

# Bitext parsing

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# Context-Free Grammars

A **CFG** grammar  $G$  is denoted by

- a set of **nonterminal** symbols  $N$
- a set of **terminal** symbols  $\Sigma$  with  $\Sigma \cap N = \emptyset$
- a set  $R$  of **rules** of the form  $X \rightarrow \alpha$  where
  - $X \in N$  and  $\alpha \in (\Sigma \cup N)^*$
- $S \in N$  a distinguished **start** symbol

Let  $\varepsilon$  denote an **empty** string

# Example CFG

$S \rightarrow NP VP$

$Vi \rightarrow \text{sleeps}$

$VP \rightarrow Vi$

$Vt \rightarrow \text{saw}$

$VP \rightarrow Vt NP$

$NN \rightarrow \text{man}$

$VP \rightarrow VP PP$

$NN \rightarrow \text{dog}$

$NP \rightarrow DT NN$

$NN \rightarrow \text{telescope}$

$NP \rightarrow NP PP$

$DT \rightarrow \text{the}$

$PP \rightarrow IN NP$

$IN \rightarrow \text{with}$

# Generative Device

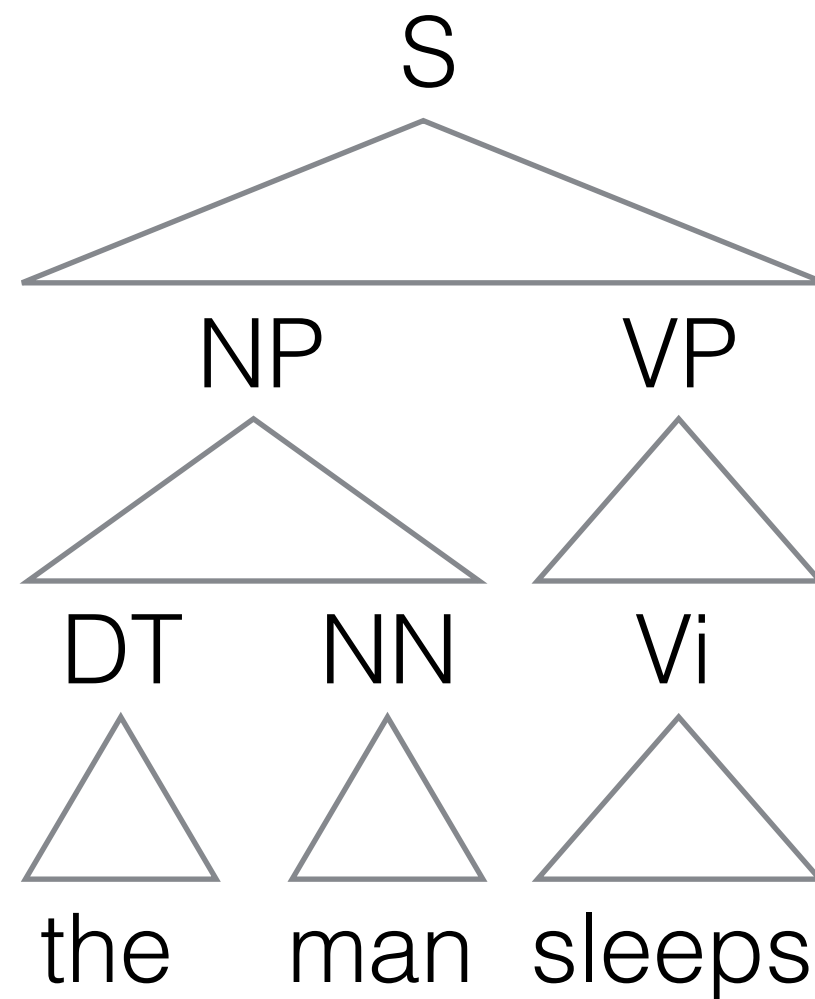
Left-most derivation

- sequence of strings  $\mathbf{s}_1 \dots \mathbf{s}_n$ 
  - $\mathbf{s}_1 = S$
  - $\mathbf{s}_n \in \Sigma^*$
  - $\mathbf{s}_{i \geq 2}$  derived from  $\mathbf{s}_{i-1}$  by picking the left-most nonterminal  $X$ 
    - replacing it by some  $\alpha$  such that  $X \rightarrow \alpha \in R$

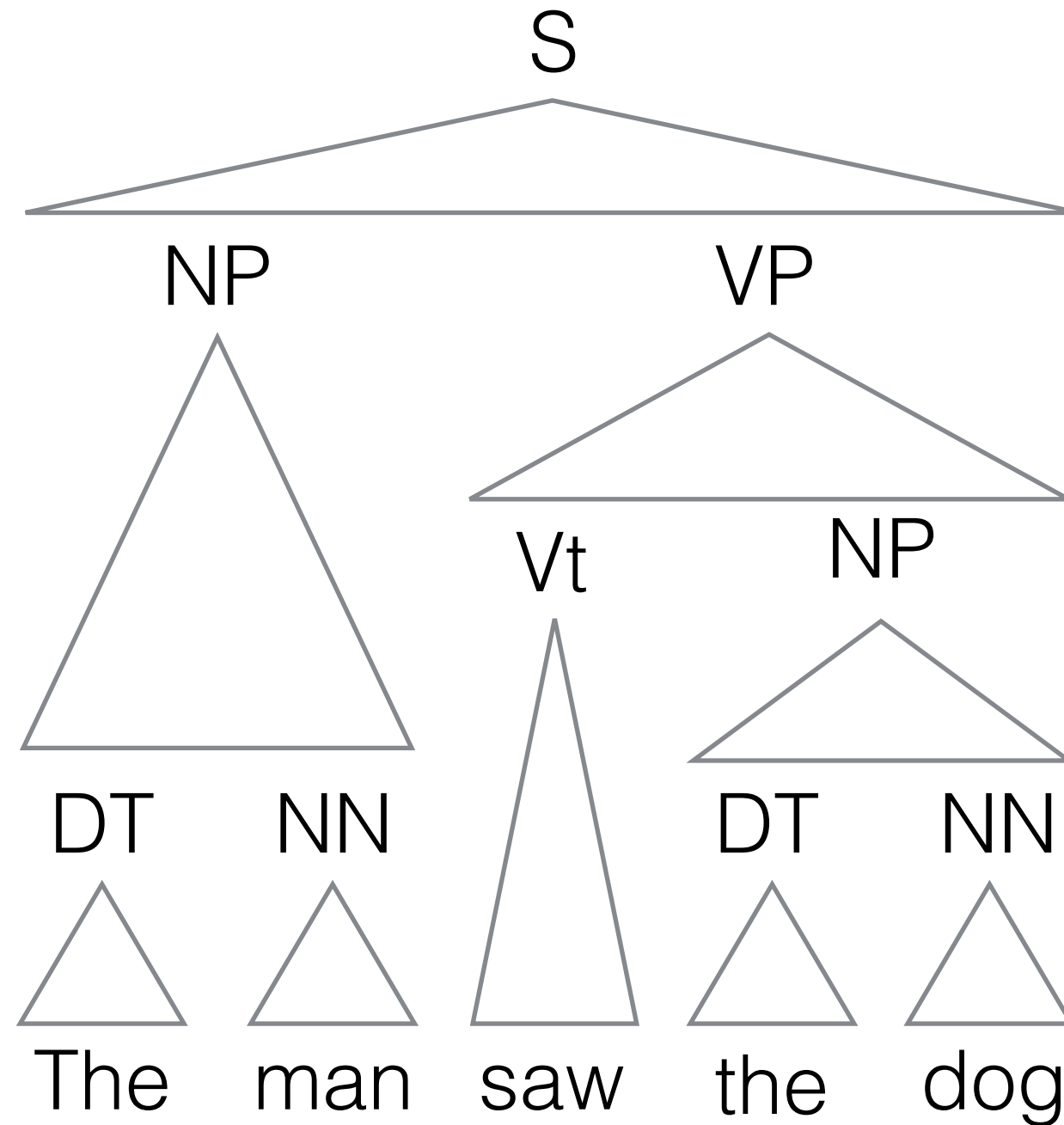
# Example of Derivation

		Substitution
<b>s</b> <sub>1</sub> =	S	S → NP VP
<b>s</b> <sub>2</sub> =	NP VP	NP → DT NN
<b>s</b> <sub>3</sub> =	DT NN VP	DT → the
<b>s</b> <sub>4</sub> =	the NN VP	NN → man
<b>s</b> <sub>5</sub> =	the man VP	VP → Vi
<b>s</b> <sub>6</sub> =	the man Vi	Vi → sleeps
<b>s</b> <sub>7</sub> =	the man sleeps	
<b>s</b> <sub>7</sub> =	S ⇒* the man sleeps	

# Example of Generation



# Example of Recognition



# Language

A string  $\mathbf{s} = s_1 \dots s_n$  is generated/accepted by  $G$  if

$$S \Rightarrow^* \mathbf{s}$$

$\Rightarrow^*$  denotes a sequence of rule applications

Language of  $G$

$$L(G) = \{\mathbf{s} : S \Rightarrow^* \mathbf{s}\} \subseteq \Sigma^*$$



# Chomsky Normal Form

Every CFG is weakly equivalent to another such that

- $X \rightarrow YZ$  where  $X, Y, Z \in N$
- $X \rightarrow w$  where  $w \in \Sigma$
- and possibly  $S \rightarrow \varepsilon$

[Hopcroft and Ullman, 1979]

# Parsing as Deduction

Deductive process to prove claims about grammaticality  
[Shieber et al., 1995]

- focus on strategy rather than implementation
- soundness/completeness easier to prove
- complexity determined by inspection
- dynamic program follows directly
- generality

# Deductive systems

**Item:** a statement / intermediate sound result

- formula or schemata expressed with variables

**Inference rule:** statement derived from existing items

- $\frac{A_1 \dots A_m}{B}$  (condition) where  $A_i$  and  $B$  are items
  - $A_i$  are called antecedents
  - $B$  is called consequent

# Deductive program

**Axioms:** trivial items

- do not depend on previous statements

**Goal:** states that a proof exists

**Proof:**

- start from axioms
- exhaustively deduce items
  - never twice under the same premises
- accept if goal is proven

# Shift-Reduce Example



Input: *the man sleeps*

Rule	Condition	Statement	Queue
Axiom		1 [•,0]	1
Shift: [1]		2 [the•,1]	2
Reduce: [2]	DT → the	3 [DT•,1]	3
Shift: [3]		4 [DT man •, 2]	4
Reduce: [4]	NN → man	5 [DT NN •, 2]	5
Reduce: [5]	NP → DT NN	6 [NP •, 2]	6
Shift: [6]		7 [NP sleeps •, 3]	7
Reduce: [7]	Vi → sleeps	8 [NP Vi •, 3]	8
Reduce: [8]	VP → Vi	9 [NP VP •, 3]	9
Reduce: [9]	S → NP VP	10 [S •, 3]	10
GOAL: [10]			∅

S → NP VP

VP → Vi

VP → Vt NP

VP → VP PP

NP → DT NN

NP → NP PP

PP → IN NP

Vi → sleeps

Vt → saw

NN → man

NN → dog

NN → telescope

DT → the

IN → with

# Shift-Reduce

**Input:**  $G$  and  $w_1 \dots w_n$

**Item form:**  $[\alpha \bullet, j]$   
 asserts that  $\alpha \Rightarrow^* w_1 \dots w_j$  or  
 that  $\alpha w_{j+1} \dots w_n \Rightarrow^* w_1 \dots w_j$

**Axiom:**  $[\bullet, 0]$

**Goal:**  $[S \bullet, n]$

**Scan (shift)**

asserts that  $\alpha w_{j+1} \Rightarrow^* w_1 \dots w_j w_{j+1}$

**Complete (reduce)**

asserts that  $\alpha B \Rightarrow^* w_1 \dots w_j$

$$\text{SHIFT} \quad \frac{[\alpha \bullet, j]}{[\alpha w_{j+1}, j + 1]}$$

$$\text{REDUCE} \quad \frac{[\alpha \gamma \bullet, j]}{[\alpha B \bullet, j]} \quad B \rightarrow \gamma \in R$$

# Top-Down recognition

**Input:**  $G$  and  $w_1 \dots w_n$

**Item form:**  $[\bullet\beta, j]$   
 asserts that  $S \Rightarrow^* w_1 \dots w_j \beta$

**Axiom:**  $[\bullet S, 0]$

**Goal:**  $[\bullet, n]$

**Scan**

asserts that  $S \Rightarrow^* w_1 \dots w_j w_{j+1} \beta$

$$\text{SCAN} \quad \frac{[\bullet w_{j+1} \beta, j]}{[\bullet \beta, j+1]}$$

**Predict**

asserts that  $S \Rightarrow^* w_1 \dots w_j B \beta$

$$\text{PREDICT} \quad \frac{\bullet B \beta, j}{[\bullet \gamma \beta, j]} \quad B \rightarrow \gamma \in R$$

# Top-Down Example

Input: *the man sleeps*

Rule	Condition	Statement	Queue
Axiom		1 [• S, 0]	1
Predict: [1]	$S \rightarrow NP VP$	2 [• NP VP, 0]	2
Predict: [2]	$NP \rightarrow DT NN$	3 [• DT NN VP, 0]	3
Predict: [3]	$DT \rightarrow the$	4 [• the NN VP, 0]	4
Scan: [4]		5 [• NN VP, 1]	5
Predict: [5]	$NN \rightarrow man$	6 [• man VP, 1]	6
Scan: [6]		7 [• VP, 2]	7
Predict: [7]	$VP \rightarrow Vi$	8 [• Vi, 2]	8, 9, 10
	$VP \rightarrow Vt NP$	9 [• Vt NP, 2]	
Predict: [8]	$Vi \rightarrow sleeps$	10 [• sleeps, 2]	9, 10
		11	10
Scan: [10]		13 [•, 3]	13
GOAL: [13]			∅

$S \rightarrow NP VP$

$VP \rightarrow Vi$

$VP \rightarrow Vt NP$

~~$VP \rightarrow VP PP$~~

$NP \rightarrow DT NN$

$NP \rightarrow NP PP$

$PP \rightarrow IN NP$

$Vi \rightarrow sleeps$

$Vt \rightarrow saw$

$NN \rightarrow man$

$NN \rightarrow dog$

$NN \rightarrow telescope$

$DT \rightarrow the$

$IN \rightarrow with$



# CKY - CNF only

**Input:**  $G$  and  $s = w_1 \dots w_n$       **Item form:**  $[i, X, j]$   
asserts that  $X \Rightarrow^* w_{i+1} \dots w_j$

**Axioms:**  $[i, X, i+1] \quad X \rightarrow w_i \in R$

**Goal:**  $[0, S, n]$

**Merge:**  
asserts that 
$$\frac{[i, A, k] \quad [k, B, j]}{[i, C, j]} \quad C \rightarrow AB \in R$$
  
 $w_{i+1} \dots w_k w_{k+1} \dots w_j \Rightarrow^* w_{i+1} \dots w_j$

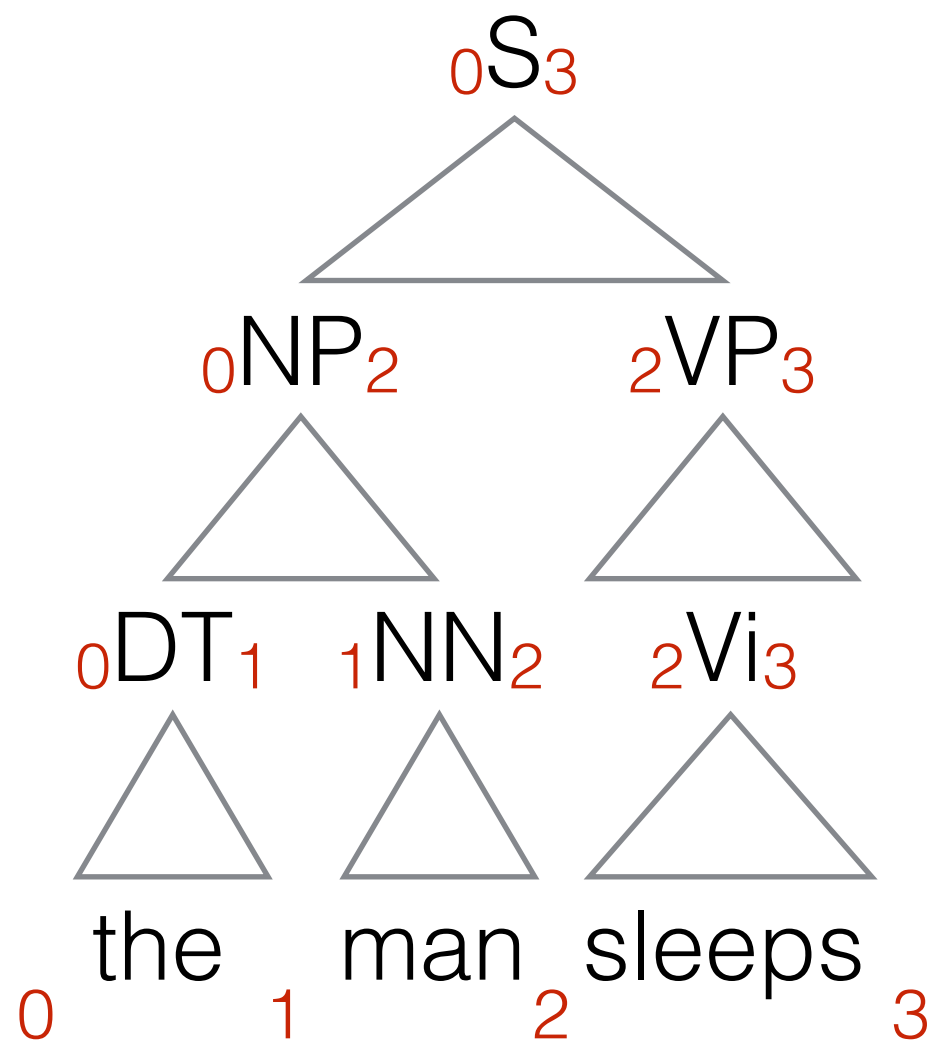
# CKY Example

Input: *the man saw the dog*

<del>S</del> → NP VP	Vi → sleeps
<del>VP</del> → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Rule	Condition	Statement	Queue	Passive
Axiom	DT → the	1 [0, DT, 1]	1	
	NN → man	2 [1, NN, 2]	1, 2	
	Vt → saw	3 [2, Vt, 3]	1, 2, 3	
	DT → the	4 [3, DT, 4]	1, 2, 3, 4	
	NN → dog	5 [4, NN, 5]	1, 2, 3, 4, 5	
			2, 3, 4, 5	1
Merge: [1][2]	NP → DT NN	6 [0, NP, 2]	3, 4, 5, 6	2
			4, 5, 6	3
			5, 6	4
Merge: [4][5]	NP → DT NN	7 [3, NP, 5]	6, 7	5
			7	6
Merge: [3][7]	VP → Vt NP	8 [2, VP, 5]	8	7
Merge: [6][8]	S → NP VP	9 [0, S, 5]	9	8
GOAL: [9]			∅	9

# Rule Segmentation: "Split Points"



${}^0S_3 \rightarrow {}^0NP_2 {}^2VP_3$

${}^0NP_2 \rightarrow {}^0DT_1 {}^1NN_2$

${}^2VP_3 \rightarrow {}^2Vi_3$

${}^0DT_1 \rightarrow \text{the}$

${}^1NN_2 \rightarrow \text{man}$

${}^2Vi_3 \rightarrow \text{sleeps}$

# "Dotted items"

Parsing a CNF grammar is easy because we know the shape of rules

When that's not the case, we have to **scan rules symbol by symbol** using a general mechanism:

**Item form:**  $[i, X \rightarrow \alpha \blacksquare \bullet \beta \square, j]$  where  $X \rightarrow \alpha \beta \in R$  is a rule

- In general, we segment rules with respect to the input  $w_1 \dots w_n$
- The dot represents progress through the rule's right-hand side (RHS)
- The prefix  $\alpha$  has already been parsed and we are waiting for  $\beta$
- The filled box represents a segmentation of  $[0 .. j]$  into  $|\alpha|$  adjacent parts
- The empty box has no actual role, it's just a reminder that the segmentation beyond  $j$  is unknown

# CKY+

**Input:**  $G$  and  $s = w_1 \dots w_n$

**Item form:**  $[i, X \rightarrow \alpha \blacksquare \bullet \beta \square, j]$   
asserts that  $X \Rightarrow^* w_{i+1} \dots w_j \beta$

**Axioms:**  $[i, X \rightarrow w_i \bullet \alpha \square, i+1]$   $X \rightarrow w_i \alpha \in R$   
 $[i, X \rightarrow \varepsilon \bullet, i]$   $X \rightarrow \varepsilon \in R$

**Goal:**  $[0, S \rightarrow \alpha \blacksquare \bullet, n]$

**Scan**

$$\frac{[i, X \rightarrow \alpha \blacksquare \bullet w_{j+1} \beta \square, j]}{[i, X \rightarrow \alpha \blacksquare w_{j+1} \bullet \beta \square, j+1]}$$

**Prefix**

$$\frac{[i, Y \rightarrow \alpha \blacksquare \bullet, j]}{[i, X \rightarrow Y_{i,j} \bullet \beta \square, j]}$$

$X \rightarrow Y \beta \in R$

**Complete**

$$\frac{[i, X \rightarrow \alpha \blacksquare \bullet Y \beta \square, k] [k, Y \rightarrow \gamma \blacksquare \bullet, j]}{[i, X \rightarrow \alpha \blacksquare Y_{k,j} \bullet \beta \square, j]}$$

# CKY + Example

Input: *the man sleeps*

$S \rightarrow NP VP$	$Vi \rightarrow \text{sleeps}$
$VP \rightarrow Vi$	$Vt \rightarrow \text{saw}$
$VP \rightarrow Vt NP$	$NN \rightarrow \text{man}$
$VP \rightarrow VP PP$	$NN \rightarrow \text{dog}$
$NP \rightarrow DT NN$	$NN \rightarrow \text{telescope}$
$NP \rightarrow NP PP$	$DT \rightarrow \text{the}$
$PP \rightarrow IN NP$	$IN \rightarrow \text{with}$

Rule	Condition	Item	Active	Passive
Axiom	$DT \rightarrow \text{the}$	1 [0, $DT \rightarrow \text{the} \bullet, 1$ ]	1	
	$NN \rightarrow \text{man}$	2 [1, $NN \rightarrow \text{man} \bullet, 2$ ]	1, 2	
	$Vi \rightarrow \text{sleeps}$	3 [2, $Vi \rightarrow \text{sleeps} \bullet, 3$ ]	1, 2, 3	
Prefix: [1]	$NP \rightarrow DT NN$	4 [0, $NP \rightarrow DT_{0,1} \bullet NN, 2$ ]	2, 3, 4	1
			3, 4	2
Prefix: [3]	$VP \rightarrow Vi$	5 [2, $VP \rightarrow Vi_{2,3} \bullet, 3$ ]	4, 5	3
Complete: [4] [2]		6 [0, $NP \rightarrow DT_{0,1} NN_{1,2} \bullet, 2$ ]	5, 6	4
			6	5
Prefix: [6]	$S \rightarrow NP VP$	7 [0, $S \rightarrow NP_{0,2} \bullet VP, 2$ ]	7	6
Complete: [7] [5]		8 [0, $S \rightarrow NP_{0,2} VP_{2,3} \bullet, 3$ ]	8	7
GOAL: [8]			$\emptyset$	

# Correctness of Parsing Strategy

Soundness: if a goal item is proven for **s**

- then **s**  $\in$  L(G)

Completeness: if **s**  $\in$  L(G)

- then a goal item can be proven for **s**

# Parse Forest

Efficient representation of the whole space  $T_G(\mathbf{s})$

- each and every possible tree yielding  $\mathbf{s}$

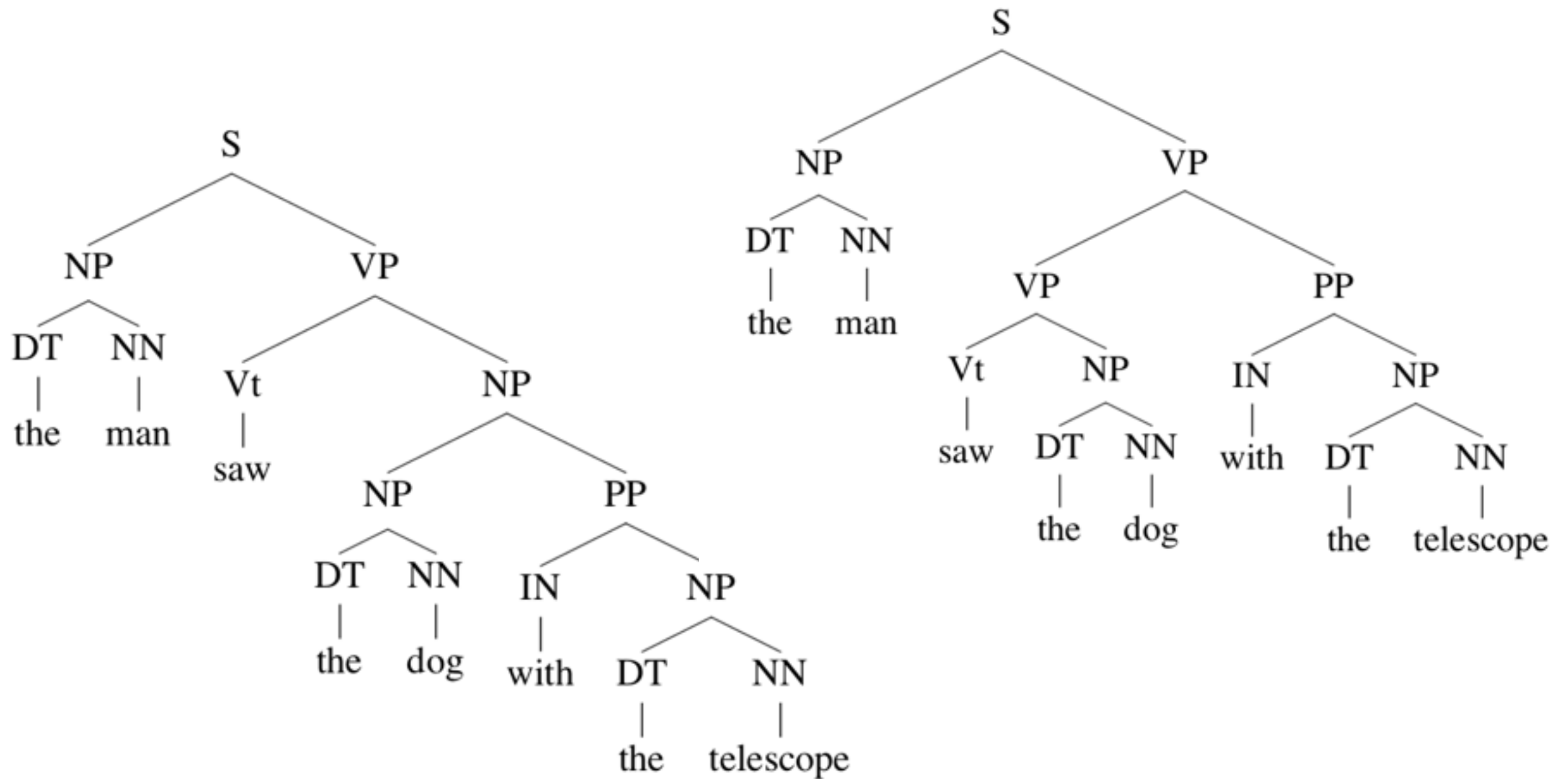
We must be able to represent partial derivations

- including alternative ones



# Ambiguity

Some strings may have more than one derivation in G



# Dealing with Ambiguity

Statistical model: weight steps in a derivation

- induces a partial ordering over derivations
- can be used to make a decision
  - e.g. best tree under the model

# Probabilistic CFG

CFG extended with parameters  $0 \leq \theta_r \leq 1$

- where  $r \in R$  and

$$\sum_{\alpha: X \rightarrow \alpha \in R} \theta_{X \rightarrow \alpha} = 1$$

# Probabilistic CFG

Distribution over trees

$$\begin{aligned} P(T = t, S = \text{yield}(t)) &= P(T = \langle r_1 \dots r_n \rangle, S = s) \\ &= \prod_{i=1}^n \theta_{r_i} = \prod_{i=1}^n \theta_{X_i \rightarrow \alpha_i} = \prod_{r \in t} \theta_r^{n(r,t)} \end{aligned}$$

and strings

$$P(S = s) = \sum_{t \in T_G(s)} P(T = t, S = s)$$

# Estimation

Let us assume the parametric form of  $\theta$  is a multinomial

- one categorical distribution per  $X \in N$

Suppose we can observe a *treebank*, then by MLE

$$\begin{aligned}\theta_{X \rightarrow \alpha} &= \frac{n(X \rightarrow \alpha)}{n(X)} \\ &= \frac{n(X \rightarrow \alpha)}{\sum_{\alpha'} n(X \rightarrow \alpha')}\end{aligned}$$

# Weighted CKY+

**Input:**  $G$  and  $s = w_1 \dots w_n$

**Item form:**  $[i, X \rightarrow \alpha \blacksquare \bullet \beta \square, j]$   
asserts that  $X \Rightarrow^* w_{i+1} \dots w_j \beta$

**Axioms:**  $[i, X \rightarrow w_i \bullet \alpha \square, i+1] : \theta_r$      $r = X \rightarrow w_i \alpha \in R$   
 $[i, X \rightarrow \varepsilon \bullet, i] : \theta_r$      $r = X \rightarrow \varepsilon \in R$

**Goal:**  $[0, S \rightarrow \alpha \blacksquare \bullet, n]$

**Scan**

$$\frac{[i, X \rightarrow \alpha \blacksquare \bullet w_{j+1} \beta \square, j] : \theta_1}{[i, X \rightarrow \alpha \blacksquare w_{j+1} \bullet \beta \square, j+1] : \theta_1}$$

**Prefix**

$$\frac{[i, Y \rightarrow \alpha \blacksquare \bullet, j] : \theta_1}{[i, X \rightarrow Y_{i,j} \bullet \beta \square, j] : \theta_r} \quad r = X \rightarrow Y \beta \in R$$

**Complete**

$$\frac{[i, X \rightarrow \alpha \blacksquare \bullet Y \beta \square, k] : \theta_1 \quad [k, Y \rightarrow \gamma \blacksquare \bullet, j] : \theta_2}{[i, X \rightarrow \alpha \blacksquare Y_{k,j} \bullet \beta \square, j] : \theta_1}$$

# Joint Distribution

${}_0S_3 \rightarrow {}_0NP_2 {}_2VP_3$

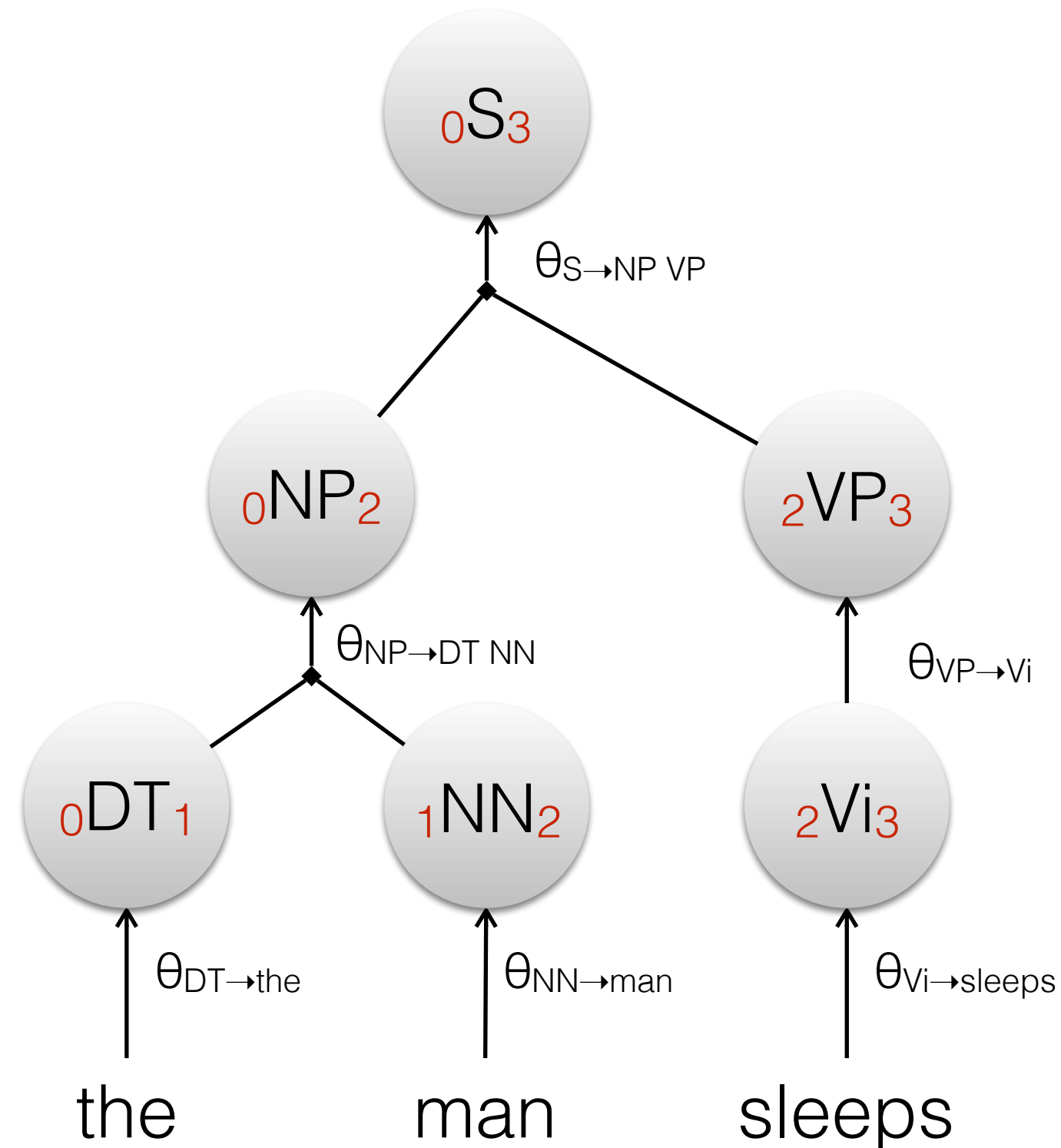
${}_0NP_2 \rightarrow {}_0DT_1 {}_1NN_2$

${}_2VP_3 \rightarrow {}_2Vi_3$

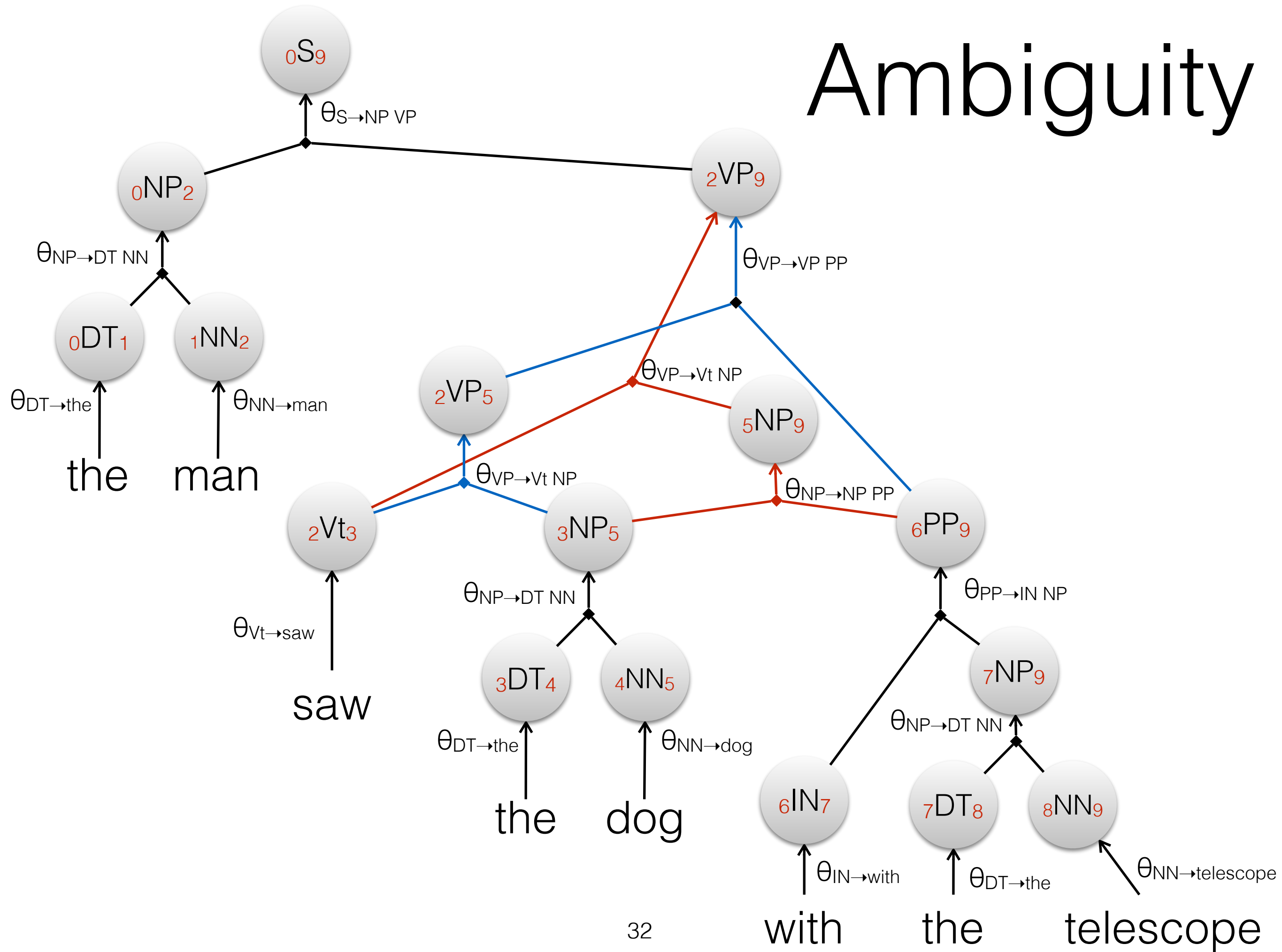
${}_0DT_1 \rightarrow \text{the}$

${}_1NN_2 \rightarrow \text{man}$

${}_2Vi_3 \rightarrow \text{sleeps}$



# Ambiguity





# Complexity

**Item form:**  $[i, X \rightarrow \alpha \blacksquare \bullet \beta \square, j]$

- Each rule segments the input  $w_1 \dots w_n$

Every CFG can be written in CNF (max arity = 2)

- In total we get up to 3 indices ranging from 0 .. n
- $O(n^3)$  annotated rules

# Bitext Parsing

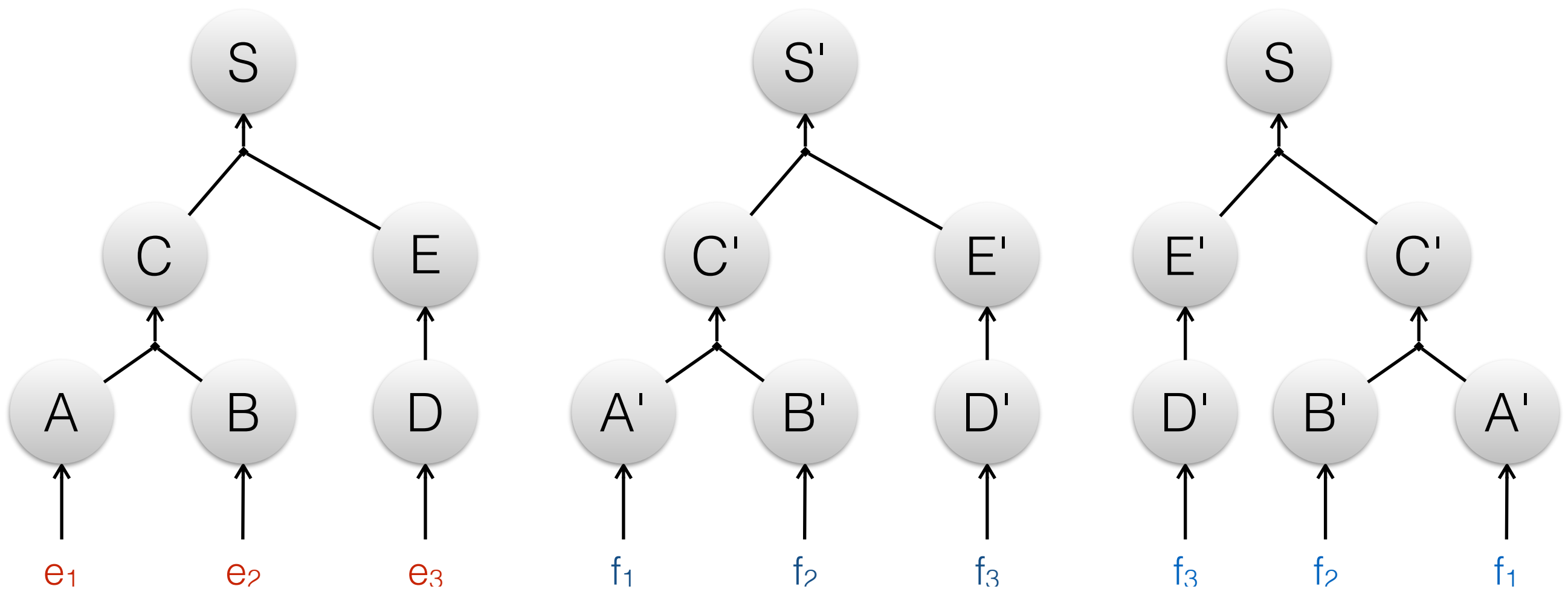
Imagine we have **two** streams of text

the man sleeps  $\Leftrightarrow$  dort l' homme

We want to parse both strings **simultaneously**  
such that their trees are **isomorphic**

- same structure up to
- relabelling and permutation of siblings

# Isomorphic trees



# Synchronous Grammar

A CFG **paired** with another

- RHS symbols map **one-to-one**

English    French		
$X \rightarrow A$	A	copy
$X \rightarrow B C$	B C	copy
	C B	invert
$X \rightarrow e$	f	transduce

# Parse E

Parse with the English side of the grammar

${}_0S_3 \rightarrow {}_0NP_2 {}_2VP_3$

${}_0NP_2 \rightarrow {}_0DT_1 {}_1NN_2$

${}_2VP_3 \rightarrow {}_2Vi_3$

${}_0DT_1 \rightarrow \text{the}$

${}_1NN_2 \rightarrow \text{man}$

${}_2Vi_3 \rightarrow \text{sleeps}$

# Projection

	English	French
${}_0S_3 \rightarrow$	${}_0NP_2 {}_2VP_3$	${}_0NP_2 {}_2VP_3$ ${}_2VP_3 {}_0NP_2$
${}_0NP_2 \rightarrow$	${}_0DT_1 {}_1NN_2$	${}_0DT_1 {}_1NN_2$ ${}_1NN_2 {}_0DT_1$
${}_2VP_3 \rightarrow$	${}_2Vi_3$	${}_2Vi_3$
${}_0DT_1 \rightarrow$	the	le
		la
		l'
${}_1NN_2 \rightarrow$	man	homme
${}_2Vi_3 \rightarrow$	sleeps	dort

# French Grammar

French

${}_0S_3 \rightarrow$   ${}_0NP_2$   ${}_2VP_3$

${}_2VP_3$   ${}_0NP_2$

${}_0NP_2 \rightarrow$   ${}_0DT_1$   ${}_1NN_2$

${}_1NN_2$   ${}_0DT_1$

${}_2VP_3 \rightarrow$   ${}_2Vi_3$

${}_0DT_1 \rightarrow$  le

la

l'

${}_1NN_2 \rightarrow$  homme

${}_2Vi_3 \rightarrow$  dort

# Parse F

French

${}_0S_3 \rightarrow {}_0NP_2 {}_2VP_3$

${}_2VP_3 {}_0NP_2$

${}_0NP_2 \rightarrow {}_0DT_1 {}_1NN_2$

${}_1NN_2 {}_0DT_1$

${}_2VP_3 \rightarrow {}_2Vi_3$

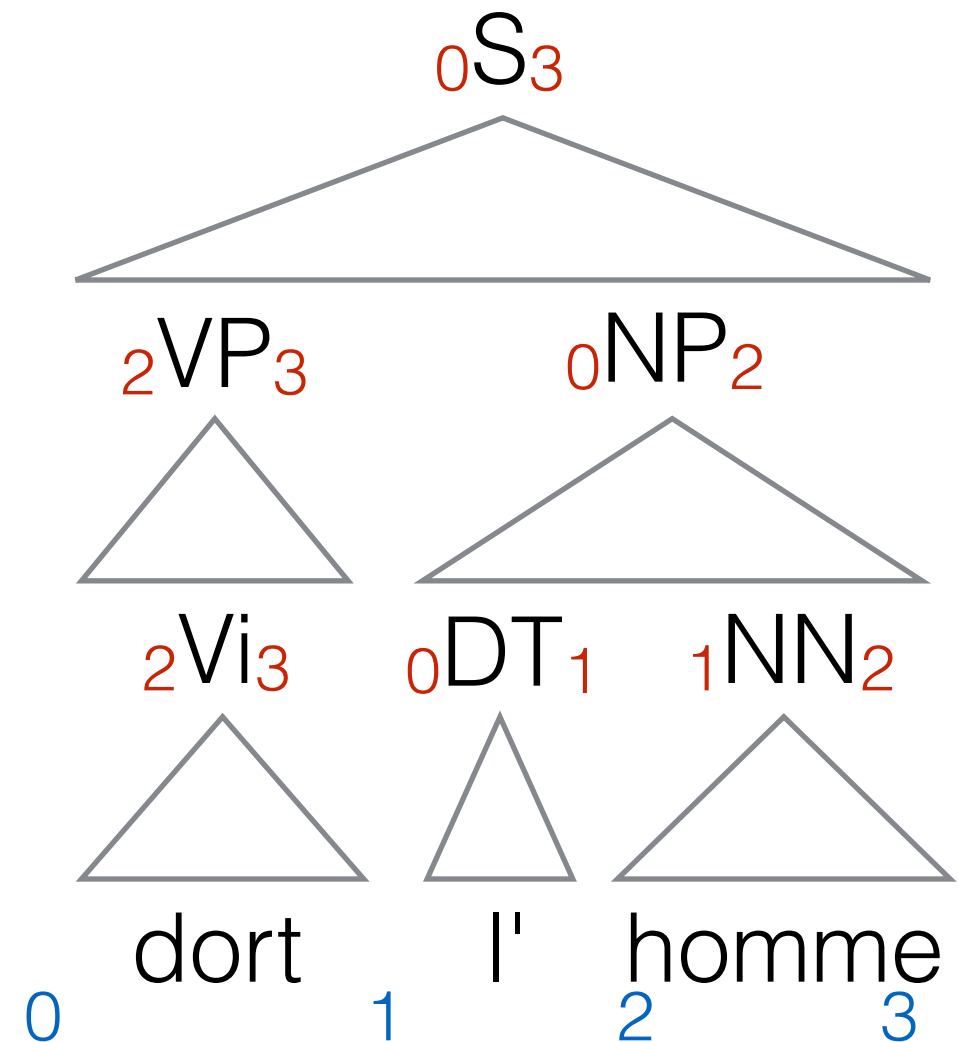
${}_0DT_1 \rightarrow$  le

la

l'

${}_1NN_2 \rightarrow$  homme

${}_2Vi_3 \rightarrow$  dort





# Cascade of Monolingual Parsers

CFG parsing can be seen as intersecting a CFG and an FSA [Bar-Hillel, 1961; Billot and Lang, 1989]

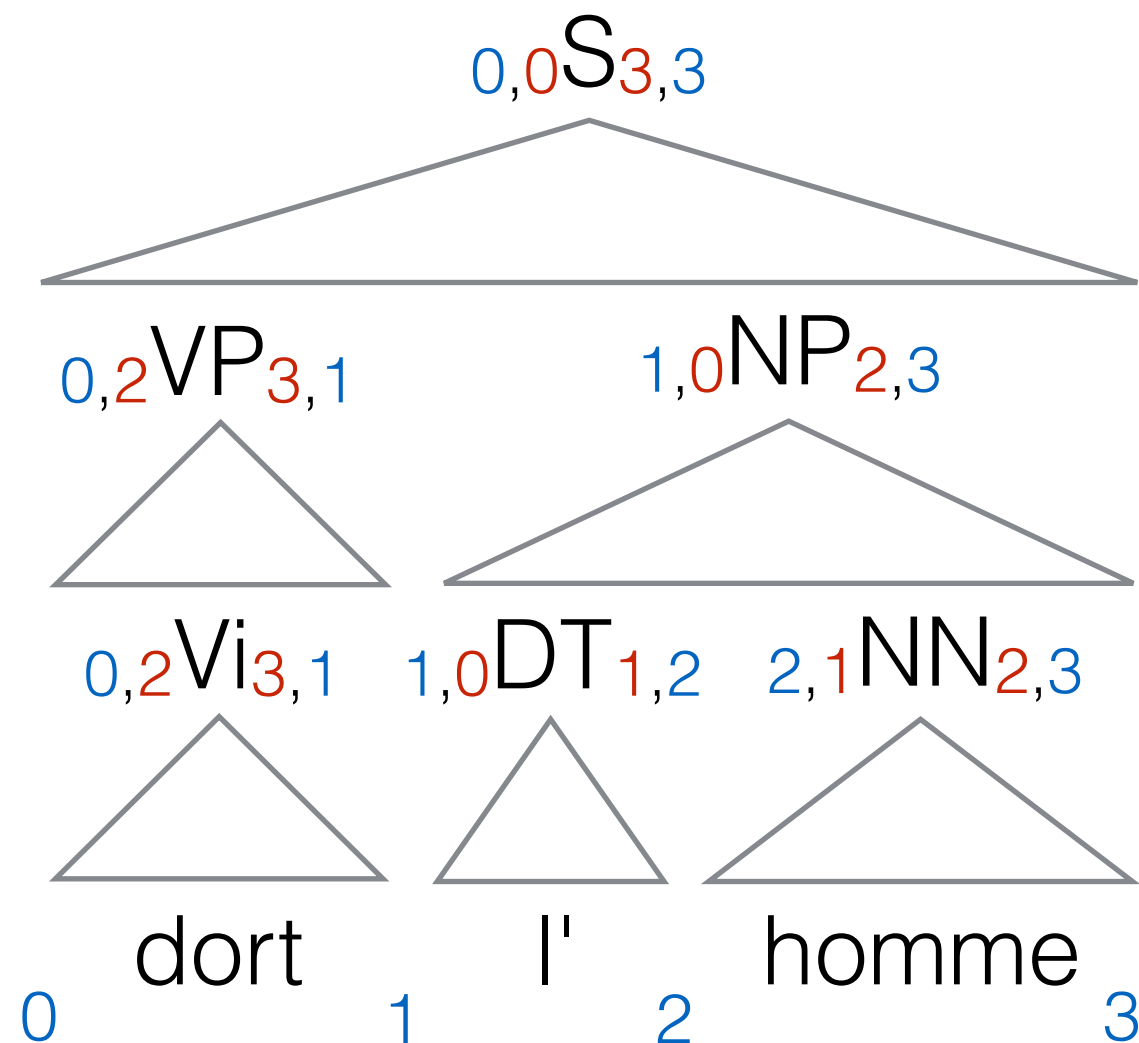
CFGs are closed under intersection [Hopcroft and Ullman, 1979]

- $L(\text{CFG}) \cap L(\text{FSA})$  is a context-free language

This neat property makes cascading intersection operations (parsers) appealing [Dyer, 2010]

- e.g. bitext parsing

# Biproduct: alignments



French

$0S_3 \rightarrow 0NP_2 \ 2VP_3$

$2VP_3 \ 0NP_2$

$0NP_2 \rightarrow 0DT_1 \ 1NN_2$

$1NN_2 \ 0DT_1$

$2VP_3 \rightarrow 2Vi_3$

$0DT_1 \rightarrow$  le

la

l'

$1NN_2 \rightarrow 1NN_2 \rightarrow$  homme

$2Vi_3 \rightarrow 2Vi_3 \rightarrow$  dort

# Complexity

- $O(l^3 \times m^3)$ 
  - where  $l$  is the length of the English string
  - and  $m$  is the length of the French string
- Joint parsing or cascade of parsers has the same theoretical complexity
- Can cascading be more efficient on average?  
Why?

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