

# Latent-variable CRF for SMT

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# Conditional random field

Lafferty et al. [2001]

$$P(y|x, w) = \frac{\exp(w^\top \phi(x, y))}{\sum_{y' \in \mathcal{Y}(x)} \exp(w^\top \phi(x, y'))} \quad (1)$$

- $\phi$  is a feature function mapping  $(x, y)$  to  $\mathbb{R}^d$
- $w$  is a feature vector in  $\mathbb{R}^d$
- $Z(x|w) = \sum_{y \in \mathcal{Y}(x)} \exp(w^\top \phi(x, y))$  must be finite

# Flexible (overlapping) features

## Examples

**As**<sub>1</sub> **meninas**<sub>2</sub> **foram**<sub>3</sub> **pra**<sub>4</sub> **lá**<sub>4</sub>  $\leftrightarrow$  **The**<sub>1</sub> **girls**<sub>2</sub> **went**<sub>3</sub> **over**<sub>4</sub> **there**<sub>4</sub>

**Apague**<sub>1</sub> **a**<sub>2</sub> **luz**<sub>3</sub>  $\leftrightarrow$  **Switch**<sub>1</sub> **the**<sub>2</sub> **light**<sub>3</sub> **off**<sub>1</sub>

Hard to account for with directed models  
(due to causality assumptions)

# Global normalisation

Local models are trained with positive context only

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What happens when we query the model with predicted contexts?

# Maximum likelihood estimation

Likelihood of an observation  $(x, y)$

$$\mathcal{L}(w|x, y) = \log P(y|x, w) \quad (2)$$

$$= w^\top \phi(x, y) - \log Z(x|w) \quad (3)$$

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Gradient-based optimisation

$$\nabla_w \mathcal{L}(w|x, y) = \phi(x, y) - \mathbb{E}_{P(Y|X=x, w)}[\phi(X, Y)] \quad (4)$$



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Expected features should match the features of the observation

# LV-CRF

## Model

$$P(y, d|x) = \frac{\exp(w^\top \phi(x, y, d))}{\sum_{y' \in \mathcal{Y}(x)} \sum_{d' \in \mathcal{D}(x, y')} \exp(w^\top \phi(x, y', d'))} \quad (5)$$

- $d$  is latent
- $Z(x, y|w) = \sum_{d \in \mathcal{D}(x, y)} \exp(w^\top \phi(x, y, d))$  must be finite

# MLE for LV-CRF

Likelihood of an observation  $(x, y)$

$$\mathcal{L}(w|x, y) = \log P(y|x, w) \tag{6}$$

$$= \log \sum_{d \in \mathcal{D}(x, y)} P(y, d|x, w) \tag{7}$$

# MLE for LV-CRF

Likelihood of an observation  $(x, y)$

$$\mathcal{L}(w|x, y) = \log P(y|x, w) \quad (6)$$

$$= \log \sum_{d \in \mathcal{D}(x, y)} P(y, d|x, w) \quad (7)$$

Gradient-based optimisation [Mann and McCallum, 2007]

$$\nabla_w \mathcal{L}(w|x, y) = \mathbb{E}_{P(D|X=x, Y=y, w)}[\phi(X, Y, D)] \quad (8)$$

$$- \mathbb{E}_{P(Y, D|X=x, w)}[\phi(X, Y, D)] \quad (9)$$

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Undirected models are considerably harder to learn

- expensive global normalisation
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- Alignment: Dyer et al. [2011]
- SMT: Blunsom et al. [2008] and Blunsom and Osborne [2008]

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Particularly hard with latent variables

- Alignment: Dyer et al. [2011]
- SMT: Blunsom et al. [2008] and Blunsom and Osborne [2008]
- Approximate techniques
  - Contrastive divergence [Hinton, 2002]
  - Contrastive estimation [Smith and Eisner, 2005]
  - Piecewise training [Sutton and McCallum, 2005]



# SMT with CRFs

Blunsom et al. [2008]

- $d$  is a derivation complying with a *hieroglyph* grammar
- $\phi$  featurises steps in a synchronous derivation

$$P(y, d|x) = \frac{\exp \left( \sum_{r_{s,t} \in d} w^\top \phi(r_{s,t}|x, y, d) \right)}{\sum_{d' \in \mathcal{D}(x)} \exp \left( \sum_{r_{s,t} \in d'} w^\top \phi(r_{s,t}|x, y', d') \right)}$$

- $\mathcal{D}(x)$  is the space of derivations over target strings aligned to the source string  $x$
- in the denominator  $y'$  is defined implicitly as  $\text{yield}(d')$
- $r_{s,t}$  is a synchronous rule decorated with a source span  $s$  and a target span  $t$

# ITG with CRFs

In Project 2, you will use an ITG

$$S \rightarrow X$$

$$X \rightarrow X X$$

$$X \rightarrow x/y \text{ for all } x \in \Sigma \text{ and } y \in \Delta$$

$$X \rightarrow \epsilon/y \text{ for all } y \in \Delta$$

$$X \rightarrow x/\epsilon \text{ for all } x \in \Sigma$$

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Normaliser may diverge for certain  $w \in \mathbb{R}^d$

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Solution: constrain strings by length

- introduce a constrain  $n$  that depends on  $x$
- make  $\mathcal{D}_n(x)$  such that  $|\text{yield}(d)| \leq n$  for  $d \in \mathcal{D}_n(x)$

- $$\sum_{d \in \mathcal{D}_n(x)} \exp \left( \sum_{r,s,t \in d} w^\top \phi(r_{s,t} | x, y, d) \right)$$

# Learning

Likelihood of an observation  $(x, y, n)$

$$\mathcal{L}(w|x, y, n) = \log P(y|x, n, w) \quad (10)$$

$$= \log \sum_{d \in \mathcal{D}(x, y)} P(y, d|x, n, w) \quad (11)$$

note that  $\mathcal{D}(x, y)$  is always finite

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Gradient-based optimisation

$$\nabla_w \mathcal{L}(w|x, y, n) = \underbrace{\mathbb{E}_{P(D|X=x, Y=y, n, w)}[\phi(X, Y, D)]}_{\text{expected features for observation } (x, y)} \quad (12)$$

$$- \underbrace{\mathbb{E}_{P(Y, D|X=x, n, w)}[\phi(X, Y, D)]}_{\text{expected features for observation } x} \quad (13)$$

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finite set of derivations over target strings that align to the source string  $x$  where the target string is no longer than  $n$  words

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set of derivations of the string pair  $(x, y)$
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finite set of derivations over target strings that align to the source string  $x$  where the target string is no longer than  $n$  words
- ➌  $\mathcal{D}(x, y)$   
set of derivations of the string pair  $(x, y)$
- ➍ expected feature vector of observations  $(x, y)$
- ➎ expected feature vector of an observation  $x$



# Notation

Using a hypergraph view

- $u, v, s$  are nodes
- $e$  is an edge
- $\text{head}(e)$  is a node
- $\text{tail}(e)$  is a sequence of nodes
- if  $u \in \text{tail}(e)$  and  $v \in \text{tail}(e)$ ,  $u$  and  $v$  are siblings
- Backward-star:  $BS(u)$  is the set of edges incoming to  $u$   
 $u$  is the head of the edge
- Forward-star:  $FS(u)$  is the set of edges outgoing from  $u$   
 $u$  is in the tail of the edge
- $w(e)$  is the weight of an edge

# Inside

The INSIDE recursion

$$I(v) = \begin{cases} \bar{1} & \text{if } BS(v) = \emptyset \\ \bigoplus_{e \in BS(v)} \omega(e) \otimes \bigotimes_{u \in \text{tail}(e)} I(u) & \text{otherwise} \end{cases} \quad (14)$$

For acyclic hypergraphs

# Outside

The OUTSIDE recursion

$$O(v) = \begin{cases} \bar{1} & \text{if } FS(v) = \emptyset \\ \bigoplus_{e \in FS(v)} \omega(e) \otimes O(\text{head}(e)) \otimes \bigotimes_{s \in \text{tail}(e) \setminus \{v\}} I(s) & \text{otherwise} \end{cases} \quad (15)$$

For acyclic hypergraphs

# Topsort

```
1: function TOPSORT( $G = \langle V, \langle E, w \rangle \rangle$ )
2:    $S = \{v \in V : BS(v) = \emptyset\}$   $\triangleright$  nodes with no dependencies
3:    $D = \{v \mapsto \{u : \exists e \in BS(v) \wedge u \in \text{tail}(e)\} : v \in V\}$ 
4:    $\triangleright$  a node depends on all of its children
5:    $L = \langle \rangle$   $\triangleright$  top-sorted nodes
6:   while  $S \neq \emptyset$  do
7:      $u \leftarrow \text{pop}(S)$   $\triangleright$  remove and return a node from  $S$ 
8:      $L \leftarrow L + \langle u \rangle$   $\triangleright$  append  $u$  to  $L$ 
9:     for  $e$  in  $FS(u)$  do  $\triangleright$  outgoing edges from  $u$ 
10:       $v \leftarrow \text{head}(e)$   $\triangleright$  parent of  $u$  in  $e$ 
11:       $D(v) \leftarrow D(v) \setminus \{u\}$   $\triangleright$  remove  $u$  from  $D(v)$ 
12:      if  $D(v) == \emptyset$  then  $\triangleright v$ 's dependencies have been sorted
13:         $S \leftarrow S \cup \{v\}$ 
14:      end if
15:    end for
16:  end while
17:  return  $L$ 
18: end function
```

# Inside

```
1: function INSIDE( $G = \langle V, \langle E, w \rangle \rangle$ )
2:   for  $v$  in TOPSORT( $G$ ) do                                ▷ visit nodes bottom-up
3:     if  $BS(v) == \emptyset$  then
4:        $I[v] \leftarrow \bar{1}$                                     ▷ leaves
5:     else
6:        $I[v] \leftarrow \bar{0}$ 
7:       for  $e \in BS(v)$  do
8:          $k \leftarrow w(e)$                                     ▷ include the edge's own weight
9:         for  $u$  in tail( $e$ ) do
10:           $k \leftarrow k \otimes I[u]$ 
11:        end for
12:         $I[v] \leftarrow I[v] \oplus k$                             ▷ accumulate for each edge
13:      end for
14:    end if
15:  end for
16:  return  $I$ 
17: end function
```

# Outside

```
1: function OUTSIDE( $G = \langle V, \langle E, w \rangle \rangle, I, \text{root}$ )
2:    $O[v] \leftarrow \bar{0}$  for  $v \in V$ 
3:    $O[\text{root}] \leftarrow \bar{1}$ 
4:   for  $v$  in REVERSE(TOPSORT( $G$ )) do
5:     for  $e \in BS(v)$  do
6:       for  $u \in \text{tail}(e)$  do
7:          $k \leftarrow w(e) \otimes O[v]$ 
8:         for  $s$  in tail( $e$ ) do
9:           if  $u \neq s$  then
10:             $k \leftarrow k \otimes I[s]$ 
11:          end if
12:        end for
13:         $O[u] \leftarrow O[u] \oplus k$ 
14:      end for
15:    end for
16:  end for
17:  return  $O$ 
18: end function
```

▷ this is the goal node

▷ visit nodes top-down

▷  $q$ 's incoming edges

▷ children of  $v$  in  $e$

▷ siblings of  $u$  in  $e$

▷  $u$  itself is excluded

▷ accumulate it for  $u$

# Expected features

```
1: function EXPECTEDFEATURES( $G = \langle V, \langle E, w \rangle \rangle, I, O, \phi$ )
2:    $\bar{\phi} \leftarrow 0$ 
3:   for  $e \in E$  do                                     ▷ these are edges
4:      $k \leftarrow O[\text{head}(e)]$ 
5:     for  $u$  in  $\text{tail}(e)$  do
6:        $k \leftarrow k \otimes I[u]$ 
7:     end for
8:      $\bar{\phi} \leftarrow \bar{\phi} + k\phi(e)$ 
9:   end for
10:  return  $\bar{\phi}$                                            ▷ expected feature vector
11: end function
```

# Traversals

## Viterbi derivation

- 1 start from the goal (root)
- 2 recursively rewrite every symbol  $v$  by solving

$$e^{\star} = \arg \max_{e \in BS(v)} w(e) \bigotimes_{u \in \text{tail}(e)} I(u)$$



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## Sampling

- 1 start from the goal (root)
- 2 recursively rewrite every symbol  $v$  by solving

$$E \sim P(e|v) = \begin{cases} \bar{0} & \text{if } e \notin BS(v) \\ \frac{w(e) \bigotimes_{u \in \text{tail}(e)} I(u)}{I(v)} & \text{otherwise} \end{cases}$$

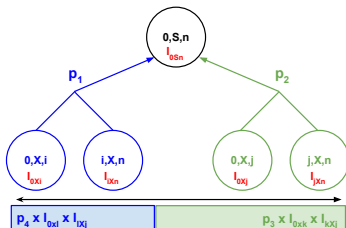
# Viterbi

Use Inside computed in the Max-Times semiring!



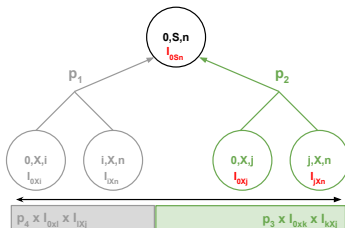
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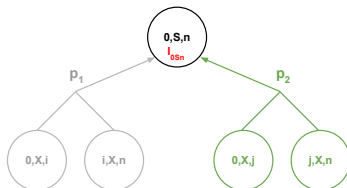
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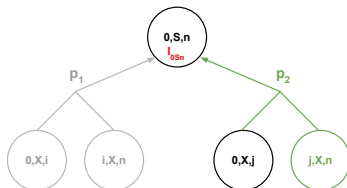
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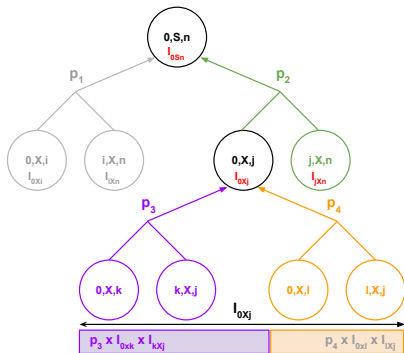
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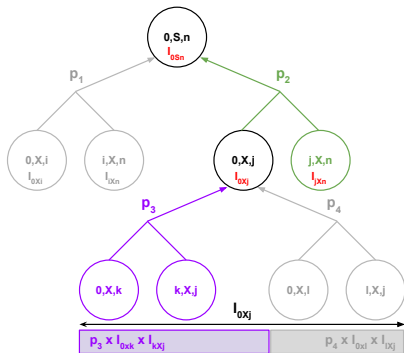
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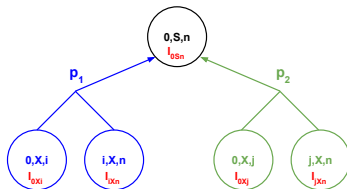
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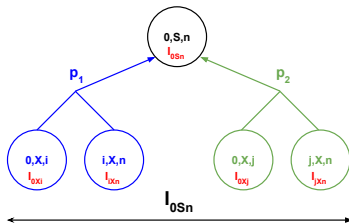
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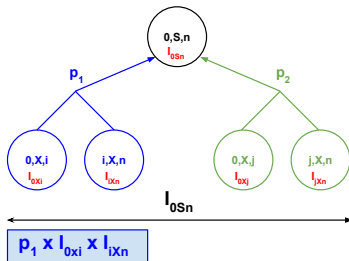
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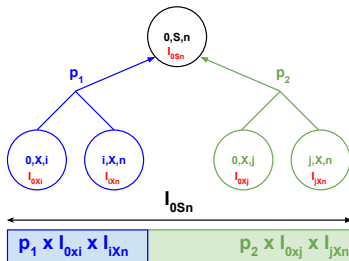
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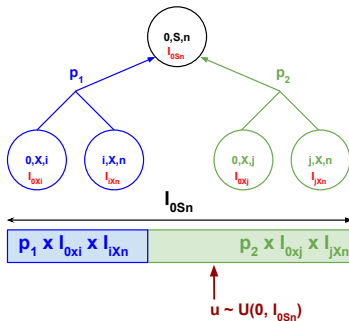
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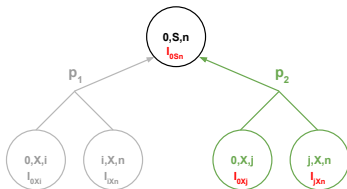
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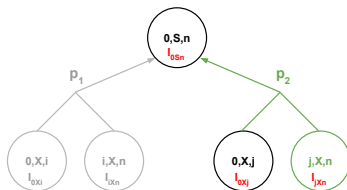
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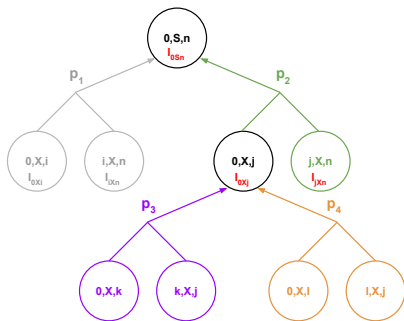
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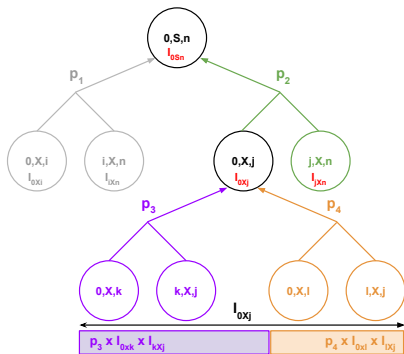
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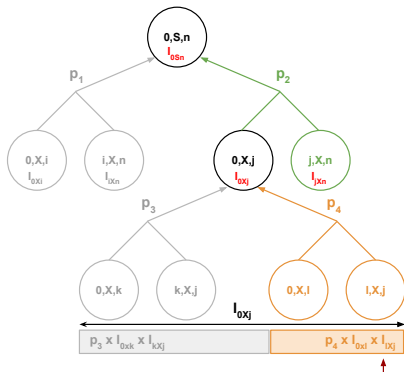
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# Roadmap to project 2

- ① understand the parser
  - check notes and notebook
- ② implement the constrained forest  $\mathcal{D}_n(x)$
- ③ implement feature functions
- ④ implement hypergraph algorithms
  - topsort, inside, outside, expected features
- ⑤ iterate over mini-batches of data making gradient updates

Questions?

# References I

- Phil Blunsom and Miles Osborne. Probabilistic inference for machine translation. In *Proceedings of the 2008 Conference on Empirical Methods in Natural Language Processing*, pages 215–223, Honolulu, Hawaii, October 2008. Association for Computational Linguistics. URL <http://www.aclweb.org/anthology/D08-1023>.
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