Lexical alignment: IBM models 1 and 2

MLE via EM for categorical distributions

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Translation data

Let's assume we are confronted with a new language
and luckily we managed to obtain some sentence-aligned data

the black dog	$\Box$ $\circledast$
the nice dog	$\Box$ $\cup$
the black cat	$\boxdot$ $\circledast$
a dog chasing a cat	$\boxdot$ $\triangleleft$ $\Box$

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Is there anything we could say about this language?

Translation by analogy

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the nice dog	$\square \; \cup$
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A few hypotheses:

Translation by analogy

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  and perhaps this language is OVS

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- *chasing* may be expressed by $\triangleleft$
  and perhaps this language is OVS
- or perhaps *chasing* is realised by a verb with swapped arguments

Probabilistic lexical alignment models

This lecture is about operationalising this intuition

- through a probabilistic learning algorithm
- for a non-probabilistic approach see for example [Lardilleux and Lepage, 2009]

Content

Word-to-word alignments

Imagine you are given a text

the black dog	o cão preto
the nice dog	o cão amigo
the black cat	o gato preto
a dog chasing a cat	um cão perseguindo um gato

Word-to-word alignments

Now imagine the French words were replaced by placeholders

the black dog	$F_1\ F_2\ F_3$
the nice dog	$F_1\ F_2\ F_3$
the black cat	$F_1\ F_2\ F_3$
a dog chasing a cat	$F_1\ F_2\ F_3\ F_4\ F_5$

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and suppose our task is to have a model explain the original data *by generating each French word from exactly one English word*

Generative story

For each sentence pair independently,

1. observe an English sentence $e_1, \cdots, e_m$
   and a French sentence length n
2. for each French word position j from 1 to n
   2.1 select an English position a_j
   2.2 conditioned on the English word e_{a_j}, generate f_j

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We have introduced an alignment
which is not directly visible in the data

Data augmentation

Observations:

$$\text{the black dog} \mid \text{o cão preto}$$

Imagine data is made of pairs: (a_j, f_j) and $e_{a_j} \to f_j$

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Data augmentation

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the black dog $\Big|$ $(1, E_{A_1} \to F_1)$ $(A_2, E_{A_2} \to F_2)$ $(A_3, E_{A_3} \to F_3)$

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Imagine data is made of pairs: (a_j, f_j) and $e_{a_j} \to f_j$

$$\text{the black dog} \mid (1, \text{the} \to \text{o})\ (3, E_{A_2} \to F_2)\ (A_3, E_{A_3} \to F_3)$$

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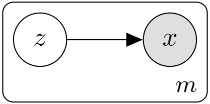
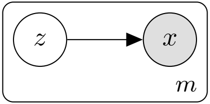
Content

Mixture models



- c mixture components
- each defines a distribution over the same data space $\mathcal{X}$
- plus a distribution over components themselves

Mixture models



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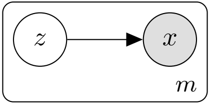
Generative story

1. select a mixture component $z \sim P(Z)$
2. generate an observation from it $x \sim P(X|Z = z)$

Mixture models



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Marginal likelihood

$$
\begin{aligned}
P(x_1^m) &= \prod_{i=1}^{m} \sum_{z=1}^{c} P(X = x_i, Z = z) &(1)\\
&= \prod_{i=1}^{m} \sum_{z=1}^{c} P(Z = z) P(X = x_i | Z = z) &(2)
\end{aligned}
$$

Interpretation

Missing data

- Let Z take one of c mixture components
- Assume data consists of pairs (x, z)
- x is always observed
- y is always missing

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Inference: posterior distribution over possible Z for each x

$$P(Z = z|X = x) = \frac{P(Z = z, X = x)}{\sum_{z'=1}^{c} P(Z = z', X = x)} \tag{3}$$

$$= \frac{P(Z = z)P(X = x|Z = z)}{\sum_{z'=1}^{c} P(Z = z')P(X = x|Z = z')} \tag{4}$$

Non-identifiability

Different parameter settings, same distribution

Suppose $\mathcal{X} = \{a, b\}$ and $c = 2$
and let $P(Z = 1) = P(Z = 2) = 0.5$

Z	$X = a$	$X = b$
1	0.2	0.8
2	0.7	0.3
$P(X)$	0.45	0.55

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Problem for parameter estimation by hillclimbing

Maximum likelihood estimation

Suppose a dataset $\mathcal{D} = \{x^{(1)}, x^{(2)}, \cdots, x^{(m)}\}$

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Maximum likelihood estimation

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the score function is

$$l(\theta) = \sum_{i=1}^{m} \log P_\theta(X = x^{(i)})$$

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the score function is

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then we choose

$$\theta^\star = \arg\max_\theta l(\theta)$$

MLE for categorical: estimation from fully observed data

Suppose we have **complete data**

- $\mathcal{D}_{\text{complete}} = \{(x^{(1)}, z^{(1)}), \ldots, (x^{(m)}, z^{(m)})\}$

MLE for categorical: estimation from fully observed data

Suppose we have **complete data**

- $\mathcal{D}_{\text{complete}} = \{(x^{(1)}, z^{(1)}), \ldots, (x^{(m)}, z^{(m)})\}$

Then, for a **categorical distribution**

$$P(X = x | Z = z) = \theta_{z,x}$$

and $n(z, x | \mathcal{D}_{\text{complete}}) =$ *count of* (z, x) *in* $\mathcal{D}_{\text{complete}}$

MLE solution:

$$P(X = x | Z = z) = \theta_{z,x} = \frac{n(z, x | \mathcal{D}_{\text{complete}})}{\sum_{x'} n(z, x' | \mathcal{D}_{\text{complete}})}$$

MLE for categorical: estimation from incomplete data

Expectation-Maximisation algorithm

E-step:

- for every observation x, imagine that every possible latent assignment z happened with probability $P_\theta(Z = z|X = x)$

$$\mathcal{D}_{\text{completed}} = \{(x, Z = 1), \ldots, (x, Z = c) : x \in \mathcal{D}\}$$

MLE for categorical: estimation from incomplete data

Expectation-Maximisation algorithm

M-step:

- reestimate θ as to climb the likelihood surface
- for categorical distributions $P(X = x|Z = z) = \theta_{z,x}$
  z and x are categorical
  $0 \leq \theta_{z,x} \leq 1$ and $\sum_{x \in X} \theta_{z,x} = 1$

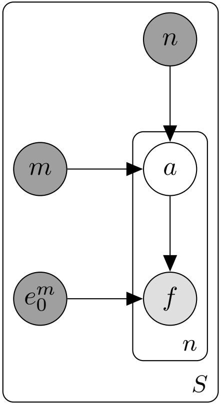
$$\theta_{z,x} = \frac{\mathbb{E}[n(z \to x|\mathcal{D}_{\text{completed}})]}{\sum_{x'} \mathbb{E}[n(z \to x'|\mathcal{D}_{\text{completed}})]} \tag{5}$$

$$= \frac{\sum_{i=1}^{m} \sum_{z'} P(z'|x^{(i)}) \mathbb{1}_z(z') \mathbb{1}_x(x^{(i)})}{\sum_{i=1}^{m} \sum_{x'} \sum_{z'} P(z'|x^{(i)}) \mathbb{1}_z(z') \mathbb{1}_{x'}(x^{(i)})} \tag{6}$$

$$= \frac{\sum_{i=1}^{m} P(z|x^{(i)}) \mathbb{1}_x(x^{(i)})}{\sum_{i=1}^{m} \sum_{x'} P(z|x^{(i)}) \mathbb{1}_{x'}(x^{(i)})} \tag{7}$$

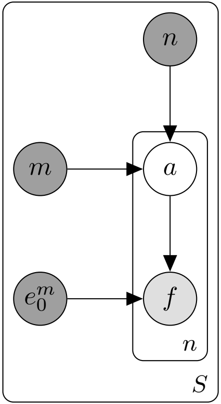
Content

IBM1: a constrained mixture model



Constrained mixture model

IBM1: a constrained mixture model



Constrained mixture model

- mixture components are English words

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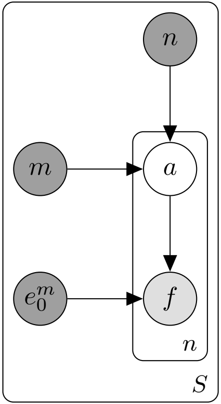

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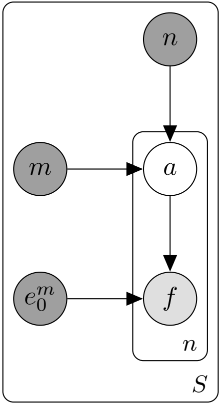


Constrained mixture model

- mixture components are English words
- but only English words that appear in the English sentence can be assigned
- a_j acts as an indicator for the mixture component that generates French word f_j
- e_0 is occupied by a special NULL component

Parameterisation

Alignment distribution: uniform

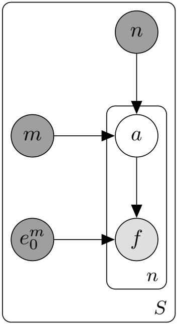
$$P(A|M = m, N = n) = \frac{1}{m+1} \tag{8}$$

Lexical distribution: categorical

$$P(F|E = e) = \mathrm{Cat}(F|\theta_e) \tag{9}$$

- where $\theta_e \in \mathbb{R}^{v_F}$
- $0 \leq \theta_{e,f} \leq 1$
- $\sum_f \theta_{e,f} = 1$

IBM1: marginal likelihood



Marginal likelihood

$$P(f_1^n|e_0^m) = \sum_{a_1=0}^{m} \cdots \sum_{a_n=0}^{m} P(f_1^n, a_1^n|e_{a_j}) \tag{10}$$

$$= \sum_{a_1=0}^{m} \cdots \sum_{a_n=0}^{m} \prod_{j=1}^{n} P(a_j|m,n)P(f_j|e_{a_j}) \tag{11}$$

$$= \prod_{j=1}^{n} \sum_{a_j=0}^{m} P(a_j|m,n)P(f_j|e_{a_j}) \tag{12}$$

IBM1: posterior

Posterior

$$P(a_1^n | f_1^n, e_0^m) = \frac{P(f_1^n, a_1^n | e_0^m)}{P(f_1^n | e_0^m)} \tag{13}$$

Factorised

$$P(a_j | f_1^n, e_0^m) = \frac{P(a_j | m, n) P(f_j | e_{a_j})}{\sum_{i=0}^{m} P(i | m, n) P(f_j | e_i)} \tag{14}$$

MLE via EM

E-step:

$$\mathbb{E}[n(\mathsf{e} \to \mathsf{f}|A_1^n)] = \sum_{a_1=0}^{m} \cdots \sum_{a_n=0}^{m} P(a_1^n|f_1^n, e_0^m) n(\mathsf{e} \to \mathsf{f}|A_1^n) \tag{15}$$

$$= \sum_{a_1=0}^{m} \cdots \sum_{A_n=0}^{m} \prod_{j=1}^{n} P(a_j|f_1^n, e_0^m) \mathbb{1}_{\mathsf{e}}(e_{a_j}) \mathbb{1}_{\mathsf{f}}(f_j) \tag{16}$$

$$= \prod_{j=1}^{n} \sum_{i=0}^{m} P(A_j = i|f_1^n, e_0^m) \mathbb{1}_{\mathsf{e}}(e_i) \mathbb{1}_{\mathsf{f}}(f_j) \tag{17}$$

M-step:

$$\theta_{e,f} = \frac{\mathbb{E}[n(e \to f|A_1^n)]}{\sum_{f'} \mathbb{E}[n(e \to f'|A_1^n)]} \tag{18}$$

EM algorithm

Repeat until convergence to a local optimum

1. For each sentence pair
    1.1 compute posterior per alignment link
    1.2 accumulate fractional counts
2. Normalise counts for each English word

Content

Alignment distribution

Positional distribution

$P(A_j|M = m, N = n) = \mathrm{Cat}(A|\lambda_{j,m,n})$

- one distribution for each tuple (j, m, n)
- support must include length of longest English sentence
- extremely over-parameterised!

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- extremely over-parameterised!

Jump distribution [Vogel et al., 1996]

- define a jump function $\delta(a_j, j, m, n) = a_j - \left\lfloor j\frac{m}{n} \right\rfloor$
- $P(A_j|m, n) = \mathrm{Cat}(\Delta|\lambda)$
- Δ takes values from $-$longest to $+$longest

Content

Limitations of IBM1-2

- too strong independence assumptions
- categorical parameterisation suffers from data sparsity
- EM suffers from local optima

Extensions

Dirichlet priors and posterior inference
[Mermer and Saraclar, 2011]

Log-linear distortion parameters and variational Bayes
[Dyer et al., 2013]

First-order dependency (HMM)
[Vogel et al., 1996]

- E-step requires dynamic programming
  [Baum and Petrie, 1966]

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