Tilts Active
5.13	1.05	2.4581	1.22	2.5435	854	Optimized Styles
5.14	0.93	2.5211	1.00	2.5312	101	Specific Risk
Average	1.0350	2.4829	1.0712	2.4975	146	

Table 5.6: Summary of monthly horizon mean bias statistics and Q-statistics for JPE3L and JPE4L Models, entire sample period (January 01, 1985 to Apr 30, 2013).

Figures	Mean B JPE4L	Q JPE4L	Mean B JPE3L	Q JPE3L	Q Diff (bp)	Portfolio Type
5.9	1.03	2.5247	1.02	2.5111	-136	Pure Factors
5.10	1.00	2.4807	0.96	2.4556	-251	Random Active
5.11	1.03	2.4773	1.07	2.4825	52	Factor Tilts Long
5.12	1.08	2.5061	1.08	2.4891	-170	Factor Tilts Active
5.13	1.05	2.4822	1.20	2.5406	584	Optimized Styles
5.14	0.92	2.5414	1.00	2.5312	-102	Specific Risk
Average	1.0198	2.5021	1.0541	2.5017	-4	

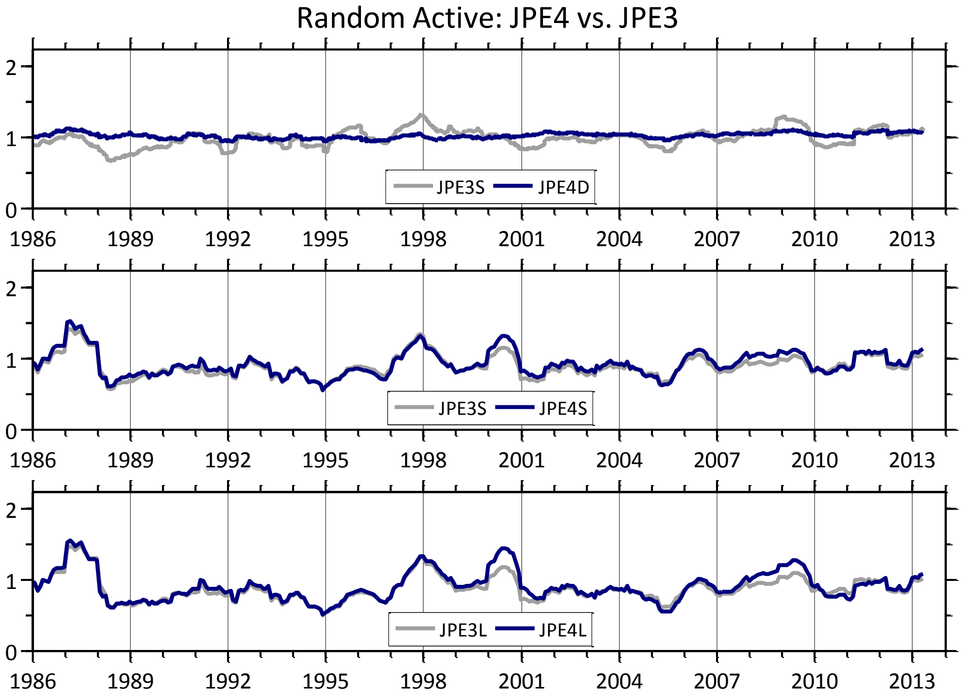
In **Figure 5.9** we plot 252-day rolling-window bias statistics for the JPE4D pure factors.

Figure 5.9: Comparison of JPE4 and JPE4 Models for pure factors.



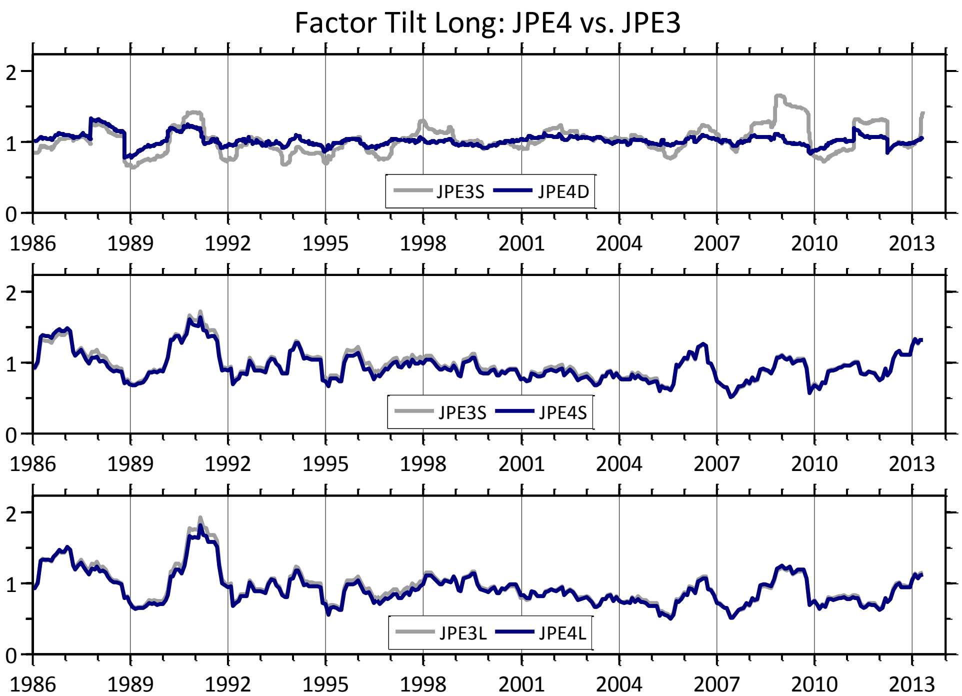
In **Figure 5.10** we plot 252-day bias statistics for 100 random active portfolios. The random active portfolios were constructed by going long 500 cap-weighted randomly selected stocks and shorting the cap-weighted JPE4 estimation universe. The list of stocks used to construct the portfolios was held fixed unless a stock dropped out of the estimation universe, in which case it was replaced by randomly selecting another stock. The portfolios used to test the two models were identical.

Figure 5.10: Comparison of JPE3 and JPE4 Models for 100 random active portfolios.



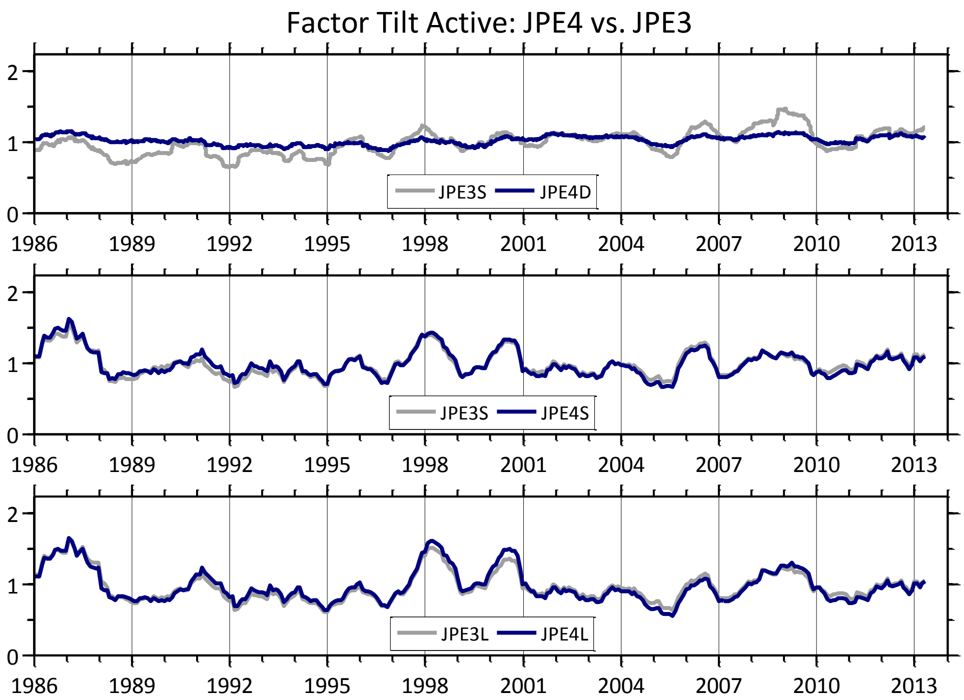
In **Figure 5.11** we plot 252-day rolling-window bias statistics for long-only factor-tilt portfolios. The portfolios were constructed by cap-weighting the JPE4 industries, and the top and bottom quintiles for each of the JPE4 style factors.

Figure 5.11: Comparison of JPE3 and JPE4 Models for industry and style-tilt long only portfolios.



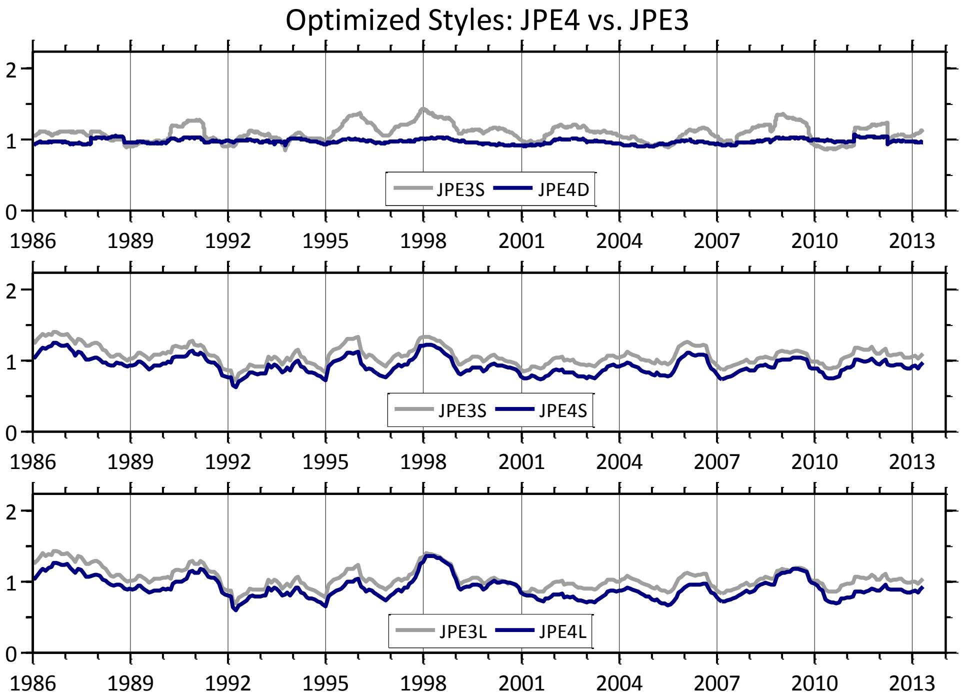
In **Figure 5.12** we plot 252-day rolling-window bias statistics for the active factor-tilt portfolios. The portfolios were constructed by going long the factor-tilt portfolios of Figure 5.11 and shorting the JPE4 estimation universe.

Figure 5.12: Comparison of JPE3 and JPE4 Models for industry and style-tilt active portfolios.



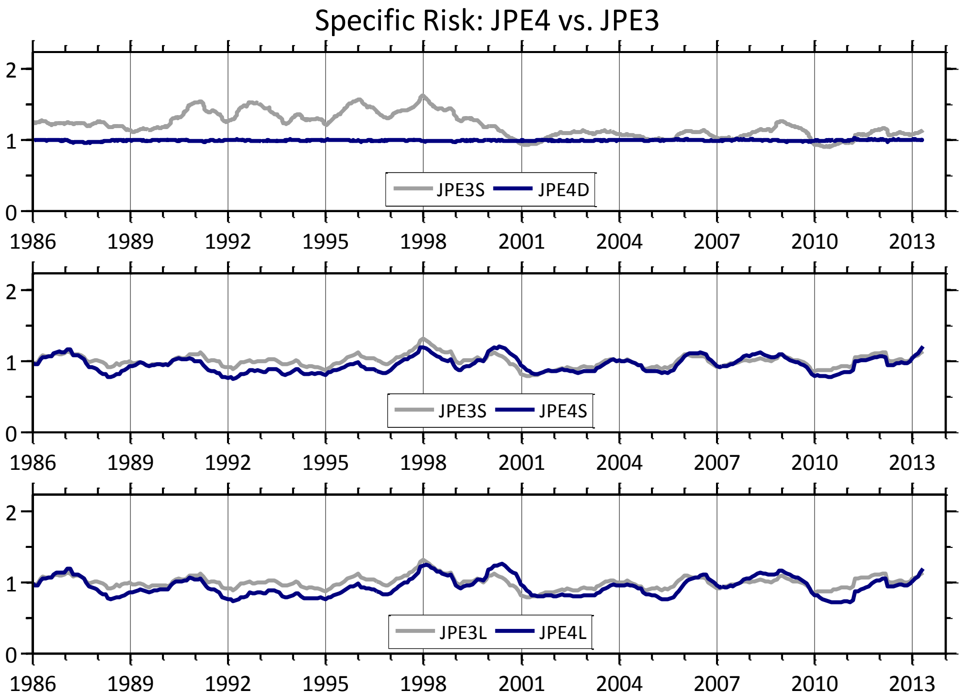
In **Figure 5.13** we plot 252-day rolling-window bias statistics for optimized style-tilt portfolios. The optimized portfolios were constructed by using the JPE3 and JPE4 style factors as “alpha signals” and then forming the minimum volatility portfolio (with alpha equal to 1) for 10 draws of 500 randomly selected stocks.

Figure 5.13: Comparison of JPE3 and JPE4 Models for optimized style-tilt portfolios.



In **Figure 5.14** we plot a 252-day rolling-window bias statistics for the specific returns of all stocks in the ` estimation universe. The mean is cap-weighted.

Figure 5.14: Comparison of JPE3 and JPE4 Models for specific risk.



6. Conclusion

The JPE4 Model incorporates newly introduced Systematic Equity Strategy (SES) factors and many methodological innovations and advances designed to address long-standing problems in risk modeling. For instance, the Optimization Bias Adjustment addresses the issue of underestimation of risk for optimized portfolios, and leads to better conditioning of the covariance matrix. The Volatility Regime Adjustment in JPE4D calibrates volatilities to current market levels and represents a key determinant of risk forecasts, especially during times of market turmoil. The introduction of the Country factor leads to more intuitive attribution of portfolio risk and return, while also providing timelier forecasts of industry correlations. Another enhancement is the use of a Bayesian adjustment technique which aims to reduce biases in specific risk forecasts.

This document provided a thorough empirical analysis of the JPE4 Model. The factor structure has been presented in detail, for both industries and styles. Key metrics were reported at the individual factor level, including statistical significance, performance, volatility, and correlation.

We also compared the explanatory power of the JPE4 Model with the JPE3 Model. Moreover, we decomposed cross-sectional dispersion into contributions from factors and stock-specific, and further decomposed the factor contribution into Country, industry, and style components.

We systematically compared the forecasting accuracy of the JPE4S and JPE4L Models versus their JPE3 counterparts over a backtesting window. We also presented the results for JPE4D, the first member of a new class of daily models. We considered several types of portfolios, including pure factors, random active portfolios, factor-tilt portfolios (both long-only and dollar-neutral), and optimized portfolios. We also demonstrated the accuracy of specific risk forecasts of the three new models as well as the legacy model.

Lastly, we backtested the model on real-life scenarios of constructing minimum volatility portfolios with and without beta constraints, index tracking portfolios with 100 and 150 names, and an active strategy portfolio composed of SES factor exposures.

Appendix A: Descriptors by Style Factor

Beta

HBETA (Historical Beta)

Computed as the slope coefficient in a time-series regression of stock return against the cap-weighted return of the estimation universe. The regression uses a trailing 252 trading days of returns with a half-life of 63 trading days. We apply Vasicek shrinkage on historical beta towards industry beta.

Earnings Yield

EPIBS (Forward earnings-to-price ratio)

Given by the 12-month forward-looking earnings divided by the market capitalization. Forward-looking earnings are defined as a weighted average between the average analyst-predicted earnings for the current and next fiscal years.

HPRFT (Earnings before interest and taxes-to-enterprise value ratio)

This descriptor measures the historical profitability. It is computed by the trailing 12-month earnings before interests and taxes divided by the enterprise value.

Financial Leverage

DTOA (Debt-to-assets ratio)

Computed as

$$DTOA = \frac{TD}{TA}, \tag{A1}$$

where TD is the book value of total debt (long-term debt and current liabilities), and TA is the book value of total assets.

BLEV (Book leverage)

Computed as

$$BLEV = \frac{BE + PE + LD}{BE}, \tag{A2}$$

where BE is the most recent book value of common equity, PE is the most recent book value of preferred equity, and LD is the most recent book value of long-term debt.

MLEV (Market leverage)

Computed as

$$MLEV = \frac{ME + PE + LD}{ME}, \qquad \text{(A3)}$$

where ME is the market value of common equity on the last trading day, PE is the most recent book value of preferred equity, and LD is the most recent book value of long-term debt.

Foreign Sensitivity

FXSEN (Currency sensitivity)

Computed as the sensitivity of asset residual return (in historical beta regression) to changes in USD/JPY exchange rate. The estimation is over the trailing window of 252 trading days with a half-life of 12 weeks.

FSTOS (Foreign sales-to-total sales ratio)

The ratio of sales in foreign countries to total sales.

Growth

EGRO (Earnings growth rate over last five years)

Annual reported earnings per share are regressed against time over the past five fiscal years. The slope coefficient is then divided by the average annual earnings per share to obtain the earnings growth.

SGRO (Sales growth rate over last five years)

Annual reported sales per share are regressed against time over the past five fiscal years. The slope coefficient is then divided by the average annual sales per share to obtain the sales growth.

EGIBS (Analyst-predicted earnings growth)

Long-term (3-5 years) earnings growth forecasted by analysts.

Industry Momentum

CRSLNK(Crosslink ratio)

Crosslink ratio measures industry relative strength. It is computed as the weighted average 6 months cumulative log return of all peer stocks in the same GICS sub-industry.

Liquidity

STOA (Annual share turnover)

Computed as the log of the median daily turnover during the last year.

STOM (Monthly share turnover)

Computed as the log of the median daily trading-volume-to-outstanding-shares during the previous month.

STOQ (Quarterly share turnover)

Computed as the log of the median daily turnover during the last quarter.

FSTOA (Annual share turnover adjusted for cross-holdings)

Computed as the log of the median daily trading-volume-to-outstanding-shares during the previous year, adjusted by cross-holding ratios.

FSTOM (Monthly share turnover adjusted for cross-holdings)

Computed as the log of the median daily trading-volume-to-outstanding-shares during the previous month, adjusted by cross-holding ratios.

FSTOQ (Quarterly share turnover adjusted for cross-holdings)

Computed as the log of the median daily trading-volume-to-outstanding-shares during the previous quarter, adjusted by cross-holding ratios.

Long-Term Reversal

LTREV (Long term reversal)

Computed as the sum of log returns over the trailing 2 years with a lag of 1 year, multiplied by -1.

Macro Sensitivity

IRSEN (Interest rate sensitivity)

Computed as the sensitivity of asset residual return (in historical beta regression) to changes in Nomura government bond price. The estimation is over the trailing window of 252 trading days with a half-life of 12 weeks.

USMSEN (US market sensitivity)

Computed as the sensitivity of asset residual return (in historical beta regression) to US equity market return. The estimation is over the trailing window of 252 trading days with a half-life of 12 weeks.

OILSEN (Oil sensitivity)

Computed as the sensitivity of asset residual return (in historical beta regression) to changes crude oil price. The estimation is over the trailing window of 252 trading days with a half-life of 12 weeks.

GOLDSEN (Gold sensitivity)

Computed as the sensitivity of asset residual return (in historical beta regression) to changes in gold spot price. The estimation is over the trailing window of 252 trading days with a half-life of 12 weeks.

Management

MGMTBIAS (Management forecast bias)

The difference between realized operating profit and last available management forecast divided by market capitalization.

MGMT_DT (Difference in market cap and returns)

The difference between the cumulative return in the previous year and the contemporaneous change of market capitalization.

Membership in NK255 Indicator

NK225 (Indicator of NK225 membership)

Exposure of 1 if asset is a member of Nikkei 225, and 0 otherwise

Momentum

RSTR (Relative strength)

This descriptor measures stock momentum. It is the sum of log returns over the trailing 252 trading days with a lag of 21 trading days.

Non-linear Size

NLSIZE (Non-linearity in Logarithm of market capitalization)

First, the standardized Size exposure (i.e., log of market cap) is cubed. The resulting factor is then orthogonalized to the Size factor on a regression-weighted basis. Finally, the factor is winsorized and standardized.

Quality

ACCR (Accruals)

The negation of Accrual anomaly.

CETOE (Cash earnings-to-earnings ratio)

The difference between cash-earnings-to-price and earnings-to-price.

Residual Volatility

HSIGMA (Historical sigma)

Computed as the volatility of residual returns in the time-series regression of historical beta. The volatility is estimated over the trailing window of 252 trading days with a half-life of 63 trading days.

Sentiment

NETPRF_CHG (Net profit forecast revisions)

The monthly change of Toyo Kezai net profit forecast, divided by firm market cap.

RATING_CHG (Analyst rating change)

Computed as the monthly change of analyst revision ratios which is defined as the number of up revisions minus the number of down revisions, divided by the total number of revisions.

EPIBS_CHG (Change in analyst-predicted earnings-to-price)

The monthly change of forecast earnings-to-price from I/B/E/S and Toyo Kezai.

Short-Term Reversal

STREV (Short term reversal)

Computed as the sum of log returns over the trailing 20 trading days with a lag of 1 trading day, multiplied by -1.

Size

LNCAP (Logarithm of market capitalization)

The logarithm of the market capitalization of the firm.

Value

BTOP (Book-to-price ratio)

Book value of common equity divided by market capitalization.

YIELD (Dividend Yield)

Trailing 12-month dividend per share divided by asset price.

STOP (Sales-to-price ratio)

Total sales divided by market capitalization.

Appendix B: Historical Beta Estimation

To make estimation of stock betas more robust, we estimate stock betas with respect to markets in Japan using (i) Bayesian shrinkage of stock beta estimates to the estimates of industry betas with respect to market (ii) shrinking Bayesian estimates of beta to the value of one (Vasicek shrinkage). The amount of Bayesian shrinkage of individual stock betas to the industry betas depends on how accurately we estimate stock betas relative to the industry betas. If the individual stock betas are estimated accurately, the amount of shrinkage to the industry beta is minimal. The amount of shrinkage of Bayesian estimates of beta to one depends on the degree of cross-sectional dispersion of betas. For periods when the cross-sectional dispersion of betas is high, the amount of shrinkage to one is minimal.

Technically, estimation of betas is done in four steps:

Step 1: Estimate the beta of each industry with respect to the market, the so-called industry betas. This is done by running a time-series regression of industry returns on the market return using a 252-day estimation window and 63-day half-life:

$$r_{industry,i,t} = \alpha + \beta_{industry,i,t} r_{mt} + e_{industry,i,t}$$

where $r_{industry,i,t}$ represents the industry return in period t, r_{mt} represents the market return, and $\beta_{industry,i,t}$ is the industry beta estimate at period t. For each industry beta, we compute the standard error around the estimate, $SE(\beta_{industry,i,t})$, which gives us an indication of how accurately we estimate industry betas.

Step 2: Estimate the beta of each individual stock with respect to the market. Similar to the estimation of industry betas, we use a time series regression with a 252-day estimation window and a 63-day half-life:

$$r_{i,t} = \alpha + \beta_{i,t} r_{mt} + e_{ry,i,t}$$

where $r_{i,t}$ represents stock i return in period t and $\beta_{i,t}$ is the individual stock beta estimate at period t. For each stock beta, we compute the standard error around the estimate, $SE(\beta_{i,t})$.

Step 3: Compute the Bayesian estimates of betas by shrinking the estimates of individual stock betas from Step 2 with estimates of industry betas from Step 1. This is done using a Bayesian updating (shrinkage) formula:

$$\beta_{i,Bayes,t} = \left(\frac{1}{SE(\beta_{i,t})} + \frac{1}{\tau SE(\beta_{industry,i,t})}\right)^{-1} \left(\frac{\beta_{i,t}}{SE(\beta_{i,t})} + \frac{\beta_{industry,i,t}}{\tau SE(\beta_{industry,i,t})}\right)$$

where τ is the calibrated parameter for scaling up the standard error estimates on industry betas. The choice of τ determines the degree of shrinkage. The higher the value of τ, the smaller is the amount of shrinkage to industry beta.

Looking at the formula, one can see that the Bayesian estimate of beta is a weighted average of the individual stock beta and the industry beta. The amount of shrinkage of individual stock betas to industry betas (or the weight that we put on the industry beta) depends on how large the stock beta standard error, $SE(\beta_{i,t})$, is with respect to industry beta standard error, $SE(\beta_{industry,i,t})$. If $SE(\beta_{i,t})$ is relatively small, we put a large weight on the stock estimate of beta. If $SE(\beta_{i,t})$ is relatively large which may happen when stock has a short history or sparse returns, we put a large weight on the industry beta estimate.

Step 4: Calculate the final estimate of individual stock beta by shrink Bayesian estimates of beta to value of one using Vasicek shrinkage:

$$\beta_{i,final,t} = w_{i,t}\beta_{i,Bayes,t} + \left(1 - w_{i,t}\right)1$$

$$w_{i,t} = \frac{VAR(\beta_t)_{CS}}{VAR\left(\beta_{i,t}\right) + VAR(\beta_t)_{CS}}$$

where $VAR(\beta_t)_{CS}$ is the cross sectional variance of the estimated Bayesian betas in Step 3.

Prior to this step, the stock beta estimates should be not "too far away" from one. To make this notion concrete, we follow Vasicek's suggestion and look at the cross-sectional dispersion of betas as a measure of distance of how far away betas are likely to be from one. If the cross-section dispersion of betas is low for a given time period, then we know that beta estimates that are much different from one are likely to be outliers caused by estimation noise. These beta estimates will be shrunk to one.

Appendix C. Covariance Matrix Estimation

Estimation of the JPE4 factor covariance matrix follows a multi-step process. The first step is to compute the factor correlation matrix from the set of daily factor returns. We employ exponentially weighted averages, characterized by the factor correlation half-life parameter τ_{ρ}^{F}. This approach gives more weight to recent observations and is an effective method for dealing with data non-stationarity.

For the JPE4S and JPE4L models the prediction horizon is one month. The factor correlation matrix, however, is estimated from daily factor returns. We must therefore account for the possibility of serial correlation in factor returns, as these may affect risk forecasts over a longer horizon.

We employ the Newey-West methodology (1987) to account for serial-correlation effects. A key parameter in this approach is the number of lags L_{ρ}^{F} over which the serial correlation is deemed important. For instance, $L_{\rho}^{F} = 2$ implies that the return of any factor may be correlated with the return of any other factor within a two-day time span.

Another complication in estimating the factor correlation matrix arises from the case of missing factor returns. In the JPE4 model, missing factor returns arise from using time series of differing lengths. For instance, the Internet factor and Sentiment factor appear in the model only after the start date of the cross-sectional regressions. The industry factors proxy early history using the returns of the parent industry. For style factors, we use the EM algorithm of Dempster (1977) to estimate the correlation matrix for the case of missing factor returns. This method employs an iterative procedure to estimate the correlation matrix. The EM algorithm also refines the correlation forecasts as new information flows into the model.

With the correlation matrix thus computed, the next step is to calculate the factor volatilities. We use exponentially weighted averages, with half-life parameter τ_{σ}^{F}. In estimating monthly factor volatilities, we also scale daily volatility by a ratio of volatility estimates calculated using overlapping monthly returns and daily returns.

Next, we construct the initial covariance matrix by combining the factor volatilities and correlations. That is, the covariance between factors i and j is given by

$$F_{ij}^{0}=\rho_{ij}\,\sigma_i\sigma_j$$

where σ_i and σ_j are the factor volatilities and ρ_{ij} is their respective correlation.

Appendix D: Optimization Bias Adjustment

Let F_0 denote the KxK sample factor correlation matrix (FCM),

$$F_0 = cor(f, f) \tag{D1}$$

where f is the KxT matrix of realized factor returns, K is the number of factors and T is the number of periods.

The sample FCM can be expressed in diagonal form as

$$D_0 = U_0' F_0 \ U_0 \tag{D2}$$

where U_0 is the KxK rotation matrix whose columns are given by the eigenvectors of F_0 . The j^{th} element of the k^{th} column of U_0 gives the weight of pure factor j in eigenfactor k . The predicted eigenvalues of the eigenfactors are given by the diagonal elements of D_0. The fact that D_0 is diagonal indicates that the eigenfactors are mutually uncorrelated.

Although the true FCM is unobservable, we suppose for simulation purposes that the sample FCM F_0 governs the "true" return-generating process. We generate a set of factor returns for simulation m as

$$f_m = U_0 \ b_m \tag{D3}$$

where b_m is a KxT matrix of simulated eigenfactor returns. The elements of row k of b_m are drawn from a random normal distribution with mean zero and eigenvalues given by the diagonal element $D_0(k)$ of matrix D_0. It can be easily verified that the simulated returns in Equation B3 have a true FCM given by F_0. Due to sampling error, however, the *estimated* FCM

$$F_m = cor(f_m, f_m) \tag{D4}$$

will differ from the true FCM F_0 . Nevertheless, F_m is unbiased in the sense that $\mathrm{E}[F_m] = F_0$. We diagonalize the simulated FCM

$$D_m = U_m' F_m U_m \tag{D5}$$

where U_m denotes the simulated eigenfactors with estimated eignevalues given by the diagonal elements of D_m, i.e. $D_m(k)$.

Since we know the true distribution that governs the simulated factor returns, we can compute the true FCM of the simulated eigenfactors,

$$\widetilde{D}_m = U_m' F_m U_m \tag{D6}$$

Note that since U_m is not composed of the "true" eigenfactors, the matrix $\widetilde{D}_m$ is not diagonal. Nevertheless, our current focus is on the diagonal elements of the matrix. We compute the *simulated* eigenvalue biases according to

$$v^2(k) = \frac{1}{M} \sum_m \frac{\widetilde{D}_m(k)}{D_m(k)} \tag{D7}$$

where M is the total number of simulations. The simulated eigenvalue bias is computed daily, and the average over the entire sample period.

We now assume that the sample FCM F_0 , which uses the same correlation estimator as the simulated FCM F_m , also suffers from the same biases. Let $\widetilde{D}_0$ denote the diagonal FCM whose eigenvalues have been adjusted

$$\widetilde{D}_0 = v^2 D_0 \tag{D8}$$

where v^2 is a diagonal matrix whose elements are given by $v^2(k)$. The FCM in Equation D9 is now rotated from the diagonal basis to the pure factor basis using the sample eigenfactors. That is,

$$\tilde{F}_0 = U_0 \widetilde{D}_0 U_0' \tag{D9}$$

Where $\tilde{F}_0$ denotes the eigen-adjusted factor correlation matrix.

For further details, please refer to Menchero, Wang, and Orr (2011).

Appendix E: Volatility-Regime Adjustment

Let f_{kt} be the return to factor *k* on day *t* , and let σ_{kt} be the one-day volatility forecast for the factor at the start of the day. The standardized return of the factor is given by the ratio f_{kt}/σ_{kt}, and should have standard deviation close to 1 if the risk forecasts are accurate. Normally, as described in Appendix H, we compute the *time-series* standard deviation to investigate whether an individual factor is unbiased across time.

Alternatively, we can compute the *cross-sectional* standard deviation to investigate whether the factor volatility forecasts are collectively unbiased at a given point in time. We define the factor cross-sectional bias statistic B_t^F on day *t* as

$$B_t^F = \sqrt{\frac{1}{K}\sum_k \left(\frac{f_{kt}}{\sigma_{kt}}\right)^2} \tag{E1}$$

where *K* is the total number of factors. This quantity represents an instantaneous measure of factor risk bias. For instance, if the risk forecasts were too small on a particular day, then $B_t^F > 1$. By observing the cross-sectional bias statistics over time, we can determine the extent to which volatility forecasts should be adjusted to remove these biases.

We define the *factor volatility multiplier* λ_F as an exponentially weighted average

$$\lambda_F = \sqrt{\sum_t (B_t^F)^2\, w_t} \tag{E2}$$

where w_t is an exponential weight with Volatility Regime Adjustment half-life τ_{VRA}^F. This parameter serves as the primary determinant of model responsiveness for factor risk. The Volatility Regime Adjustment forecasts are given by

$$\tilde{\sigma}_k = \lambda_F \sigma_k \tag{E3}$$

This is equivalent to multiplying the entire factor covariance matrix by a single number, λ_F^2 . As a result, the Volatility Regime Adjustment has no effect on factor correlations.

Appendix F: Specific Risk Bayesian Shrinkage

One potential problem with using a pure time-series approach is that specific volatilities may not fully persist out-of-sample. In particular, stocks with either extremely low or extremely high specific volatility forecasts tend to revert to the mean.

To remove this bias, we shrink our estimates toward the cap-weighted mean specific volatility for the size decile s_n to which the stock belongs. More precisely, the shrunk estimate σ_n^{SH} is given by

$$\sigma_n^{SH} = v_n \bar{\sigma}(s_n) + (1 - v_n)\,\hat{\sigma}_n \tag{F1}$$

where $\hat{\sigma}_n$ is the original forecast and v_n is the shrinkage intensity that determines the weight given to the Bayesian prior, also known as the shrinkage target,

$$\bar{\sigma}(s_n) = \sum_{n \in S_n} w_n\,\hat{\sigma}_n \tag{F2}$$

where w_n is the capitalization weight of stock n with respect to the size decile. The shrinkage intensity is given by

$$v_n = \frac{q|\hat{\sigma}_n - \bar{\sigma}(s_n)|}{\Delta_\sigma(s_n) + q|\hat{\sigma}_n - \bar{\sigma}(s_n)|} \tag{F3}$$

where q is an empirically determined shrinkage parameter and

$$\Delta_\sigma(s_n) = \sqrt{\frac{1}{N(s_n)} \sum_{n \in S_n} (\hat{\sigma}_n - \bar{\sigma}(s_n))^2} \tag{F4}$$

is the standard deviation of specific risk forecasts within the size decile. The intuition behind this approach is straightforward: the more $\hat{\sigma}_n$ deviates from the mean, the greater the weight we assign to the Bayesian prior $\bar{\sigma}(s_n)$.

Appendix G: Decomposing RMS Returns

We decompose excess stock returns r_n into a systematic component, due to factors, and a stock-specific component u_n. The factor returns f_k are estimated each period by cross-sectional regression

$$r_n = \sum_k X_{nk} f_k + u_n \ , \tag{G1}$$

where X_{nk} is the exposure of stock n to factor k. The specific returns are assumed to be uncorrelated with one another as well as to the other factors.

The total *R*-squared of a regression measures the cross-sectional variation explained by the factors,

$$R_T^2 = 1 - \frac{\sum_n v_n u_n^2}{\sum_n v_n r_n^2} \ , \tag{G2}$$

where v_n is the regression weight of stock n (proportional to square-root of market capitalization). The root mean square (RMS) return, computed as

$$RMS = \sqrt{\sum_n v_n r_n^2} \, , \tag{G3}$$

measures the cross-sectional dispersion from zero return. As described by Menchero and Morozov (2011), the RMS return can be exactly decomposed into the return sources of Equation B1 using a cross-sectional version of the *x-sigma-rho* formula,

$$RMS = \sum_k f_k \sigma(X_k) \rho(X_k, r) + \sigma(u) \rho(u, r) \ , \tag{G4}$$

where $\sigma(X_k)$ is the RMS dispersion of factor k, and $\rho(X_k, r)$ is the cross-sectional correlation between factor k and the asset returns. The last term in Equation B4 represents the contribution to RMS coming from stock-specific sources.

Appendix H: Review of Bias Statistics

H1. Single-Window Bias Statistics

A commonly used measure to assess a risk model's accuracy is the bias statistic. Conceptually, the bias statistic represents the ratio of realized risk to forecast risk.

Let R_{nt} be the return to portfolio n over period t, and let σ_{nt} be the beginning-of-period volatility forecast. Assuming perfect forecasts, the *standardized* return,

$$b_{nt} = \frac{R_{nt}}{\sigma_{nt}}, \qquad \text{(H1)}$$

has an expected standard deviation of 1. The bias statistic for portfolio n is the *realized* standard deviation of standardized returns,

$$B_n = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}\left(b_{nt} - \bar{b}_n\right)^2}, \qquad \text{(H2)}$$

where T is the number of periods in the observation window.

Assuming normally distributed returns and perfect risk forecasts, for sufficiently large T the bias statistic B_n is approximately normally distributed about 1, and roughly 95 percent of the observations fall within the confidence interval,

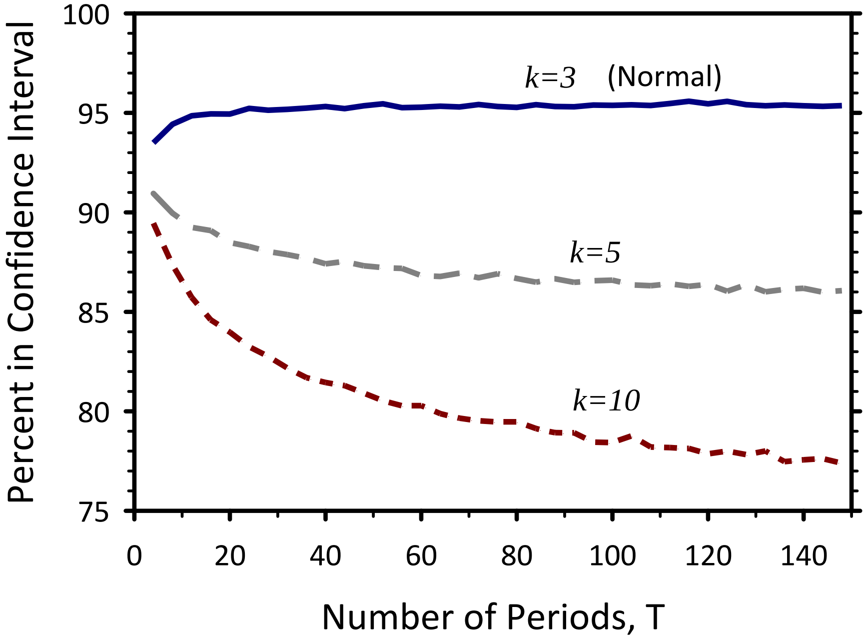
$$B_n \in \left[1-\sqrt{2/T}, 1+\sqrt{2/T}\right]. \qquad \text{(H3)}$$

If B_n falls outside this interval, we reject the null hypothesis that the risk forecast was accurate.

If returns are not normally distributed, however, then fewer than 95 percent of the observations will fall within the confidence interval, even for perfect risk forecasts. In **Figure H1**, we show simulated results for the percentage of observations actually falling within this interval, plotted versus observation window length T, for several values of kurtosis k.

For the normal case (kurtosis $k = 3$), except for the smallest values of T, the confidence interval indeed captures about 95 percent of the observations. As the kurtosis increases, however, the percentage falling within the interval drops significantly. For instance, at a kurtosis level of 5, only 86 percent of bias statistics fall inside the confidence interval for an observation window of 120 periods.

Figure H1. Percent of observations falling within the confidence interval $1 \pm \sqrt{2/T}$, where T is the number of periods in the observation window. Results were simulated using a normal distribution $k = 3$, and using a *t*-distribution with kurtosis values $k = 5$ and $k = 10$. The standard deviations were equal to 1 in all cases. For the normal distribution, the percentage of observations inside the confidence interval quickly approaches 95 percent. As kurtosis is increased, however, the proportion within the confidence interval declines considerably.



H2. Rolling-Window Bias Statistics

The purpose of bias-statistic testing is to assess the accuracy of risk forecasts, typically over a long sample period. One possibility is to select the entire sample period as a single window, and to compute the bias statistic as in Equation H2. This would be a good approach if financial data were stationary, as sampling error is reduced by increasing the length of the window. In reality, however, financial data are not stationary. It is possible to significantly overpredict risk for some years, and underpredict it for others, while ending up with a bias statistic close to 1.

Often, a more relevant question is to study the accuracy of risk forecasts over a window of k observations. For this purpose, we define the rolling window bias statistic for portfolio n,

$$B_n^\tau = \sqrt{\frac{1}{k}\sum_{t=\tau-k+1}^{\tau}\left(b_{nt} - \bar{b}_n\right)^2}, \quad \text{(H4)}$$

where τ denotes the last observation of the window. The windows are rolled forward one observation at a time until reaching the end of the sample period. If T is the number of observations in the sample period, then each portfolio will have $T - k + 1$ (overlapping) k-observation windows.

It is useful to consider, for a collection of N portfolios, the mean of the rolling window bias statistics,

$$\bar{B}^\tau = \frac{1}{N}\sum_n B_n^\tau . \quad \text{(H5)}$$

We also define $B^\tau(5\%)$ and $B^\tau(95\%)$ to be the 5-percentile and 95-percentile values for the rolling window bias statistics at a given point in time.

H3. Q-Statistic

The Q-Statistic is defined as $Q_{nt} = b_{nt}^2 - \ln b_{nt}^2$, where b_{nt} is a standardized return introduced in (H1). The Q-statistic penalizes both under and over forecast and is not prone to "error cancellation" when averaged across time and/or test portfolios. For averaging, we define the mean of Q-statistic as follows:

$$\bar{Q} = \sum_{n=1}^{N}\sum_{t=1}^{T} Q_{nt} \quad \text{(H6)}$$

where N is a number of portfolios and T is a sample size. Further information on Q-Statistic can be found in Patton (2011).

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[1] As of September 30, 2012, as published by eVestment, Lipper and Bloomberg on January 31, 2013 *Apr 2013*