Machine Learning Home work I — Hasan F. Durkaya - 504241526

Q1-A) $P(Z|X=0,Y=0) = \dfrac{P(X=0,Y=0,Z)}{P(X=0,Y=0)}$

$\rightarrow P(X=0,Y=0,Z=1)$: $X=0,\ Y=0,\ Z=1$ is 0.06 (from table)

$\rightarrow P(X=0,Y=0,Z=2)$: $X=0,\ Y=0,\ Z=2$ is 0.09 "

$0{,}06 + 0{,}09 = 0{,}15$ $\qquad P(X=0,Y=0) = 0{,}15$

$P(Z|X=0,Y=0)$: for $Z=1$:

$[Z=1]$ $P(Z=1|X=0,Y=0) = \dfrac{P(X=0,Y=0,Z=1)}{P(X=0,Y=0)} = \dfrac{0{,}06}{0{,}15} = 0{,}4$

$[Z=2]$ $P(Z=2|X=0,Y=0) = \dfrac{P(X=0,Y=0,Z=2)}{P(X=0,Y=0)} = \dfrac{0{,}09}{0{,}15} = 0{,}6$

therefore;
$$P(Z|X=0,Y=0) = \begin{Bmatrix} 0{,}4 & \text{if } Z=1 \\ 0{,}6 & \text{if } Z=2 \end{Bmatrix}$$

Q1-B) Marginalizing over X, we must find $P(Z,Y)$ values for each X.

$P(Z,Y) = \sum_x P(X,Y,Z)$

$(Z=1,Y=-1) \Rightarrow P(Z=1,Y=-1) = P(X=0,Y=-1,Z=1) + P(X=1,Y=-1,Z=1) = 0{,}06+0{,}06$ $= 0{,}12$

$(Z=1,Y=0) \Rightarrow P(X=0,Y=0,Z=1) + P(X=1,Y=0,Z=1) = 0{,}06 + 0{,}02 = 0{,}08$

$(Z=1,Y=1) \Rightarrow P(X=0,Y=1,Z=1) + P(X=1,Y=1,Z=1) = 0{,}08 + 0{,}02 = 0{,}10$

$(Z=2,Y=-1) \Rightarrow P(X=0,Y=-1,Z=2) + P(X=1,Y=-1,Z=2) = 0{,}09 + 0{,}24 = 0{,}33$

$(Z=2,Y=0) \Rightarrow P(X=0,Y=0,Z=2) + P(X=1,Y=0,Z=2) = 0{,}09 + 0{,}08 = 0{,}17$

$(Z=2,Y=1) \Rightarrow P(X=0,Y=1,Z=2) + P(X=1,Y=1,Z=2) = 0{,}12 + 0{,}08 = 0{,}20$

$P(Z) = \sum_y (P(Z,Y))$ $\rightarrow$ $Z=1$, $P(Z=1,Y=-1) + P(Z=1,Y=0) + P(Z=1,Y=1) = 0{,}12 + 0{,}08 + 0{,}1 = 0{,}3$

$\hookrightarrow$ $Z=2$, $P(Z=2,Y=-1) + P(Z=2,Y=0) + P(Z=2,Y=1) = 0{,}33 + 0{,}17 + 0{,}2 = 0{,}7$

$P(Y) = \sum_z (P(Z,Y)) \rightarrow$ $Y=-1$, $P(Z=1,Y=-1) + P(Z=2,Y=-1) = 0{,}12 + 0{,}33 = 0{,}45$

$Y=0$, $P(Z=1,Y=0) + P(Z=2,Y=0) = 0{,}08 + 0{,}17 = 0{,}25$

$Y=1$, $P(Z=1,Y=1) + P(Z=2,Y=1) = 0{,}1 + 0{,}2 = 0{,}30$

Independence; Z and Y to be independent, $P(Z,Y)$ must be $P(Z) \cdot P(Y)$ for all comb.

$\rightarrow Z=1, Y=-1$ $\qquad P(Z=1,Y=-1) = 0{,}12$, $P(Z=1) \cdot P(Y=-1) = 0{,}12 \neq 0{,}3 \cdot 0{,}45$

Since $P(Z=1,Y=-1) \neq P(Z=1) \cdot P(Y=-1)$, Z and Y are not independent, they are dependent

Q2-) Discussion questions ⇒

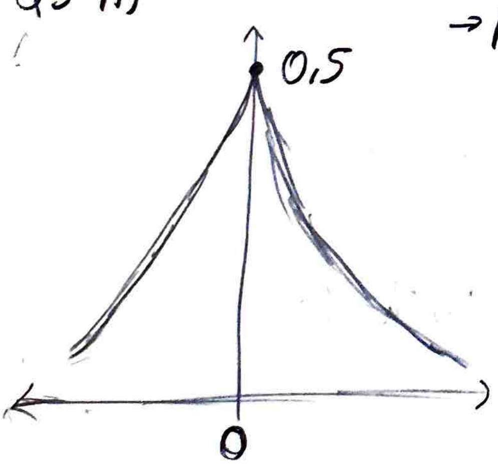
→ What happens if folds are different for different models?

If the folds differ for each model. Models will be trained and validated on different data points. This reduces the reliability of comparison. If models will work on diffent data subset loss and error calculations are hard to comment because of inconsistency. Each model performance are directly relateth to data samples. Therefore if the folds are different comparison is less meaningfull

→ What happens if the folds are obtained without shuffling?

Without shuffling data may not be randomly distributed accross folds. Therefore biased distrubution will appear in training and validation sets. If different folds has patterns because of the nature of dataset. This case may cause overfitting or underfitting, depends on the fold splits

Q5-A)



$\rightarrow P(x>2) = \int_2^{\infty} p(x)dx \rightarrow p(x) = \frac{1}{2}\exp(-|x|)$, $|x| = x$, for $x>0$;

$$P(x>2) = \int_2^{\infty} \frac{1}{2}\exp(-x)\,dx$$

$$\rightarrow \int \exp(-x)dx = -\exp(-x)$$

$$P(x>2) = 0 - \left(-\frac{1}{2}\exp(-2)\right) = \frac{1}{2}\exp(-2)$$

So, $P(x>2) \approx \frac{1}{2} \cdot .01353 = 0,0677$

% 6,77 chance of x to be >2

Machine Learning ~~Homework II~~

3-) A-)

with virus ⇒ %90 accuracy +

without virus ⇒ %90 accuracy −

%2 of population sick

actually sick, tested positive → $\frac{9}{10}$

$P(\text{Positive} \mid \text{Sick}) = 0.9$

$P(\text{Negative} \mid \text{Healthy}) = 0.9$

$P(\text{Sick}) = 0.02$

$P(\text{Sick} \mid \text{Positive})$

$P(\text{Healthy}) = 1 - P(\text{Sick}) = 0.98$

$P(\text{Positive} \mid \text{Sick}) = 0.9$

$P(\text{Positive} \mid \text{Healthy}) = 0.1$

$$P(\text{Sick} \mid \text{Positive}) = \frac{P(\text{Positive} \mid \text{Sick}) \cdot P(\text{Sick})}{P(\text{Positive})}$$

$$P(\text{Positive}) = P(\text{Positive} \mid \text{Sick}) \cdot P(\text{Sick}) + P(\text{Positive} \mid \text{Healthy}) \cdot P(\text{Healthy})$$

$$0.9 \cdot 0.02 + 0.1 \cdot 0.98$$

$$P(\text{Positive}) = 0.018 + 0.098 = 0.116$$

$$P(\text{Sick} \mid \text{Positive}) = \frac{0,9 \cdot 0,02}{0,116} = \frac{0,018}{0,116} = \underline{\underline{0,155}}$$

%15 is a relatively low probability so there is a high chance of false positives due to the %2 of population, test is untrustable

Q5-B

$E[x] = \sum_{x=0}^{n} x\,P(x) = \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x}$

Simplify with binomial distribution into a sum of Bernoulli Random variables

$E[x_1 + x_2 \ldots x_n] = E[x_1] + E[x_2] \ldots + E[x_n]$

if x_i is a Bernoulli Random variable $E[x_i] = p$

therefore

$E[x] = \underbrace{p + p + p \ldots p}_{n} = \underline{\underline{np}}$

$x^2 \to x(x-1) + x$

$\to E[x^2] = \sum_{x=0}^{n} x^2 p(x)$ substitute $p(x) \to$

$E[x^2] = \sum_{x=0}^{n} x^2 \binom{n}{x} p^x (1-p)^{n-x}$

$E[x^2] = \sum_{x=0}^{n} x(x-1) \cdot \binom{n}{x} p^x (1-p)^{n-x} + \underbrace{\sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x}}$ $\to np$

$\to \sum_{x=0}^{n} x(x-1) \cdot \binom{n}{x} p^x (1-p)^{n-x}$ factor out $n(n-1)$, choosing 2

$\sum_{x=2}^{n} \frac{n!}{(x-2)!(n-x)!} p^x (1-p)^{n-x} \to n(n-1) \sum_{x=2}^{n} \binom{n-2}{x-2} p^x (1-p)^{n-x}$

$y = x-2$

$n(n-1) \sum_{y=0}^{n-2} \binom{n-2}{y} p^{y+2} (1-p)^{(n-2)-y} \to n(n-1)p^2 \sum_{y=0}^{n-2} \binom{n-2}{y} p^y (1-p)^{(n-2)-y}$

Summation expands to $(p + (1-p))^{n-2} = 1$ thus;

$\sum_{x=0}^{n} x(x-1)\,p(x) = n(n-1)p^2$.

$E[x^2] = n(n-1)p^2 + np$