# Latent-variable CRF for SMT

Wilker Aziz

Universiteit van Amsterdam
w.aziz@uva.nl

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# Conditional random field

Lafferty et al. [2001]

$$P(y|x, w) = \frac{\exp(w^\top \phi(x, y))}{\sum_{y' \in \mathcal{Y}(x)} \exp(w^\top \phi(x, y'))} \tag{1}$$

- $\phi$ is a feature function mapping $(x, y)$ to $\mathbb{R}^d$
- $w$ is a feature vector in $\mathbb{R}^d$
- $Z(x|w) = \sum_{y \in \mathcal{Y}(x)} \exp(w^\top \phi(x, y))$ must be finite

# Flexible (overlapping) features

Examples

**As**$_1$ menin**as**$_2$ for**am**$_3$ pra$_4$ lá$_4$ $\leftrightarrow$ The$_1$ girls$_2$ went$_3$ over$_4$ there$_4$

Apague$_1$ a$_2$ luz$_3$ $\leftrightarrow$ Switch$_1$ the$_2$ light$_3$ off$_1$

Hard to account for with directed models
(due to causality assumptions)

# Global normalisation

Local models are trained with positive context only

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What happens when we query the model with predicted contexts?

# Maximum likelihood estimation

Likelihood of an observation $(x, y)$

$$\mathcal{L}(w|x,y) = \log P(y|x,w) \tag{2}$$
$$= w^\top \phi(x,y) - \log Z(x|w) \tag{3}$$

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Gradient-based optimisation

$$\nabla_w \mathcal{L}(w|x, y) = \phi(x, y) - \mathbb{E}_{P(Y|X=x, w)}[\phi(X, Y)] \qquad (4)$$

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Expected features should match the features of the observation

# LV-CRF

Model

$$P(y, d|x) = \frac{\exp(w^\top \phi(x, y, d))}{\sum_{y' \in \mathcal{Y}(x)} \sum_{d' \in \mathcal{D}(x,y')} \exp(w^\top \phi(x, y', d'))} \tag{5}$$

- $d$ is latent
- $Z(x, y|w) = \sum_{d \in \mathcal{D}(x,y)} \exp(w^\top \phi(x, y, d))$ must be finite

# MLE for LV-CRF

Likelihood of an observation $(x, y)$

$$\mathcal{L}(w|x,y) = \log P(y|x,w) \tag{6}$$

$$= \log \sum_{d \in \mathcal{D}(x,y)} P(y,d|x,w) \tag{7}$$

# MLE for LV-CRF

Likelihood of an observation $(x, y)$

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$$= \log \sum_{d \in \mathcal{D}(x,y)} P(y, d|x, w) \tag{7}$$

Gradient-based optimisation [Mann and McCallum, 2007]

$$\nabla_w \mathcal{L}(w|x, y) = \mathbb{E}_{P(D|X=x, Y=y, w)}[\phi(X, Y, D)] \tag{8}$$

$$- \mathbb{E}_{P(Y,D|X=x, w)}[\phi(X, Y, D)] \tag{9}$$

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Undirected models are considerably harder to learn

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- Alignment: Dyer et al. [2011]
- SMT: Blunsom et al. [2008] and Blunsom and Osborne [2008]

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- Alignment: Dyer et al. [2011]
- SMT: Blunsom et al. [2008] and Blunsom and Osborne [2008]
- Approximate techniques
    - Contrastive divergence [Hinton, 2002]
    - Contrastive estimation [Smith and Eisner, 2005]
    - Piecewise training [Sutton and McCallum, 2005]

# SMT with CRFs

Blunsom et al. [2008]

- $d$ is a derivation complying with a *hiero* grammar
- $\phi$ featurises steps in a synchronous derivation

$$P(y, d|x) = \frac{\exp\left(\sum_{r_{s,t} \in d} w^\top \phi(r_{s,t}|x, y, d)\right)}{\sum_{d' \in \mathcal{D}(x)} \exp\left(\sum_{r_{s,t} \in d} w^\top \phi(r_{s,t}|x, y', d')\right)}$$

- $\mathcal{D}(x)$ is the space of derivations over target strings aligned to the source string $x$
- in the denominator $y'$ is defined implicitly as yield$(d')$
- $r_{s,t}$ is a synchronous rule decorated with a source span $s$ and a target span $t$

# ITG with CRFs

In Project 2, you will use an ITG

$$
\begin{array}{lcl}
S & \rightarrow & X \\
X & \rightarrow & X\ X \\
X & \rightarrow & x/y \text{ for all } x \in \Sigma \text{ and } y \in \Delta \\
X & \rightarrow & \epsilon/y \text{ for all } y \in \Delta \\
X & \rightarrow & x/\epsilon \text{ for all } x \in \Sigma
\end{array}
$$

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Solution: constrain strings by length

- introduce a constrain $n$ that depends on $x$
- make $\mathcal{D}_n(x)$ such that $|\text{yield}(d)| \leq n$ for $d \in \mathcal{D}_n(x)$
- $$\sum_{d \in \mathcal{D}_n(x)} \exp\left(\sum_{r_{s,t} \in d} w^\top \phi(r_{s,t} | x, y, d)\right)$$

# Learning

Likelihood of an observation $(x, y, n)$

$$\mathcal{L}(w|x,y,n) = \log P(y|x,n,w) \tag{10}$$

$$= \log \sum_{d \in \mathcal{D}(x,y)} P(y,d|x,n,w) \tag{11}$$

note that $\mathcal{D}(x, y)$ is always finite

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Gradient-based optimisation

$$\nabla_w \mathcal{L}(w|x, y, n) = \underbrace{\mathbb{E}_{P(D|X=x, Y=y, n, w)}[\phi(X, Y, D)]}_{\text{expected features for observation } (x,y)} \quad (12)$$

$$- \underbrace{\mathbb{E}_{P(Y, D|X=x, n, w)}[\phi(X, Y, D)]}_{\text{expected features for observation } x} \quad (13)$$

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3. $\mathcal{D}(x, y)$
   set of derivations of the string pair $(x, y)$
4. expected feature vector of observations $(x, y)$
5. expected feature vector of an observation $x$

# Notation

Using a hypergraph view

- $u, v, s$ are nodes
- $e$ is an edge
- $\mathrm{head}(e)$ is a node
- $\mathrm{tail}(e)$ is a sequence of nodes
- if $u \in \mathrm{tail}(e)$ and $v \in \mathrm{tail}(e)$, $u$ and $v$ are siblings
- Backward-star: $BS(u)$ is the set of edges incoming to $u$
  $u$ is the head of the edge
- Forward-star: $FS(u)$ is the set of edges outgoing from $u$
  $u$ is in the tail of the edge
- $w(e)$ is the weight of an edge

# Inside

The INSIDE recursion

$$I(v) = \begin{cases} \bar{1} & \text{if } BS(v) = \emptyset \\ \bigoplus_{e \in BS(v)} w(e) \bigotimes_{u \in \text{tail}(e)} \beta(u) & \text{otherwise} \end{cases} \tag{14}$$

For acyclic hypergraphs

# Outside

The OUTSIDE recursion

$$O(v) = \begin{cases} \bar{1} & \text{if } FS(v) = \emptyset \\ \bigoplus_{e \in FS(v)} O(\text{head}(e)) \bigotimes_{s \in tail(e) \setminus \{v\}} I(s) & \text{otherwise} \end{cases} \quad (15)$$

For acyclic hypergraphs

# Topsort

**function** $\textsc{TopSort}(G = \langle V, \langle E, w \rangle\rangle)$
    $S = \{v \in V : BS(v) = \emptyset\}$ ▷ nodes with no dependencies
    $D = \{v \mapsto \{u : \exists e \in BS(v) \wedge u \in \mathrm{tail}(e)\} : v \in V\}$
    ▷ a node depends on all of its children
    $L = \langle\rangle$ ▷ top-sorted nodes
    **while** $S \neq \emptyset$ **do**
        $u \leftarrow$ pop$(S)$ ▷ remove and return a node from $S$
        $L \leftarrow L + \langle u \rangle$ ▷ append $u$ to $L$
        **for** $e$ in $FS(u)$ **do** ▷ outgoing edges from $u$
            $v \leftarrow \mathrm{head}(e)$ ▷ parent of $u$ in $e$
            $D(v) \leftarrow D(v) \setminus \{u\}$ ▷ remove $u$ from $D(v)$
            **if** $D(v) == \emptyset$ **then** ▷ $v$'s dependencies have been sorted
                $S \leftarrow S \cup \{v\}$
            **end if**
        **end for**
    **end while**
    **return** $L$
**end function**

# Inside

```
function Inside(G = ⟨V, ⟨E, w⟩⟩)
    for v in TopSort(G) do                                  ▷ visit nodes bottom-up
        if BS(v) == ∅ then
            I[v] ← 1̄                                        ▷ leaves
        else
            I[v] ← 0̄
            for e ∈ BS(v) do
                k ← w(e)                                    ▷ include the edge's own weight
                for u in tail(e) do
                    k ← k ⊗ I[u]
                end for
                I[v] ← I[v] ⊕ k                             ▷ accumulate for each edge
            end for
        end if
    end for
    return I
end function
```

# Outside

**function** $\textsc{Outside}(G = \langle V, \langle E, w \rangle \rangle, I, \text{root})$<br>
    $O[v] \leftarrow \bar{0}$ **for** $v \in V$<br>
    $O[\text{root}] \leftarrow \bar{1}$ ▷ this is the goal node<br>
    **for** $v$ in $\textsc{reverse}(\textsc{TopSort}(G))$ **do** ▷ visit nodes top-down<br>
        **for** $e \in BS(v)$ **do** ▷ $q$'s incoming edges<br>
            **for** $u \in \text{tail}(e)$ **do** ▷ children of $v$ in $e$<br>
                $k \leftarrow w(e) \otimes O[v]$<br>
                **for** $s$ in $\text{tail}(e)$ **do** ▷ siblings of $u$ in $e$<br>
                    **if** $u \neq s$ **then** ▷ $u$ itself is excluded<br>
                        $k \leftarrow k \otimes I[s]$<br>
                    **end if**<br>
                **end for**<br>
                $O[u] \leftarrow O[u] \oplus k$ ▷ accumulate it for $u$<br>
            **end for**<br>
        **end for**<br>
    **end for**<br>
    **return** $O$<br>
**end function**

# Expected features

```
function ExpectedFeatures(G = ⟨V, ⟨E, w⟩⟩, I, O, φ)
    φ̄ ← 0
    for e ∈ E do                                   ▷ these are edges
        k ← O[head(e)]
        for u in tail(e) do
            k ← k ⊗ I[u]
        end for
        φ̄ ← φ̄ + kφ(e)
    end for
    return φ̄                                       ▷ expected feature vector
end function
```

# Traversals

Viterbi derivation

1. start from the goal (root)
2. recursively rewrite every symbol $v$ by solving

$$e^\star = \arg\max_{e \in BS(v)} w(e) \bigotimes_{u \in \mathrm{tail}(e)} I(u)$$

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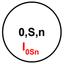
Sampling

1. start from the goal (root)
2. recursively rewrite every symbol $v$ by solving

$$E \sim P(e|v) = \begin{cases} \bar{0} & \text{if } e \notin BS(v) \\ \frac{w(e) \bigotimes_{u \in \text{tail}(e)} I(u)}{I(v)} & \text{otherwise} \end{cases}$$

# Viterbi



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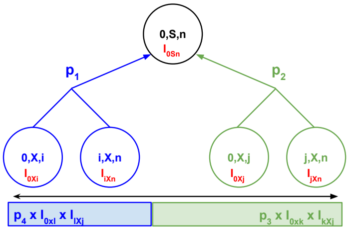
0,S,n
I0Sn
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p4 x I0xl x IlXj
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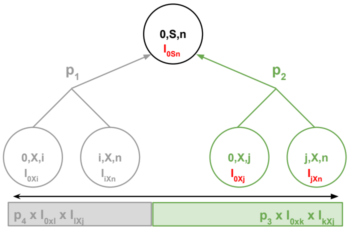

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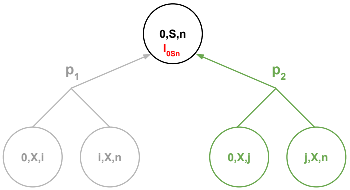

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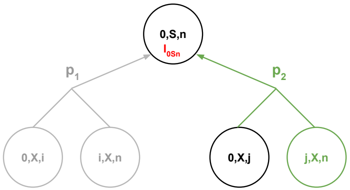

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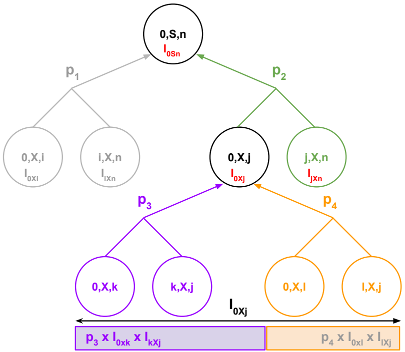
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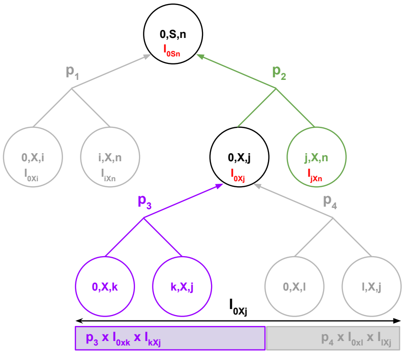
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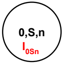

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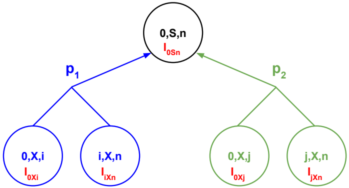

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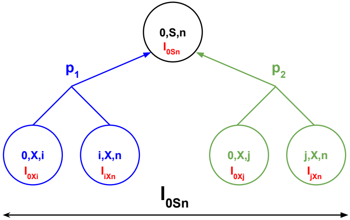

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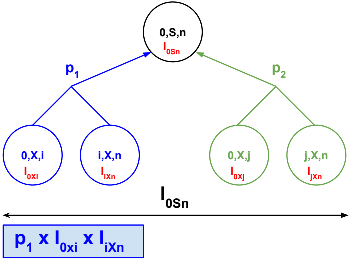
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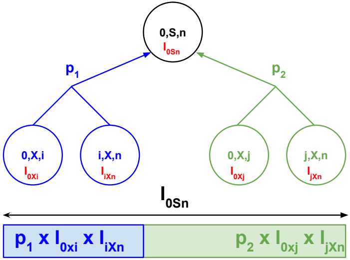
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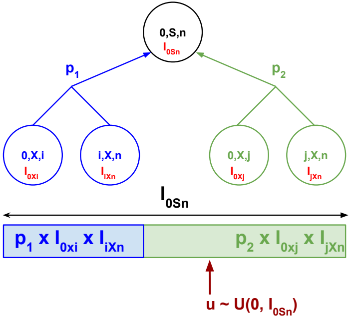
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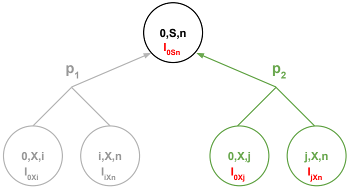

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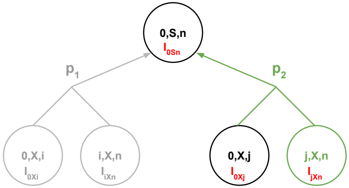

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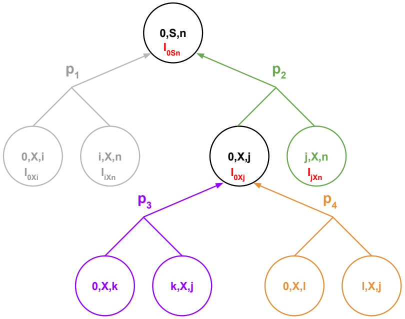
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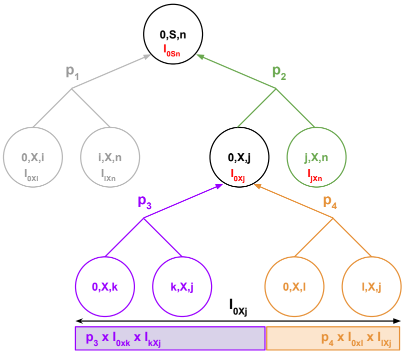

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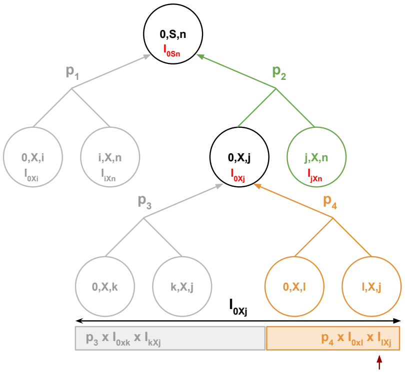
0,X,i
l0Xi
i,X,n
liXn
0,X,j
l0Xj
j,X,n
ljXn
p3
p4
0,X,k
k,X,j
0,X,l
l,X,j
l0Xj
p3 x l0xk x lkXj
p4 x l0xl x llXj

# Sampling


0,S,n
$I_{0Sn}$
$p_1$
$p_2$
0,X,i
$I_{0Xi}$
i,X,n
$I_{iXn}$
0,X,j
$I_{0Xj}$
j,X,n
$I_{jXn}$
$p_3$
$p_4$
0,X,k
k,X,j
0,X,l
l,X,j
$I_{0Xj}$
$p_3 \times I_{0xk} \times I_{kXj}$
$p_4 \times I_{0xl} \times I_{lXj}$

# Roadmap to project 2

1. understand the parser
   - check notes and notebook
2. implement the constrained forest $\mathcal{D}_n(x)$
3. implement feature functions
4. implement hypergraph algorithms
   - topsort, inside, outside, expected features
5. iterate over mini-batches of data making gradient updates

# Questions?

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